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Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2024 Mains Test Series

Electrical Engineering Test No : 11

Section-A

Q.1 (a) Solution:

(i) An inductive load is lagging pf, capacitive load is leading pf and resistive load is unity pf.

For load 1:

$$\theta_1 = \cos^{-1}(0.28) = 73.74° \text{ lagging}$$

The load complex powers are:

$$S_1 = 125\angle 73.74 \text{ kVA}$$
$$= 35 \text{ kW} + j120 \text{ kVAR}$$
$$S_2 = 10 \text{ kW} - j40 \text{ kVAR}$$
$$S_3 = 15 \text{ kW} + j0 \text{ kVAR}$$

Total apparent power is

$$S = P + jQ = S_1 + S_2 + S_3$$
$$S = (35 + j120) + (10 - j40) + 15 + j0$$
$$S = 60 \text{ kW} + j80 \text{ kVAR}$$
$$S = 100\angle 53.13° \text{ kVA}$$

Total kW = 60

kVAR = 80

kVA = 100

supply power factor = cos 53.13 = 0.6 lagging

The total current is

$$S^* = V^* I$$

$$I = \frac{S^*}{V^*} = \frac{100\angle -53.13\text{ kVA}}{1400\ \angle 0°} = 71.43\ \angle -53.13°\text{ A}$$

(ii) Total real power = P = 60 kW at the new power factor of 0.8 lagging results in new reactive power Q'.

$$\theta' = \cos^{-1}(0.8) = 36.87°$$

$$Q' = 60\tan(36.87) = 45\text{ kVAR}$$

Therefore, the required capacitor kVAR is

$$Q_C = 80 - 45 = 35\text{ kVAR}$$

and

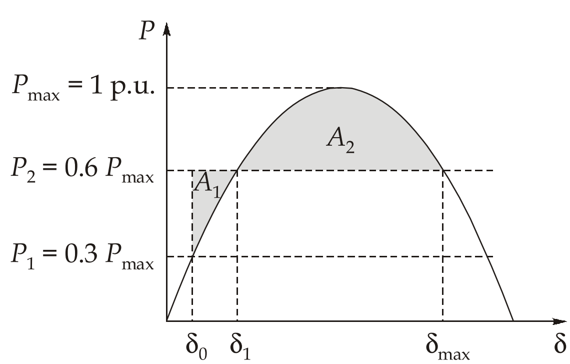
$$X_C = \frac{|V|^2}{S_C^*} = \frac{1400^2}{j35,000}$$

$$X_C = -j56\ \Omega$$

$$C = \frac{10^6}{2\pi(60)(56)} = 47.37\ \mu\text{f}$$

Q.1 (b) Solution:

Power angle curve of motor



Initially,

$$P_1 = P_{max}\sin\delta_o$$

$$\delta_o = \sin^{-1}\left[\frac{P_1}{P_{emax}}\right] = \sin^{-1}(0.3) = 17.45°$$

Also,

$$P_2 = P_{max}\sin\delta_1$$

$$0.6P_{max} = P_{max}\sin\delta_1$$

$$\delta_1 = \sin^{-1}(0.6) = 36.87°$$

And

$$\delta_{max} = 180° - \delta_1 = 143.13°$$

Motor will be in synchronism as long as $A_2 > A_1$.

Calculation of A_1,

$$A_1 = \int_{\delta_0}^{\delta_1} (P_2 - P_{max}\sin\delta)d\delta$$

$$= 0.6(\delta_1 - \delta_0) + \cos\delta\Big|_{\delta_0}^{\delta_1}$$

$$= 0.6 \times [36.87 - 17.45] \times \frac{\pi}{180} + \cos(36.87) - \cos(17.45)$$

$$A_1 = 0.049$$

Calculation of Area A_2,

$$A_2 = \int_{\delta_1}^{\delta_{max}} (P_{max}\sin\delta - P_2)d\delta$$

$$= \cos\delta_1 - \cos\delta_{max} - 0.6(\delta_{max} - \delta_1)$$

$$= \cos(36.87) - \cos(143.13) - 0.6(143.13 - 36.87) \times \frac{\pi}{180}$$

$$A_2 = 0.4872$$

Since $A_2 > A_1$, therefore motor will be in synchronism.

Now, calculation of excursion angle :

$$A_2 = A_1$$

$$\int_{\delta_1}^{\delta_2} (P_{max}\sin\delta - 0.6P_{max})d\delta = 0.049$$

$$\cos\delta_1 - \cos\delta_2 + 0.6\delta_1 - 0.6\delta_2 = 0.049$$

$$\cos(36.87) - \cos\delta_2 + 0.6 \times 36.87 \times \frac{\pi}{180} - 0.6\delta_2 = 0.049$$

$$0.6\delta_2 + \cos\delta_2 - 1.137 = 0$$

$$\delta_2 = 58.04°$$

Therefore, excursion in rotor angle about new steady state rotor position is

$$\delta_2 - \delta_1 = 58.04 - 36.87 = 21.17°$$

Q.1 (c) Solution:

For a factor of safety of 2, the permitted values are $I_p = \frac{200}{2} = 100$ A

$$\left(\frac{di}{dt}\right)_{max} = \frac{50}{2} = 25\text{A}/\mu\text{s}$$

$$\left(\frac{dv}{dt}\right)_{max} = \frac{200}{2} = 100 \text{ V}/\mu\text{s}$$

In order to restrict the rate of rise of current beyond specified value, (di/dt) inductor must be inserted in series with thyristor.

$$L = \frac{V_s}{(di/dt)_{max}} = \frac{400 \times 10^{-6}}{25} = 16 \,\mu\text{H}$$

$$R_s = \frac{L}{V_s} \cdot \left(\frac{dv}{dt}\right)_{max} = \frac{16 \times 10^{-6}}{400} \times \frac{100}{10^{-6}} = 4 \,\Omega$$

Before thyristor is turned on, C_s is charged to 400 V. When thyristor is turned on, the peak current through the thyristor is

$$\frac{400}{10} + \frac{400}{4} = 140 \text{ A}$$

As this peak current through SCR is more than the permissible peak current of 100 A, the magnitude of R_s must be increased. Taking R_s as 8 Ω, the peak current through the

SCR = $\frac{400}{10} + \frac{400}{8} = 90$ A, less than the allowable peak current. So choose R_s = 8 Ω.

Also,
$$C_s = \left(\frac{2\xi}{R_s}\right)^2 L = \left(\frac{1.3}{8}\right)^2 \times 16 \times 10^{-6} = 0.4225 \,\mu\text{F}$$

At the instant switch S is closed, thyristor is open circuited and current through C_s is given by

$$C_s \frac{dv}{dt} \cong \frac{V_s}{R_s + R_L}$$

or
$$0.3 \times 10^{-6} \frac{dv}{dt} = \frac{400}{10+8}$$

or
$$\frac{dv}{dt} = \frac{400}{18} \times \frac{1}{0.3 \times 10^{-6}} = 74.07 \text{ V}/\mu\text{s}$$

Since designed value of (dv/dt) is less than the specified maximum value of 100 V/μs. Value of C_S chosen is correct.

So choose L = 16 μH, R_S = 8 Ω

and C_S = 0.3 μF

Q.1 (d) Solution:

By taking z-transform on both side,

$$Y(z) = A[z^{-1}y(z) + y(-1)] + X(z)$$

$$Y(z) = A[z^{-1}y(z) + 1] + X(z)$$

$$Y(z) - Az^{-1}Y(z) = A + X(z)$$

$$Y(z)[1 - Az^{-1}] = A + X(z)$$

given, step response $x[n] = u[n]$

$$X(z) = z[u(n)] = \frac{1}{1-z^{-1}}$$

$$\therefore \quad Y(z)[1 - Az^{-1}] = A + \frac{1}{1-z^{-1}}$$

$$Y(z) = \frac{A}{1-Az^{-1}} + \frac{1}{(1-z^{-1})(1-Az^{-1})}$$

Using partial fraction expansion,

$$Y(z) = \frac{A}{1-Az^{-1}} + \frac{B_1}{1-z^{-1}} + \frac{B_2}{1-Az^{-1}}$$

$$B_1 = \left.\frac{1}{1-Az^{-1}}\right|_{z=1} = \frac{1}{1-A}$$

$$B_2 = \left.\frac{1}{1-z^{-1}}\right|_{z^{-1}=\frac{1}{A}} = \frac{1}{1-\frac{1}{A}} = \frac{A}{A-1} = \frac{-A}{1-A}$$

$$\therefore \quad Y(z) = \frac{A}{1-Az^{-1}} + \frac{\left(\frac{1}{1-A}\right)}{1-z^{-1}} + \frac{\frac{-A}{1-A}}{1-Az^{-1}}$$

by taking inverse z-transform,

$$Y(n) = A^{n+1}u(n) + \frac{1}{(1-A)}u(n) + \frac{-1}{(1-A)}A^{n+1}u(n)$$

$$= A^{n+1}u(n) + \left[\frac{1-A^{n+1}}{1-A}\right]u(n)$$

$$\therefore \quad Y(n) = \frac{1}{1-A}\left[1-A^{n+2}\right]u(n)$$

Q.1 (e) Solution:

Energy loss during turn on process,

$$E_{ON} = \int_0^{t_{ON}} i_C v_{CE} dt$$

$$E_{ON} = \int_0^{40\mu} \left(\frac{100}{50}\times 10^6 t\right)\left(200 - \frac{200}{40}\times 10^6 t\right)dt$$

$$E_{ON} = 0.1067 \text{ watt sec} \quad \text{...(i)}$$

Energy loss during turn off process,

$$E_{OFF} = \int_0^{15\mu} (100)\left(\frac{200\times 10^6}{75}t\right)dt + \int_0^{60\mu}\left(100 - \frac{100\times 10^6}{60}t\right)\left(40 + \frac{200-40}{60\times 10^{-6}}t\right)dt$$

$$E_{OFF} = 3 \times 10^{-2} + 0.28$$

$$E_{OFF} = 0.31 \text{ watt-sec}$$

So, total energy loss in one cycle

$$E = (0.1067 + 0.31) \text{ watt-sec}$$

$$E = 0.4167 \text{ watt-sec}$$

Average power loss in transistor = (Switching frequency × Energy loss in one cycle)

$$P = Ef_s$$

$$f_s = \frac{P}{E} = \frac{300}{0.4167} = 719.94 \text{ Hz}$$

Q.2 (a) Solution:

(i) The maximum torque is

$$T_{em} = \frac{m}{2\pi n_s}\cdot\frac{V_e^2}{\left[R_e + \sqrt{R_e^2 + X^2}\right]}$$

with negligible resistance, $R_e = 0$

$$T_{em} = \frac{m}{2\pi n_s}\cdot\frac{V_e^2}{X}$$

Both X and n_s are proportional to supply frequency f,

$$\therefore \quad T_{em} \propto \left(\frac{V_e}{f}\right)^2$$

$$T_{em} = k\left(\frac{V_1}{f}\right)^2$$

For 440V, 50 Hz source, $T_{em1} = k\left(\frac{440}{50}\right)^2$

For 400 V, 40 Hz source, $T_{em2} = k\left(\frac{400}{40}\right)^2$

$$\therefore \quad \frac{T_{em_1}}{T_{em_2}} = \left(\frac{440}{50}\right)^2 \times \left(\frac{40}{400}\right)^2$$

$$T_{em2} = 3T_{efL}\left(\frac{50}{440}\right)^2 \times \left(\frac{400}{40}\right)^2 = 3.874\ T_{efL}$$

(ii) At 50 Hz operation, $s_{mT1} = \frac{1500 - 1200}{1500} = 0.2$

and $\frac{r_2}{s_{mT_1}} = X = 2\pi(50)L$

$$\therefore \quad \frac{r_2}{2\pi L} = (50)(0.2) = 10$$

At 40 Hz operation $\frac{r_2}{s_{mT_2}} = (2\pi)(40)L$

$$\therefore \quad s_{mT2} = \frac{r_2}{2\pi L} \times \frac{1}{40} = 0.25$$

For 400 V, 40 Hz operation, therefore, the speed at which maximum torque would occur is given by

$$\frac{120f}{P}(1 - s_{mT_2}) = \frac{120 \times 40}{4}(1 - 0.25) = 900 \text{ rpm.}$$

Q.2 (b) Solution:

(i) For a lossless line,

phase constant $(\beta) = \omega\sqrt{LC} = 2\pi \times 60\sqrt{0.97 \times 10^{-3} \times 0.0115 \times 10^{-6}}$

$= 0.001259$ rad/km

and surge impedance $(Z_c) = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.97 \times 10^{-3}}{0.0115 \times 10^{-6}}} = 290.43\ \Omega$

Velocity of propagation $(V_p) = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.97\times10^{-3}\times0.0115\times10^{-6}}}$

$$V_p = 2.994 \times 10^5 \text{ km/s}$$

and the line wavelength

$$\lambda = \frac{V}{f} = \frac{1}{60}\times2.994\times10^5 = 4990 \text{ km}$$

(ii) The receiving end voltage per phase

$$V_R = \frac{500\angle0°}{\sqrt{3}} = 288.675\angle0° \text{ kV}$$

receiving end current, $I_R = \frac{S_R}{\sqrt{3}\cdot V_{R(\text{Line})}} = \frac{P_R}{\sqrt{3}(V_{R(Line)})\cdot\cos\phi_R} = \frac{800\times10^3}{\sqrt{3}\times500\times0.8}$

$$I_R = 1154.7\angle-36.87° \text{ Amp}$$

From the sending end voltage,

$$V_S = \cos\beta l V_R + jZ_c \sin\beta l \cdot I_R \qquad \text{...(i)}$$

$$\beta l = 0.001259 \times 300 = 0.3777 \text{ rad} = 21.641°$$

From equation (1)

$$V_S = \cos(21.641°) \times 288.675\angle0° + j \times (290.43)\sin(21.641°)$$
$$(1154.7\angle-36.87°) \times 10^{-3}$$
$$= 0.9295 \times 288.675\angle0° + j(290.43)(0.3688)$$
$$(1154.7\angle-36.87°) \times 10^{-3}$$
$$V_S = 356.53\angle16.1° \text{ kV}$$

The sending end line-to line voltage magnitude is

$$|V_{S(\text{L-L})}| = \sqrt{3}|V_S| = 617.53 \text{ kV}$$

Sending end current, $I_S = j\cdot\frac{1}{Z_c}\sin\beta l V_R + \cos\beta l\cdot I_R$

$$= j\frac{1}{290.43}\sin(21.641)(288.675\angle0°)$$
$$+ (0.9295)(1154.7\angle-36.87) \times 10^{-3}$$
$$I_S = 902.3\angle-17.9° \text{ A}$$

Sending end power, $S_{S(3\phi)} = 3V_S.I_S^*$

$$= 3 \times (356.53\angle16.1°)(902.3\angle-17.9)^*$$

$$S_{S(3\phi)} = 965.1\angle 34° \text{ MVA}$$

Voltage regulation, $$\%V_R = \frac{V_{R(NL)} - V_{R(FL)}}{V_{R(\text{rated})}} = \frac{\left|\frac{V_S}{A}\right| - V_{R(Fl)}}{V_{R(\text{rated})}}$$

$$= \frac{\left(\frac{356.53}{0.9295}\right) - 288.675}{288.675} \times 100 = 32.87\%$$

Q.2 (c) Solution:

Given, Input voltage, V_s = 200 V (DC)

$$R = 20\ \Omega$$

$$L = 0.06 \text{ H}$$

First two dominant harmonics in output voltage are 3rd and 5th harmonic

For two symmetrical pulses per half cycle - (N = 2)

$$2d = 0.5 \times 180° = 90°$$

$$d = 45°$$

The output voltage expression is given by

$$V_0(t) = N \times \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin n\gamma \sin \frac{nd}{N} \sin\left(n\omega_0 t - \frac{nd}{2}\right)$$

$$V_0(t) = \sum_{n=1,3,5}^{\infty} \frac{8V_s}{n\pi} \sin n\gamma \sin \frac{nd}{2} \sin\left(n\omega_0 t - \frac{nd}{2}\right)$$

Where, $$\gamma = \frac{\pi - 2d}{N+1} + \frac{d}{N}$$

$$\gamma = \frac{180° - 90°}{2+1} + \frac{45°}{2} = 52.5°$$

Rms value of fundamental output voltage is

$$V_{01} = \frac{8V_S}{\pi\sqrt{2}} \sin(52.5) \sin\left(\frac{45°}{2}\right)$$

$$= \frac{8 \times 200}{\pi\sqrt{2}} \sin(52.5) \sin(22.5°)$$

$$V_{01} = 109.335 \text{ volt}$$

Rms value of 3rd harmonic voltage is

$$V_{03} = \frac{8\times 200}{3\pi\times\sqrt{2}}\times\sin(3\times 52.5°)\sin\left(\frac{3\times 45°}{2}\right) = 42.44 \text{ volt}$$

Rms value of 5th harmonic voltage is

$$V_{05} = \frac{8\times 200}{5\pi\times\sqrt{2}}\sin(5\times 52.5)\sin\left(5\times\frac{45°}{2}\right)$$

$$V_{05} = 65.973 \text{ volt}$$

Load impedances,

fundamental load impedance = $|Z_1| = \sqrt{R^2 + (2\pi f_0 L)^2}$

$$= \sqrt{20^2 + (2\pi\times 50\times 0.06)^2} = 27.485\ \Omega$$

3rd harmonic load impedance = $|Z_3| = \sqrt{R^2 + (2\pi\times 3 f_0 L)^2}$

$$= \sqrt{20^2 + (2\pi\times 3\times 50\times 0.06)^2} = 59.981\Omega$$

5th harmonic load impedance = $|Z_5| = \sqrt{20^2 + (2\pi\times 5\times 50\times 0.06)^2} = 96.346\ \Omega$

fundamental load current, $I_{01} = \frac{V_{01}}{|Z_1|} = \frac{109.335}{27.482} = 3.978 \text{ A}$

3rd harmonic load current $I_{03} = \frac{V_{03}}{|Z_3|} = \frac{42.44}{59.981} = 0.707 \text{ A}$

5th harmonic load current $I_{05} = \frac{V_{05}}{|Z_5|} = \frac{65.973}{96.346} = 0.6847 \text{ A}$

load RMS current $I_{or} = \sqrt{I_{01}^2 + I_{03}^2 + I_{05}^2}$

$$= \sqrt{3.978^2 + 0.707^2 + 0.6847^2}$$

$$= 4.098 \text{ A}$$

Power delivered to load

$$P_L = I_{or}^2\cdot R = 4.098^2\times 20$$

$$P_L = 335.862 \text{ W}$$

Q.3 (a) (i) Solution:

Given signal, $$f(t) = \begin{cases} t\,; & 0 < t < \pi \\ \pi\,; & \pi < t < 2\pi \end{cases}$$

$$a_0 = \frac{2}{T}\int_0^T f(t)dt = \frac{2}{2\pi}\int_0^{2\pi} f(t)dt$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} t\,dt + \int_{\pi}^{2\pi} \pi\cdot dt\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{t^2}{2}\right)^{\pi} + \pi[t]_{\pi}^{2\pi}\right] = \frac{1}{\pi}\left[\frac{\pi^2}{2} + \pi[2\pi - \pi]\right]$$

$$= \frac{1}{\pi}\times\frac{\pi^2}{2} + \frac{1}{\pi}\times\pi^2 = \frac{\pi}{2} + \pi$$

$\therefore$ $$a_0 = \frac{3\pi}{2}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt$$

where, $$T = 2\pi \Rightarrow \omega_0 = \frac{2\pi}{T} = 1$$

$$a_n = \frac{2}{2\pi}\left[\int_0^{\pi} t\cos nt\,dt + \int_{\pi}^{2\pi} \pi\cdot\cos nt\,dt\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n}(\pi\sin n\pi - 0\cdot\sin n0) - \left[\frac{-\cos nt}{n^2}\right]_0^{\pi}\right.$$

$$\left. + \frac{1}{n}(\sin n\times 2\pi - \sin n\pi)\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n}(0-0) + \left(\frac{\cos n\pi}{n^2} - \frac{\cos 0}{n^2}\right)\right] + \frac{1}{n}(0-0)$$

$$= \frac{1}{n^2\pi}(\cos n\pi - 1)$$

$$a_n = \frac{1}{n^2\pi}\left((-1)^n - 1\right)$$

$$b_n = \frac{2}{T}\int_0^{2\pi} f(t)\sin n\omega_0 t\,dt = \frac{2}{2\pi}\int_0^{2\pi} f(t)\sin nt\,dt$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} t\sin nt\,dt + \int_{\pi}^{2\pi} \pi\cdot\sin nt\,dt\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{-\pi\cos n\pi}{n}+0\right)+\left(\frac{\sin nt}{n^2}\right)_0^{\pi}\right]-\frac{1}{n}(\cos 2n\pi - \cos n\pi)$$

$$= \frac{1}{\pi}\left[\frac{-\pi(-1)^n}{n}+\left(\frac{\sin n\pi - \sin 0}{n^2}\right)\right]-\frac{1}{n}(1-(-1)^n)$$

$$b_n = \frac{-1}{n}(-1)^n - \frac{1}{n}(1-(-1)^n)$$

∴ The trigonometric Fourier series coefficients as

$$a_0 = \frac{3\pi}{2};\ a_n = \begin{cases} 0 & ;\ n \text{ even} \\ \dfrac{-2}{n^2\pi} & ;\ n \text{ odd}, b_n = \dfrac{-1}{n};\ \ n \text{ even}\end{cases}$$

Q.3 (a) (ii) Solution:

Given,

$$X(e^{j\omega}) = \frac{-\frac{1}{4}e^{-j\omega}+3}{-\frac{1}{16}e^{-j2\omega}+1} = \frac{3-\frac{1}{4}e^{-j\omega}}{1-\frac{1}{16}e^{-j2\omega}}$$

$$= \frac{3-\frac{1}{4}e^{-j\omega}}{1-\frac{1}{16}(e^{-j\omega})^2} = \frac{3-\frac{1}{4}e^{-j\omega}}{(1)^2-\left(\frac{1}{4}e^{-j\omega}\right)^2}$$

$$= \frac{3-\frac{1}{4}e^{-j\omega}}{\left(1+\frac{1}{4}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}$$

$$\frac{3-\frac{1}{4}e^{-j\omega}}{\left(1+\frac{1}{4}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)} = \frac{A}{1-\frac{1}{4}e^{-j\omega}}+\frac{B}{1+\frac{1}{4}e^{-j\omega}}$$

$$A = \left.\frac{3-\frac{1}{4}e^{-j\omega}}{1+\frac{1}{4}e^{-j\omega}}\right|_{e^{-j\omega}=+4} = \frac{3-\frac{1}{4}(4)}{1+\frac{1}{4}(+4)} = 1$$

$$B = \left.\frac{3-\frac{1}{4}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}}\right|_{e^{-j\omega}=-4} = \frac{3-\frac{1}{4}(-4)}{1-\frac{1}{4}(-4)} = 2$$

$$\therefore \quad X(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}} + \frac{2}{1-\left(\frac{-1}{4}\right)e^{-j\omega}}$$

by taking inverse DTFT,

$$x(n) = \left(\frac{1}{4}\right)^n u(n) + 2\left(\frac{-1}{4}\right)^n u(n)$$

Q.3 (a) (iii) Solution:

Given,

$$x(n) = 3(0.8)^{|n|}\cos(0.1\pi n)$$

$$= 3(0.8)^n u(n)\cos(0.1\pi n) + 3(0.8)^{-n} u(-n-1)\cos(0.1\pi n)$$

$$= 3(0.8)^n u(n)\left[\frac{e^{j0.1\pi n}+e^{-j0.1\pi n}}{2}\right] +$$

$$3(0.8)^{-n} u(-n-1)\left[\frac{e^{j0.1\pi n}+e^{-j0.1\pi n}}{2}\right]$$

$$= \frac{3}{2}0.8^n \times e^{j0.1\pi n}u(n) + \frac{3}{2}1.25^n e^{j0.1\pi n}u(-n-1) +$$

$$\frac{3}{2}0.8^n \times e^{-j0.1\pi n}u(n) + \frac{3}{2}1.25^n \times e^{-j0.1\pi n}u(-n-1)$$

$$= \frac{3}{2}\left(0.8\times e^{j0.1\pi}\right)^n u(n) + \frac{3}{2}\left(1.25\times e^{j0.1\pi}\right)^n u(-n-1) +$$

$$\frac{3}{2}\left(0.8\times e^{-j0.1\pi}\right)^n u(n) + \frac{3}{2}\left(1.25\times e^{-j0.1\pi}\right)^n u(-n-1)$$

by the definition of DTFT,

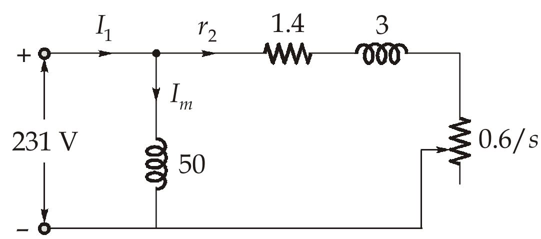
$$\text{DTFT}\{a^n u(n)\} = \sum_{n=0}^{+\infty} a^n e^{-j\omega n} = \frac{e^{j\omega}}{e^{j\omega}-a} \quad \text{if } |a| > 1$$

$$\text{DTFT}\{-a^n u(-n-1)\} = \sum_{n=-1}^{-\infty} a^n e^{-j\omega n} = \sum_{n=0}^{+\infty} a^{-n} e^{j\omega n} - 1 = \frac{-e^{j\omega}}{e^{j\omega} - a} \quad \text{if } |a| < 1$$

$$\therefore \quad X(e^{j\omega}) = \frac{e^{j\omega}}{e^{j\omega} - 0.8e^{j0.1\pi}} + \frac{e^{j\omega}}{e^{j\omega} - 0.8e^{-j0.1\pi}} - \frac{e^{j\omega}}{e^{j\omega} - 1.25e^{j0.1\pi}} - \frac{e^{j\omega}}{e^{j\omega} - 1.25e^{-j0.1\pi}}$$

Q.3 (b) Solution:

The approximate circuit is drawn in figure



(i) Slip = 0.03

$$\frac{R_2'}{s} = \frac{0.6}{0.03} = 20\,\Omega$$

$$\bar{I}_2' = \frac{231\angle 0^\circ}{(1.4+20)+j3} = 10.69\angle{-8^\circ}\text{ A} = 10.58 - j1.49\text{ A}$$

$$\bar{I}_m = \frac{231\angle 0^\circ}{50\angle 90^\circ} = -j4.62\text{ A}$$

$$\bar{I}_1 = \bar{I}_m + \bar{I}_2' = 10.58 - j1.49 - j4.62$$

$$= 10.58 - j6.11 = 12.22\angle{-30^\circ}\text{ A}$$

$$I_1 = 12.22\text{ A, pf} = \cos 30^\circ = 0.866 \text{ lagging}$$

$$\text{Power input} = \sqrt{3}\times 400\times 12.22\times 0.866 = 7.33\text{ kW}$$

$$P_G = \frac{3\times(10.69)^2\times 0.6}{0.03} = 6.86\text{ kW}$$

Mechanical output (gross) = (1 – 0.03) × 6.86 = 6.65 kW

Rotational loss = 0.275 kW

Mechanical output (net) = 6.65 – 0.275 = 6.37 kW

n_s = 1000 rpm,

ω_s = 104.72 rad/sec

$$\text{Torque (net)} = \frac{6370}{104.72(1-0.03)} = 62.22 \text{ Nm}$$

$$\eta = \frac{6.37}{7.33}\times 100 = 86.9\%$$

(ii) Slip = –0.3

$$\frac{R_2'}{s} = -20\ \Omega \text{ (negative resistance)}$$

$$= \frac{231\angle 0^\circ}{(1.4-20)+j3} = 12.26\angle -171.3^\circ \text{ A}$$

$$= -12.12 - j1.85 \text{ A}$$

$$\bar{I}_m = -j4.62$$

$$\bar{I}_1 = (-j4.62) + (-12.12 - j1.85)$$

$$= -12.12 - j6.47 = 13.73\angle -151.9^\circ \text{ A}$$

$$\bar{I}_1(\text{out}) = -\bar{I}_1 = 13.73\angle 28.1^\circ \text{ A (machine is generating)}$$

$$I_1(\text{out}) = 13.73 \text{ A, pf} = \cos 28.1^\circ = 0.882 \text{ leading}$$

$$\text{Power input (elect)} = \sqrt{3}\times 400\times 13.73\times 0.882$$

$$= 8.39 \text{ kW}$$

$$\text{Mechanical power output} = (1-s)\left(\frac{3I_s'^2 R_2'}{s}\right) = 1.03\times\frac{3\times(12.26)^2\times 0.6}{-0.03}$$

$$= -9.20 \text{ kW}$$

$$\text{Mechanical power input (net)} = 9.20 \text{ kW}$$

$$\text{Mechanical power input (gross)} = 9.20 + 0.275 \text{ (rotational loss)} = 9.4 \text{ kW}$$

$$\text{Shaft torque (gross)} = \frac{9480}{104.72(1+0.03)} = 87.9 \text{ Nm}$$

$$\eta \text{ (gen)} = \frac{8.39}{9.48}\times 100 = 88.5\%$$

(iii) Slip = 1.2

$$\frac{R_2'}{s} = \frac{0.6}{1.2} = 0.5\,\Omega$$

$$\bar{I}_2' = \frac{231\angle 0^\circ}{(1.4+0.5)+j3} = 65.1\angle -57.7^\circ \text{ A}$$

$$= 34.79 - j55.02 \text{ A}$$

$$\bar{I}_m = -j4.62$$

$$\bar{I}_1 = (-j4.62) + (34.79 + j55.02)$$

$$= 34.79 - j59.64 \text{ A}$$

$$= 69.05\angle -59.7° \text{ A}$$

$$I_1 = 69.05 \text{ A},$$

$$\text{pf} = \cos 59.7° = 0.505 \text{ lagging}$$

$$\text{Power input (elect)} = \sqrt{3} \times 400 \times 69.05 \times 0.505$$

$$= 24.16 \text{ kW}$$

$$P_G = 3 \times \frac{(65.1)^2 \times 0.5}{1.2} = 5.30 \text{ kW}$$

$$\text{Mechanical power output} = (1 - s)P_G$$

$$= (1 - 1.2) \times 5.30 = -1.06 \text{ kW}$$

or Mechanical power absorbed (net)

$$= 1.06 \text{ kW}$$

Rotational loss can be ignored as motor speed is

$$\eta = 1500 \times (1 - 1.2) = -200 \text{ rpm or } -20.94 \text{ rad/s}$$

Note: Motor runs in opposite direction of the air-gap field - absorbing mechanical power (braking action)

$$\text{Torque developed} = \frac{-1060}{-20.94} = 50.62 \text{ Nm}$$

This torque acts in direction opposite to that of the rotating field.

Total power dissipated (in the motor)

$$= 24.16 \text{ (elect)} + 1.06 \text{ (mech)}$$

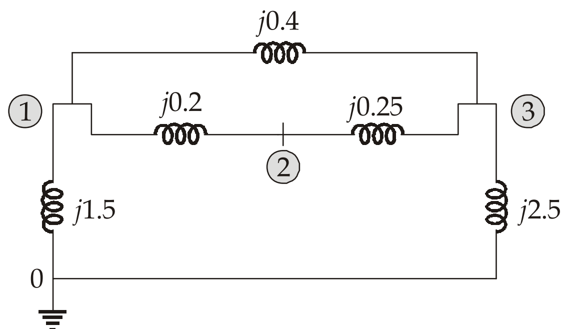
$$= 25.22 \text{ kW}$$

Q.3 (c) Solution:

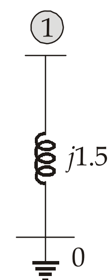
(i) For the Z_{Bus} Matrix, all independent current sources and voltage source should be open circuited and short circuited respectively.

As asked in part (ii), fault current at point *P*, consider point *P* as Bus-2 of power system, similarly *A* or Bus-1 and *B* as Bus-3.

So equivalent network is



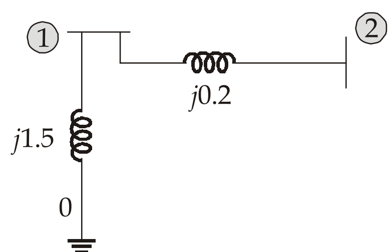
Now from node '0' to node '1'.



$$[Z_{Bus}] = \begin{matrix} & 1 \\ 1 & [j1.5] \end{matrix}$$

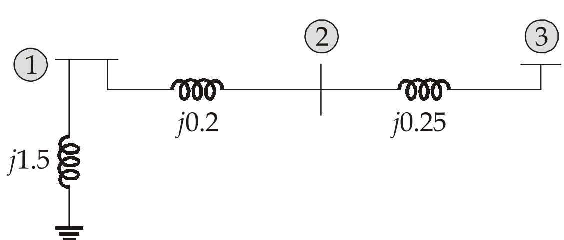
From node '1' to node '2'

$$Z_{Bus} = \begin{matrix} & 1 & 2 \\ 1 & j1 & j1 \\ 2 & j1 & j1.7 \end{matrix}$$



Now from node '2' and node '3'

$$Z_{Bus} = \begin{bmatrix} j1.5 & j1.5 & j1.5 \\ j1.5 & j1.7 & j1.7 \\ j1.5 & j1.7 & j1.95 \end{bmatrix}$$



From node '3' to node '0'

Modification from old Bus 'k' to reference bus Z_b.

$$(Z_{\text{Bus}})_{\text{new}} = (Z_{Bus})_{old} - \frac{1}{z_{33}+z_k}\begin{bmatrix} z_{13} \\ z_{23} \\ z_{33} \end{bmatrix}\begin{bmatrix} z_{13} & z_{23} & z_{33} \end{bmatrix}$$

$$= (Z_{Bus})_{old} - \frac{1}{j1.95+j2.5}\begin{bmatrix} j1.5 \\ j1.7 \\ j1.95 \end{bmatrix}\begin{bmatrix} j1.5 & j1.7 & j1.95 \end{bmatrix}$$

$$= j\begin{bmatrix} 1.5 & 1.5 & 1.5 \\ 1.5 & 1.7 & 1.7 \\ 1.5 & 1.7 & 1.95 \end{bmatrix} - \frac{1}{j4.45}\begin{bmatrix} -2.25 & -2.55 & -2.925 \\ -2.55 & -2.89 & -3.315 \\ -2.925 & -3.315 & -3.8025 \end{bmatrix}$$

$$= j\begin{bmatrix} 1.5 & 1.5 & 1.5 \\ 1.5 & 1.7 & 1.7 \\ 1.5 & 1.7 & 1.95 \end{bmatrix} - \begin{bmatrix} j0.505 & j0.573 & j0.6573 \\ j0.573 & j0.6494 & j0.745 \\ j0.6573 & j0.745 & j0.8545 \end{bmatrix}$$

$$(Z_{\text{Bus}})_{\text{new}} = \begin{bmatrix} j0.995 & j0.927 & j0.8427 \\ j0.927 & j1.0506 & j0.955 \\ j0.8427 & j0.955 & j1.0955 \end{bmatrix}\text{pu}$$

From node (1) to node (3)

$$Z_k = j0.4$$

$$(Z_{\text{Bus}})_{\text{new}} = (Z_{Bus})_{old} - \frac{1}{z_{11}+z_{33}-2z_{13}+z_k}\begin{bmatrix} z_{11}-z_{13} \\ z_{12}-z_{23} \\ z_{13}-z_{33} \end{bmatrix}\begin{bmatrix} z_{11}-z_{13} & z_{12}-z_{23} & z_{13}-z_{33} \end{bmatrix}$$

$$= (Z_{\text{Bus}})_{\text{old}} - \frac{1}{j0.995+j1.0955-2\times(j0.8427)+j0.4}\begin{bmatrix} j0.1523 \\ -j0.028 \\ -j0.2528 \end{bmatrix}\begin{bmatrix} j0.1523 & -j0.028 & -j0.2528 \end{bmatrix}$$

$$= (Z_{Bus})_{old} - \frac{1}{j0.8051}\begin{bmatrix} -0.0232 & 0.00426 & 0.0385 \\ 0.00426 & -0.000784 & -0.007078 \\ 0.0385 & -0.007078 & -0.0639 \end{bmatrix}$$

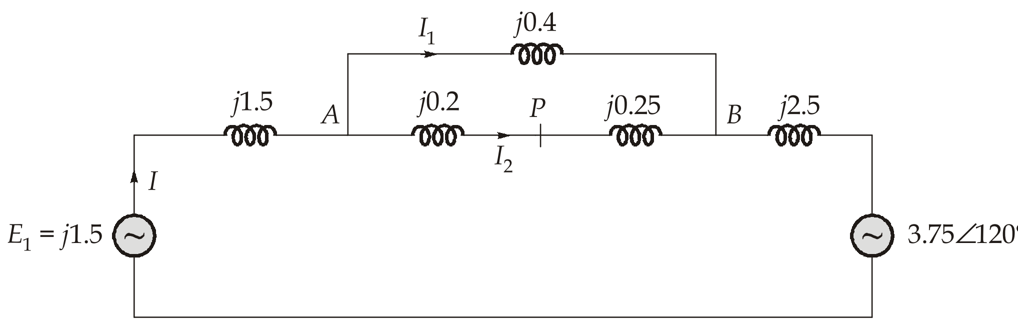
$$= (Z_{Bus})_{old} + j\begin{bmatrix} -0.0288 & 0.005291 & 0.0478 \\ 0.005291 & 0.0009738 & -0.008791 \\ 0.0478 & -0.008791 & -0.07937 \end{bmatrix}$$

$$= \begin{bmatrix} j0.995 & j0.927 & j0.8427 \\ j0.927 & j1.0506 & j0.955 \\ j0.8427 & j0.955 & j1.0955 \end{bmatrix} - \begin{bmatrix} -0.0288j & 0.005291j & 0.00478j \\ 0.005291j & 0.0009738j & -0.008791j \\ 0.0478j & -0.008791j & -0.07937j \end{bmatrix}$$

$$(Z_{Bus})_{new} = j\begin{bmatrix} 1.0238 & 0.9217 & 0.83792 \\ 0.9217 & 1.04962 & 0.96379 \\ 0.83792 & 0.96379 & 1.17487 \end{bmatrix} \text{pu}$$

(ii) For fault at point P

changing current source into equivalent voltage source.



Calculate prefault voltage at point P

$$\text{Current } I = \frac{j1.5 - 3.75\angle 120°}{j1.5 + j2.5 + [j0.45 \,||\, j0.4]}$$

$$I = 0.6085 \angle -132.985° \text{ pu}$$

From current division,

$$I_2 = \frac{j0.4}{j0.4 + j0.2 + j0.25} \times I = 0.28638\angle -132.985°$$

Now,

$$V_p = E_1 - j1.51 - j0.2I_2$$

$$= j1.5 - j1.5(0.6085 \angle -132.985) - j0.2[0.28638 \angle -132.985°]$$

$$V_p = 2.274 \angle 108.185° \text{ pu}$$

fault current,

$$I_f = \frac{V_p}{Z_{22}} = \frac{2.274\angle 108.185°}{j1.04962}$$

$$I_f = 2.167 \angle 18.18° \text{ pu}$$

(iii) Post fault voltage at Bus (1)

$$V_A f = V_p\left(1 - \frac{z_{12}}{z_{22}}\right) = 2.274\angle 108.185\left(1 - \frac{0.927}{1.04962}\right)$$

$$V_A f = 0.2656 \angle 108.185 \text{ pu}$$

Q.4 (a) Solution:

Given, plant transfer function,

$$G_p(s) = \frac{K}{s(s+4)}, \text{ peak overshoot, } \%M_p = 25\%$$

We know that, peak overshoot

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$\xi = \frac{(-\ln(M_p))}{\sqrt{\pi^2 + (\ln(M_p))^2}}$$

$$\xi = \frac{-\ln(0.25)}{\sqrt{\pi^2 + (\ln(0.25))^2}} = 0.404$$

$$\xi \cong 0.4$$

Characteristic equation,

$$s^2 + 4s + K = 0$$

Standard second order characteristic equation,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

On comparing, $\omega_n^2 = K, \quad 2\xi\omega_n = 4 \quad \Rightarrow \quad \xi\omega_n = 2$

$$\omega_n = \frac{2}{\xi} = \frac{2}{0.40} = 5 \text{ rad/sec}$$

$\therefore$ $K = \omega_n^2 = 5^2 = 25$

$\therefore$ Settling time, $T_s = \frac{4}{\xi\omega_n} = \frac{4}{2} = 2$

$\therefore$ The plant transfer function,

$$G_p(s) = \frac{25}{s(s+4)}$$

Let the lead compensator is having transfer function,

$$G_c(s) = \frac{K_c(s+a)}{(s+b)}$$

Peak overshoot will remain same after addition of compensator

i.e., $\xi' = \xi = 0.4$

Settling time is reduced by a factor of 2 after addition of compensator

i.e., $$T_s' = \frac{T_s}{2} = 1 \text{ sec}$$

$$\frac{4}{\xi'\omega_n'} = 1 \Rightarrow \omega_n' = \frac{4}{0.4} = 10 \text{ rad/sec}$$

Let us select a zero directly below the dominant pole for a lead compensator,

i.e, $$a = \xi'\omega_n' = 4$$

$$G_c(s) = \frac{K_c(s+4)}{(s+b)}$$

The open loop transfer function after adding lead compensator,

$$G_p(s)G_c(s) = \frac{25}{s(s+4)} \times \frac{K_c(s+4)}{(s+b)}$$

$$G_p(s)G_c(s) = \frac{25K_c}{s(s+b)}$$

The characteristic equation,

$$s^2 + bs + 25K_c = 0$$

$$\therefore \quad \omega_n'^2 = 25\,K_c \Rightarrow K_c = \frac{100}{25} = 4$$

$$2\xi'\omega_n' = b \Rightarrow b = 2 \times 0.4 \times 10 = 8$$

Transfer function of lead compensator,

$$G_c(s) = \frac{4(s+4)}{(s+8)}$$

Overall open-loop transfer function of a system,

$$G_p(s)G_c(s) = \frac{25}{s(s+4)} \times \frac{4(s+4)}{(s+8)}$$

$$G_p(s)G_c(s) = \frac{100}{s(s+8)}$$

Overall transfer function, $$T(s) = \frac{\frac{100}{s(s+8)}}{1+\frac{100}{s(s+8)}}$$

$$\therefore \quad T(s) = \frac{100}{s^2+8s+100}$$

Q.4 (b) Solution:

(i) For the field converter with $\alpha = 0°$

$$\text{field voltage } V_f = \frac{2V_m}{\pi} = \frac{2\times\sqrt{2}\times 400}{\pi} = 360 \text{ Volt}$$

$$\text{field current, } I_f = \frac{V_f}{R_f} = \frac{3600}{100} = 3.6 \text{ A}$$

with magnetic saturation neglected

$$\phi = K_f \cdot I_f$$

and $E_a = K_a \cdot \phi\omega_m = K_a \cdot K_f I_f \cdot \omega_m = K \cdot I_f \cdot \omega_m$

where K has the unit at V-sec/A-rad

Similarly $$T_e = K_a\phi \cdot I_a = K_a K_f \cdot I_f \cdot I_a$$

$$T_e = KI_f \cdot I_a$$

Given, $$T_e = 90 \text{ N-m}$$

$$90 = 0.5 \times I_a \times 3.6$$

$$\text{Rated armature current } I_a = \frac{90}{0.5\times 3.6} = 50 \text{ Amp}$$

(ii) Here,

$$V_t = V_0 = \frac{2V_m}{\pi}\cos\alpha = E_a + I_a \cdot r_a$$

$$= K \cdot I_f \cdot \omega_m + I_a r_a$$

$$\frac{2\times\sqrt{2}\times 400}{\pi}\cos\alpha = 0.5\times 3.6\times\frac{2\pi\times 1500}{60} + 50\times 0.1$$

$$360 \cos\alpha = 287.743$$

$$\alpha = 36.938°$$

(iii) At the same firing angle $\alpha = 36.938°$, motor emf at no load

$$E_a = V_t = V_0 = \frac{2V_m}{\pi}\cos\alpha = \frac{2\sqrt{2}\times 400}{\pi}\cos 36.938$$

$$K \cdot I_f \cdot \omega_{m0} = 287.743$$

$$\omega_{m0} = \frac{287.743}{0.5\times 3.6} = 159.857 \text{ rad/sec}$$

$$N = \frac{60\times 159.857}{2\pi} = 1526.524 \text{ rpm}$$

% speed regulation at full load

$$= \frac{\text{No load speed – full load speed}}{\text{full load speed}} \times 100$$

$$= \frac{1526.524 - 1500}{1500} \times 100 = 1.768\%$$

(iv) Input power factor of armature

$$= \frac{V_t \cdot I_a}{V_s \cdot I_{ar}} = \frac{287.743 \times 50}{400 \times 50} = 0.72 \text{ lag}$$

Rms value of current in armature converter

$$I_{ar} = I_a = 50 \text{ A}$$

Rms value of current in field current

$$I_{fr} = I_f = 3.6 \text{ A}$$

Total RMS current taken from source

$$I_{sr} = \sqrt{I_{ar}^2 + I_{fr}^2} = \sqrt{50^2 + 3.6^2} = 50.13 \text{ A}$$

$$\text{Input } VA = V_{sn} \cdot I_{sn} = 400 \times 50.13 = 20.051 \text{ kVA}$$

With no loss in the converter, total power input to the motor and field circuit

$$= V_t \cdot I_a + V_f \cdot I_f$$

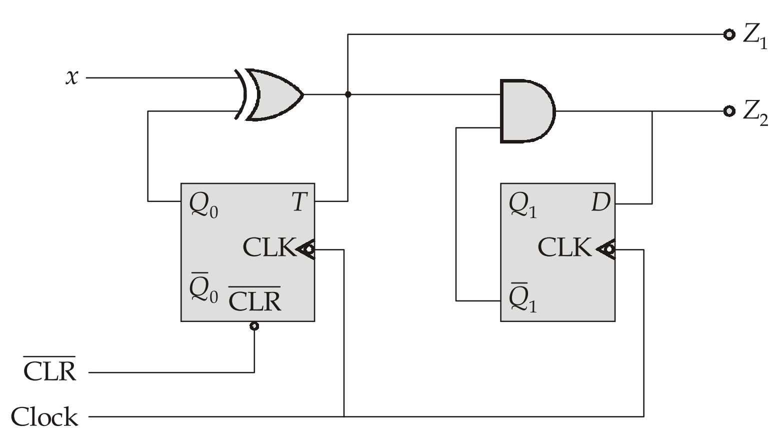
$$= 287.743 \times 50 + 360 \times 3.6$$

$$= 15.683 \text{ kW}$$

$$\text{Input power factor of the drive} = \frac{\text{Power input (kW)}}{\text{Power input (VA)}} = \frac{15.683}{20.051} = 0.782 \text{ lag}$$

Q.4 (c) Solution:

(i)

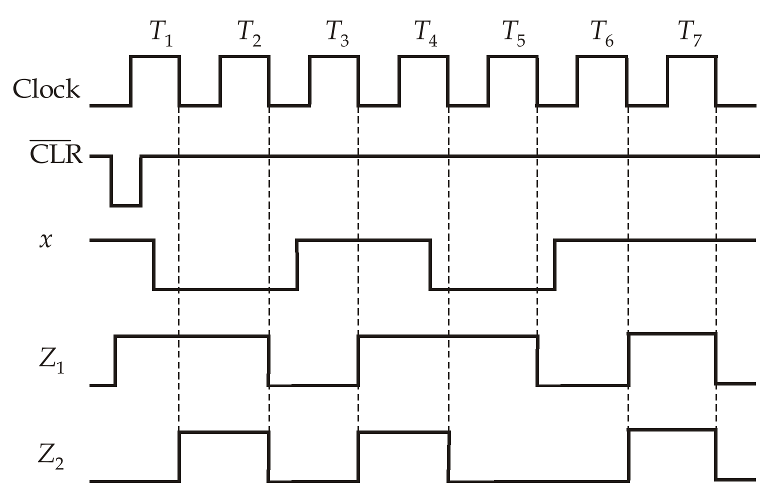


$$Q_{0(n+1)} = T\overline{Q}_{0(n)} + \overline{T}Q_{0(n)}$$

$$\bar{Q}_{1(n+1)} = \bar{D}$$
$$Z_1 = x \oplus Q_0 = T$$
$$Z_2 = Z_1 \cdot \bar{Q}_1 = D$$

n^{th} cycle	x	T	Q_0	Z_1	D	Q_1	Z_2
0	1	1	0	1	0	0	0
↓ T_1	0	1	1	1	1	0	1
↓ T_2	0	0	0	0	0	1	0
↓ T_3	1	1	0	1	1	0	1
↓ T_4	0	1	1	1	0	1	0
↓ T_5	0	0	0	0	0	0	0
↓ T_6	1	1	0	1	1	0	1
↓ T_7	1	0	1	0	0	1	0

Both flip-flops are negative edge triggered:



(ii) D to JK excitation table:

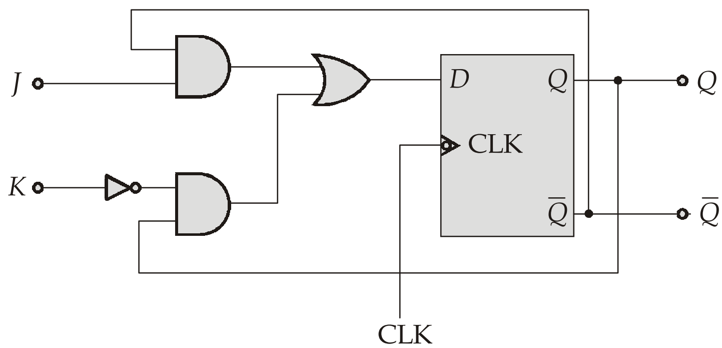
J	K	Q_n (Present state)	Q_{n+1} (Next state)	D
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	0
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

K-map:

$J \backslash KQ_n$	00	01	11	10
0	0	1	0	0
1	1	1	0	1

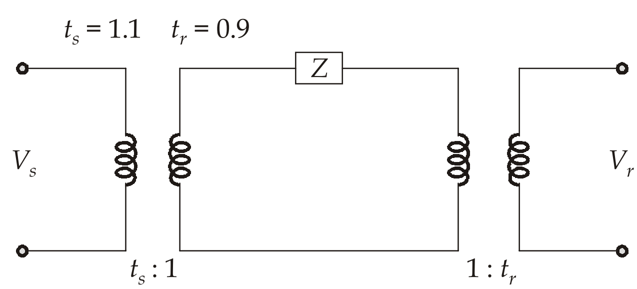
$D = J\bar{Q}_n + \bar{K}Q_n$

Logic diagram:



Section-B

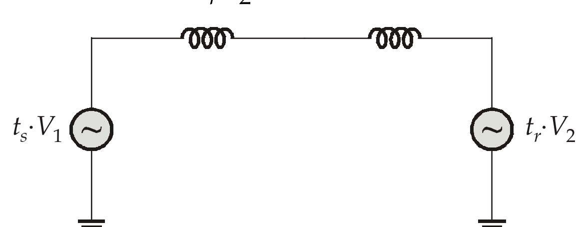
Q.5 (a) Solution:



impedance diagram is given as

$$\Delta V = V_1 t_s - t_r V_2$$

$$= \frac{t_r V_2}{t_r V_2} I(X \sin\theta \times R\cos\theta)$$



$$P_R = t_r V_2 I \cos\phi$$

$$Q_R = t_r V_2 I \sin\theta$$

$$V_1 t_s - V_2 t_r = \frac{PR+QX}{t_r \cdot V_2}$$

Since, $$t_s \cdot t_r = 1$$

$$t_s \cdot V_1 = t_r \cdot V_2 + \frac{PR+QX}{t_r \cdot V_2}$$

$$t_s = \frac{1}{V_1}\left[\frac{V_2}{t_s} + \frac{PR+QX}{V_2 / t_s}\right]$$

$$t_s^2 = \frac{V_2}{V_1} + \frac{PR+QX}{V_1 \cdot V_2} \cdot t_s^2$$

$$t_s^2\left[1 - \frac{PR+QX}{V_1 \cdot V_2}\right] = \frac{V_2}{V_1}$$

Given, $$V_1 = V_2 = 1 \text{ pu}$$

$R = 0$ $$X = 2 \times 0.2 = 0.4 \text{ pu}$$

$$t_s^2\left[1 - \frac{0.4 \times Q}{1 \times 1}\right] = 1$$

$$1 - 0.4Q = \left(\frac{1}{1.1}\right)^2$$

$$0.4Q = 0.1735$$

$$Q = 0.4338 \text{ pu}$$

$$Q = 0.4338 \times 100 = 43.38 \text{ MVAR}$$

Q.5 (b) Solution:

Given, $$G(s) = \frac{ke^{-0.5s}}{(s+1)}$$

put $$s = j\omega$$

$$G(j\omega) = \frac{ke^{-0.5j\omega}}{(1+j\omega)}$$

$\therefore$ $$|G(j\omega)| = \left|\frac{k}{1+j\omega}\right| = \frac{k}{\sqrt{\omega^2+1}} \qquad (\because |e^{-j0.5\omega}| = 1)$$

$$\angle G(j\omega) = -\tan^{-1}\omega - 0.5\,\omega$$

$$= -(\tan^{-1}\omega + 0.5\,\omega)$$

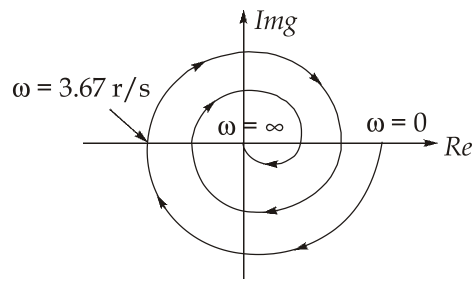
As $\omega \to 0$ $$|G(j\omega)| = k$$

$$\angle G(j\omega) = 0°$$

$\omega \to \infty \qquad |G(j\omega)| = 0$

$\angle G(j\omega) = -\infty$

As ω varies from $\omega = 0$ to $\omega = \infty$, $|G(j\omega)|$ decreases and $\angle G(j\omega)$ changes continuously from 0° to ∞. The Nyquist plot takes the shape of a spiral intersecting real and imaginary axis many time.



The intersection points of the plot with the negative real axis of $G(s)H(s)$-plane are determined using the relation given below:

$$-(\tan^{-1}\omega + 0.5\,\omega) = -180\,(2k + 1)$$

$$k = 0, 1, 2, ...$$

For $\omega > 0$, at the first instant, the said intersection is given by considering $k = 0$. The frequency at this intersection point is the phase crossover frequency ω_2.

Thus, $$-(\tan^{-1}\omega_2 + 0.5\omega_2) = -180°$$

$$\tan^{-1}\omega_2 + 0.5\omega_2 = 180° \times \frac{(\pi)}{180°} = \pi$$

using trial and error, this equation is satisfied at $\omega_2 = 3.67$ rad/sec.

For stability $|G(j\omega_2)| < 1$

$$|G(j\omega_2)| = \left|\frac{ke^{-j0.5\omega_2}}{j\omega_2 + 1}\right|$$

$\because$ $$\left|e^{-j0.5\omega_2}\right| = 1$$

$$|G(j\omega_2)| = \frac{k}{\sqrt{1+\omega_2^2}}$$

$$\omega_2 = 3.67 \text{ rad/sec}$$

$$|G(j3.67)| = \frac{k}{\sqrt{1+(3.67)^2}} = \frac{k}{3.8}$$

For stability, the critical point $(-1 + j0)$ should not be encircled by the Nyquist plot

$$\frac{k}{3.8} < 1$$

$$k < 3.8$$

Q.5 (c) Solution:

Battery terminal voltage is

$$V_0 = \frac{3V_{ml}}{\pi}\cos\alpha = E + I_0R$$

$$\frac{3\sqrt{2}\times 220}{\pi}\cos\alpha = 200 + (15 \times 0.3) = 204.5$$

$$\cos\alpha = 0.688$$

Firing angle, $\alpha = 46.50°$

As given load current is constant, the supply current of any phase is quasi-square wave of amplitude 15 A. Each phase current flows for 120° over every half cycle of 180° rms value of supply current I_s over π-radians is

$$I_{sr} = 15\sqrt{\frac{\frac{2\pi}{3}}{\pi}} = 15\sqrt{\frac{2}{3}} = 12.247 \text{ A}$$

Rms value of output current, $I_{0r} = I_0$ (constant)

Power delivered to load is

$$P_L = EI_0 + I_{0r}^2 \cdot R = 200 \times 15 + 15^2 \times 0.3 = 3067.5 \text{ W}$$

Now, $\sqrt{3}V_{sr}\cdot I_{sr}\cdot\cos\phi = P_L$

$$\sqrt{3}\times 220\times 12.247\times\cos\phi = 3067.5$$

Supply power factor $= \cos\phi = 0.657$ lag.

Q.5 (d) Solution:

Given, $X = Ax$

time response is given by $X(t) = \phi(t)\,x(0)$

where, $\phi(t) = L^{-1}\left[(sI - A)^{-1}\right]$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} s & -1 \\ 2 & s \end{bmatrix}^{-1}$$

$$[sI - A]^{-1} = \frac{\text{Adj}(sI - A)}{|sI - A|} = \frac{\begin{bmatrix} s & 1 \\ -2 & s \end{bmatrix}}{\begin{vmatrix} s & -1 \\ 2 & s \end{vmatrix}}$$

$$\therefore \quad [sI - A]^{-1} = \frac{\begin{bmatrix} s & 1 \\ -2 & s \end{bmatrix}}{s^2+2} = \begin{bmatrix} \frac{s}{s^2+2} & \frac{1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{bmatrix}$$

$\therefore$ The state transition matrix $\phi(t)$ is

$$\phi(t) = L^{-1}[\phi(s)] = L^{-1}\begin{bmatrix} \frac{s}{s^2+2} & \frac{1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{bmatrix}$$

$$\therefore \quad \phi(t) = \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\frac{2}{\sqrt{2}}\sin\sqrt{2}\,t & \cos\sqrt{2}t \end{bmatrix}$$

$$= \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}$$

$$X(t) = \phi(t)\,x(0)$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x_1(t) = \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}\,t$$

$$x_2(t) = -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t$$

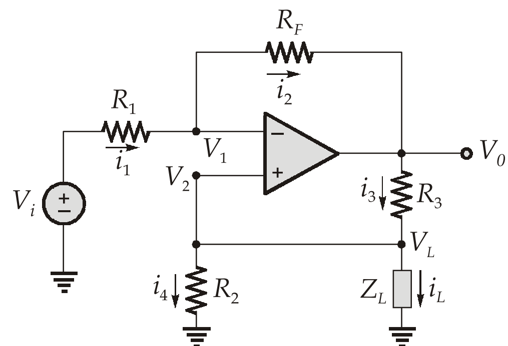
the time response $y(t) = x_1(t) - x_2(t)$

$$y(t) = \left[\cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t\right] - \left[-\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t\right]$$

$$\therefore \quad y(t) = \frac{3}{\sqrt{2}}\sin\sqrt{2}t$$

Q.5 (e) Solution:

The circuit can be redrawn by showing the currents,



From virtual short concept, $V_1 = V_2$ and also, we know that

$$V_1 = V_2 = V_L = i_L z_L$$

and

$$i_1 = i_2$$

$$\frac{V_i - i_L z_L}{R_1} = \frac{i_L z_L - V_0}{R_F} \quad \text{...(i)}$$

Taking the sum of currents in the non-inverting terminal,

$$i_3 = i_4 + i_L$$

$$\frac{V_0 - i_L Z_L}{R_3} = i_L + \frac{i_L z_L}{R_2} \quad \text{...(ii)}$$

From equation (i) and (ii) solving for $(V_0 - i_L z_L)$

$$\frac{R_F}{R_1 R_3}(i_L z_L - V_i) = i_L + \frac{i_L z_L}{R_2}$$

Combining terms in i_L, we get,

$$i_L\left(\frac{R_F z_L}{R_1 R_3} - \frac{z_L}{R_2} - 1\right) = \frac{V_i R_F}{R_1 R_3} \quad \text{...(iii)}$$

In order to make i_L independent of z_L, we can design the circuit such that the coefficient of z_L is zero.

i.e.
$$\frac{R_F}{R_1 R_3} = \frac{1}{R_2}$$

$\Rightarrow$
$$R_F = \frac{R_1 R_3}{R_2}$$

then equation (iii) reduces to

$$i_L = \frac{-V_i (R_F)}{R_1 R_3} = \frac{-V_i}{R_2}$$

Which means that load current is proportional to input voltage and is independent of the load impedance Z_L.

Q.6 (a) Solution:

(i) The relationship between the dc supply V_s and dc machine back emf is given by,

$$I_o = \frac{E - V_o}{R} = \frac{E - V_s(1-\alpha)}{R}$$

$$10 = \frac{150 - 200(1-\alpha)}{1}$$

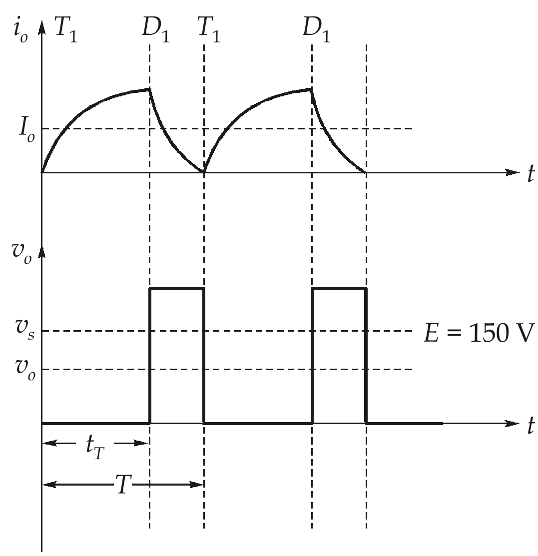
$$200(1 - \alpha) = 150 - 10$$

$$1 - \alpha = \frac{140}{200} = 0.7$$

$$\alpha = 0.3 \text{ or } 30\% \text{ duty cycle}$$

Load time constant, $\tau = \frac{L}{R} = \frac{1}{1}\text{ ms} = 1 \text{ msec}$

Waveforms :



The expression for average dc machine output current is based on continuous armature inductance current. Therefore, the switching period must be shorter than the time t_x, given by below expression, for the current to reach zero, before the next switch on-period. That is $t_x = T$ and $\alpha = 0.30$.

$$t_x = t_T + \tau \ln\left[1 + E\left(1 - e^{-\frac{t_T}{T}}\right)\right]$$

$$1 = 0.3 + \frac{1\,\text{mS}}{T}\ln\left[1 + \frac{150}{50}\left(1 - e^{\frac{-0.3T}{1\,\text{mS}}}\right)\right]$$

$$e^{0.7T} = 4 - 3e^{-0.3T}$$

On solving for T,

$$T = 0.494 \text{ msec}$$

Therefore, switching frequency must be greater than $f_s = \frac{1}{T} = 2.024$ kHz, else machine output current discontinuous.

(ii) The operational boundary giving by equation,

$$\frac{E}{V_s} = \frac{1 - e^{-\frac{T+t_T}{\tau}}}{1 - e^{-\frac{T}{\tau}}}$$

$$\frac{150}{200} = \frac{1 - e^{\frac{(\alpha-1)\times 1\text{ msec}}{1\text{ msec}}}}{1 - e^{-\frac{1\text{ mS}}{1\text{ mS}}}}$$

On solving for α;

$$\alpha = 0.357$$

Therefore, on-state duty cycle must be at least 35.7%. For continuous machine output current at a switching frequency of 1 kHz,

$$I_o = \frac{E - V_o}{R} = \frac{150 - 200(1 - 0.357)}{1} = 21.4 \text{ A}$$

$$V_o = 150 - 21.4 \times 1 = 128.6 \text{ Volt}$$

(iii) At an increased switching frequency of 5 kHz, the duty cycle would be expected to be much lower that the 35.7% as at 1 kHz. The operational boundary between continuous and discontinuous armature current is given by equation

$$\frac{E}{V_s} = \frac{1 - e^{\frac{-T+t_T}{\tau}}}{1 - e^{-\frac{T}{\tau}}}$$

$$\frac{150}{200} = \frac{1 - e^{(-1+\alpha)\times\frac{0.2}{1}}}{1 - e^{\frac{-0.2}{1}}} \Rightarrow \alpha = 26.9\%$$

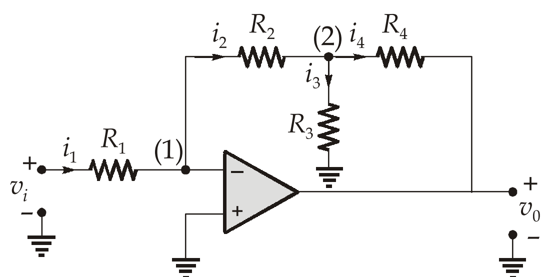
Machine average output current,

$$I_o = \frac{E - V_o}{R} = \frac{150 - 200 \times (1 - 0.269)}{1} = 3.8 \text{ A}$$

and average output voltage,

$$V_o = (1 - \alpha)V_s = 146.2 \text{ V}$$

Q.6 (b) Solution:



Assume opamp to be ideal: $R_i \to \infty$, $R_o \to 0$, $A_{OL} \to \infty$

Virtual Ground Theory:

$$v_+ = v_- = 0$$

$\Rightarrow$ $$i_1 = \frac{v_i - v_-}{R_1} = \frac{v_i}{R_1}$$

At node (1) $$i_- = 0 \ (\because R_i \to \infty)$$

$\Rightarrow$ $$i_2 = i_1 = \frac{v_i}{R_1}$$

$$v_2 = v_- - i_2 R_2 = -v_i \frac{R_2}{R_1}$$

KCL at node (ii)

$$i_2 = i_3 + i_4$$

$$\frac{v_i}{R_1} = \frac{v_2}{R_3} + \frac{v_2 - v_o}{R_4}$$

$$\frac{v_i}{R_1} = -v_i \frac{R_2}{R_1 R_3} - v_i \frac{R_2}{R_1 R_4} - \frac{v_o}{R_4}$$

$\Rightarrow$ $$\frac{v_o}{v_i} = -\frac{R_2}{R_1}\left(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right)$$

Now, Input resistance = 1 MΩ

$$\frac{v_i}{i_1} = R_1 = 1\text{ M}\Omega \text{ (Maximum limit)}$$

Voltage gain = 100

$\because R_1$ is at maximum possible value choose R_2 also at maximum value,

$$R_2 = 1\text{ M}\Omega$$

Now,
$$1+\frac{R_4}{R_2}+\frac{R_4}{R_3} = 100$$

$$R_4\left(\frac{1}{R_2}+\frac{1}{R_3}\right) = 99$$

If we choose R_4 at maximum value = 1 MΩ

Then, R_3 = 10.2 kΩ < 1 MΩ

Q.6 (c) Solution:

Given : f_m = 3 kHz, n = 8

$$\overline{x^2_{(t)}} = \text{Mean square value of message signal}$$
$$= 2\text{ V}$$

(i) Normalized power for quantization noise

$$N_q = \frac{\Delta^2}{12} \text{ where } \Delta = \text{step size}$$

$$\Delta = \frac{V_H - V_L}{L} = \frac{V_H - V_L}{2^n} = \frac{5-(-5)}{2^8} = 0.0390625$$

$$\Delta = 39.0625\text{ mV}$$

So,
$$N_q = \frac{\Delta^2}{12} = \frac{(39.0625\times10^{-3})^2}{12}$$

$$N_q = 127.1566 \times 10^{-6}\text{ W}$$

(ii) Bit transmission rate

$$R_b = nf_s$$
$$f_s = 2f_m$$
$$= 2 \times 3 \times 10^3 = 6\text{ kHz}$$
$$R_b = 8 \times 6 \times 10^3 = 48 \times 10^3\text{ bps}$$
$$R_b = 48\text{ kbps}$$

(iii) The normalized signal power,

$$P = \frac{\text{Mean square value of signal}}{1\ \Omega}$$

$$P = 2 \text{ Watts}$$

Signal to quantization noise ratio :

$$SN_qR = \frac{P}{N_q} = \frac{2}{127.1566 \times 10^{-6}}$$

$$SN_qR = 15728.64$$

$$\text{In dB} = 10 \log(15728.64)$$

$$SN_qR \text{ in dB} = 41.967 \text{ dB}$$

Q.7 (a) Solution:

$$P_{L1} = P_m \sin \delta_1 = P_m \sin 30°$$

$$P_{L1} = 0.5P_m$$

Now,
$$P_{L2} = \sqrt{2}P_{L1}$$

$$P_{L2} = \sqrt{2} \times 0.5P_m \quad \text{...(i)}$$

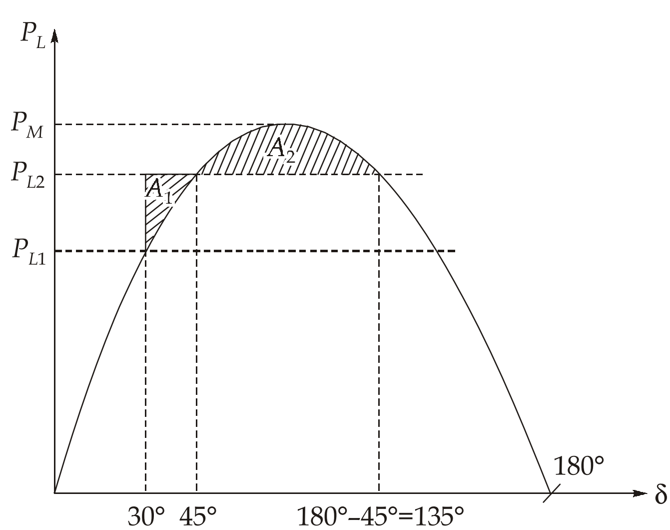
$$P_{L2} = P_m \sin \delta_2 \quad \text{...(ii)}$$

Comparing eqn. (i) and (ii),

$$\sqrt{2} \times 0.5 = \sin \delta_2$$

$$\delta_2 = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

Now power angle curve



Using equal area criterion for transient stability

$$\text{Area, } A_1 = \text{Area, } A_2$$

$$\text{Area, } A_1 = \int_{\pi/6}^{\pi/4} \left(\sqrt{2}P_{L2} - P_m \sin\delta\right) d\delta$$

$$A_1 = P_m \int_{\pi/6}^{\pi/4} (0.707 - \sin\delta)\, d\delta$$

$$A_1 = 0.026 P_m$$

$$\text{Area, } A_2 = \int_{\pi/4}^{3\pi/4} \left(P_m \sin\delta - \sqrt{2}P_{L1}\right) d\delta$$

$$= P_m \left[-\cos\delta \Big|_{\pi/4}^{3\pi/4} - 0.707\delta \Big|_{\pi/4}^{3\pi/4} \right]$$

$$A_2 = 0.3036 P_m$$

Since $A_2 > A_1$, therefore, the system is stable.

Let the power angle corresponding to safe load P_{Ls} be δ_s.

$$\text{Area, } A_1 = \int_{\pi/6}^{\delta_s} (P_{Ls} - P_m \sin\delta)\, d\delta$$

$$= (\delta_s - 30°) \times \frac{\pi}{180°} P_{Ls} + P_m (\cos\delta_s - \cos 30°)$$

But, we know that

$$P_{Ls} = P_m \sin\delta_s$$

Therefore,

$$A_1 = P_m \left[\frac{\pi}{180°} (\delta_s - 30°) \sin\delta_s + (\cos\delta_s - \cos 30°) \right] \quad \text{...(1)}$$

Area, A_2 :

$$A_2 = \int_{\delta_s}^{(180° - \delta_s)} (P_m \sin\delta - P_s)\, d\delta$$

$$= P_m \left[2\cos\delta_s - \frac{\pi}{180°} (180 - 2\delta_s) \sin\delta_s \right] \quad \text{...(ii)}$$

For drive to remain stable,

$$A_1 = A_2$$

Equating eqn. (i) and (ii),

$$\frac{\pi}{180°}(150 - \delta_s)\sin\delta_s = 0.866 + \cos\delta_s$$

Solving by trial and error method, we get

$$\delta_s = 60.50°$$

Hence, the maximum safe load $= P_m \sin 60.5°$

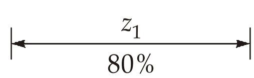
$$P_{Ls} = 1.74 P_{L1}$$

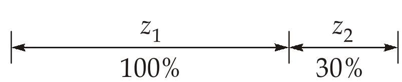
So, additional load that can be thrown suddenly on the shaft = 0.74 × rated load.

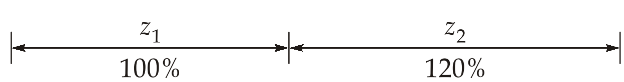
Q.7 (b) Solution:

CT ratio : 500/1

PT ratio : 132 kV/110 V

for Zone 1. 

for Zone 2. 

for Zone 3. 

(i) Reactance Relay:

For Zone (1): 80% of section-1,

$$Z_1 = (1.5 + j4) \times 0.8 = (1.2 + j3.2)\ \Omega$$

Reactance: $X_1 = 3.2\ \Omega \rightarrow$ primary side

$$X_1(\text{secondary}) = \left(\frac{\text{CTR}}{\text{PTR}}\right) \times X_1(\text{Primary})$$

$$= \frac{500/1}{132000/110} \times 3.2 = \frac{4}{3}\ \Omega$$

Therefore, setting for reactance relay for zone-1 : = 1.33 Ω

For zone (2): 100% of section-1 + 30% of section-2

$$Z_2 = (1.5 + j4) \times 1 + 0.3 \times (3.5 + j7) = (2.55 + j6.1)\Omega$$

$$X_2 = 6.1\ \Omega$$

$$X_2(\text{secondary}) = \frac{\text{CTR}}{\text{PTR}} \times X_2(\text{Primary})$$

$$= \frac{500 \times 110}{132000} \times 6.1 = 2.541\ \Omega$$

Therefore, setting of reactance relay for zone-2 = 2.541 Ω

For zone (3): 100% of section 1 + 120% of section-2

$$Z_3 = (1.5 + j4) + 1.2\,(3.5 + j7) = (5.7 + j12.4)\Omega$$

$$X_3 = 12.4\ \Omega$$

$$X_3(\text{secondary}) = \frac{\text{CTR}}{\text{PTR}} \times X_3(\text{Primary})$$

$$= \frac{500 \times 110}{132000} \times 12.4 = 5.1667\ \Omega$$

Therefore, setting of reactance relay for zone-3 = 5.1667 Ω

(ii) Mho relay:

$$\alpha = 60°$$

For zone 1: z_1 = 80% of section 1

$$z_1 = (1.2 + j3.2)\ \Omega$$

$$z_1(\text{secondary}) = \frac{\text{CTR}}{\text{PTR}} \times Z_1(\text{Primary})$$

$$= \frac{500 \times 110}{132000} \times (1.2 + j3.2) = (0.5 + j1.33)\ \Omega$$

$$z_1 = 1.424 \angle 69.44° \qquad \phi = 69.44°$$

Therefore, setting of Mho relay for zone-1

$$k_1 = \frac{z_1}{\cos(\phi - \alpha)} = \frac{1.424}{\cos(69.44 - 60°)} = 1.4435\ \Omega$$

For zone 2: z_1 = 100% of section 1 + 30% of section 2

$$= (2.55 + j6.1)\ \Omega = 6.611 \angle 67.313°$$

$$z_2(\text{secondary}) = \frac{\text{CTR}}{\text{PTR}} \times Z_2(\text{Primary})$$

$$= \frac{500 \times 110}{132000} \times (6.611\angle 67.313°) = 2.754\angle 67.31°$$

Therefore, setting of Mho relay for zone-2,

$$k_2 = \frac{2.754}{\cos(67.31 - 60°)} = 2.776\ \Omega$$

For zone 3: z_3 = 100% of section 1 + 120% of section 3

$$= (1.5 + j4) + 1.2(3.5 + j7)$$

$$z_3 = (5.7 + j12.4)\Omega = 13.647 \angle 65.31°$$

$$z_3(\text{secondary}) = \frac{500 \times 110}{132000} \times (13.647\angle 65.31°)$$

$$= 2.375 + j5.1666 = 5.686\ \angle 65.31°$$

Therefore, setting of Mho relay for zone-3,

$$k_3 = \frac{5.686}{\cos(5.31)} = 5.71\ \Omega$$

Q.7 (c) (i) Solution:

The initial slope of the plot is –20 dB/dec, and its intersection with the 0 dB axis is located at ω_4, hence the system is type 1 and $k = \omega_4$.

At ω_1 the slope of the plot changes by –20 dB/dec, hence the corresponding term of the transfer function is $\dfrac{1}{(1+sT_1)}$

where, $T_1 = \dfrac{1}{\omega_1}$

At ω_2 the slope of the plot changes by +20 dB/dec, hence the corresponding term of the transfer function is $1 + sT_2$.

where, $T_2 = 1/\omega_2$

At ω_3, the slope of the plot change by –20 dB/dec, hence the corresponding term of the transfer function is $\dfrac{1}{(1+sT_3)}$

where, $T_3 = \dfrac{1}{\omega_3}$

for ω_1: $$\log_{10}\left(\frac{8}{\omega_1}\right) = \frac{24.1}{40}$$

$$\frac{8}{\omega_1} = \text{antilog}_{10}\frac{24.1}{40} = 4$$

$\therefore$ $$\omega_1 = 2 \text{ rad/sec}$$

and $$T_1 = \frac{1}{\omega_1} = \frac{1}{2} = 0.5 \text{ sec}$$

for ω_2: $$\log_{10}\left(\frac{\omega_2}{\omega_1}\right) = \frac{12.05+24.1}{40} = \frac{36.15}{40}$$

$$\frac{\omega_2}{\omega_1} = \text{antilog}_{10}\left(\frac{36.15}{40}\right) = 8$$

$\therefore$ $$\omega_2 = 8\omega_1 = 8 \times 2 = 16 \text{ rad/sec}$$

$\therefore$ $$T_2 = \frac{1}{\omega_2} = \frac{1}{16} = 0.0625 \text{ sec}$$

for ω_3: $$\log_{10}\left(\frac{\omega_3}{\omega_2}\right) = \frac{20.05 - 12.05}{20} = \frac{8}{20}$$

$$\frac{\omega_3}{\omega_2} = \text{antilog}_{10}\left(\frac{8}{20}\right) = 2.5$$

$$\omega_3 = 2.5\omega_2 = 2.5 \times 16 = 40 \text{ rad/sec}$$

and $$T_3 = \frac{1}{\omega_3} = \frac{1}{40} = 0.025 \text{ sec}$$

for ω_4: $$\log_{10}\left(\frac{\omega_4}{\omega_1}\right) = \frac{24.1}{20}$$

$$\frac{\omega_4}{\omega_1} = \text{antilog}_{10}\left(\frac{24.1}{20}\right) = 16$$

$$\frac{\omega_4}{\omega_1} = 16 \quad \Rightarrow \quad \omega_4 = 16\,\omega_1 = 16 \times 2 = 32 \text{ rad/sec}$$

and $$K = \omega_4 = 32$$

$\therefore$ $$G(s)H(s) = \frac{K(sT_2+1)}{s(sT_1+1)(sT_3+1)}$$

$\therefore$ $$G(s)H(s) = \frac{32(0.0625s+1)}{s(0.5s+1)(0.025s+1)}$$

Q.7 (c) (ii) Solution:

According to Mason's gain formula,

$$\frac{C}{R} = \sum_{i=1}^{N} \frac{P_i \Delta_i}{\Delta}$$

$$P_i = \text{Gain of } i^{th} \text{ forward path}$$

Δ = 1 – Σ (Individual loop gains) + Σ (Multiplication of gains of two non touching loops) – Σ (Multiplication of gains of three non touching loops) + ...

Δ_i = Δ calculated by considering the loops non touching to the i^{th} forward path

N = total number of forward paths

Forward path gains of the given SFG:

$$P_1 = G_1G_2G_3G_4G_5$$
$$P_2 = G_1G_4G_5G_6$$
$$P_3 = G_1G_2G_7$$

Individual loop gains of the given SFG:

$$L_1 = -G_4H_1$$
$$L_2 = -G_2G_3G_4G_5H_2$$
$$L_3 = -G_4G_5G_6H_2$$
$$L_4 = -G_2G_7H_2$$

Two non-touching loop gains of the given SFG:

$$L_1L_4 = G_2G_4G_7H_1H_2$$

There are no three non-touching loops in the given SFG.

So, $$\Delta = 1 + G_4H_1 + (G_2G_3 + G_6)\, G_4G_5H_2 + G_2G_7H_2(1 + G_4H_1)$$
$$= (1 + G_4H_1)(1 + G_2G_7H_2) + (G_2G_3 + G_6)\, G_4G_5H_2$$

Only forward path-3 has non touching loop to it.

So, $$\Delta_3 = 1 - L_1 = 1 + G_4H_1$$
$$\Delta_1 = \Delta_2 = 1$$

The overall transfer function,

$$\frac{C}{R} = \frac{G_1G_4G_5(G_6 + G_2G_3) + G_1G_2G_7(1 + G_4H_1)}{(1 + G_4H_1)(1 + G_2G_7H_2) + (G_2G_3 + G_6)(G_4G_5H_2)}$$

Q.8 (a) Solution:

(i) Given, $$y(n) = \frac{1}{N}[x(n-1) + ... + X(n-N)]$$

for $N = 4$,

$$y(n) = \frac{1}{4}[x(n-1) + x(n-2) + x(n-3) + x(n-4)]$$

by taking z-transform,

$$Y(z) = \frac{1}{4}\left[z^{-1}X(z) + z^{-2}X(z) + z^{-3}X(z) + z^{-4}X(z)\right]$$

$$Y(z) = \frac{1}{4}X(z)\left[z^{-1} + z^{-2} + z^{-3} + z^{-4}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{1}{4}\left[z^{-1} + z^{-2} + z^{-3} + z^{-4}\right]$$

$$H(z) = \frac{1}{4}\left[\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\frac{1}{z^4}\right]$$

$$H(z) = \frac{1}{4}\left[\frac{z^3+z^2+z+1}{z^4}\right]$$

∴ The above transfer function has four poles at $z = 0$.

The zeros from the solution $z^3 + z^2 + z + 1 = 0$

i.e.,
$$\frac{1-z^4}{1-z} = 0$$

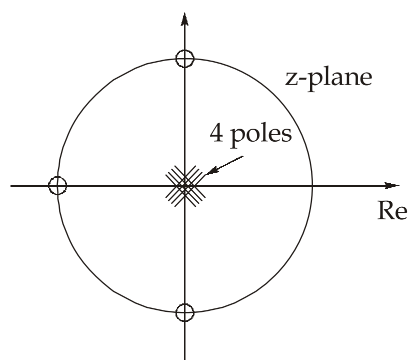
∴ Zeros must be such that $z^4 = 1$ with exclusion of $z = 1$.

i.e., $z_4 = e^{jk2\pi}$ for $K = 1, 2, 3$

∴ $z = j^k$ for $K = 1, 2, 3$

∴ $z = j, -1, -j$

The pole-zero plot in z-plane.



(ii) Given,
$$y(n) = \frac{1}{N}[x(n-1)+.....+x(n-N)]$$

$$y(n-1) = \frac{1}{N}\left[x((n-1)-1)+....+x((n-1)-N)\right]$$

$$y(n-1) = \frac{1}{N}[x(n-2)+...+x(n-N-1)]$$

by comparing $y(n)$ and $y(n - 1)$,

$$y(n) = y(n-1)+\frac{1}{N}x(n-1)-\frac{1}{N}x(n-N-1)$$

by taking z-transform,

$$Y(z) = z^{-1}Y(z)+\frac{1}{N}X(z)\cdot z^{-1}-\frac{1}{N}z^{-N-1}X(z)$$

$$Y(z) - z^{-1}Y(z) = \frac{1}{N}X(z)\left[z^{-1} - z^{-N-1}\right]$$

$$Y(z)[1 - z^{-1}] = \frac{1}{N}X(z)\left[z^{-1} - z^{-N-1}\right]$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{1}{N}\frac{z^{-1} - z^{-N-1}}{1 - z^{-1}}$$

$\therefore$ $H(z) = \dfrac{1}{N}\dfrac{1 - z^N}{z^N(1 - z)}$ is the general form of transfer function of zeros and poles for any N.

Q.8 (b) Solution:

(i) The system's characteristic equation is given by $|\lambda I - A| = 0$

$$|\lambda I - A| = \left|\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -2 \end{bmatrix}\right| = \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 0 & 3 & \lambda + 2 \end{vmatrix}$$

$$= \lambda[\lambda(\lambda + 2) + 3]$$

$$= \lambda[\lambda^2 + 2\lambda + 3]$$

$$\therefore \quad q(s) = \lambda(\lambda^2 + 2\lambda + 3) = 0$$

As there is a root at origin ($\lambda = 0$), the system is limitedly stable or marginally stable.

(ii) Due to state feedback, the characteristic equation changes as A changes.

Stability is now to be determined from the closed loop characteristic equation. The state equation of the closed-loop system is given by

$$\dot{x}(t) = Ax(t) + B[-Kx(t)]$$

$$\dot{x}(t) = [A - BK]\,x(t)$$

$$\dot{x}(t) = A'x(t)$$

Hence, $A' = A - BK$

$$A' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_1 & -(K_2 + 3) & -(K_3 + 2) \end{bmatrix}$$

Now characteristic equation becomes

$$q(s) = |\lambda I - A'| = 0$$

$$\left|\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_1 & -(K_2+3) & -(K_3+2) \end{bmatrix}\right| = 0$$

$$\begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ K_1 & (K_2+3) & \lambda+(K_3+2) \end{vmatrix} = 0$$

$$\lambda[\lambda(\lambda+K_3+2)+K_2+3]+1[K_1] = 0$$

$$\lambda^2(\lambda + K_3 + 2) + K_2\lambda + 3\lambda + K_1 = 0$$

$$\lambda^3 + (K_3 + 2)\lambda^2 + (K_2 + 3)\lambda + K_1 = 0 \quad \text{...(i)}$$

Now applying Routh-Hurwitz criterion

λ^3	1	(K_2+3)
λ^2	(K_3+2)	K_1
λ^1	$\dfrac{(K_3+2)(K_2+3)-K_1}{(K_3+2)}$	
λ^0	K_1	

Conditions for the system to be stable are

(a) $K_1 > 0$

(b) $K_3 > -2$

(c) $[(K_3 + 2)(K_2 + 3) - K_1] > 0$

$$(K_3 + 2)(K_2 + 3) > K_1$$

$$K_3K_2 + 3K_3 + 2K_2 + 6 > K_1$$

$$K_2[K_3 + 2] > K_1 - 3K_3 - 6$$

$$K_2 > \frac{(K_1-3K_3-6)}{(K_3+2)}$$

(iii) Desired closed loop positions are

$$s = -1, s = -1 \pm j2$$

Hence the desired characteristic equation is

$$(s + 1)(s + 1 + j2)(s + 1 - j2) = 0$$

$$(s + 1)(s^2 + 2s + 5) = 0$$

$$s^3 + 2s^2 + 5s + s^2 + 2s + 5 = 0$$

$$s^3 + 3s^2 + 7s + 5 = 0 \qquad \text{...(ii)}$$

Comparing equation (i) and (ii), we get

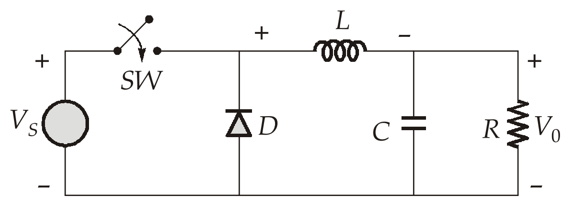
$K_3 + 2 = 3,$ $\quad K_2 + 3 = 7,$ $\quad \boxed{K_1 = 5}$

$\boxed{K_3 = 1}$ $\quad \boxed{K_2 = 4}$

Hence the feedback gain matrix $K = [5, \; 4, \; 1]$.

Q.8 (c) Solution:

(i) Given below converter is buck converter.



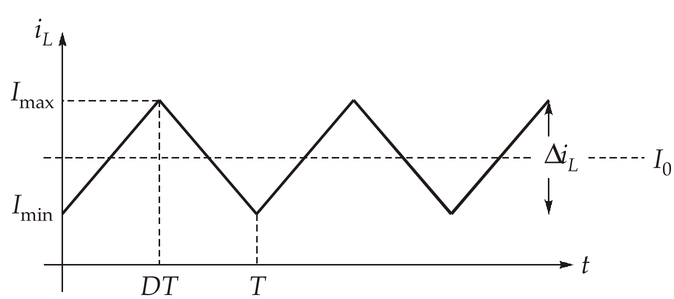
Assuming inductor current is in continuous conduction mode:

When SW → ON, Diode D → open

$$V_{L\,ON} = V_S - V_0 = \frac{L \cdot d_{iL}}{dt}$$

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s - V_0}{L}$$

$$(\Delta i_L)_{closed} = \left(\frac{V_s - V_0}{L}\right) DT$$



Average inductor current must be same as the average current in the load resistor.

$$I_L = I_0 = \frac{V_0}{R}$$

So minimum value of current

$$I_{min} = I_L - \frac{\Delta i_L}{2}$$

$$I_{min} = \frac{V_0}{R} - \frac{1}{2}\left[\left(\frac{V_s - V_0}{L}\right)DT\right]$$

when switch is off

Diode-ON

$$V_{LOFF} = -V_0 = \frac{L \cdot di}{dt}$$

for inductor, average value of inductor voltage = 0

$$V_{LON} \cdot T_{ON} + V_{LOFF} \times T_{OFF} = 0$$

$$(V_s - V_0)(DT) + (-V_0)(1 - D)T = 0$$

$$(V_s - V_0)DT = V_0(1 - D)T$$

$$V_s \cdot D - V_0 D = V_0 - V_0 D$$

$$V_0 = DV_s$$

Now, minimum current in inductor

$$I_{Lmin} = \frac{V_0}{R} - \frac{1}{2}\left[\left(\frac{V_s - V_0}{L}\right)DT\right]$$

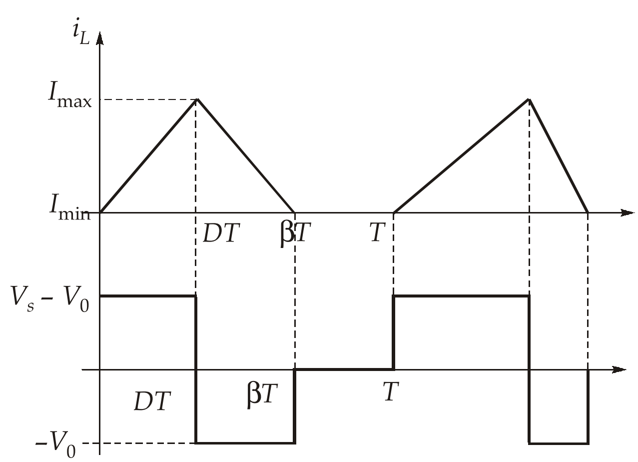
On putting the values

$$I_{Lmin} = \frac{0.4 \times 24}{10} - \frac{1}{2}\left[\frac{24 - (0.4 \times 24)}{100 \times 10^{-6}} \times 0.4 \times \frac{1}{10 \times 10^3}\right]$$

$$= -1.92 < 0$$

Since negative current is not possible, inductor current must be discontinuous.

(ii) Output voltage V_0



average inductor voltage is always zero.

$$(V_s - V_0) = (\beta - D)T \cdot V_D$$

$$V_0 = \left(\frac{D}{\beta}\right)V_s$$

and $$I_L = \frac{1}{2}I_{max}\beta = \frac{V_0}{R} \quad \text{...(i)}$$

when switch '*s*' is closed

$$\frac{di_L}{dt} = \frac{V_s - V_0}{L}$$

$$I_{max} = \Delta i_L = \left(\frac{V_s - V_0}{L}\right)DT$$

$$I_{max} = \Delta i_L = \frac{V_s\left[1-\frac{D}{\beta}\right]DT}{L} \quad \text{...(ii)}$$

from equation (i) and (ii),

$$\frac{1}{2}\cdot\frac{V_s(1-D/\beta)DT\times\beta}{L} = \frac{D}{\beta}\cdot\frac{V_s}{R}$$

$$\frac{1}{2}\frac{(\beta-D)T\times\beta}{L} = \frac{1}{R}$$

$$\beta(\beta - D) = \frac{2L}{RT}$$

$$\beta(\beta - 0.4) = \frac{2\times100\times10^{-6}}{10\times\frac{1}{10\times10^3}}$$

$$\beta^2 - 0.4\beta - 0.2 = 0$$

$$\beta = 0.6898 \approx 0.69$$

So, output voltage, $$V_0 = \frac{DV_s}{\beta}$$

$$= \frac{0.4\times24}{0.69}$$

$$V_0 = 13.91 \text{ Volt}$$

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