MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

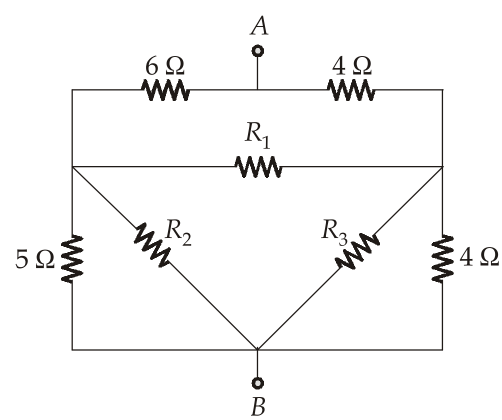
ESE-2024 Mains Test Series

E & T Engineering Test No : 10

Section A

Q.1 (a) Solution:

Converting the star network formed by resistances of 3 Ω, 5 Ω and 8 Ω into an equivalent delta network, we get

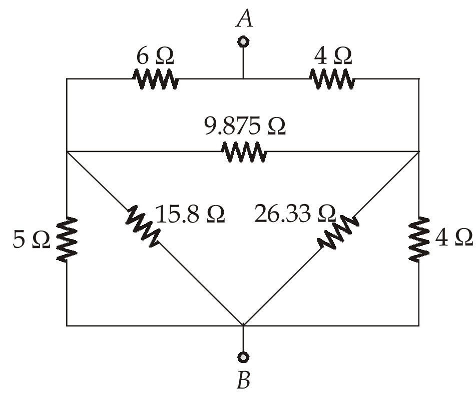


$$R_1 = 3 + 5 + \frac{3 \times 5}{8} = 9.875\ \Omega$$

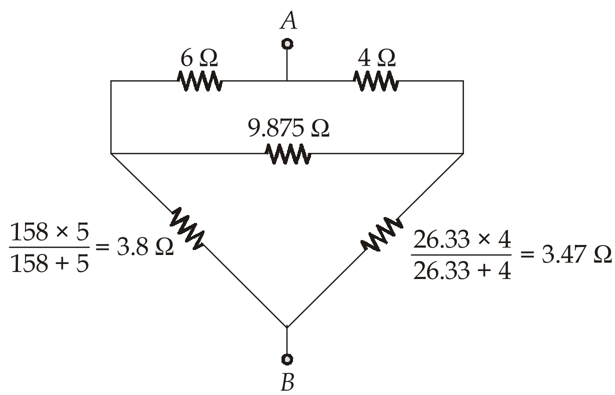
$$R_2 = 3 + 8 + \frac{3 \times 8}{5} = 15.8\ \Omega$$

$$R_3 = 5 + 8 + \frac{5 \times 8}{3} = 26.33\ \Omega$$

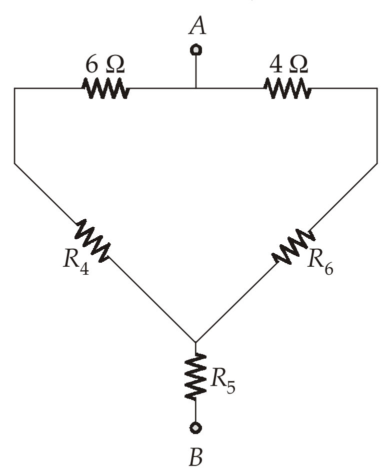
The network can be redrawn as follows:



The resistances of 15.8 Ω and 5 Ω and resistances of 26.33 Ω and 4 Ω are in parallel.



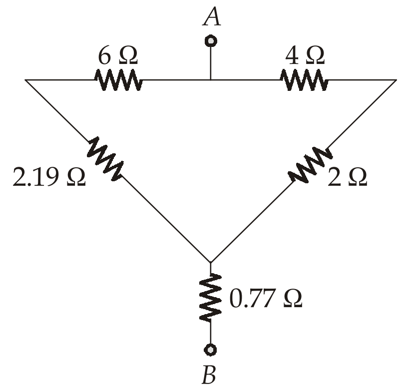
Converting the delta network into star network,



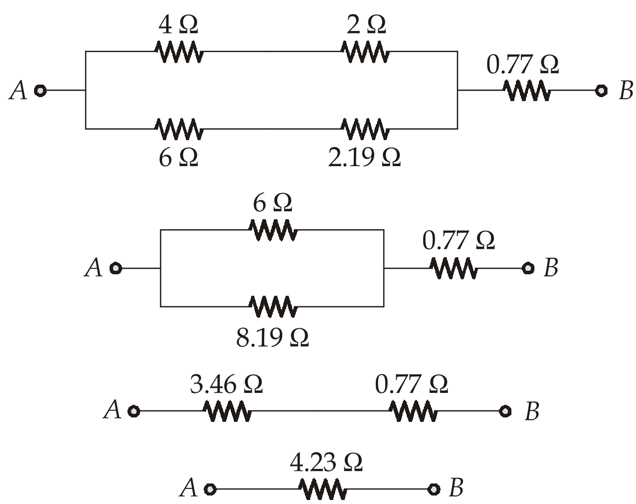
$$R_4 = \frac{3.8 \times 9.875}{3.8 + 9.875 + 3.47} = 2.19\ \Omega$$

$$R_5 = \frac{3.8 \times 3.47}{3.8 + 9.875 + 3.47} = 0.77\ \Omega$$

$$R_6 = \frac{3.47 \times 9.875}{3.8 + 9.875 + 3.47} = 2\ \Omega$$



The network can be simplified as follows:



Hence, $R_{AB} = 4.23\ \Omega$

Q.1 (b) Solution:

(i) In the solar cell, the current flows from n to p when there is illumination given as

$$I = qA\left(\frac{L_p}{\tau_p}p_n + \frac{L_n}{\tau_n}n_p\right)\left(e^{qV/kT} - 1\right) - I_{op}$$

where, optical generation current

$$I_{op} = q\,Ag_{op}(L_p + L_n + W)$$

$$\therefore \quad I = qA\left(\frac{L_p}{\tau_p}p_n + \frac{L_n}{\tau_n}n_p\right)\left(e^{qV/kT} - 1\right) - Aq\,g_{op}(L_P + L_n + W)$$

We can write above equation as,

$$I = I_{th}\left(e^{\frac{qV}{kT}} - 1\right) - I_{sc} \qquad (\because \text{for solar cell } I_{sc} = I_{op})$$

The power dissipation by solar cell,

$$P = V \cdot I$$

$$P = I_{th}\left(e^{qV/kT} - 1\right) \cdot V - I_{sc} \cdot V$$

For maximum power dissipation,

$$\frac{dP}{dV} = 0$$

$$\therefore \quad \frac{dP}{dV} = I_{th}\left(e^{\frac{qV}{kT}} - 1\right) + I_{th}V \cdot \frac{q}{kT} e^{qV/kT} - I_{sc} = 0$$

at $V = V_{mp}$, $$e^{\frac{qV_{mp}}{kT}}\left(V_{mp} \cdot \frac{q}{kT} + 1\right) = 1 + \frac{I_{sc}}{I_{th}}$$

(ii) Assuming, $I_{sc} >> I_{th}$ and $V_{mp} >> \frac{kT}{q}$, we can write

$$e^{\frac{qV_{mp}}{kT}} \cdot V_{mp} \cdot \frac{q}{kT} = \frac{I_{sc}}{I_{th}}$$

Taking natural log on both sides,

$$\ln\left[e^{\frac{qV_{mp}}{kT}} \cdot V_{mp} \cdot \frac{q}{kT}\right] = \ln\left[\frac{I_{sc}}{I_{th}}\right]$$

$$\ln\left[V_{mp} \cdot \frac{q}{kT}\right] + \frac{q \cdot V_{mp}}{kT} = \ln\left[\frac{I_{sc}}{I_{th}}\right]$$

$$\ln\left[V_{mp} \cdot \frac{q}{kT}\right] = \ln\left(\frac{I_{sc}}{I_{th}}\right) - \frac{qV_{mp}}{kT}$$

Let, $$x = V_{mp} \cdot \frac{q}{kT}$$

∴ The above equation can be written as,

$$\ln x = c - x$$

where, $$c = \ln\left(\frac{I_{sc}}{I_{th}}\right) = \ln\left(\frac{100 \times 10^{-3}}{1.5 \times 10^{-9}}\right) = 18$$

$$\therefore \quad \ln x = 18 - x$$

Q.1 (c) Solution:

Composite: These are artificially produced multiphase materials having a desirable combination of the best properties of the constituent phases. Usually, one phase (the matrix) is continuous and completely surrounds the other (the dispersed phase).

(i) Carbon-Carbon Composites:

Carbon-Carbon composite i.e., carbon fibre reinforced carbon matrix composite is one of the most advanced and promising engineering material. Here, both reinforcement and matrix phase are carbon.

These materials have high tensile modulii and tensile strengths that are retained to temperatures in excess of 2000°C, resistance to creep, and relatively large fracture toughness values.

These material have low co-efficients of thermal expansion and relatively high thermal conductivities. These characteristics of these materials, coupled with high strengths, give rise to a relatively low susceptibility to thermal shock. The major drawback of these materials is a propensity to high temperature oxidation. These materials are relatively new and expensive, and therefore are not in wide use.

These materials are employed in rocket motors, as friction materials in aircraft and high performance automobiles, for hot pressing molds, in components for advanced turbine engine etc.

(ii) Hybrid composites:

The composites obtained by using two or more different kinds of fibers in a single matrix are termed as hybrid.

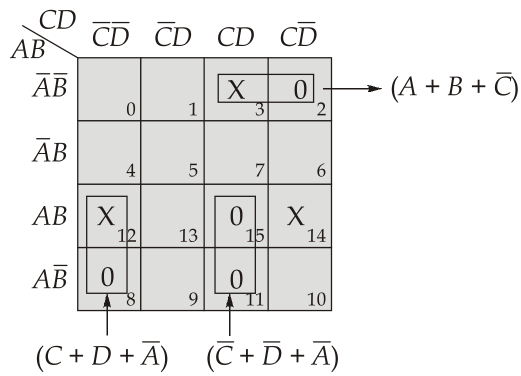
Hybrid composites have a better all-round combination of properties than composites containing only a single fibre type. Although a variety of fibre combinations and matrix materials are used, but in the most common system, both carbon and glass fibres are incorporated into a polymeric resin. The carbon fibres are expensive, but they are strong and relatively stiff and provide a low density reinforcement. Glass fibres lack the stiffness of carbon, but they are inexpensive. The glass-carbon hybrid may be produced at lower cost than either of the comparable all carbon or all glass reinforced plastics and also it is stronger and tougher.

The properties of hybrid composites are anisotropic. When these are stressed in tension, failure is usually non-catastrophic i.e., does not occur suddenly.

Hybrid composites find application in light weight land, water and air transport structural components, light weight orthopedic components and sporting goods.

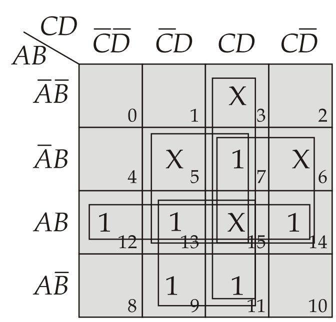
Q.1 (d) Solution:

(i) 1. $f(A, B, C, D) = \pi M(2, 8, 11, 15) + d(3, 12, 14)$



$$f(A, B, C, D) = (A + B + \overline{C}) \cdot (\overline{A} + \overline{C} + \overline{D}) \cdot (\overline{A} + C + D)$$

2. $f(A, B, C, D) = \Sigma m(7, 9, 11, 12, 13, 14) + d(3, 5, 6, 15)$



$$f(A, B, C, D) = AB + AD + CD$$

or $$f(A, B, C, D) = AB + AD + BD$$

or $$f(A, B, C, D) = AD + AB + BC$$

(ii) To perform excess 3 code subtraction, both the numbers in excess 3 code are subtracted using binary subtraction. Subtract '0011' from each BCD four-bit group in the answer if the subtraction operation of the relevant four-bit groups required a borrow from the next higher adjacent four-bit group. Add '0011' to the remaining four-bit groups, if any, in the result. The result, so obtained is in the excess -3 format.
11001011 – 01001001 in excess 3 format:

```
     1 1 0 0 1 0 1 1
     0 1 0 0 1 0 0 1
    -----------------
     1 0 0 0 0 0 1 0  → Add 0011 to both nibbles (No borrow from the higher nibble)
 (+) 0 0 1 1 0 0 1 1
    -----------------
     1 0 1 1 0 1 0 1  → Final result in excess 3 format
    -----------------
```

Converting the given excess-3 numbers in decimal format (for checking)

$$1\,1\,0\,0\,1\,0\,1\,1 \xrightarrow[\text{BCD}]{\text{To 8421}} 1\,0\,0\,1\,1\,0\,0\,0 \xrightarrow{\text{to decimal}} (98)_{10}$$

$$0\,1\,0\,0\,1\,0\,0\,1 \xrightarrow[\text{BCD}]{\text{To 8421}} 0\,0\,0\,1\,0\,1\,1\,0 \xrightarrow{\text{to decimal}} (16)_{10}$$

$$(98)_{10} - (16)_{10} = (82)_{10} \xrightarrow[\text{BCD}]{\text{To 8421}} 1\,0\,0\,0\,0\,0\,1\,0 \xrightarrow{\text{to excess 3}} 1\,0\,1\,1\,0\,1\,0\,1$$

Q.1 (e) Solution:

We know that,

$$\text{Pole pitch} = \text{distance between two adjacent poles}$$

$$= \frac{\text{Periphery of the armature}}{\text{number of poles of the generator}} = \frac{\pi D}{P}$$

$$= \frac{\pi \times 0.35}{4}$$

$$\text{Pole pitch} = 0.275 \text{ m}$$

Given $$\frac{\text{Pole arc}}{\text{Pole pitch}} = 0.7$$

$$\text{Pole arc} = 0.7 \times \text{Pole pitch}$$

$$\text{Pole arc} = 0.7 \times 0.275$$

$$= 0.193 \text{ m}$$

Now, $$\text{area of pole face} = \text{Pole arc} \times \text{axial length}$$

$$= 0.193 \times 0.2$$

$$= 0.039 \text{ m}^2$$

We have,

$$\text{Generated emf, } E = \frac{NP\phi Z}{60A}$$

Substituting the given values,

$$250 = \frac{500 \times 4 \times \phi \times 1200}{60 \times 4}$$

[For a lap-connected winding, $A = P = 4$]

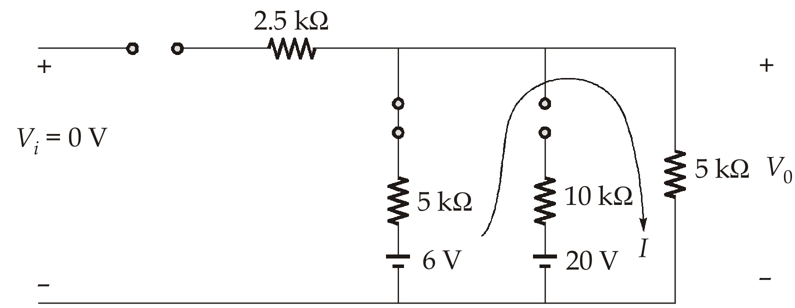
$$\phi = 0.025 \text{ Wb}$$

This flux is uniformly distributed over the pole area. Hence, the mean flux density in the air gap

$$B = \frac{\text{flux per pole}}{\text{area of pole shoe}} = \frac{0.025}{0.039} = 0.64 \text{ T}$$

Q.2 (a) Solution:

Case I: When $V_i = 0$, then diode D_3 is OFF, D_2 is OFF and D_1 is ON. The equivalent circuit is obtained as below:



Applying KVL in loop, we get

$$-6 + 5kI + 5kI = 0$$

$$I = 0.6 \text{ mA}$$

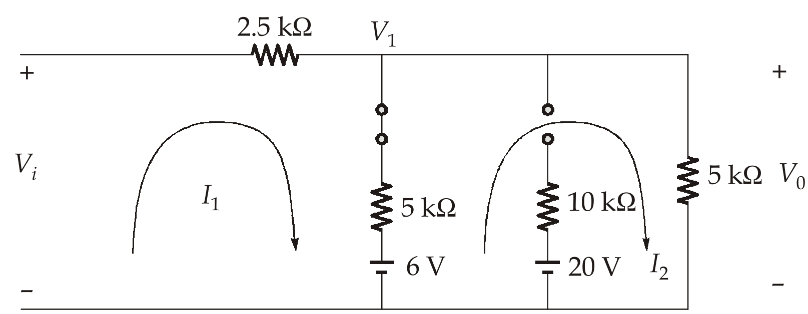
$$V_0 = 5k \times I$$

$$V_0 = 3 \text{ V} \quad \text{...(i)}$$

For $0 < V_i < 3$ V, V_0 remains at 3 V, with only diode D_1 conducting.

Case II: When, the input attains 3 V, both diodes D_1 and D_3 are conducting.

Thus, the equivalent circuit can be constructed as



Applying KVL in loop 1, we get

$$-V_i + 2.5\, kI_1 + 5k(I_1 - I_2) + 6 = 0$$

$$V_i = 7.5kI_1 - 5kI_2 + 6 \quad \text{...(1)}$$

On applying KVL in loop 2, we get

$$-6 + 10kI_2 - 5kI_1 = 0$$

$$-5kI_1 + 10kI_2 = 6 \quad \text{...(2)}$$

$$V_0 = 5kI_2 \quad \text{...(3)}$$

On solving (1), (2) and (3) we get,

$$V_0 = 0.5V_i + 1.5;$$ D_1 and D_3 both are in conducting state.

$$V_0 = 0.5V_i + 1.5 \text{ is valid till } 3 \le V_1 < 6 \text{ V}$$

Here

$$V_1 = V_0$$

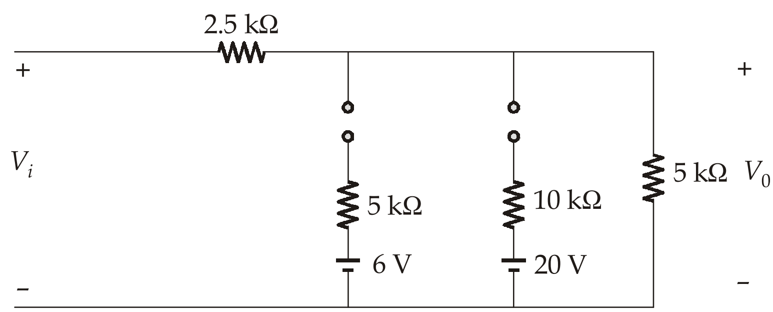
$$3 = 0.5V_i + 1.5; \qquad 6 = 0.5V_i + 1.5$$

$$V_i = 3 \text{ Volt} \qquad V_i = 9 \text{ Volt}$$

Hence, $V_0 = 0.5V_i + 1.5$ for $3 \text{ V} \le V_i < 9 \text{ V}$ with both diodes D_1 and D_3 conducting.

Case III: When $V_i = 9$ Volt

Diode D_1 moves to non-conducting state as V_i reaches to 9 volt.



$$V_0 = \frac{V_i \times 5k}{7.5k} = \frac{2V_i}{3}$$

When V_0 reaches to 20 V, Diode D_2 start conducting i.e. for input voltage,

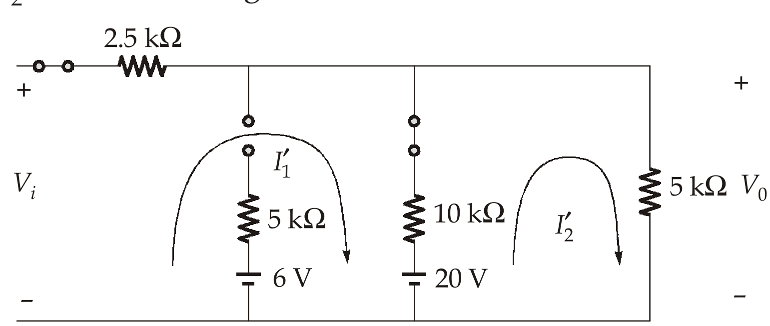
$$20 = \frac{2V_i}{3}$$

$$V_i = 30 \text{ Volt}$$

for $9 \text{ V} < V_i < 30 \text{ V}$; $V_0 = \frac{2V_i}{3}$; with only diode D_3 conducting

Case IV: When $V_i = 30$ Volt

Diode, D_3 and D_2 start conducting.



$$-V_i + 12.5kI_1' - 10kI_2' + 20 = 0$$

$$V_i = 12.5kI_1' - 10kI_2' + 20 \qquad \text{...(4)}$$

$$-20 + 10kI_2' - 10kI_1' + 5kI_2' = 0$$

$$-10kI_1' + 15kI_2' = 20 \quad ...(5)$$

$$5kI_2' = V_0 \quad ...(6)$$

On solving (4), (5) and (6) we get,

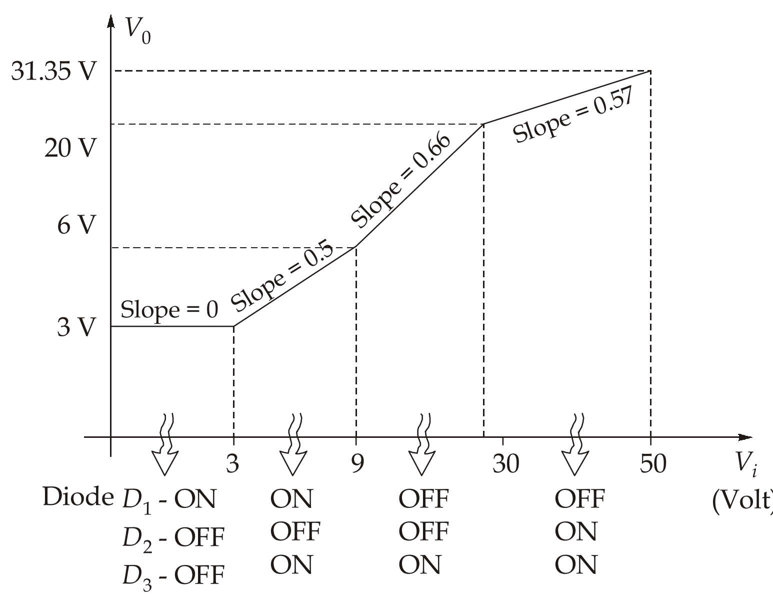
$$V_0 = 0.57V_i + 2.85 \text{ for } 30 \text{ V} < V_i$$

When $V_i > 30$ V diode D_3 and D_2 are ON.

On combining all the cases, we get output as

$$V_0 = \begin{cases} 3\,V \quad ; \quad 0 \le V_{in} < 3\text{ V} & \Rightarrow \text{In this input voltage range only Diode } D_1 \text{ is conducting.} \\ 0.5V_i + 1.5 \;;\quad 3\text{ V} \le V_{in} < 9\text{ V} & \Rightarrow \text{In this input voltage range; Diode } D_1 \text{ and } D_3 \text{ are in conducting state.} \\ 0.66V_i \quad ;\quad 9\text{ V} \le V_{in} < 30\text{ V} & \Rightarrow \text{In this input voltage range, only Diode } D_3 \text{ is conducting.} \\ 0.57V_i + 2.85;\quad 30 \le V_{in} \le 50\text{ V} & \Rightarrow \text{In this input voltage range, Diode } D_3 \text{ and } D_2 \text{ are in conducting state.} \end{cases}$$

The V_0 versus V_i plot is obtained as below:



Q.2 (b) Solution:

In the pn junction, diffusion capacitance occurs when the diode is forward biased. The depletion capacitance and diffusion capacitances will be equal at some forward bias voltage.

For applied voltage V_a, the junction capacitance is

$$C_j = \frac{\epsilon_s A}{W}$$

where width of depletion region,

$$W = \left\{\frac{2\in_s (V_{bi} - V_a)}{q}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{\frac{1}{2}}$$

$$C_j = \frac{\in_s A}{\left\{\frac{2\in_s (V_{bi} - V_a)}{q}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{\frac{1}{2}}}$$

$$C_j = A\left[\frac{q\in_s N_a N_d}{2(V_{bi} - V_a)(N_a + N_d)}\right]^{\frac{1}{2}} \quad ...(i)$$

The diffusion capacitance is given as,

$$C_d = \frac{\tau I_f}{\eta V_t}$$

where, $I_f = I_p + I_n$

$$\therefore \quad C_d = \frac{I_p \tau_{p0} + I_n \tau_{n0}}{2V_t} \quad ...(ii)$$

where,

$$I_p = \frac{Aqn_i^2}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\exp\left(\frac{V_a}{V_t}\right)$$

$$I_n = \frac{Aqn_i^2}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}\exp\left(\frac{V_a}{V_t}\right)$$

Here,

$$D_p = \mu_p V_t = 320 \times 0.0259 = 8.29\ \text{cm}^2/\text{s}$$

$$D_n = \mu_n V_t = 1250 \times 0.0259 = 32.375\ \text{cm}^2/\text{s}$$

$$V_{bi} = V_t \ln\left[\frac{N_a N_d}{n_i^2}\right] = 0.0259 \ln\left[\frac{10^{17} \times 5 \times 10^{15}}{(1.5 \times 10^{10})^2}\right]$$

$$\therefore \quad V_{bi} = 0.7363\ \text{V}$$

The junction capacitance, using equation (i)

$$C_j = (10^{-4})\left[\frac{1.6 \times 10^{-19} \times 11.7 \times 8.85 \times 10^{-14}}{2(0.7363 - V_a)} \times \frac{(5 \times 10^{15}) \times 10^{17}}{(5 \times 10^{15}) + 10^{17}}\right]^{\frac{1}{2}}$$

$$\therefore \quad C_j = 10^{-4}\left[\frac{3.945 \times 10^{-16}}{0.7363 - V_a}\right]^{1/2} \quad ...(iii)$$

We have,

$$I_p = \frac{(10^{-4})(1.6\times10^{-19})(1.5\times10^{10})^2}{10^{17}}\sqrt{\frac{8.29}{10^{-7}}}\exp\left(\frac{V_a}{V_t}\right)$$

$$\therefore \quad I_p = 3.278\times10^{-16}\exp\left(\frac{V_a}{V_t}\right)$$

and

$$I_n = \frac{(10^{-4})(1.6\times10^{-19})(1.5\times10^{10})^2}{5\times10^{15}}\sqrt{\frac{32.375}{10^{-6}}}\exp\left(\frac{V_a}{V_t}\right)$$

$$I_n = 4.097\times10^{-15}\exp\left(\frac{V_a}{V_t}\right)$$

From equation (ii),

$$C_d = \frac{1}{2(0.0259)}\left[(3.278\times10^{-16})(10^{-7})+(4.097\times10^{-15})(10^{-6})\right]\exp\left(\frac{V_a}{V_t}\right)$$

$$C_d = 7.972\times10^{-20}\exp\left(\frac{V_a}{V_t}\right) \quad \text{...(iv)}$$

Given $C_j = C_d$. Hence, from equation (iii) and (iv),

$$(10^{-4})\left[\frac{3.945\times10^{-16}}{0.7363-V_a}\right]^{1/2} = 7.972\times10^{-20}\exp\left(\frac{V_a}{0.0259}\right)$$

$$\exp\left(\frac{V_a}{0.0259}\right) = \frac{10^{-4}}{7.972\times10^{-20}}\left[\frac{3.945\times10^{-16}}{0.7363-V_a}\right]^{1/2}$$

$$\exp\left(\frac{V_a}{0.0259}\right) = 1.254\times10^{15}\left[\frac{3.945\times10^{-16}}{0.7363-V_a}\right]^{1/2}$$

$$\exp\left(\frac{V_a}{0.0259}\right) = \frac{24.9\times10^{6}}{\sqrt{0.7363-V_a}}$$

by taking 'ln' on both sides,

$$\frac{V_a}{0.0259} = \ln(24.9\times10^6)-\frac{1}{2}\ln(0.7363-V_a)$$

$$= 17.03-\frac{1}{2}\ln(0.7363-V_a)$$

$$77.22\,V_a + \ln(0.7363 - V_a) = 34.06$$

by trial and error, $V_a = 0.4575$ V

The diffusion capacitance, from equation (iv) can thus be calculated as

$$C_d = 7.972\times10^{-20}\exp\left(\frac{V_a}{0.0259}\right) = 7.972\times10^{-20}\exp\left(\frac{0.4575}{0.0259}\right) = 3.74\text{ pF} = C_j$$

Q.2 (c) Solution:

We have, full load output = 20 kW at 500 V

$$\eta = 90\%$$

We know that,
$$\eta = \frac{\text{output}}{\text{input}}$$

$$\text{input} = \frac{20 \times 10^3}{0.9} = 22.22 \text{ kW}$$

Hence, Losses = input – output = 22.22 kW – 20 kW = 2.22 kW

Now,

according to question,

armature copper losses are 40% of full load losses;

$$\text{Armature copper losses} = 0.4 \times 2.22$$

$$= 0.888 \text{ kW} = I_a^2 R_a$$

$$\text{Full load line current} = \frac{\text{input power}}{\text{source voltage}}$$

$$\text{Full load line current} = \frac{22.22 \text{ kW}}{500} = 44.44 \text{ amp}$$

$$\text{Full current} = \frac{V}{R} = \frac{500}{250} = 2 \text{ Amp}$$

Full load armature current, $I_a = I_L - I_f = 44.44 - 2 = 42.44$ Amp

$$\text{Armature resistance, } R_a = \frac{888}{(42.44)^2} = 0.49\ \Omega$$

Starting current refers to line current,

Minimum starting line current = 1.2 × 44.44

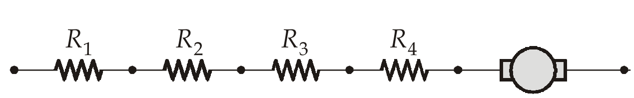
= 53.33 A

Minimum starting armature current = 53.33 – 2 = 51.33 A

Maximum starting line current = 2 × 44.44 = 88.88 A

Maximum starting armature current = 88.88 – 2 = 86.88 A

A starter having 4 sections is as follows:



$$K = \frac{I_{a2}}{I_{a1}} = \frac{51.33}{86.88} = 0.591$$

Also, $\frac{500}{R_{total}} = 86.88 \text{ A}$

$R_{total} = 5.76\ \Omega$

Resistance of 1st section, $R_1 = R_{total}(1 - K)$

$R_1 = 5.76(1 - 0.591)$

$R_1 = 2.36\ \Omega$

Resistance of 2nd section, $R_2 = KR_1 = (0.591)(2.36)$

$R_2 = 1.39\ \Omega$

Resistance of 3rd section, $R_3 = K^2R_1 = 0.82\ \Omega$

Resistance of 4th section, $R_4 = K^3R_1 = 0.486\ \Omega$

Q.3 (a) Solution:

(i)

B_2	B_1	B_0	A_1	A_0	P_4	P_3	P_2	P_1	P_0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	1
0	0	1	1	0	0	0	0	1	0
0	0	1	1	1	0	0	0	1	1
0	1	0	0	0	0	0	0	0	0
0	1	0	0	1	0	0	0	1	0
0	1	0	1	0	0	0	1	0	0
0	1	0	1	1	0	0	1	1	0
0	1	1	0	0	0	0	0	0	0
0	1	1	0	1	0	0	0	1	1
0	1	1	1	0	0	0	1	1	0
0	1	1	1	1	0	1	0	0	1
1	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0
1	0	0	1	1	0	1	1	0	0
1	0	1	0	0	0	0	0	0	0
1	0	1	0	1	0	0	1	0	1
1	0	1	1	0	0	1	0	1	0
1	0	1	1	1	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0
1	1	0	0	1	0	0	1	1	0
1	1	0	1	0	0	1	1	0	0
1	1	0	1	1	1	0	0	1	0
1	1	1	0	0	0	0	0	0	0
1	1	1	0	1	0	0	1	1	1
1	1	1	1	0	0	1	1	1	0
1	1	1	1	1	1	0	1	0	1

K map simplification

(for P_0)

$B_2 = 0$

B_1B_0 \ A_1A_0	00	01	11	10
00				
01		1	1	
11		1	1	
10				

$B_2 = 1$

B_1B_0 \ A_1A_0	00	01	11	10
00				
01		1	1	
11		1	1	
10				

$$P_0 = B_0A_0$$

For P_1

$B_2 = 0$

B_1B_0 \ A_1A_0	00	01	11	10
00				
01			1	1
11		1		1
10		1	1	

$B_2 = 1$

B_1B_0 \ A_1A_0	00	01	11	10
00				
01			1	1
11		1		1
10		1	1	

$$P_1 = \overline{B}_1B_0A_1 + B_0A_1\overline{A}_0 + B_1\overline{B}_0A_0 + B_1\overline{A}_1A_0$$

For P_2

$B_2 = 0$

B_1B_0 \ A_1A_0	00	01	11	10
00				
01				
11				1
10			1	1

$B_2 = 1$

B_1B_0 \ A_1A_0	00	01	11	10
00		1	1	
01		1	1	
11		1	1	1
10		1		1

$$P_2 = \overline{B}_2B_1\overline{B}_0A_1 + B_1A_1\overline{A}_0 + B_2\overline{A}_1A_0 + B_2\overline{B}_1A_0 + B_2B_0A_0$$

For P_3

$B_2 = 0$

$B_1B_0 \backslash A_1A_0$	00	01	11	10
00				
01				
11			1	
10				

$B_2 = 1$

$B_1B_0 \backslash A_1A_0$	00	01	11	10
00			1	1
01			1	1
11				1
10				1

$$P_3 = \overline{B}_2 B_1 B_0 A_1 A_0 + B_2 \overline{B}_1 A_1 + B_2 A_1 \overline{A}_0$$

For P_4:

$B_2 = 0$

$B_1B_0 \backslash A_1A_0$	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$B_2 = 1$

$B_1B_0 \backslash A_1A_0$	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	1	0
10	0	0	1	0

$$P_4 = B_2 B_1 A_1 A_0$$

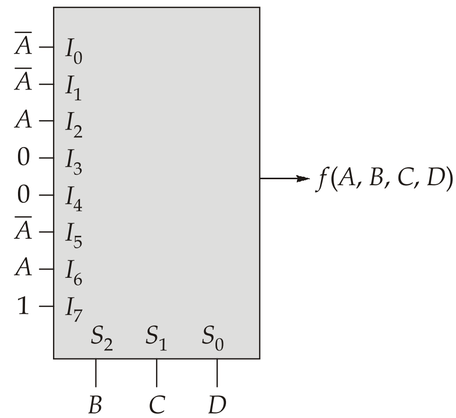
(ii) $$f(A, B, C, D) = \Sigma m(0, 1, 5, 7, 10, 14, 15)$$

Consider B, C, D as select lines S_2, S_1, S_0 respectively for 8 × 1 MUX.

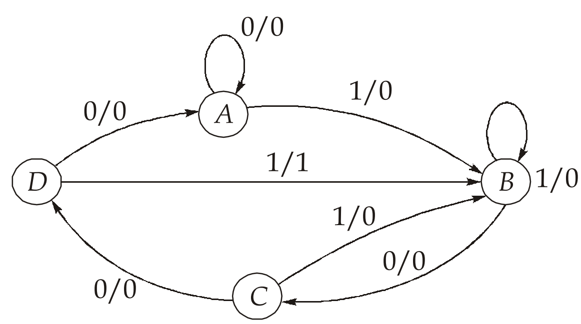
So, we need to find input I_0 to I_7.

	I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7
$\overline{A}$	(0)	(1)	2	3	4	(5)	6	(7)
A	8	9	(10)	11	12	13	(14)	(15)
	$\overline{A}$	$\overline{A}$	A	0	0	$\overline{A}$	A	1

Hence, the given boolean function can be implemented using 8 × 1 MUX as below:



(iii) State diagram:



State table:

Present State	Next State $X = 0$	Next State $X = 1$	Output $X = 0$	Output $X = 1$
A	A	B	0	0
B	C	B	0	0
C	D	B	0	0
D	A	B	0	1

Q.3 (b) Solution:

(i) The value of N_{cu} - the number of atomic sites per cubic meter for copper, can be determined from its atomic weight A_{cu}, its density ρ, and Avogadro's number N_A, according to

$$N_{cu} = \frac{N_A \rho}{A_{cu}}$$

$$N_{cu} = \frac{(6.022 \times 10^{23} \text{ atoms/mol})(8.4 \text{ g/cm}^3)(10^6 \text{ cm}^3/\text{m}^3)}{63.5 \text{ g/mol}}$$

$$= 8 \times 10^{28} \text{ atoms/m}^3$$

Thus, the number of vacancies at 1000°C (1273 K) is equal to

$$N_V = N\exp\left(\frac{-Q_v}{kT}\right)$$

$$N_V = (8\times10^{28})\exp\left(\frac{-0.9}{8.62\times10^{-5}\times1273}\right)$$

$$N_V = 2.2 \times 10^{25} \text{ vacancies/m}^3$$

(ii) **1.** The saturation magnetization is the product of the number of Bohr magnetons per atom (0.60), the magnitude of the Bohr magneton μ_B, and the number N of atoms per cubic meter, or

$$M_s = 0.60\,\mu_B N$$

The number of atoms per cubic meter is related to the density ρ, the atomic weight A_{Ni}, and Avogadro's number N_A, as follows:

$$N = \frac{\rho N_A}{A_{Ni}} = \frac{8.90\times10^6\times6.022\times10^{23}}{58.71}$$

$$= 9.13 \times 10^{28} \text{ atoms/m}^3$$

Finally, $$M_s = 0.60 \times 9.27 \times 10^{-24} \times 9.13 \times 10^{28}$$

$$M_s = 5.07 \times 10^5 \text{ A/m}$$

$$M_s \cong 5.1 \times 10^5 \text{ A/m}$$

2. The saturation flux density is

$$B_s = \mu_0 M_s$$

$$B_s = 4\pi \times 10^{-7} \times 5.1 \times 10^5$$

$$B_s = 0.64 \text{ Tesla}$$

Q.3 (c) Solution:

(i) **1.** **Advantages of LVDT:**

- **High range:** For measuring displacement ranging from 1.25 mm - 250 mm with a 0.025% full scale linearity, it allows measurement down to 0.003 mm.
- **Friction and electrical isolation:** Ordinarily, there is no physical contact between the movable core and coil structure which means LVDT is a frictionless device. It implies there is no wear out and this gives LVDT larger life.
- **Immunity from external effect:** The separation between core and coils of LVDT permits isolation of media such as pressurized corrosive and caustic fluids from the coil assembly by adding a non magnetic barrier interposed between the core and inside of the coil. It makes the hermetic sealing of coil

assembly possible and eliminates the need of a dynamic sealing on moving member. The isolation also makes an LVDT an effective analog computing element without the need of a buffer amplifier.

- **High sensitivity:** It produces a high sensitivity which is typically about 40 V/mm.
- **Ruggedness:** These transducers can usually tolerate high degree of shock and vibration especially when core is spring loaded without any adverse effect. LVDT are simple in construction and are stable and easy to align.
- **Low hysteresis:** LVDT shows a low hysteresis and hence repeatability is excellent under all conditions.

Disadvantages of LVDT:

- Relatively large displacement is required for appreciable differential output.
- Sensitive to stray magnetic fields but shielding is possible.
- Transducer performance is affected by vibrations.
- The receiving instrument must be selected to operate on an ac signal or a demodulator network must be used if a dc output is required.
- The dynamic response is limited mechanically by the mass of the core and electrically by the frequency of the applied voltage.
- Temperature affects the performance of the transducer. Temperature also causes phase shifting effects.

2. Sensitivity of LVDT $= \dfrac{\text{Output voltage}}{\text{Displacement}}$

$$= \frac{3 \times 10^{-3}\ \text{V}}{0.5\ \text{mm}}$$

$$= 6 \times 10^{-3}\ \text{V/mm}$$

Sensitivity of instrument $=$ amplification factor $\times$ sensitivity of LVDT

$$= 200 \times 6 \times 10^{-3}$$

$$= 1.2\ \text{V/mm} = 1200\ \text{mV/mm}$$

$$1 \text{ scale division } = \frac{10}{150} = 66.667\ \text{mV}$$

Minimum voltage that can be read on the voltmeter

$$= \left(\frac{1}{10} \times 66.667\right) \text{mV}$$

$$= 6.6667\ \text{mV}$$

$\therefore$ Resolution of instrument $= 6.6667 \text{ mV} \times \frac{1}{1200} \text{ mV/mm}$

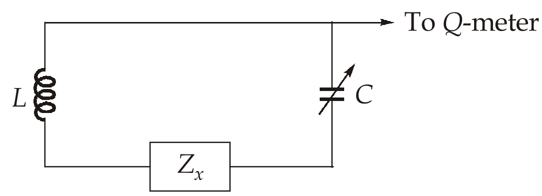
$$= 5.555 \times 10^{-3} \text{ mm}$$

(ii) 1. An unknown impedance can be measured using a Q-meter, either by a series or shunt substitution method.

- If impedance to be measured is small, series substitution method is used.
- If impedance to be measured is large, parallel substitution method is used.

Measurement of small impedance:

The circuit for measurement of small impedance is shown below:



The unknown impedance is shorted or otherwise not connected and the tuned circuit is adjusted for resonance at the oscillator frequency. The value of Q and C are noted. The unknown impedance is then connected, the capacitor is varied to obtain resonance, and new values Q' and C' are noted.

From part 1, we have,

$$\omega L = \frac{1}{\omega C}$$

From part-2, we have,

$$\omega L + X_x = \frac{1}{\omega C'}$$

Subtracting equation (ii) from (i), we have,

$$X_x = \frac{1}{\omega C'} - \frac{1}{\omega C} = \frac{1}{\omega C}\left(\frac{C - C'}{C'}\right) \quad \text{...(i)}$$

$$X_x = \frac{1}{\omega}\left(\frac{C - C'}{CC'}\right)$$

$$R' = R + R_x$$

$$R_x = R' - R$$

where, R is the resistance of auxiliary coil

$$R_x = R' - R$$

$$= \frac{\omega L}{Q'} - \frac{\omega L}{Q} = \omega L\left(\frac{Q - Q'}{QQ'}\right) \quad \text{...(ii)}$$

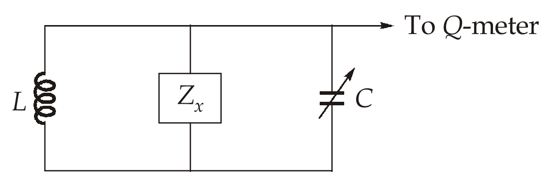
Z_x can be calculated from equations (i) and (ii),

$$Z_x = R_x + jX_x$$

$$Z_x = \omega L\left(\frac{Q-Q'}{QQ'}\right) + \frac{j}{\omega}\left(\frac{C-C'}{CC'}\right)$$

Measurement of large impedance:

To determine large impedance the following connection is used



$$Y_x = \frac{1}{Z_x} = G_x + jB_x$$

In this method, Y_x is disconnected and the capacitor C is tuned to the resonant value. At the oscillator frequency, the value of Q and C are noted.

With Y_x connected, the capacitor, is tuned again for resonance at oscillator frequency and new values of Q' and C' are noted.

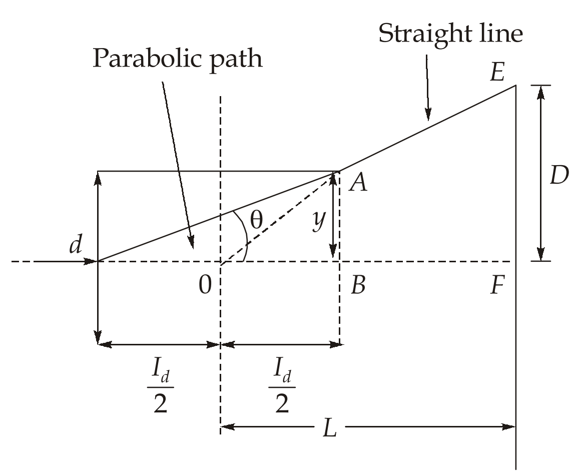
Hence, $$Y_X = G_x + jB_x$$

with $$B_x = \omega C - \omega C'$$

Also, $$G_x = \frac{1}{\omega L}\left(\frac{Q-Q'}{QQ'}\right)$$

$\therefore$ $$Y_x = \frac{Q-Q'}{\omega LQQ'} + j\omega(C-C')$$

2. Consider the deflection of a cathode ray beam given below:



The maximum deflection of the electron beam before it is shadowed by its own deflection plate can be calculated from the geometry of the CRT. We have,

$$L = \frac{2dE_a}{GI_d}$$

$E_a \rightarrow$ Accelerating voltage

$G \rightarrow$ Deflection factor of CRT.

For a deflection factor G, the accelerating voltage E_a, the distance between centre of the deflection plates and the phosphor screen, L is limited by the maximum deflection which produces a value y equal to $\frac{d}{2}$.

Any deflection greater than this produces a shadow on the CRT screen due to electron beam striking its own deflection plates.

The geometry of the electron beam produces two similar right triangles: one ΔOAB at the deflection plates consisting of two sides, $\frac{d}{2}$ and $\frac{I_d}{2}$ and the second ΔOEF between the centre of the deflection plates and the phosphor screen with two sides D and L. This geometry produces the following relationship:

$$\frac{L}{D} = \frac{I_d}{d}$$

Substituting this result into preceding equation produces the following relationship

$$L^2 = \frac{2DE_a}{G}$$

Given:

$$G = 120 \text{ V/cm} = 12 \times 10^3 \text{ V/m}$$

$$E_a = 2000 \text{ V}$$

$$D = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$$

$$L^2 = \frac{2 \times 5 \times 10^{-2} \times 2000}{12 \times 10^3} = 0.0166$$

$$L = 0.1288 \text{ m}$$

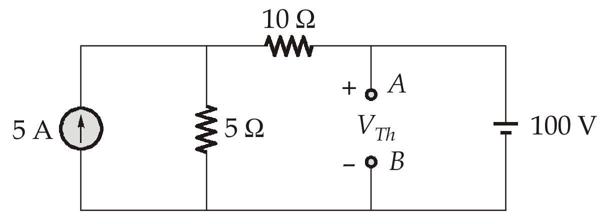
$$L = 12.88 \text{ cm}$$

Thus, minimum distance from deflection plates to the oscilloscope tube screen must be 12.88 cm.

Q.4 (a) Solution:

(i) Step 1: *Calculation of V_{th}*

Removing the 20 Ω resistor from the network, the circuit can be drawn as below:

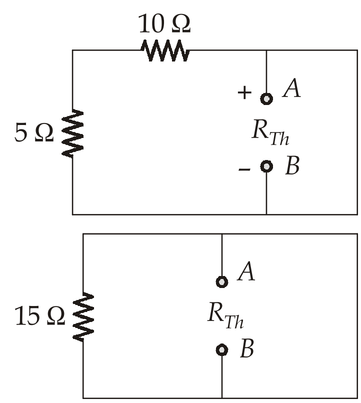


From fig.

$$V_{Th} = 100 \text{ V}$$

Step II: *Calculation of* R_{Th}

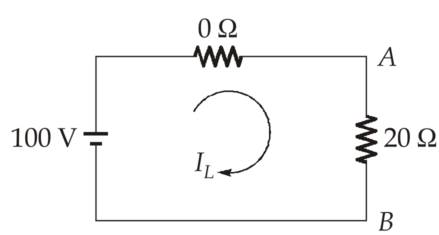
Replacing the voltage source by a short circuit and the current source by an open circuit,



$$R_{Th} = 0$$

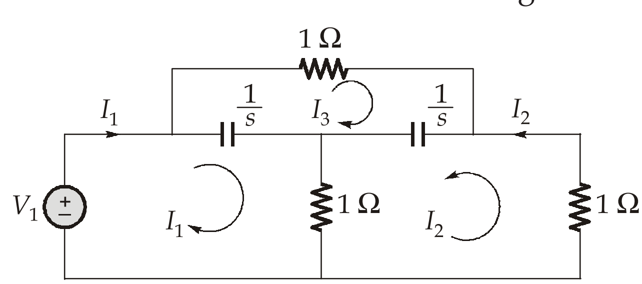
Step III: *Calculation of* I_L

The Thevenin equivalent circuit can be drawn as below:



$$I_L = \frac{100}{20} = 5 \text{ A}$$

(ii) The transformed network in s-domain is shown in figure below:



Applying KVL to Mesh 1,

$$V_1 = \left(\frac{1}{s}+1\right)I_1 + I_2 - \frac{1}{s}I_3 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$0 = I_1 + \left(2+\frac{1}{s}\right)I_2 + \frac{1}{s}I_3 \quad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$0 = -\frac{1}{s}I_1 + \frac{1}{s}I_2 + \left(\frac{2}{s}+1\right)I_3 \quad \text{...(iii)}$$

Writing these equations in matrix form,

$$\begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{s}+1 & 1 & \frac{-1}{s} \\ 1 & 2+\frac{1}{s} & \frac{1}{s} \\ \frac{-1}{s} & \frac{1}{s} & \frac{2}{s}+1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

By Cramer's rule, $I_1 = \frac{\Delta_1}{\Delta}$

where,

$$\Delta = \begin{vmatrix} 1+1 & 1 & \frac{-1}{s} \\ 1 & 2+\frac{1}{s} & \frac{1}{s} \\ \frac{-1}{s} & \frac{1}{s} & \frac{2}{s}+1 \end{vmatrix}$$

$$= \left(1+\frac{1}{s}\right)\left[\left(2+\frac{1}{s}\right)\left(1+\frac{2}{s}\right)-\frac{1}{s^2}\right] - 1\left[(1)\left(1+\frac{2}{s}\right)+\frac{1}{s^2}\right]$$

$$-\frac{1}{s}\left[(1)\left(\frac{1}{s}\right)+\left(\frac{1}{s}\right)\left(2+\frac{1}{s}\right)\right]$$

$$= \frac{s^2+5s+2}{s^2}$$

$$\Delta_1 = \begin{vmatrix} V_1 & 1 & \frac{-1}{s} \\ 0 & 2+\frac{1}{s} & \frac{1}{s} \\ 0 & \frac{1}{s} & \frac{2}{s}+1 \end{vmatrix}$$

$$= V_1\left[\left(2+\frac{1}{s}\right)\left(1+\frac{2}{s}\right)-\frac{1}{s^2}\right] = V_1\left(\frac{2s^2+5s+1}{s^2}\right)$$

$$I_1 = V_1\left(\frac{2s^2+5s+1}{s^2+5s+2}\right)$$

Driving-point admittance

$$Y_{11}(s) = \frac{I_1}{V_1} = \frac{2s^2+5s+1}{s^2+5s+2}$$

$$I_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta_2 = \begin{vmatrix} \frac{1}{s}+1 & V_1 & \frac{-1}{s} \\ 1 & 0 & \frac{1}{s} \\ \frac{-1}{s} & 0 & \frac{2}{s}+1 \end{vmatrix}$$

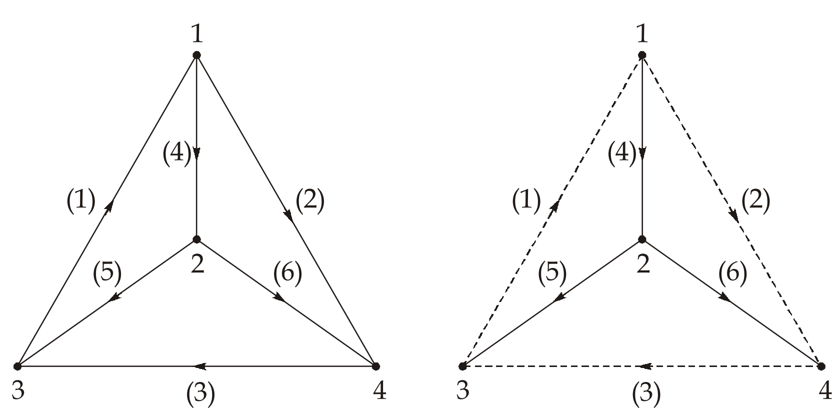
$$= -V_1\left[\frac{2}{s}+1+\frac{1}{s^2}\right] = -V_1\left(\frac{s^2+2s+1}{s^2}\right)$$

$$I_2 = -V_1\left(\frac{s^2+2s+1}{s^2+5s+2}\right)$$

Transfer admittance, $Y_{21}(s) = \frac{I_2}{V_1} = -\frac{s^2+2s+1}{s^2+5s+2}$

Q.4 (b) Solution:

(i) The oriented graph for the given network and one of its trees are shown in figure below:



Tieset 1: [1, 4, 5]
Tieset 2: [2, 4, 6]
Tieset 3: [3, 5, 6]

Tieset Matrix (B)

$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

The KVL equation in matrix form is given by

$$B\,Z_b B^T I_l = B\,V_s - B\,Z_b I_s$$

Here, $I_s = 0$

$$B\,Z_b B^T I_l = B\,V_s$$

$$Z_b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \quad B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} \quad V_s = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B\,Z_b = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & 0 & -2 & 2 \end{bmatrix}$$

$$B\,Z_b B^T = \begin{bmatrix} 1 & 0 & 0 & 2 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & 0 & -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & -2 \\ -2 & 5 & -2 \\ -2 & -2 & 5 \end{bmatrix}$$

$$B\,V_s = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

The KVL equation in matrix form is given by

$$\begin{bmatrix} 5 & -2 & -2 \\ -2 & 5 & -2 \\ -2 & -2 & 5 \end{bmatrix} \begin{bmatrix} I_{l_1} \\ I_{l_2} \\ I_{l_3} \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

Solving this matrix equation using Carner's rule, we get

$$I_{l_1} = \frac{6}{7}\text{A}$$

$$I_{l_2} = \frac{4}{7}\text{A}$$

$$I_{l_3} = \frac{4}{7}\text{A}$$

The branch currents are given by

$$I_b = B^T I_l$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 6/7 \\ 4/7 \\ 4/7 \end{bmatrix} = \begin{bmatrix} 6/7 \\ 4/7 \\ 4/7 \\ 2/7 \\ 2/7 \\ 0 \end{bmatrix}$$

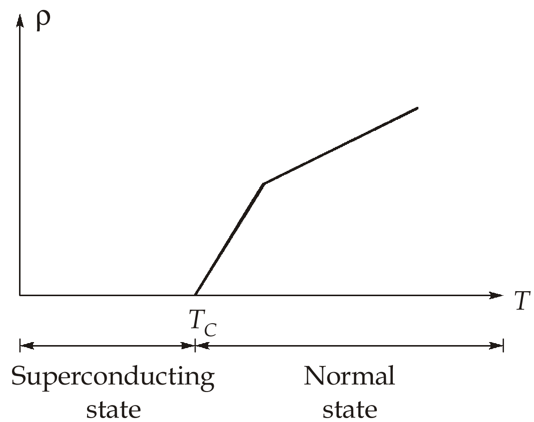
(ii) Superconductivity: A state of material in which it has zero resistivity is called superconductivity. At the state of superconductivity, material shows diamagnetic property.

For superconducting materials,

1. Resistivity should be zero ($\rho = 0$)
2. Perfect diamagnetism ($\mu_r = 0$) should exist

and both conditions exists independently.

The temperature at which transition of state occurs from superconducting to normal state or vice versa is known as critical or transition temperature, TC as shown below:



Properties in superconductivity:

1. **Frequency:** With the increase in frequency, resistivity increases due to skin effect thus superconductivity decreases with increase in frequency. It is observed upto RF frequency.
2. **Entropy:** Entropy increases from superconducting state to normal state.
3. **Thermal conductivity:** Thermal conductivity increases from superconducting state to normal state.
5. **Isotope effect:** The transition temperature is inversely proportional to the square root of the isotopic mass of a superconducting material. As isotopic mass increases transition temperature decreases.

$$T_c \propto \frac{1}{\sqrt{\text{Isotopic Mass}}}$$

Applications of superconductivity:

1. Electronic switching devices.
2. Generator for motors.
3. Magnets for nuclear fusion.
4. Magnets for high energy physics.
5. Magnetically levitated transportation.
6. Superconducting magnets for energy storage.

Q.4 (c) Solution:

(i) Since, $V_{GS(\text{off})} = -4\text{ V}; V_p = 4\text{ V}$

The minimum value of V_{DS} for the JFET to be in its constant-current region is

$$V_{DS} = V_P = 4\text{ V}$$

In the constant-current region with $V_{GS} = 0$ V,

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = I_{DSS} = 12 \text{ mA}$$

The drop across the drain resistor is

$$V_{RD} = I_D R_D = (12 \text{ mA})(560) = 6.72 \text{ V}$$

Apply Kirchhoff's law around the drain circuit,

$$V_{DD} = V_{DS} + V_{RD}$$
$$V_{DD} = 4 + 6.72$$
$$V_{DD} = 10.72 \text{ V}$$

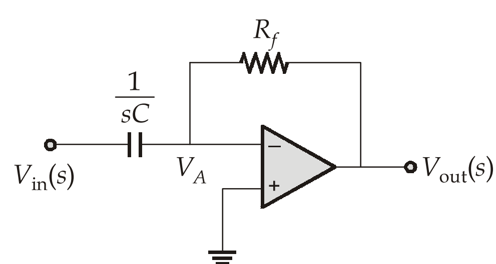
For $V_{DD} = 10.72$ V, V_{DS} become equal to V_P and put the device in constant current region.

(ii) Starting at $t = 0$, the input voltage is a positive-going ramp ranging from –5 V to +5 V (a +10 V change) in 5 μsec. Then it changes to a negative-going ramp ranging from +5 V to –5 V (a –10 V change) in 5 μs.

The time constant is

$$R_f C = (2.2K)(0.001 \times 10^{-6})$$
$$R_f C = 2.2 \text{ μsec}$$

The circuit in s-domain can be drawn as below:



Now from the virtual ground concept,

$$V_A = 0 \text{ V}$$

Applying KCL at the inverting terminal, we get

$$\frac{0 - V_{in}(s)}{\left(\frac{1}{sC}\right)} + \frac{0 - V_{out}(s)}{R_f} = 0$$

$$V_{out}(s) = -V_{in}(s) R_f C s$$

Now on taking inverse Laplace transform, we get

$$V_{out}(t) = -R_f C \frac{dV_{in}(t)}{dt}$$

For 0 to 5 μsec; $\quad \frac{dV_{in}(t)}{dt} = \frac{10 \text{ V}}{5 \text{ μs}} = 2 \text{ V/μs}$

$$V_{out}(t) = -2.2\times10^{-6}\times\frac{2}{10^{-6}}$$

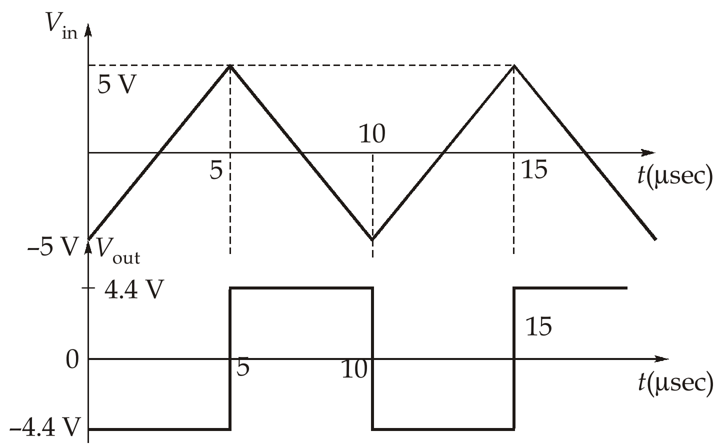
$$V_{out}(t) = -4.4\text{ V}$$

For 5 to 10 μsec; $$\frac{dV_{in}(t)}{dt} = \frac{-10\text{ V}}{5\text{ μs}} = -2\text{ V/μs}$$

∴ $$V_{out}(t) = -2.2\times10^{-6}\times\frac{(-2)}{10^{-6}}$$

$$V_{out}(t) = 4.4\text{ V}$$

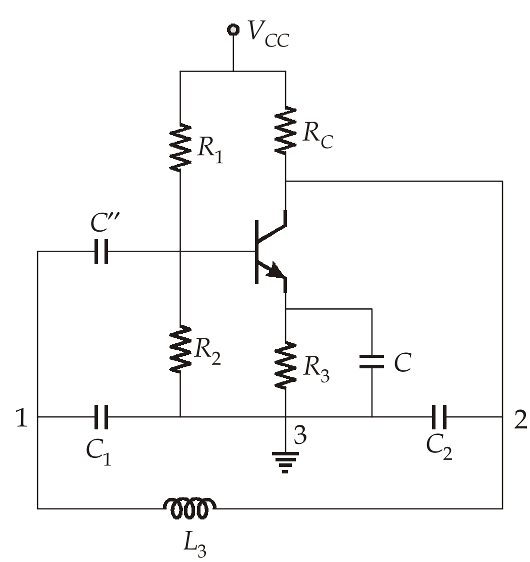
Hence, $$V_{out}(t) = \begin{cases}-4.4\text{ V}; 0 < t < 5\text{ μs}\\ 4.4\text{ V}; 5\text{ μs} < t < 10\text{ μs}\end{cases}$$ and so on.....



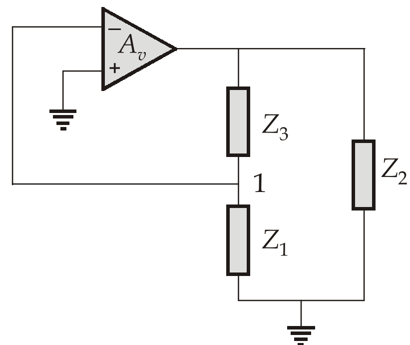
Section B

Q.5 (a) Solution:

- The circuit diagram of a Colpitt's oscillator using transistor is drawn below:



- Colpitt's oscillator using op-amp is drawn below:



$$Z_1 = jX_1;\ Z_2 = jX_2;\ Z_3 = jX_3$$

where, $$X_1 = \frac{-1}{\omega C_1};\ X_2 = \frac{-1}{\omega C_2};\ X_3 = \omega L_3$$

Load impedance,

$$Z_L = Z_2 \parallel (Z_1 + Z_3)$$

The gain without feedback is

$$A = \frac{A_V Z_L}{Z_L + R_0}$$

Feedback factor, $$\beta = \frac{-Z_1}{Z_1 + Z_3}$$

Loop gain, $$A\beta = \frac{-A_V Z_1 Z_L}{(Z_L + R_0)(Z_1 + Z_3)}$$

On putting, $$Z_L = Z_2 \parallel (Z_1 + Z_3)$$

$$Z_L = \frac{Z_2 \cdot (Z_1 + Z_3)}{Z_2 + Z_1 + Z_3}$$

We get, $$A\beta = \frac{-A_V Z_1 Z_2}{R_0(Z_1 + Z_2 + Z_3) + Z_2(Z_1 + Z_3)}$$

For loop gain to be real, $X_1 + X_2 + X_3 = 0$ at $\omega = \omega_0$

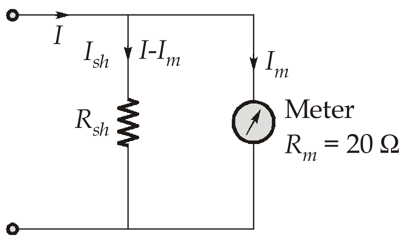
(Zero phase shift as per Barkhausen criteria)

$$\frac{-1}{\omega_0 C_1} + \omega_0 L_3 - \frac{1}{\omega_0 C_2} = 0$$

On solving we get, $$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{L_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}$$

Q.5 (b) Solution:

(i)



Multiplying factor of shunt, $m = \dfrac{I}{I_m} = \dfrac{100}{2} = 50$

$\therefore$ Resistance of shunt, $R_{sh} = \dfrac{R_m}{m-1} = \dfrac{20}{49} = 0.408\ \Omega$

Instrument resistance for a 15°C rise in temperature,

Coil resistance, $R_{mt} = R_m(1 + \alpha_{cu}\Delta T) = 20(1 + 15 \times 0.005) = 21.5\ \Omega$

Shunt resistance, $R_{sht} = R_{sh}(1 + \alpha_m \Delta T)$

where, $\alpha_m = 0.0002/°C$

$= 0.408(1 + 0.0002 \times 15) = 0.409224\ \Omega$

Current I_{mt} through meter for 100 mA in the main circuit for 15°C rise in temperature.

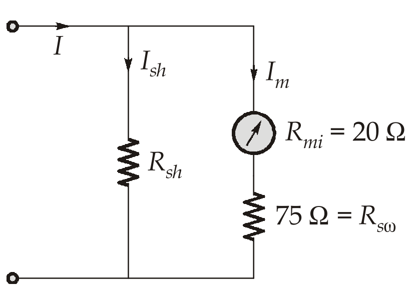
$$I_{mt} = \frac{IR_{sht}}{R_{mt} + R_{sht}} = \frac{100 \times 0.40922}{21.5 + 0.40922} = 1.8677 \text{ mA}$$

Normal meter current = 2 mA

Error due to 15°C rise in temperature

$$= \frac{(1.8677 - 2)}{2} \times 100 = -6.615\%$$

(ii)



With a 75 Ω manganin resistance used in series with the instrument moving coil,

Total resistance in meter circuit

$$R_m = R_{mi} + R_{s\omega}$$
$$= 20 + 75 = 95\ \Omega$$

Shunt resistance required to extend the range of ammeter,

$$R_{sh} = \frac{R_m}{m-1} = \frac{95}{49} = 1.938\ \Omega$$

Resistance for 15°C rise in temperature,

$$R_{mt} = R_{mi}(1 + \alpha_m \Delta T) + R_{s\omega}(1 + \alpha_{s\omega}\Delta T)$$

Given,

$$\alpha_m = 0.005/°C$$

$$\alpha_{s\omega} = 0.0002/°C$$

$$R_{mt} = 20(1 + 0.005 \times 15) + 75\,(1 + 0.0002 \times 15)$$

$$= 96.725\ \Omega$$

$$R_{sht} = R_{sh}(1 + \alpha_m \Delta T) = 1.938(1 + 15 \times 0.0002)$$

$$= 1.943\ \Omega$$

Current through meter for $I = 100$A,

$$I_{mt} = \frac{100 \times 1.943}{96.725 + 1.943} = 1.969\ \text{mA}$$

$$\%\ \text{error} = \frac{1.969 - 2}{2} \times 100 = -1.55\%$$

Conclusion: The improvement in error from –6.615% to –1.55% has been obtained by the use of additional series swamping resistance of 4 times as compared to meter resistance. We could have obtained better correction by increasing the ratio of swapping resistance as compared to meter resistance. However the disadvantage of using swamping resistors is a reduction in the full scale sensitivity as a higher voltage across the instrument is necessary to sustain the full scale current.

Q.5 (c) Solution:

(i)

$$F(s) = \frac{N(s)}{D(s)} = \frac{s^2+1}{s^3+4s} = \frac{(s+j1)(s-j1)}{s(s+j2)(s-j2)}$$

The function $F(s)$ has poles at $s = 0$, $s = -j2$ and $s = j2$ and zeros at $s = -j1$ and $s = j1$. Thus, all the poles and zeros are on the $j\omega$ axis.

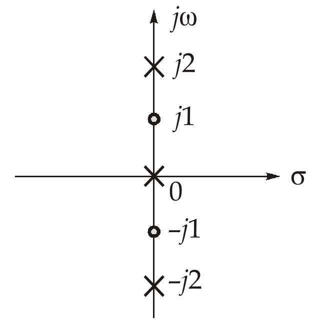
(ii) The poles on the $j\omega$ axis are simple. Hence, residue test is carried out.

$$F(s) = \frac{s^2+1}{s^3+4s} = \frac{s^2+1}{s(s^2+4)}$$

By partial-fraction expansion, we have

$$F(s) = \frac{K_1}{s} + \frac{K_2}{s+j2} + \frac{K_2^*}{s-j2}$$

The constants K_1, K_2 and K_2^* are called residues.



$$K_1 = sF(s)|_{s=0}$$

$$= \left.\frac{s^2+1}{s^2+4}\right|_{s=0} = \frac{1}{4}$$

$$K_2 = (s+j2)F(s)|_{s=-j2}$$

$$= \left.\frac{s^2+1}{s(s-j2)}\right|_{s=-j2}$$

$$= \frac{-4+1}{(-j2)(-j2-j2)} = \frac{3}{8}$$

$$K_2^* = K_2 = \frac{3}{8}$$

Thus, residues are real and positive.

(iii) Even part of $N(s) = m_1 = s^2 + 1$

Odd part of $N(s) = n_1 = 0$

Even part of $D(s) = m_2 = 0$

Odd part of $D(s) = n_2 = s^3 + 4s$

$$A(\omega^2) = m_1m_2 - n_1n_2|_{s=j\omega}$$

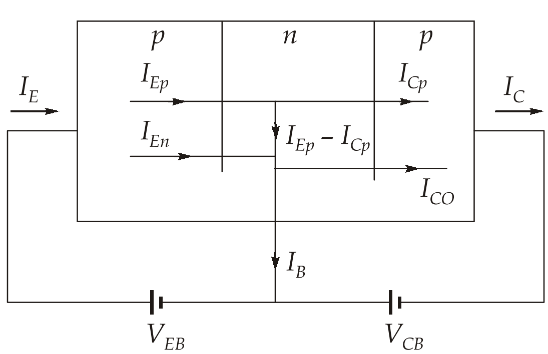
$$= (s^2+1)(0) - (0)(s^3+4s)|_{s=j\omega} = 0$$

$A(\omega^2) \geq$ for all $\omega \geq 0$.

Since all the three conditions are satisfied, the given function is positive real.

Q.5 (d) Solution:

Given, p-n-p BJT with $N_E > N_B > N_C$, the current components are shown as below:



(i) Base transport factor, $\beta^* = \frac{I_{Cp}}{I_{Ep}} = \frac{9.8 \text{ mA}}{10 \text{ mA}} = 0.98$

(ii) Emitter efficiency, $\gamma = \dfrac{I_{Ep}}{I_{Ep} + I_{En}} = \dfrac{10 \text{ mA}}{10 \text{ mA} + 0.1 \text{ mA}} = 0.99$

(iii) Common base current gain, $\alpha = \beta^* \gamma$

$\therefore \quad \alpha = (0.98)(0.99) = 0.97$

(iv) Common emitter current gain, $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.97}{1-0.97} \simeq 32$

(v) Given, minority stored base charge,

$$Q_B = 4.9 \times 10^{-11} \text{ C}$$

$$\text{Base transit time, } \tau_t = \frac{Q_B}{I_{cp}}$$

$$\tau_t = \frac{4.9 \times 10^{-11}}{9.8 \times 10^{-3}} = 5 \text{ nsec}$$

$$\text{Base life time, } \tau_p = \frac{Q_B}{(1-\beta^*)I_{Ep}} = \frac{4.9 \times 10^{-11}}{(1-0.98) \times 10 \times 10^{-3}} = 245 \text{ nsec}$$

Q.5 (e) Solution:

We know that,

$$f = \frac{PN_s}{120}$$

$$f = \frac{16 \times 375}{120} = 50 \text{ Hz}$$

Angle by which the coil in short-pitched,

$$\alpha = 180° - 150° = 30°$$

Pitch factor,

$$K_c = \cos\left(\frac{\alpha}{2}\right) = \cos\left(\frac{30°}{2}\right)$$

$$K_c = 0.97$$

We have,

$$m = \text{slots per pole per phase} = \frac{\text{slots}}{\text{poles} \times \text{phases}}$$

$\therefore$

$$\text{slots} = \text{m} \times \text{poles} \times \text{phases}$$

$$= 2 \times 16 \times 3 = 96$$

Total number of conductors = slots × conductors per slot

$$= 96 \times 4 = 384$$

Number of conductors per phase;

$$Z_p = \frac{384}{3} = 128$$

Angular displacement between adjacent slots,

$$\beta = \frac{180°}{\text{Slots per pole}} = \frac{180° \times 16}{96} = 30°$$

Distribution factor, $$K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\sin\frac{2\times 30°}{2}}{2\sin\frac{30°}{2}} = 0.97$$

Since the flux is sinusoidally distributed,

$$K_f = 1.11$$

The generated voltage per phase is given by

$$E_p = 2\,K_f K_c K_d f \phi Z_p$$

$$E_p = 2 \times 1.11 \times 0.97 \times 0.97 \times 50 \times 0.06 \times 128$$

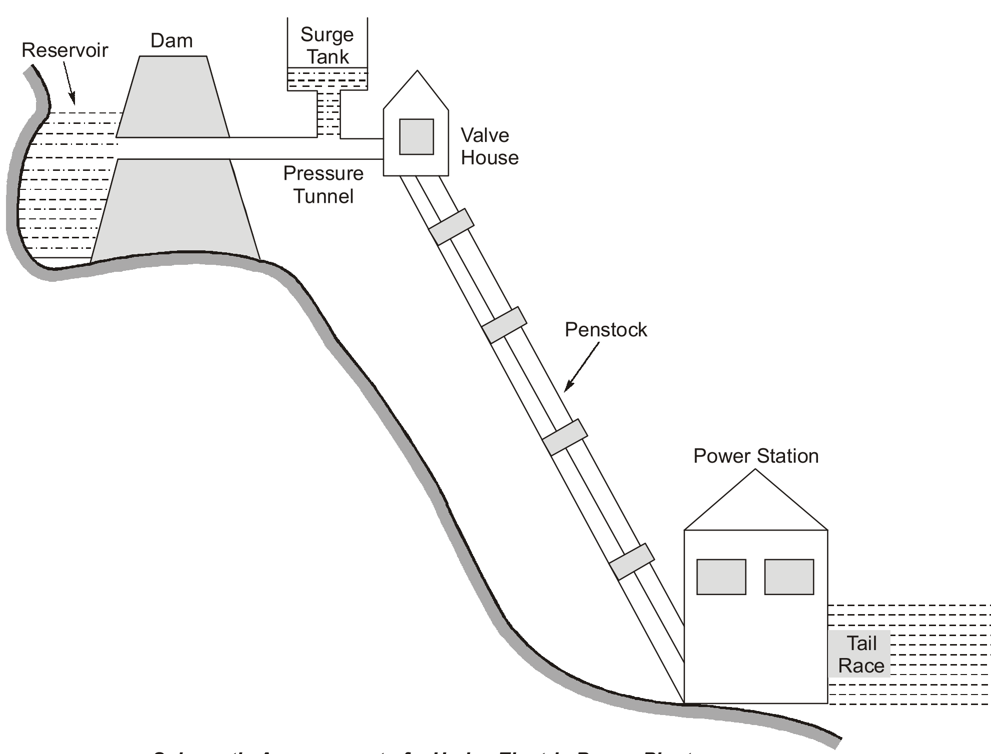
$$E_p = 802.1 \text{ V}$$

Generated line voltage, $$E_L = \sqrt{3}E_p = \sqrt{3}\times 802.1 = 1389.3 \text{ V}$$

Q.6 (a) Solution:

Hydroelectric power is the power obtained from the energy of falling water. The hydroelectric power plant utilizes the potential energy of water at a high level for the generation of electrical energy.

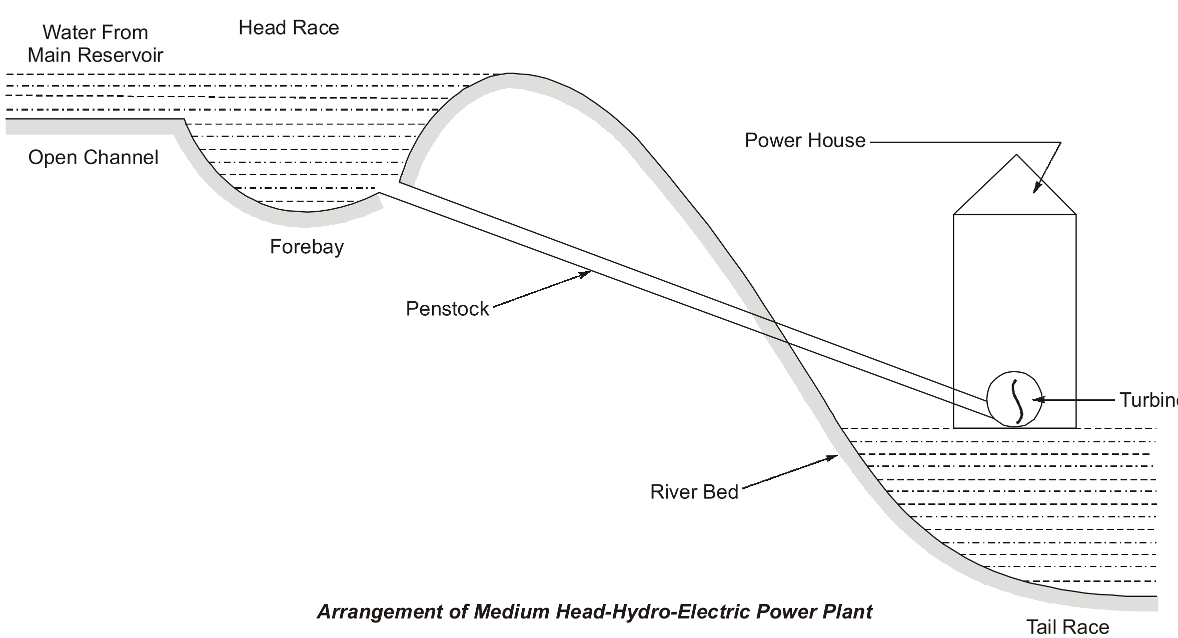
Construction:



Schematic Arrangement of a Hydro-Electric Power Plant

The elements of hydroelectric power plant are as follows:

1. **Storage Reservoir**
   - Its purpose is to store water during excess flow periods (i.e. rainy season) and supply the same during lean flow periods (i.e. dry season).
   - A reservoir can be natural or artificial. A natural reservoir is a lake in high mountains and an artificial reservoir is made by constructing a dam across the river.
   - Low head plants require very large storage reservoir. The capacity of reservoir depends on the difference between runoffs during high and lean flows.
2. **DAM**
   - A dam is the most expensive and important part of hydro-project.
   - The function of dam is not only to raise the water surface of the stream to create an artificial head but also to provide the storage.
3. **Forebay**
   - It serves as a regulating space, storing water temporarily during light load period and providing the same for initial increase on account of increasing load during which water in the canal is being accelerated.
   - This may be a pond behind the diversion dam or canal spread out.
   - Where the hydroelectric plants are located just at the base of the dam, no fore bay is required because the reservoir itself serves the purpose of the forebay. However where the plants are situated away from the storage reservoir a forebay is provided.



Arrangement of Medium Head-Hydro-Electric Power Plant

4. **Spillway**

This is constructed to act as a safety valve. It discharges the overflow of water to the down stream side when the reservoir is full, a condition mainly arising during flood periods. These are generally constructed of concrete and provided with water discharge opening shut off by metal control gates. By changing the degree to which the gates are opened, the discharge of the head water to the tail race can be regulated in order to maintain the water level in the reservoir.

5. **Surge Tank**

A reduction in load on the generator causes the governor to close the turbine gates and thus create an increased pressure in the penstock. This may result in water hammer phenomenon and may need pipe of extraordinary strength to withstand it otherwise the penstock may burst.

To avoid this positive water hammer pressure, some means are required to be provided for taking rejected flow. This may be accomplished by providing a small storage reservoir or tank (open at the top) for receiving the rejected flow and thus relieving the pipe of excessive water hammer pressure. This storage reservoir is called surge tank.

6. **Penstock**

It is closed conduit which connects the forebay or surge tank to the scroll case of the turbine. Penstocks are built of steel or reinforced concrete.

7. **Tail Race**

The water after having done its useful work in the turbine is discharged to the tail race which may lead it to the same stream or to another one.

8. **Draft Tubes**

An air tight pipe of suitable diameter attached to the runner outlet and conducting water down from the wheel and discharging it under the surface of the water in the tail race is known as draft tube. This creates a negative pressure head at the runner exit. By installing draft tube, the operating head is increased by an amount equal to the height of the runner outlet above the tailrace. which makes it possible to install the turbine above the tailrace without loss of head.

9. **Prime Movers or Water Turbine**

Water turbines are used as prime movers and their function is to convert the kinetic energy of water into mechanical energy which is further utilised to drive the alternators generating electrical energy.

The classification of turbines on various basis is given below:

Axial Flow → Kaplan turbine → propeller turbines → low head and high flow (Head < 30 m)

Mixed Flow → Francis turbine → reaction turbine → medium head and medium flow (30 m < Head < 200 m)

Tangential Flow → Pelton wheel turbine → impulse turbine → high head and low flow (Head > 200 m)

Note: The electrical power given by,

$$P = WQH\eta \times 9.81 \times 10^{-3} \text{ kW}$$

W = specific weight of water in kg/m^3

Q = rate of flow of water in m^3/s

H = height of water fall in meter (head)

η = overall efficiency of operation

Advantages:

- No fuel is required by such plants as water is the source of energy. Hence operating costs are low and there are no problems of handling and storage of fuel and disposal of ash.
- The plant is highly reliable and it is cheapest in operation and maintenance.
- The plant can be run up and synchronized in a few minutes.
- The load can be varied quickly and the rapidly changing load demands can be met without any difficulty.
- Very acute governing is possible with water turbines so such power plants have constant speed and hence constant frequency.
- There are no standby losses in such plants.
- Such plants are robust and have got longer life (around 50 years).
- The efficiency of such plants does not fall with the age.
- It is very neat and clean plant because no smoke or ash is produced.
- Highly skilled engineers are required only at the time of construction but later on only a few experienced persons will be required.
- Such plants in addition to generation of electric power also serve other purposes such as irrigation, flood control and navigation.
- Hydroelectric plants are usually located in remote areas where land is available at cheaper rates.

Disadvantages:

- It requires large area.

- Its construction cost is enormously high and takes a long time for erection (owing to involvement of huge civil engineering works).
- Long transmission lines are required as the plants are located in hilly areas which are quite away from the load centre.
- The output of such plants is never constant owing to vagaries of monsoons and their dependence on the rate of water flow in a river. Long dry season may affect the power supply.
- The firm capacity of hydroelectric plants is low and so backup by steam plants is essential.
- Hydroelectric power plant reservoir submerges huge areas, uproots large population and creates social and other problems.

Q.6 (b) Solution:

Since the switch is kept closed at the position 1 for a long time, the circuit has reached steady state. At steady state, inductor acts as short circuit. The steady-state circuit at $t = 0^-$ with the switch in the position 1 is shown in figure (b). The steady-state current at $t = 0^-$ is

$$i(0^-) = \frac{10}{5} = 2\text{ A}$$

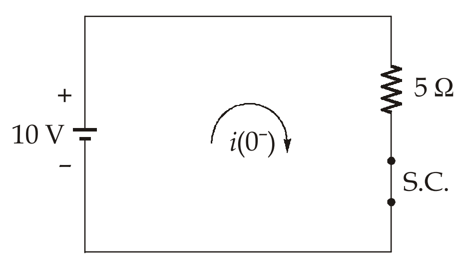


Fig. (b)

When the switch is moved from the position 1 to the position 2, the inductor doesn't allow sudden change in current, therefore

$$i(0^-) = i(0^+) = 2\text{ A}$$

The RL circuit with the switch at the position 2 is shown in fig. (c).

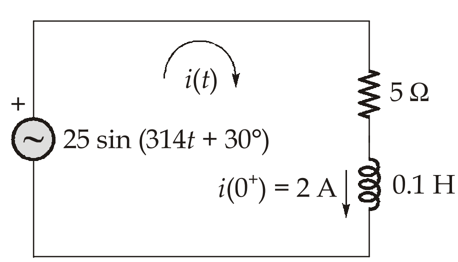


Fig. (c)

Applying KVL to the circuit, we get

$$5i(t) + 0.1\frac{di(t)}{dt} = 25\sin(314t + 30°)$$

$$= 25(\sin 314t \cos 30° + \cos 314t \sin 30°)$$

$$= 25 \times \left(\sin 314 \frac{\sqrt{3}}{2} + \cos 314t \times \frac{1}{2}\right)$$

$$= 25 \times \frac{\sqrt{3}}{2} \times \sin 314t + 25 \times \frac{1}{2} \times \cos 314t$$

Taking Laplace transform, we get

$$5I(s) + 0.1[sI(s) - i(0^+)] = 25 \times \frac{\sqrt{3}}{2} \times \frac{314}{s^2 + 314^2} + 25 \times \frac{1}{2} \times \frac{s}{s^2 + 314^2}$$

$$I(s)(5 + 0.1s) - 0.2 = \frac{6798.3}{s^2 + 314^2} + \frac{12.5s}{s^2 + 314^2}$$

$$I(s) \times 0.1(s + 50) = \frac{12.5s + 6798.3}{s^2 + 314^2} + 0.2$$

$$I(s) = \frac{12.5s + 6798.3}{0.1(s + 50)(s^2 + 314^2)} + \frac{0.2}{0.1(s + 50)}$$

$$= \frac{125s + 67983}{(s + 50)(s^2 + 314^2)} + \frac{2}{(s + 50)}$$

Let $$\frac{125s + 67983}{(s + 50)(s^2 + 314^2)} = \frac{A_1}{s + 50} + \frac{A_2 s + A_3}{s^2 + 314^2}$$

Therefore, $$A_1 = \left.\frac{125s + 67985}{(s^2 + 314^2)}\right|_{s=-50} = \frac{125 \times (-50) + 67985}{(-50)^2 + 314^2} = 0.6107$$

From equation (i), we get

$$125s + 67983 = A_1(s^2 + 314^2) + (A_2 s + A_3)(s + 50)$$

$$125s + 67983 = A_1 s^2 + A_1 314^2 + A_2 s^2 + 50A_2 s + A_3 s + 50A_3$$

$$125s + 67983 = (A_1 + A_2)s^2 + (50A_2 + A_3)s + (A_1\, 314^2 + 50A_3)$$

Equating coefficients of s^2 term of the equation, we get

$$A_1 + A_2 = 0$$

Therefore, $$A_1 = -A_2 = -0.6107$$

Equating coefficients of 's' term of the above equation, we get

$$50A_2 + A_3 = 125$$

Therefore, $A_3 = 125 - 50A_2$

$$= 125 - 50 \times (-0.6107) = 155.535$$

Thus, $$I(s) = \frac{0.6107}{s+50} + \frac{-0.6107s + 155.535}{s^2+314^2} + \frac{2}{(s+50)}$$

$$= \frac{2.6107}{s+50} - \frac{0.6107s}{s^2+314^2} + \frac{155.535}{s^2+314^2}$$

$$= 2.6107 \times \frac{1}{s+50} - 0.6107 \times \frac{s}{s^2+314^2} + \frac{155.535}{314} \times \frac{314}{s^2+314^2}$$

$$= 2.6107 \times \frac{1}{s+50} - 0.6107 \times \frac{s}{s^2+314^2} + 0.4953 \times \frac{314}{s^2+314^2}$$

Taking inverse Laplace transform we get

$$L^{-1}[I(s)] = L^{-1}\left[2.6107 \times \frac{1}{s+50} - 0.6107 \times \frac{s}{s^2+314^2} + 0.4953 \times \frac{314}{s^2+314^2}\right]$$

$$L^{-1}[I(s)] = 2.6107 L^{-1}\left[\frac{1}{s+50}\right] - 0.6107\, L^{-1}\left[\frac{s}{s^2+314^2}\right] + 0.4953\, L^{-1}\left[\frac{314}{s^2+314^2}\right]$$

Therefore, $i(t) = 2.6107e^{-50t} - 0.6107 \cos 314t + 0.4953 \sin 314t$

$$= 2.6107\, e^{-50t} + [\sin 314t \times 0.4953 - \cos 314t \times 0.6107]A \qquad \text{...(ii)}$$

A right -angled triangle is constructed with 0.4953 and 0.6107 as two sides as shown in fig. (d).

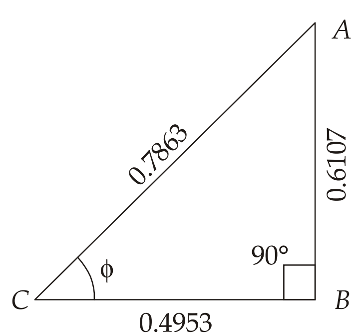


Fig. (d)

$$\tan\phi = \frac{0.6107}{0.4953} = 1.233$$

Therefore, $\phi = \tan^{-1} 1.233 = 51°$

We get, $AC = \dfrac{0.4953}{\cos(\phi)} = 0.7863$

We can write,

$$0.4953 = 0.7863 \cos\phi = 0.7863 \cos 51°$$

Also, $\sin\phi = \dfrac{0.6107}{0.7863}$

i.e., $0.6107 = 0.7863 \sin\phi = 0.7863 \sin 51°$

From equation (ii),

$$i(t) = 2.6107e^{-50t} + [\sin 314t \times 0.7863 \cos 51° - \cos 314t \times 0.7863 \sin 51°]$$

$$= 2.6107e^{-50t} + 0.7863[\sin 314t \cos 51° - \cos 314t \sin 51°]$$

$$= 2.6107e^{-50t} + 0.7863 \sin(314t - 51°)A \quad \text{for } t > 0$$

In the above current equation, the first term is the transient response and the second term is the steady-state response or forced response.

Q.6 (c) Solution:

(i) **1.** Change in stress,

$$\Delta S = \frac{1}{10} \text{ (elastic limit stress)}$$

$$= \frac{1}{10} \times 450 \times 10^6$$

$$= 45 \times 10^6 \text{ N/m}^2 = 45 \text{ MN/m}^2$$

Change in strain, $\dfrac{\Delta L}{L} = \dfrac{\Delta S}{E}$

$$E = \text{Modulus of elasticity}$$

$$= 230 \times 10^9 \text{ N/m}^2$$

$$\frac{\Delta L}{L} = \frac{45 \times 10^6}{230 \times 10^9} = 195.652 \times 10^{-6}$$

Change in resistance, $\Delta R = G_f \times \dfrac{\Delta L}{L} \times R$

$$= 2.5 \times 195.652 \times 10^{-6} \times 150$$

$$= 0.07336\ \Omega$$

$$\Delta R = 0.07336\ \Omega$$

2. Change in resistance due to change in temperature,

$$\Delta R = \alpha\Delta\theta.R$$

$$\alpha = 18 \times 10^{-6}/°C$$

Given, $\Delta\theta = 25°$

$\therefore$ $$\Delta R = 18 \times 10^{-6} \times 25 \times 150$$

$$= 0.0675\ \Omega$$

3. Strain due to differential expansion of the gauge metal and steel is

$$\text{strain} = (15 - 10) \times 10^{-6} \times 25$$

$$\text{strain} = \frac{\Delta L}{L} = 125 \times 10^{-6} = 125 \text{ micro strain}$$

Change in resistance,

$$\Delta R = G_f \times \frac{\Delta L}{L} \times R$$

$$= 2.5 \times (125) \times 150 \times 10^{-6} = 0.0468\ \Omega$$

$$\Delta R = 0.0468\ \Omega$$

(ii) The CsCl structure has one cation (Cs^+) and one anion (Cl^-) in the unit cell. A *Cs* cation is located at the center of the cube with eight chloride ions occupying the eight corners. Given the lattice parameter $a = 0.412 \times 10^{-9}$ m, the number of ion pairs, N_i per unit volume is $\frac{1}{a^3} = \frac{1}{(0.412\times10^{-9})^3} = 1.43 \times 10^{28}\ \text{m}^{-3}$. N_i is also the concentration of cations and anions individually.

From the Clausius-Mossotti equation,

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{1}{3\varepsilon_0}\left[N_i\alpha_e(Cs^+) + N_i\,\alpha_e(Cl^-) + N_i\,\alpha_i\right]$$

where a_e is the electronic polarizability and ai is the ionic polarizability.

That is, $$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{(1.43\times10^{28})[3.35\times10^{-40} + (3.40\times10^{-40} + 6\times10^{-40})]}{3(8.85\times10^{-12})}$$

Solving for ε_r, we get

$$\varepsilon_r = 7.56$$

At high frequencies that is near optical frequencies, the ionic polarization is too sluggish to allow ionic polarization to contribute to ε_r. The ε_{rop}, relative permittivity at optical frequencies is given by

$$\frac{\varepsilon_{rop} - 1}{\varepsilon_{rop} + 2} = \frac{1}{3\varepsilon_0}\left[N_i \alpha_e (Cs^+) + N_i \alpha_e (Cl^-)\right]$$

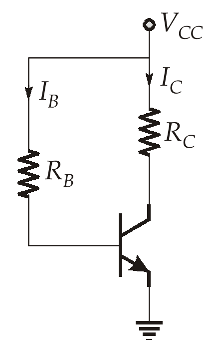
That is, $$\frac{\varepsilon_{rop} - 1}{\varepsilon_{rop} + 2} = \frac{(1.43 \times 10^{28})(3.35 \times 10^{-40} + 3.40 \times 10^{-40})}{3(8.85 \times 10^{-12})}$$

On solving, $$\varepsilon_{rop} = 2.71$$

Q.7 (a) Solution:

To achieve the desired switching or the amplification effect, a transistor must be supplied with the control amount of voltage and current through it. This technique is known as transistor biasing which helps setting transistor's DC operating voltage or current conditions to the correct level. The most commonly preferred methods for biasing transistors are

(a) Fixed Bias Circuit:



The circuit shown above is called as a "fixed base bias circuit", because the transistor base current, I_B remains constant for given values of V_{CC}, and therefore the transistor operating point must also remain fixed. This two resistor biasing network is used to establish the initial operating region of the transistor using a fixed current bias. It is also called as Base-Biased circuit or beta-dependent biasing circuits.

Advantages of fixed Base Bias Circuit:

- Simple design and construction — Fixed bias circuit are straight forward, making them easy to build and understand, even for beginners.
- Stable at low temperatures — Performance doesn't waver much when its cold, so it's reliable at low temperatures..
- Easy to implement — Setting one up doesn't require much effort or specialized knowledge, which is convenient.
- Cost-effective for mass production — Owing to requirement of less components, making lots of them doesn't cost much, so they are great for when you need many, like in consumer electronics.

Disadvantages of fixed Bias Circuit:

- Poor thermal stability — These circuit can overheat because they don't adjust to temperature variations, which might damage the components.
- No self-bias correction — Without self-bias correction, the circuit can't adjust its bias automatically, leading to potential issues with operation and reliability.
- High distortion levels — They tend to produce more unwanted noise in the signal, which can interfere with the clarity and quality of the output.
- Sensitive to supply variations — If the power supply fluctuates, these circuits are likely to be affected, causing the performance to vary and possibly compromising the device's functionality.
- Limited temperature stability — Fixed bias circuit can struggle with changes in temperature due to poor stability factor, leading to inconsistent performance as the environment heats up or cool down.

Stability Factor:

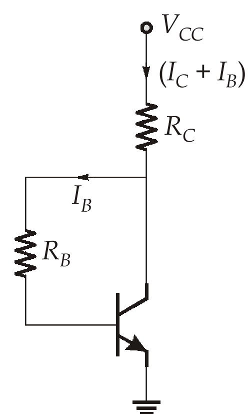
As we know that, $I_C = \beta I_B + (1+\beta)I_{C0}$

$$\frac{\partial I_C}{\partial I_{C0}} = S = (1+\beta) \qquad \therefore I_B = \text{constant}$$

$$S = (1+\beta)$$

(b) Collector to Base Bias Circuit:

The collector to base bias circuit is same as base bias circuit except that the base resistor R_B is connected to collector, rather than to V_{CC} supply as shown in the figure below.



This circuit helps in improving the stability considerably. If the value of I_C increases, the voltage across R_C increases and hence V_{CE} also increases. This in turn reduces the base current I_B. This action somewhat compensate the original increase.

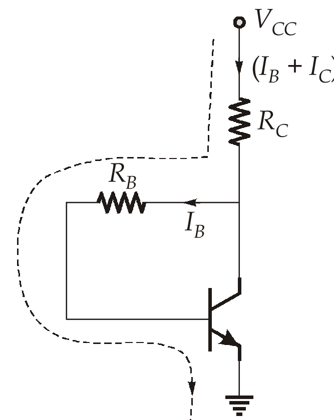
Advantages of Collector to Base Biasing:

- Simple design and construction.
- Low component count.
- Due to smaller stability factor, the circuit provides more stabilization in comparison to fixed bias circuit.

Disadvantages of Collector to Base Biasing:

- The circuit does not provide good stabilization.
- The circuit provides negative feedback.

Stability factor 'S':



$$S = \frac{\partial I_C}{\partial I_{C0}} = \frac{1+\beta}{1 - \frac{\beta \partial I_B}{\partial I_C}}$$

On applying KVL, we get

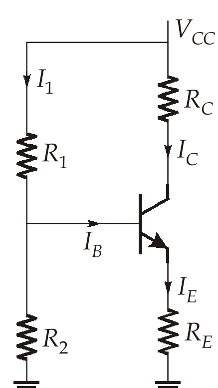
$$-V_{CC} + R_C(I_B + I_C) + I_B R_B + V_{BE} = 0$$

$$\frac{\partial I_B}{\partial I_C} = \frac{-R_C}{R_C + R_B}$$

$$\therefore \qquad S = \frac{1+\beta}{1 + \frac{\beta R_C}{R_C + R_B}}$$

(c) Self Bias Circuit or Voltage Divider Bias Circuit

In a voltage divider bias configuration, the bias voltage is provided by voltage divider network consisting of two resistors connected in series between the supply voltage and ground. The voltage across the lower resistor provides the base bias voltage for the transistor as shown below:

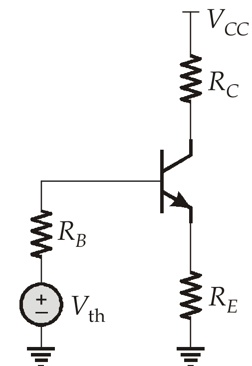


R_2 is selected such that $I_1 >>> I_B$. Hence, R_1 and R_2 acts as series connected resistance and therefore it act as a voltage divider network.

We can also represent the above circuit using Thevenin's equivalent as drawn below:

$$R_B = R_1 \parallel R_2 \qquad V_{th} = \frac{V_{CC} \times R_2}{R_1 + R_2}$$

$$R_B = \frac{R_1 R_2}{R_1 + R_2}$$



Advantage of Self Bias Method:

- Stability factor 'S' for voltage divider biasing is less as compared to fixed bias and collector to base bias. Hence, this circuit is more stable.

  'R_E' helps in keeping I_C stable and provides more stability. If collector current I_c increases due to temperature variations, it causes the voltage drop across R_E to increase which causes I_B to decrease and thereby, I_C to decrease.

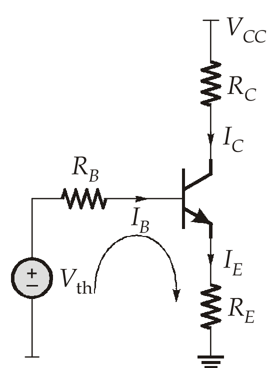
Disadvantage of Self Base Biasing Method:

- The resistor R_1 and R_2 have large resistance. The size of resistor is too large and it requires large chip area.

Stability factor 'S':

$$\Rightarrow \qquad S = \frac{\partial I_C}{\partial I_{C0}} = \frac{1+\beta}{1 - \frac{\beta \partial I_B}{\partial I_C}}$$

On applying KVL in base loop, we get



$$-V_{th} + I_B R_B + V_{BE} + I_E R_E = 0$$

$$-V_{th} + I_B R_B + V_{BE} + (I_C + I_B) R_E = 0$$

$$I_B = \frac{V_{th} - V_{BE}}{R_B + R_E} - \frac{I_C R_E}{R_E + R_B}$$

$$\frac{\partial I_B}{\partial I_C} = \frac{0 - R_E}{R_E + R_B}$$

$$S = \frac{1+\beta}{1+\frac{\beta R_E}{R_E + R_B}}$$

- For the biasing of discrete BJT transistor circuit self biasing method is used whereas in case of ICs fabrication due to larger resistor size, a large chip area is needed. Therefore in ICs instead of self biasing method current mirrors are used as biasing circuit.

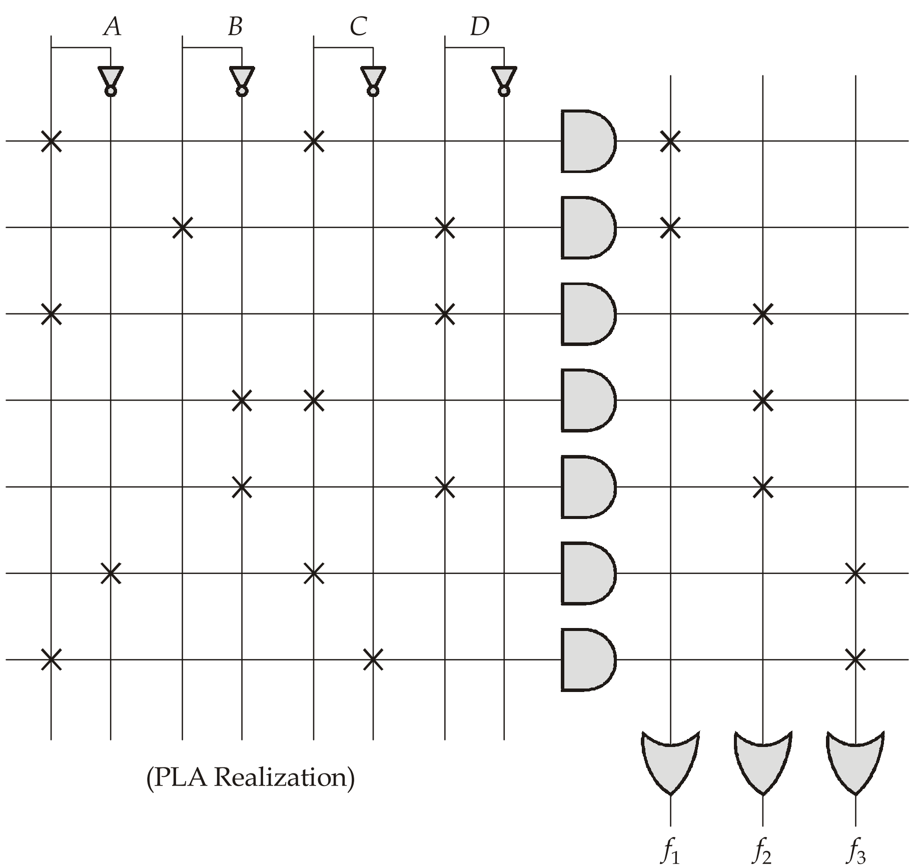
Q.7 (b) Solution:

(i) Programmable Logic Array (PLA) has a programmable AND array followed by a programmable OR array. The given boolean functions can be implemented using PLA as shown below.

$$f_1 = AC + BD$$

$$f_2 = AD + \bar{B}C + \bar{B}D$$

$$f_3 = A \oplus C = \bar{A}C + A\bar{C}$$



(PLA Realization)

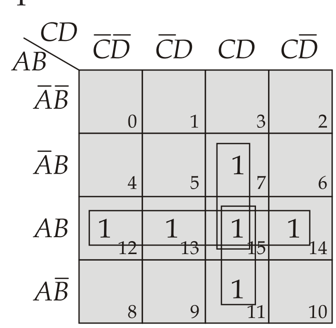
(ii) The truth table for the given logic can be obtained as below with Output = 1 when the resolution is passed:

A	*B*	*C*	*D*	**Output**
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

From above truth table,

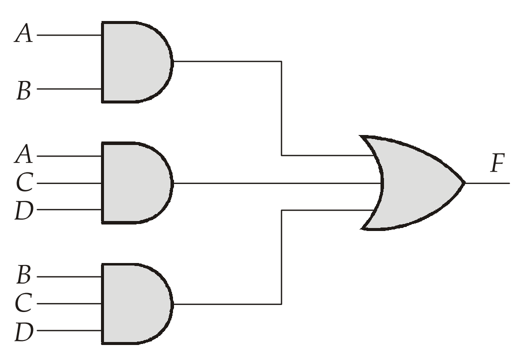
$$\text{Output} = \Sigma m(7, 11, 12, 13, 14, 15)$$

On simplifying using K-map we have



$$\text{Output} = AB + ACD + BCD$$

Logic diagram

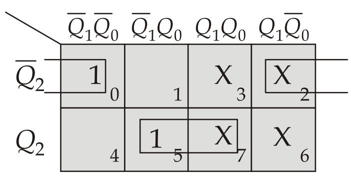


(iii) Excitable table of J-K flip-flops:

Q_2	Q_2	Q_2	Q_2^+	Q_1^+	Q_0^+	J_2	K_2	J_1	K_1	J_0	K_0
0	0	0	0	1	0	0	X	1	X	0	X
0	0	1	0	0	0	0	X	0	X	X	1
0	1	0	1	0	1	1	X	x	1	1	X
0	1	1	0	0	0	0	X	x	1	X	1
1	0	0	0	0	0	X	1	0	X	0	X
1	0	1	1	1	0	X	0	1	X	X	1
1	1	0	0	0	0	X	1	X	1	0	X
1	1	1	0	0	0	X	1	X	1	X	1

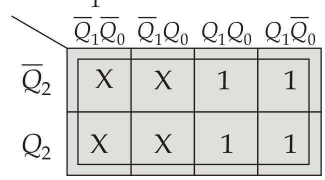
From the above table:

For J_1:



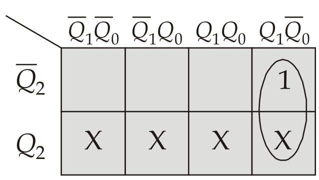
$J_1 = Q_2 \odot Q_0$

For K_1:

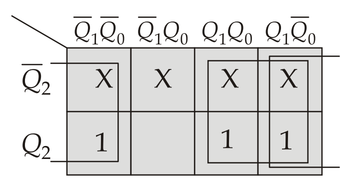


$K_1 = 1$

For J_2:



$J_2 = Q_1\overline{Q_0}$



$K_2 = Q_1 + \overline{Q_0}$

Q.7 (c) Solution:

(i) 1. Given,

$$N_d = 10^{14}\ \text{cm}^{-3}$$

$$N_a = 0$$

$$N_C = 2\times10^{19}\left(\frac{T}{300}\right)^{\frac{3}{2}}\ \text{cm}^{-3}$$

$$N_V = 1\times10^{19}\left(\frac{T}{300}\right)^{\frac{3}{2}}\ \text{cm}^{-3}$$

At $T = 300$ K,

$$N_C = 2\times10^{19}\ \text{cm}^{-3}$$

$$N_V = 1\times10^{19}\ \text{cm}^{-3}$$

$$E_g = 1.1\ \text{eV}$$

We know that,

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{KT}\right)$$

$$= 2\times10^{19}\times1\times10^{19}\exp\left(\frac{-1.1}{0.0259}\right)$$

∴ $n_i = 8.47 \times 10^9 \text{ cm}^{-3}$

Since, $N_d = 10^{14} \text{ cm}^{-3} >> n_i$

∴ $n \simeq N_d$

The current density, $J = \sigma E$

$$J = nq\mu_n E$$

$$= N_d q\mu_n E$$

$$= (1.6 \times 10^{-19})(10^{14}) \times 1000 \times 100$$

$$J = 1.6 \text{ A/cm}^2$$

2. It is given that mobility is independent of temperature, so a 5% increase is due to a 5% increase in electron concentration. Therefore, $n = 1.05 \times 10^{14} \text{ cm}^{-3}$. We have,

$$n = 1.05 \times 10^{14} = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$$1.05 \times 10^{14} = \frac{10^{14}}{2} + \sqrt{\left(\frac{10^{14}}{2}\right)^2 + n_i^2}$$

$$(1.05 \times 10^{14} - 5 \times 10^{13})^2 = (5\times10^{13})^2 + n_i^2$$

∴ $n_i^2 = 5.25 \times 10^{26} \text{ cm}^{-3}$

We have, $$n_i^2 = N_C N_V \left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{KT}\right)$$

$$5.25 \times 10^{26} = 2\times10^{19}\times1\times10^{19}\times\left(\frac{T}{300}\right)^3\exp\left(\frac{-1.1}{KT}\right)$$

$$2.625 \times 10^{-12} = \left(\frac{T}{300}\right)^3\exp\left(\frac{-1.1}{KT}\right)$$

∴ $T \approx 456$ K satisfy the above condition.

(ii) We know that,

Fermi-Dirac distribution

$$f(E) = \left[1+\exp\left(\frac{E-E_F}{KT}\right)\right]^{-1}$$

$$\frac{df(E)}{dE} = (-1)\left[1+\exp\left(\frac{E-E_F}{KT}\right)\right]^{-2} \times \left(\frac{1}{KT}\right)\exp\left(\frac{E-E_F}{KT}\right)$$

$$\frac{df(E)}{dE} = \frac{\frac{-1}{KT}\exp\left(\frac{E-E_F}{KT}\right)}{\left[1+\exp\left(\frac{E-E_F}{KT}\right)\right]^2}$$

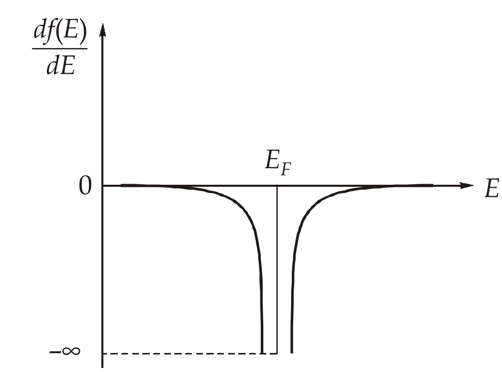
For $T \to 0$ K

$$E < E_F \Rightarrow \frac{df(E)}{dE} = \frac{\frac{-1}{K(\in)}\exp(-\infty)}{[1+\exp(-\infty)]^2} = 0 \qquad (\in \to 0)$$

$$E > E_F \Rightarrow \frac{df(E)}{dE} = \frac{\frac{-1}{K(\in)}\exp(\infty)}{[1+\exp(\infty)]^2} = 0$$

At $$E = E_F \Rightarrow \frac{df(E_F)}{dE} = \frac{\frac{-1}{K(\in)}\exp(0)}{[1+\exp(0)]^2} = -\infty$$

At T = 0K, it becomes a dirac delta function.



Q.8 (a) Solution:

(i) Digital multimeter:

1. A digital multimeter uses a numeric display to indicate the value of the measurement.

2. Superior resolution and accuracy. Least expensive digital meter can have accuracy of better than ±0.6% of the measurement quantity.
3. It gives a clear reading, thereby removing any observational errors.
4. Indicates a negative quantity when the terminal polarity is reversed.
5. Rugged and non usually not damaged by rough treatment.

Analog multimeter:

1. An analog multimeter uses a pointer that moves along a scale to indicate the value of the measurement.
2. Inferior resolution and accuracy. A good quality analog instrument has typically an accuracy of ±2% of full scale.
3. Wrong scale might be used or might be read incorrectly.
4. Pointer attempts to deflect to the left of zero when the polarity is reversed.
5. Can be irreparably damaged when dropped from bench level.

(ii) Analog instrument:

$$\text{Voltage error} = \pm 2.5\% \text{ of full scale i.e., } 30\text{ V}$$
$$= \pm(0.025)(30)$$
$$= \pm 0.75\text{ V}$$

When reading a 20V dc voltage,

$$\text{error} = \pm\frac{0.75}{20}\times 100\% = \pm 3.75\%$$

Digital Multimeter:

Accuracy of digital meter is given as ±(0.8% reading + 1 digit).
where 1 digit refers to extreme right (or least significant) digit of the display.

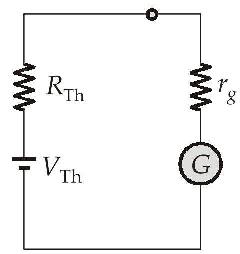
For a $3\frac{1}{2}$ digit display with 20V range, it displays from 0 to 19.99 V. Hence,

$$\text{Resolution} = 1\text{ digit} = \frac{1}{10^3}\times 10 = 0.01\text{ V}$$
$$\text{Voltage error} = \pm(0.8\% \text{ of } 20\text{ V} + 0.01\text{ V})$$
$$= \pm(0.16 + 0.01)\text{ V}$$
$$= \pm 0.17\text{ V}$$

$$\text{error} = \pm\frac{0.17}{20}\times 100\%$$

$$\text{error} = \pm 0.85\%$$

(iii) The Wheatstone bridge can be replaced with Thevenin equivalent circuit given below:



$$R_{Th} = (P)||(R) + (Q)||(S) \quad ...(i)$$

We have,
$$I_g = \frac{V_R - V_S}{R_{Th} + r_g} \quad ...(ii)$$

with
$$V_R - V_S = E_B\left(\frac{R}{R+P} - \frac{Q}{Q+S}\right) \quad ...(iii)$$

Given,
$$r_g = 2 \text{ k}\Omega$$
$$P = 4 \text{ k}\Omega$$
$$Q = 10 \text{ k}\Omega$$
$$S = 5 \text{ k}\Omega$$
$$R = 8 \text{ k}\Omega$$
$$R_{Th} = P \,||\, R + Q \,||\, S$$
$$= 4 \,||\, 8 + 10 \,||\, 5$$
$$= 6 \text{ k}\Omega$$

From equation (ii)
$$V_R - V_S = I_g(R_{Th} + r_g)$$

To produce a minimum deflection of 1 mm in Galvanometer, $I_g = 2\ \mu\text{A}$
$$V_R - V_S = (2 \times 10^{-6})\,(6 \times 10^3 + 2 \times 10^3)$$
$$= 16 \text{ mV}$$

If the resistor R changes by ΔR, we have
$$E_B\left[\frac{R+\Delta R}{R+\Delta R+P} - \frac{Q}{Q+S}\right] = 16 \times 10^{-3}$$

$$\Rightarrow \quad E_B\left[\frac{\Delta R \cdot S}{(R+\Delta R+P)\,(Q+S)}\right] = 16 \times 10^{-3}$$

Substituting the given values,

$$12\left[\frac{5\Delta R}{(12+\Delta R)(15)}\right] = 16 \times 10^{-3}$$

$$\Rightarrow \quad 60\Delta R = 2.88 + 0.24\Delta R$$

$$59.76\Delta R = 2.88$$

$$\Rightarrow \quad \Delta R = 0.0482 \text{ k}\Omega = 48.2\ \Omega$$

Q.8 (b) Solution:

(i) Odd parity generator gives the output 1 when the number of 1's in the input data bits is even, so that total number of 1's in the input data bits and parity bit together is odd.

Truth table:

A	*B*	*C*	*D*	*F*
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

$$f(A, B, C, D) = \Sigma m(0, 3, 5, 6, 9, 10, 12, 15)$$

K-Map:

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 $_{0}$	$_{1}$	1 $_{3}$	$_{2}$
$\bar{A}B$	$_{4}$	1 $_{5}$	$_{7}$	1 $_{6}$
AB	1 $_{12}$	$_{13}$	1 $_{15}$	$_{14}$
$A\bar{B}$	$_{8}$	1 $_{9}$	$_{11}$	1 $_{10}$

The output is $$F = \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + AB\bar{C}\bar{D} + ABCD + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + \bar{A}BC\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D}$$

$$F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + AB\bar{C}\bar{D} + ABCD + \bar{A}B\bar{C}D + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + \bar{A}BC\bar{D}$$

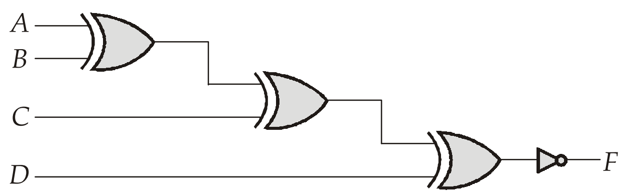
$$F = \bar{A}\bar{B}(\overline{C \oplus D}) + AB(\overline{C \oplus D}) + \bar{A}B(C \oplus D) + A\bar{B}(C \oplus D)$$

$$F = (\overline{C \oplus D})(\overline{A \oplus B}) + (C \oplus D)(A \oplus B)$$

$$F = \overline{(A \oplus B) \oplus (C \oplus D)}$$

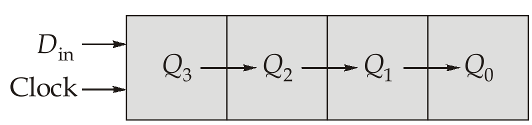
Logic diagram:

$$F = \overline{(A \oplus B) \oplus (C \oplus D)}$$



(ii) Let us assume the output of flip-flops as Q_0, Q_1, Q_2, Q_3.

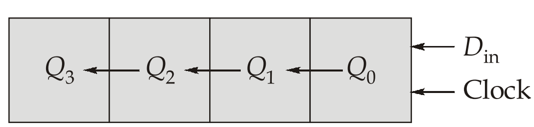
When $M = 1$, it should work as a shift right register. The equivalent model of the register is



Required excitations are,

$$D_3 = D_{in}, D_2 = Q_3 = D_1 = Q_2, D_0 = Q_1$$

When $M = 0$, it should work as a shift left register. The equivalent model of the register is



Required excitations are:

$$D_3 = Q_2, D_2 = Q_1, D_1 = Q_0, D_0 = D_{in}$$

So, combinedly the excitations for a bidirectional register can be given as,

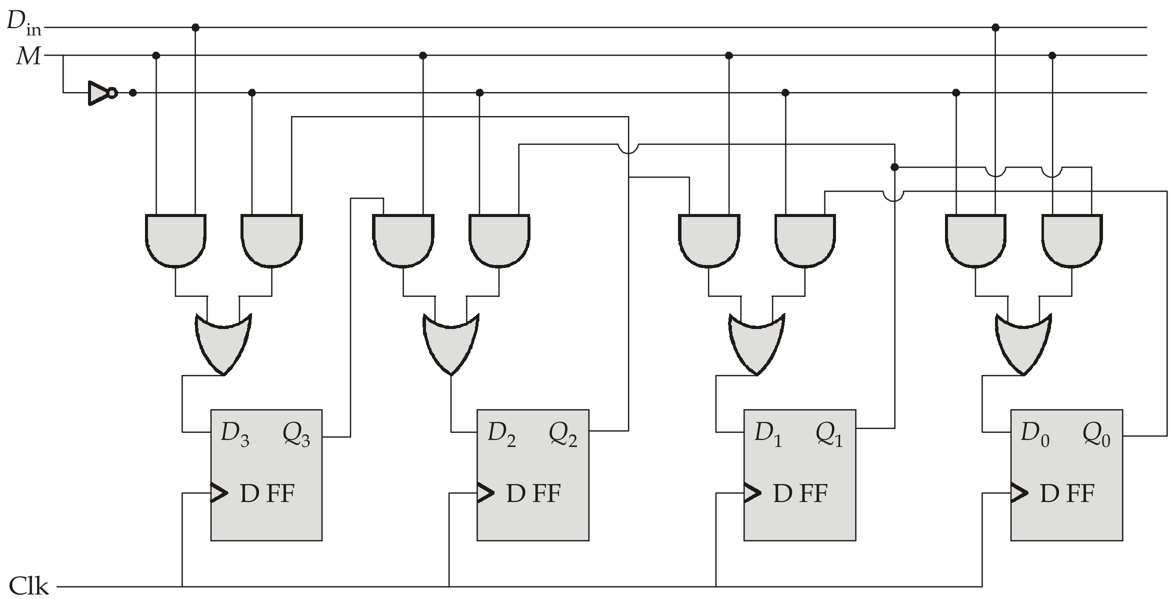
$$D_3 = \bar{M}Q_2 + MD_{in}$$

$$D_2 = \bar{M}Q_1 + MQ_3$$

$$D_1 = \bar{M}Q_0 + MQ_2$$

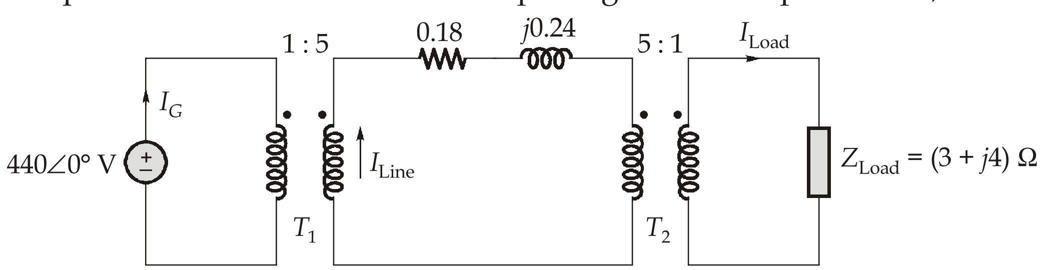
$$D_0 = \bar{M}D_{in} + MQ_1$$

Logic Circuit:

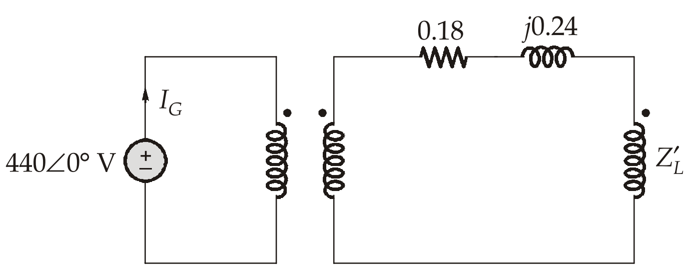


Q.8 (c) Solution:

(i) The equivalent circuit based on description given in the problem is,



Refer load impedance from secondary of T_2 to primary of T_2,

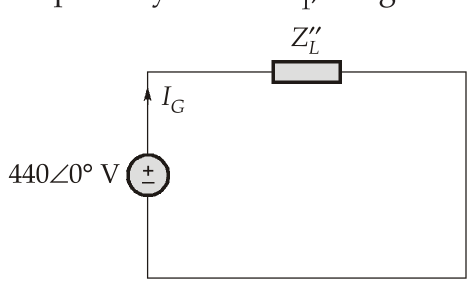


$$Z'_L = \left(\frac{5}{1}\right)^2 Z_L$$

$$Z'_L = 25(3 + j4)$$

$$Z'_L = 75 + j100\ \Omega$$

Refer the entire circuit to primary side of T_1, we get



$$Z_L'' = \left(\frac{1}{5}\right)^2 [0.18 + j0.24 + 75 + j100]$$

$$Z_L'' = (3.0072 + j4.0096)\ \Omega$$

Thus,

$$I_G = \frac{440}{3.0072 + j4.0096}$$

$$I_G = 52.67 - 70.23j = 87.79\angle -53.13°\ \text{V}$$

We know that,

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

$$\frac{1}{5} = \frac{I_{\text{Line}}}{I_G}$$

$$I_{\text{Line}} = \frac{I_G}{5}$$

$$I_{\text{Line}} = 17.558\angle(-53.13°)\ \text{A}$$

Similarly,

$$\frac{5}{1} = \frac{I_{\text{Load}}}{I_{\text{Line}}}$$

$$I_{\text{Load}} = 5 \times I_{\text{Line}}$$

$$I_{\text{Load}} = 87.79\angle -53.13°\ \text{A}$$

$$V_{\text{Load}} = I_{\text{Load}} \times Z_{\text{Load}}$$

$$V_{\text{Load}} = 88.79\angle -53.13° \times (3 + j4)$$

$$V_{\text{Load}} = 438.95\angle 0.000102°\ \text{V}$$

(ii) Given that:

Condition (1) : KVA = 1000 KVA; cos ϕ = 0.8 lagging; η = 0.985; x = half load = $\frac{1}{2}$

Condition (2) : cos ϕ = 1 (upf); η = 0.988; x = full load = 1

As we know, from condition (1)

$$\eta = \frac{x \times KVA \times \cos\phi}{x \times KVA \times \cos\phi + P_i + x^2 P_{cufL}}$$

$$0.985 = \frac{\frac{1}{2} \times 1000 \times 0.8}{\frac{1}{2} \times 1000 \times 0.8 + P_i + \frac{1}{4} P_{cufL}}$$

$$P_i + 0.25\ P_{cufL} = \frac{400}{0.985} - 400 = 6.09\ \text{KW} \qquad \text{...(1)}$$

From condition (2), ($x = 1$)

$$0.988 = \frac{1 \times 1000 \times 1}{1 \times 1000 \times 1 + P_i + P_{cufL}}$$

$$P_i + P_{cufL} = \frac{1000}{0.988} - 1000 = 12.145 \text{ kW} \qquad \text{...(2)}$$

On solving equation eq. (1) and (2)

$$\text{Iron loss, } P_i = 4.07 \text{ kW}$$

$$\text{full load Cu loss, } P_{cufL} = 8.07\text{kW}$$

At maximum efficiency of transformer

$$x^2 P_{cufL} = P_i$$

$$\text{fraction of load } (x) = \sqrt{\frac{P_i}{P_{cufL}}} = \sqrt{\frac{4.07}{8.07}} = 0.71$$

Maximum efficiency at u.p.f.,

$$\eta_{max} = \frac{0.71 \times 1000 \times 1}{0.71 \times 1000 \times 1 + 2 \times 4.07} = 0.9886$$

$$\%\eta_{max} = 98.86\%$$

❍❍❍❍