MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

Civil Engineering Test No : 9

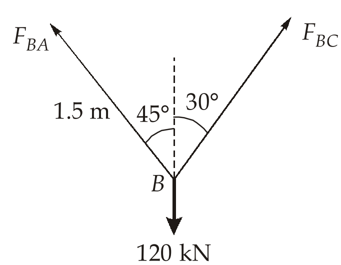
Section - A

Q.1 (a) Solution:

Given: $L_{AB} = 1.5\text{ m}, L_{BC} = 2\text{ m}$

$d_{AB} = 20\text{ mm}, d_{BC} = 35\text{ mm}$

FBD at joint *B*,



From sine rule

$$\frac{F_{BA}}{\sin(180° - 30°)} = \frac{F_{BC}}{\sin(180° - 45°)} = \frac{120}{\sin(75°)}$$

$\therefore$
$$F_{BA} = \frac{120\sin(150°)}{\sin(75°)} = 62.12\text{ kN}$$

$$F_{BC} = \frac{120\sin(135°)}{\sin 75°} = 87.85\text{ kN}$$

Stress in wire *AB* :

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{62.12\times10^3}{\frac{\pi}{4}(20)^2}$$

$\Rightarrow$ $$\sigma_{AB} = 197.734 \text{ MPa}$$

From graph, strain in wire *AB* is,

$$\in_{AB} = 0.01+\frac{(0.03-0.01)}{(220-110)}(197.734-110)$$

$\Rightarrow$ $$\frac{\Delta_{AB}}{L_{AB}} = 2.595 \times 10^{-2}$$

$\Rightarrow$ $$\Delta_{AB} = 1.5 \times 2.595 \times 10^{-2} \text{ m} = 38.925 \text{ mm}$$ **Ans.**

Stress in wire *BC* :

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{87.85\times10^3}{\frac{\pi}{4}(35)^2}$$

$\Rightarrow$ $$\sigma_{BC} = 91.309 \text{ MPa}$$

From graph, strain in wire *BC* is,

$$\in_{BC} = \frac{0.01}{110}\times 91.309$$

$\Rightarrow$ $$\frac{\Delta_{BC}}{L_{BC}} = 8.3 \times 10^{-3}$$

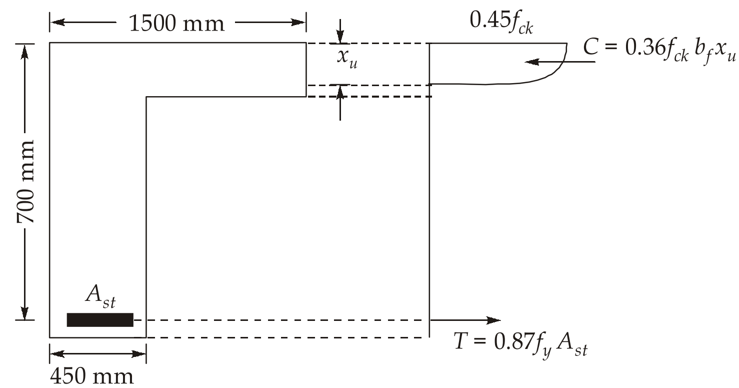
$\Rightarrow$ $$\Delta_{BC} = 8.3 \times 10^{-3} \times 2000 = 16.60 \text{ mm}$$ **Ans.**

Q.1 (b) Solution:

Given: b_f = 1500 mm, d_f = 200 mm, d = 700 mm, b_w = 450 mm

f_{ck} = 30 N/mm², f_y = 450 N/mm, M_u = 1000 kN-m

Assume: Neutral axis lies within the flange ($x_u < D_f$)



Now, $M_u = C(LA)$

$\Rightarrow$ $1000 \times 10^6 = 0.36 f_{ck}\, b_f x_u (d - 0.42\, x_u)$

$\Rightarrow$ $1000 \times 10^6 = 0.36 \times 30 \times 1500\, x_u (700 - 0.42 x_u)$

$\Rightarrow$ $61728.395 = 700 x_u - 0.42 x_u^2$

$\Rightarrow$ $0.42 x_u^2 - 700 x_u + 61728.395 = 0$

$\therefore$
$$x_u = \frac{700 \pm \sqrt{700^2 - 4 \times 0.42 \times 61728.395}}{2 \times 0.42}$$

$$x_u = 93.419 \text{ mm} < (D_f = 200 \text{ mm})$$

$\therefore$ Assumption is correct.

Now, $C = T$

$\Rightarrow$ $0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$

$\Rightarrow$
$$A_{st} = \frac{0.36 \times 30 \times 1500 \times 93.419}{0.87 \times 415}$$

$\Rightarrow$ $A_{st} = 4191.629 \text{ mm}^2$

$\therefore$ $M_u = 0.87\, f_y A_{st} \quad (d - 0.42\, x_u)$

$\Rightarrow$
$$A_{st} = \frac{1000 \times 10^6}{0.87 \times 415(700 - 0.42 \times 93.419)}$$

$$\simeq 4191.66 \text{ mm}^2$$

Q.1 (c) Solution:

Ceramic materials are polycrystalline, inorganic, non-metallic solids formed by baking natural clay and mineral admixtures at high temperatures or by sintering oxides of metals and other inorganic compounds. The word 'Ceramic' is derived from the Greek word 'Keramos', meaning potter's earth or clay.

While traditionally associated with clay based products, the modern definition of ceramics include a wide range of materials such as carbon (like graphite), boron (like boron carbide), silicon (like silicon carbide), refractory hydrides, some sulphides, and their combinations. Clay is the most common ceramic-building material.

Some properties of ceramic materials are:

1. They are usually hard and brittle.
2. They are in the form of amorphous (non-crystalline) or glassy solids.
3. They possess mixed ionic and covalent bonds.
4. Because of covalent ionic bonding, the electrons are not free which makes these materials as electrical insulators.
5. At low temperatures, ceramics behave elastically.
6. Under proper conditions of stress and temperature, they deform by viscous flow.
7. Theoretically, the tensile strength of ceramic is very high but in practice it is quite low.
8. Compressive strength of these materials is very high.
9. They have excellent corrosion and chemical resistance and are resistant to most acids, alkalis and corrosive environments.

Q.1 (d) Solution:

Eccentricity of prestressing force,

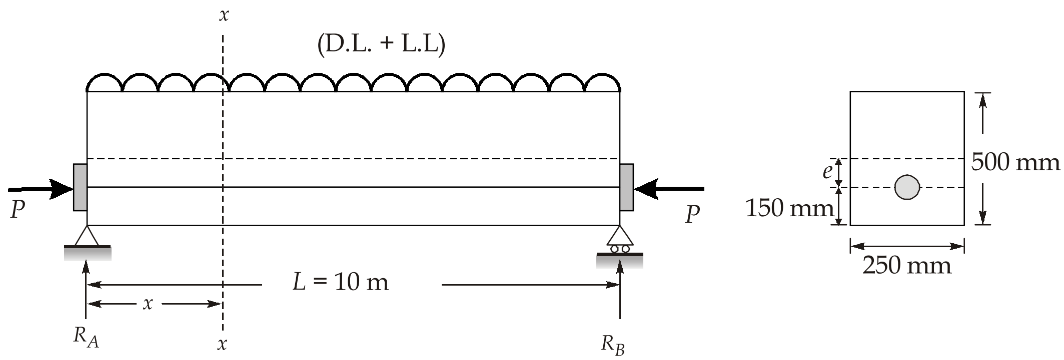
$$e = 250 - 150 = 100 \text{ mm}$$

$$\text{D.L} = 0.25 \times 0.5 \times 1 \times 24 = 3 \text{ kN/m}$$

$$\text{LL} = 10 \text{ kN/m}$$

Total UDL $w = 13 \text{ kN/m}$

Prestressing force, $P = A_{st} \times \sigma_{st} = 1800 \times 750 \text{ N} = 1350 \text{ kN}$



Reaction at each support $$R_A = R_B = \frac{wL}{2} = \frac{13\times 10}{2}$$

$\Rightarrow$ $$R_A = R_B = 65 \text{ kN}$$

Bending moment at any section distant x meters from the support is,

$$M_{xx} = R_A x - \frac{wx^2}{2} = 65x - \frac{13x^2}{2} \qquad \text{(from cable line)}$$

Shift of the pressure line at any section distant x meters from the support is,

$$a_{xx} = \frac{M_{xx}}{P}$$

$\Rightarrow$ $$a_{xx} = \frac{1}{1350}\left(65x - \frac{13x^2}{2}\right)$$

Now shift of the pressure line from centroidal axis is

$$y_{xx} = a_{xx} - e$$

$\Rightarrow$ $$y_{xx} = \frac{1}{1350}\left(65x - \frac{13x^2}{2}\right) - 0.1$$

At $x = 0$ m, $y_{0\text{ m}} = -0.1 \text{ m} = -100 \text{ mm}$

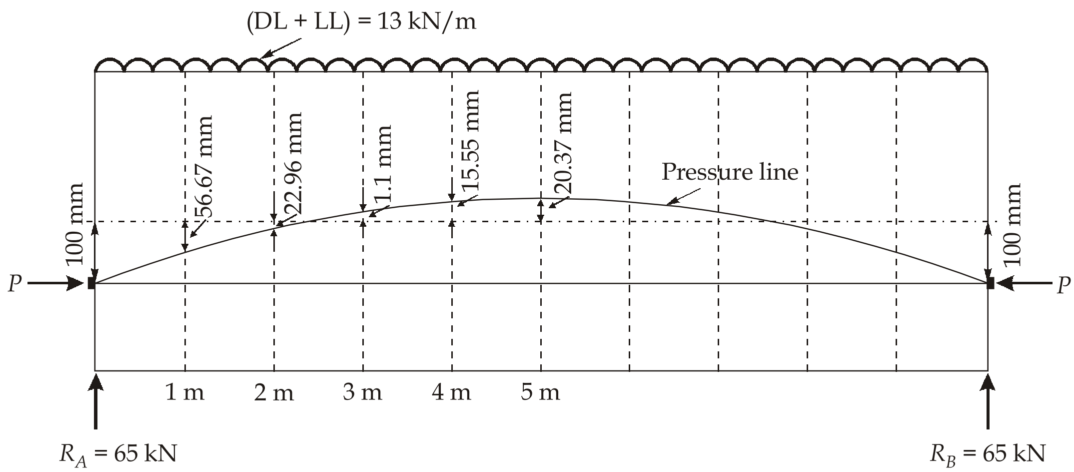
At $x = 1$ m, $y_{1\text{ m}} = -56.67 \times 10^{-3} \text{ m} = -56.67 \text{ mm}$

At $x = 2$ m, $y_{2\text{ m}} = -22.96 \times 10^{-3} \text{ m} = -22.96 \text{ mm}$

At $x = 3$ m, $y_{3\text{ m}} = +1.11 \times 10^{-3} \text{ m} = +1.11 \text{ mm}$

At $x = 4$ m, $y_{4\text{ m}} = 0.01555 \text{ m} = 15.55 \text{ mm}$

At $x = 5$ m, $y_{5\text{ m}} = 0.02037 \text{ m} = 20.37 \text{ mm}$



Q.1 (e) Solution:

l_{eff}, (effective length of column section) = 4.25 m

Moment of Inertia of built up column section in x direction,

$$(I_{xx})_{\text{combination}} = 2I_{xx} + I_{yy}$$
$$= [2 \times (35057.6) + 1706.7] \times 10^4$$
$$= 71{,}821.9 \times 10^4 \text{ mm}^4$$

Moment of inertia of built up column section in y direction,

$$(I_{yy})_{\text{combination}} = I_{xx} + 2[I_{yy} + A \times (229.6)^2]$$
$$\Rightarrow \quad (I_{yy})_{\text{combination}} = 35057.6 \times 10^4 + 2[1706.7 \times 10^4 + 101.15 \times 10^2 \times (229.6)^2]$$
$$= 1.451 \times 10^9 \text{ mm}^4$$
$$\therefore \quad (I_{xx})_{\text{combination}} < (I_{yy})_{\text{combination}}$$

$$r_{\min} \text{ (minimum radius of gyration)} = \sqrt{\frac{(I_{xx})_{\text{comb}}}{A_{\text{comb}}}} = \sqrt{\frac{0.7182 \times 10^9}{3 \times 101.15 \times 10^2}} = 153.85 \text{ mm}$$

$$\lambda \text{ (slenderness ratio)} = \frac{l_{\text{eff}}}{r_{\min}} = \frac{4.25 \times 10^3}{153.85} = 27.624$$

From given table by interpolation

$$f_1 = 223 + \frac{(204 - 223)}{(30 - 20)} \times (27.624 - 20) = 208.51 \text{ MPa}$$

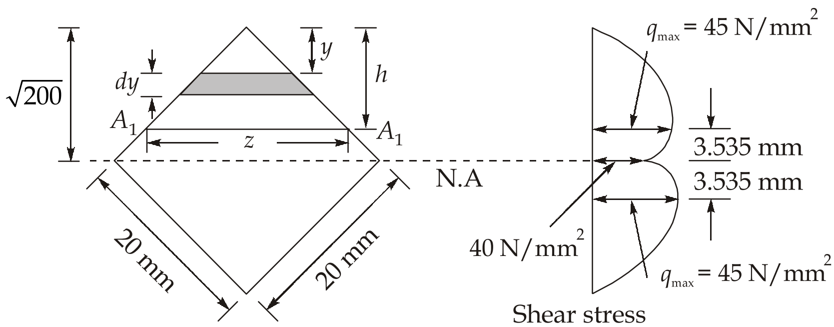
$$\therefore \quad P_{\text{Load}} \text{ (Load carrying capacity)} = f_1 \times A_{\text{comb}}$$
$$= 208.51 \times (3 \times 101.15 \times 10^2) \text{ N} = 6327.24 \text{ kN}$$

$$P_{\text{safe}} \text{ (safe load carrying capacity)} = \frac{P_{\text{Load}}}{1.5} = \frac{6327.24}{1.5} = 4218.16 \text{ kN}$$

Q.2 (a) Solution:

(i)

Consider a strip of thickness 'dy' at distance 'y' from top. Width of strip = $2y$. Area of strip = $2y \times dy$. Moment of this area about N.A. = $2y \times dy \times \left(\sqrt{2} - y\right)$.



Shear stress distribution

$$A\bar{y} = \int_0^h 2y \times dy \times \left(\sqrt{2} - y\right) = \int_0^h \left(\sqrt{2} - y^3\right) dy$$

$$\Rightarrow \quad A\bar{y} = 2\left(\sqrt{2}\frac{y^2}{2} - \frac{y^3}{3}\right)_0^h = 2\left(\frac{\sqrt{2}}{2}h^2 - \frac{h^3}{3}\right)$$

$$\Rightarrow \quad A\bar{y} = \frac{h^2}{3}(3\sqrt{2} - 2h)$$

Moment of inertia about diagonal

$$I = \frac{D^4}{12} = \frac{2^4}{12} = \frac{4}{3}\text{cm}^4$$

Now,
$$\tau = \frac{FA\bar{y}}{Iz}$$

z at $A_1A_1 = 2h$

$$\therefore \quad \tau_h = \frac{F \times \frac{h^2}{3}(3\sqrt{2} - 2h)}{\frac{4}{3} \times 2h}$$

$$\Rightarrow \quad \tau_h = \frac{F}{8}h(3\sqrt{2} - 2h) \qquad ...(1)$$

This represents a parabolic equation. Therefore, shear stress varies in a parabolic form. For maximum shear stress,

$$\frac{d\tau_h}{dh} = 0$$

$$\Rightarrow \quad \frac{F}{8}(3\sqrt{2} - 4h) = 0$$

$$\Rightarrow \quad h = \frac{3\sqrt{2}}{4}$$

$$\text{Distance from N.A.} = \sqrt{2} - \frac{3}{4}\sqrt{2} = \frac{1}{4}\sqrt{2} = 0.3535 \text{ cm}$$

$$\tau_{max} = \frac{16000}{8} \times \frac{3}{4}\sqrt{2}\left(3\sqrt{2} - 2 \times \frac{3}{4}\sqrt{2}\right) \text{ N/cm}^2$$

$$\Rightarrow \quad \tau_{max} = 15\sqrt{2} \times \frac{3}{2}\sqrt{2} = 45 \text{ N/mm}^2$$

In equation, (1) putting $h = \sqrt{2}$,

Shear stress at N.A. $\tau_{N.A.} = \dfrac{16,000 \times \sqrt{2}}{8}(3\sqrt{2} - 2\sqrt{2}) \text{ N/cm}^2 = 40 \text{ N/mm}^2$

(ii)

Given:

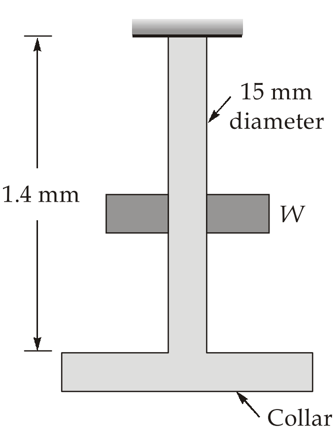
Diameter, d = 15 mm

Length of bar, L = 1.4 m = 1400 mm

Height of fall, h = 95 mm

Maximum stress, σ = 150 MPa

Young's modulus, E = 200 GPa = 200×10^3 MPa



$$\text{Area, } A = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ mm}^2$$

$$\text{Instantaneous stress, } \sigma = \frac{W}{A} + \sqrt{\left(\frac{W}{A}\right)^2 + 2\frac{W}{A}\frac{Eh}{L}}$$

Let $\dfrac{W}{A} = x$

$$\therefore \quad 150 = x + \sqrt{x^2 + 2x\frac{200 \times 10^3 \times 95}{1400}}$$

$$\Rightarrow \quad 150 - x = \sqrt{x^2 + 27142.86x}$$

Squaring both sides, $(150 - x)^2 = x^2 + 27142.86x$

$$\Rightarrow \quad 150^2 - 300x + x^2 = x^2 + 27142.86x$$

$$\Rightarrow \quad (27142.86 + 300)x = 150^2$$

$$\Rightarrow \quad x = 0.82$$

But $$\frac{W}{A} = x = 0.82$$

$$\therefore \quad W = 0.82 \times 176.71 = 144.9 \text{ N}$$

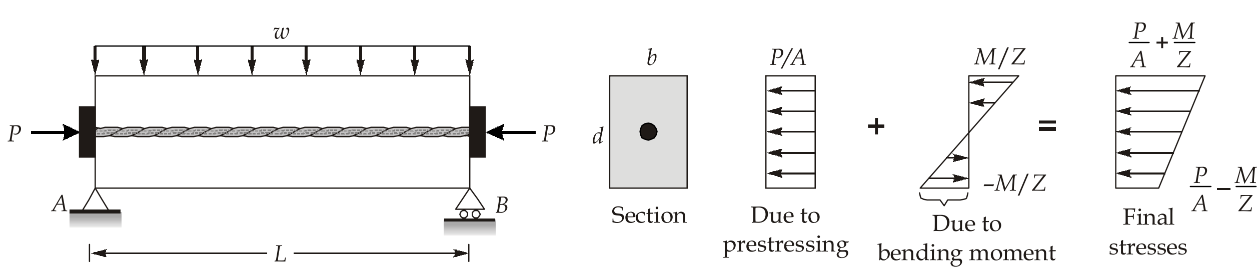
$\therefore$ The maximum weight that can be applied on collar is 144.9 N.

Q.2 (b) Solution:

(i)

The main principle of pre-stressing a concrete member consists of inducing sufficient compressive stress in concrete before a member is subjected to loads, in the zones which develop tensile stresses due to the applied load. The pre-induced compressive stress in concrete neutralizes the tensile stress developed due to the external loads. Hence, the zone ultimately will be free from any stress. In a pre-stressed member, the entire cross section becomes effective in resisting bending and the danger of cracking is minimized or even avoided.

General principle of pre-stressing is explained as under. A rectangular beam of area of cross section A is shown in figure below.



Compressive stress induced in concrete, $f_o = \frac{P}{A}$

Let M be the bending moment at the section due to dead and external loads.

The extreme stress at the section due to bending moment.

$$f_b = \pm\frac{M}{Z}$$

where Z is the section modulus of the beam section

$$\text{Resultant extreme stress} = f_c + f_b = \frac{P}{A} \pm \frac{M}{Z}$$

Final stress at the extreme top fibre = $\frac{P}{A} + \frac{M}{Z}$

Final stress at the extreme bottom fibre = $\frac{P}{A} - \frac{M}{Z}$

(ii)

Given: Diameter, D = 400 mm

f_{ck} = 20 MPa, f_g = 415 MPa

Circular column with helical reinforcement

P = 1200 kN

Factored load, P_u = 1.5 × 1200 kN = 1800 kN

For given condition, effective length, l_{eff} = 1.0 × l = 1.0 × 3 = 3000 mm

$$\frac{l_{eff}}{D} = \frac{3000}{400} = 7.5 < 12$$

∴ This is a short column

$$\text{Gross area, } A_g = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 400^2 = 125{,}663.7 \text{ mm}^2$$

$$\frac{l}{500} + \frac{D}{30} = \frac{3000}{500} + \frac{400}{30} = 19.33 \text{ mm}$$

$$e_{min} = \text{Max}\begin{Bmatrix} 19.33 \text{ mm} \\ 20 \text{ mm} \end{Bmatrix} = 20 \text{ mm}$$

$$\leq 0.05 \times 400 = 20 \text{ mm} \quad \text{(OK)}$$

Column is axially loaded.

Ultimate load carrying capacity of the column from limit state method

$$P_u = 1.05 \times (0.40 f_{ck} A_c + 0.67 f_y A_{sc})$$

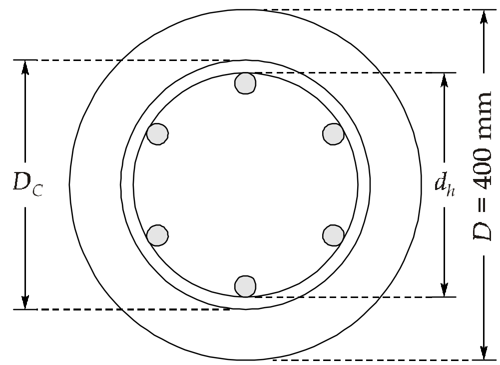
⇒ $$1800 \times 10^3 = 1.05 \{0.40 \times 20 \times (125663.7 - A_{sc}) + 0.67 \times 415 A_{sc}\}$$

⇒ $$A_{sc} = 2625.35 \text{ mm}^2$$

$$\text{Minimum reinforcement} = \frac{0.8}{100} \times \frac{\pi}{4} \times (400)^2 = 1005.31 \text{ mm}^2$$

$$A_{sc} > A_{sc,\, min.} \quad \text{(OK)}$$

Provide 6-25 ϕ bars (A = 2945 mm^2)



(Assume nominal cover = 40 mm)

Helical ties design : ϕ_{ties} = Maximum $\left\{\begin{array}{l}\frac{\phi_{min}}{4} = \frac{25}{4} = 6.25 \text{ mm} \\ 6 \text{ mm}\end{array}\right\}$ = 6.25 mm

Take ϕ_{ties} = 8 mm

$D_c = D - 2 \times 40 = 400 - 80 = 320$ mm

Volume of helical reinforcement in unit length of column

$$V_h = \left(\frac{1000}{P}\right)(\pi d_h)\left(\frac{\pi}{4}\phi_h^2\right)$$

$$= \left(\frac{1000}{P}\right)\pi \times (320-8) \times \frac{\pi}{4} \times 8^2 = \left(\frac{49269065}{P}\right)\text{mm}^3$$

Core volume, $V_C = A_c \times 1000 = \frac{\pi}{4} \times 320^2 \times 1000 = 80424.7 \times 10^3 \text{ mm}^3$

$$\therefore \quad \frac{V_h}{V_c} \geq 0.36\left(\frac{A_g}{A_c} - 1\right)\frac{f_{ck}}{f_y}$$

$$\Rightarrow \quad \frac{49269065}{80424.7 \times P \times 10^3} \geq 0.36\left(\frac{\frac{\pi}{4} \times 400^2}{\frac{\pi}{4} \times 320^2} - 1\right) \times \frac{20}{415}$$

$$\Rightarrow \quad P \leq \frac{612.61 \times 10^{-3}}{9.76 \times 10^{-3}} = 62.77 \text{ mm}$$

$$\frac{D_c}{6} = \frac{320}{6} = 53.33 \text{ mm}$$

$\therefore$ Maximum pitch = Lesser of $\frac{D_c}{6}$ and 75 mm = 53.33 mm

Minimum pitch = Greater of 25 mm and 3 × ϕ_{ties} = (3 × 8 = 24 mm) = 25 mm

$P_{calcualted}$ = 62.77 mm

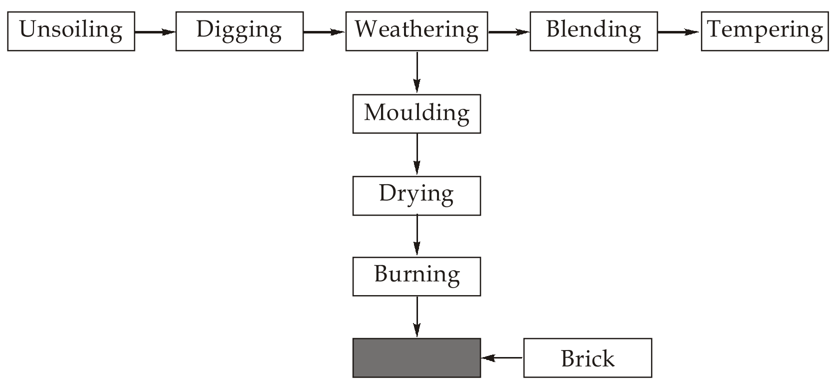
∴ Provide pitch, P = 50 mm

Ties 8 ϕ @ 50mm c/c pitch.

Q.2 (c) Solution:

(i)

Preparation of Brick Earth



Operations involved in manufacturing of clay bricks

The preparation of brick earth involves the following steps:

1. **Unsoiling:** The top 20 cm layer of soil, which contains stones, pebbles, gravel, roots, and other organic matter, is removed. This ensures the removal of impurities and unwanted materials before brick-making.
2. **Digging:** After unsoiling, suitable additives like fly ash, sandy loam, rise husk ash, stone dust, etc. are spread uniformly on the ground. The earth is then manually dug, puddled with water, and left to weather. Digging should preferably be done before the rainy season.
3. **Weathering:** The dug earth is heaped in layers of 60 – 120 cm and exposed to weather for at least a month. This process helps develop homogeneity and remove impurities like soluble salts that may cause scumming during burning. The soil is turned over twice and kept moist throughout to ensure plasticity and strength. Water is sprayed as needed.
4. **Blending:** The weathered soil is blended with sand and calcareous earth in proper proportions. A moderate amount of water is added to achieve moulding consistency. The soil is mixed uniformly using spades. Moisture is controlled to avoid moulding and drying issues. Excessive moisture may affect the shape and size of bricks.

5. **Tempering:** Tempering involves kneading the earth to make it plastic and homogeneous. It is stored in 30 cm thick layers for at least 36 hours to ensure uniformity. Tempering is usually done in a pug mill, a conical iron drum with rotating knives on a vertical shaft. The blended clay and water are fed from the top, and the shaft breaks down clods into fine particles. The well-mixed clay is discharged near the base and the pug mill yields about 1500 bricks per batch. This process is also called pugging.

(ii)

Factors that affect the selection of construction equipment:

- **Existing equipments :** The maximum utilisation of the existing equipment should be done in order to reduce the cost of production to the minimum. If certain type of equipment is already being used in the project, it is desirable to have additional equipment of same type because the existing workmen are already acquainted with the operation of such machines.
- **Availability of the equipment:** As far as practicable, the equipment which is easily available in the market should be selected for the purpose because any delay in delivery may increase the cost of construction/production substantially.
- **Standard equipment :** In general, the choice should be restricted to standard equipment because its delivery time is short, trained operators are available and spare parts can be easily available.
- **Special equipment:** If the project is very big, special equipment may be selected provided the economic analysis justifies the purchase.
- **Operating cost:** The most efficient and therefore the most economical equipment is the one whose operating cost is the minimum. The requisite economic analysis must be made.
- **Indigenous equipment :** It is always advisable to purchase equipment which is manufactured in our country because this will decrease the repair cost and downtime cost and at the same time it will be beneficial.
- **Obsolescence:** Obsolescence of the equipment should not be overlooked. Research and development going on in the design of equipment should be ascertained.
- **Economic life :** It must be analysed and should not be less than useful period of the project.
- **Cost benefit analysis :** For various alternatives, cost benefit analysis must be made and selection is based on economics only.

- **Suitability of equipment for future:** The equipment must be useful for future use also.
- **Study of site condition:** Equipment must be suited to site conditions.
- **Size of equipment:** Equipment must not be bulky else it will incur additional cost on transportation.

Q.3 (a) Solution:

Member *AB*
$$M_{AB} = 0+\frac{2EI}{5}\left(0+\theta_B-\frac{3\delta}{5}\right)=\frac{2}{5}EI\theta_B-\frac{6}{25}EI\delta$$

$$M_{BA} = 0+\frac{2EI}{5}\left(2\theta_B+0-\frac{3\delta}{5}\right)=\frac{4}{5}EI\theta_B-\frac{6}{25}EI\delta$$

Member *BC*
$$M_{BC} = 0+\frac{2EI}{3}\left(2\theta_B+\theta_C\right)=\frac{4}{3}EI\theta_B+\frac{2}{3}EI\theta_C$$

But
$$\theta_C = -\theta_B$$

∴
$$M_{BC} = \frac{4}{5}EI\theta_B-\frac{2}{3}EI\theta_B$$

⇒
$$M_{BC} = \frac{2}{3}EI\theta_B$$

$$M_{CB} = 0+\frac{2EI}{3}\left(2\theta_C+\theta_B\right)=\frac{2EI}{3}\left(-2\theta_B+\theta_B\right)$$

⇒
$$M_{CB} = -\frac{2}{3}EI\theta_B$$

Member *CD*
$$M_{CD} = 0+\frac{2EI}{5}\left(2\theta_C+0+\frac{3\delta}{5}\right)=\frac{2EI}{5}\left(-2\theta_B+\frac{3\delta}{5}\right)$$

⇒
$$M_{CD} = -\frac{4}{5}EI\theta_B+\frac{6}{25}EI\delta$$

$$M_{DC} = 0+\frac{2EI}{5}\left(0+\theta_C-\frac{3-\delta}{5}\right)=\frac{2EI}{5}\left(-\theta_B+\frac{3\delta}{5}\right)$$

∴
$$M_{DC} = -\frac{2}{5}EI\theta_B+\frac{6}{25}EI\delta$$

For the equilibrium at *B*, $M_{BA} + M_{BC} = 0$

$$\frac{4}{5}EI\theta_B-\frac{6}{25}EI\delta+\frac{2}{3}EI\theta_B = 0$$

$$\Rightarrow \quad \frac{22}{15}EI\theta_B - \frac{6}{25}EI\delta = 0$$

$$\Rightarrow \quad 110EI\theta_B - 18EI\delta = 0$$

$$\Rightarrow \quad 55EI\theta_B - 9EI\delta = 0 \quad \text{...(i)}$$

$$H_a = \frac{M_{AB} + M_{BA}}{5}$$

and
$$H_D = \frac{M_{CD} + M_{DC}}{5}$$

For horizontal equilibrium, $H_a + H_d + 1.5 = 0$

$$\Rightarrow \quad \frac{M_{AB} + M_{BA}}{5} - \frac{M_{CD} + M_{DC}}{5} + 1.5 = 0$$

$$\Rightarrow \quad [M_{AB} + M_{BA}] - [M_{CD} + M_{DC}] = -7.5$$

$$\Rightarrow \quad \left[\frac{2}{5}EI\theta_B - \frac{6}{25}EI\delta + \frac{4}{5}EI\theta_B - \frac{6}{25}EI\delta\right] - \left[-\frac{4}{5}EI\theta_B + \frac{6}{25}EI\delta - \frac{2}{5}EI\theta_B + \frac{6}{25}EI\delta\right] = -7.5$$

$$\Rightarrow \quad \frac{12}{5}EI\theta_B - \frac{24}{25}EI\delta = -7.5$$

$$\Rightarrow \quad 10EI\theta_B - 4EI\delta = -31.25 \quad \text{...(ii)}$$

Solving equations (*i*) and (*ii*), we get

$$EI\theta_B = 2.1635 \quad \text{and} \quad EI\delta = 13.2212$$

Substituting for $EI\theta_B$ and $EI\delta$,

$$M_{AB} = \frac{2}{5}(2.1635) - \frac{6}{25}(13.2212) = -2.31 \text{ kNm}$$

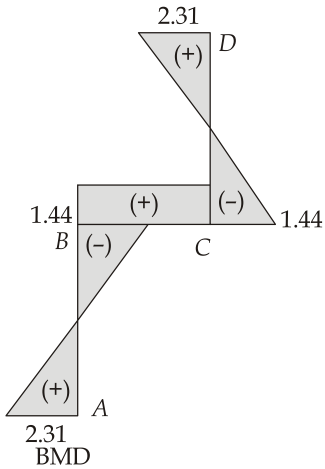
$$M_{BA} = \frac{4}{5}(2.1635) - \frac{6}{25}(13.2212) = -1.41 \text{ kNm}$$

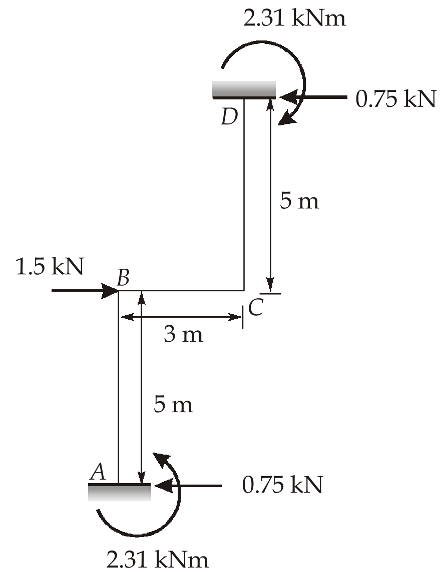
$$M_{BC} = \frac{2}{3}(2.1635) = +1.44 \text{ kNm}$$

$$M_{CB} = -\frac{2}{3}(2.1635) = -1.44 \text{ kNm}$$

$$M_{CD} = -\frac{4}{5}(2.1635) + \frac{6}{25}(13.2212) = 1.44 \text{ kNm}$$

$$M_{DC} = -\frac{2}{5}(2.1635) + \frac{6}{25}(13.2212) = +2.31 \text{ kNm}$$





Reactions

$$H_A = \frac{-2.31 - 1.44}{5} = -0.75 \text{ kNm } (\leftarrow)$$

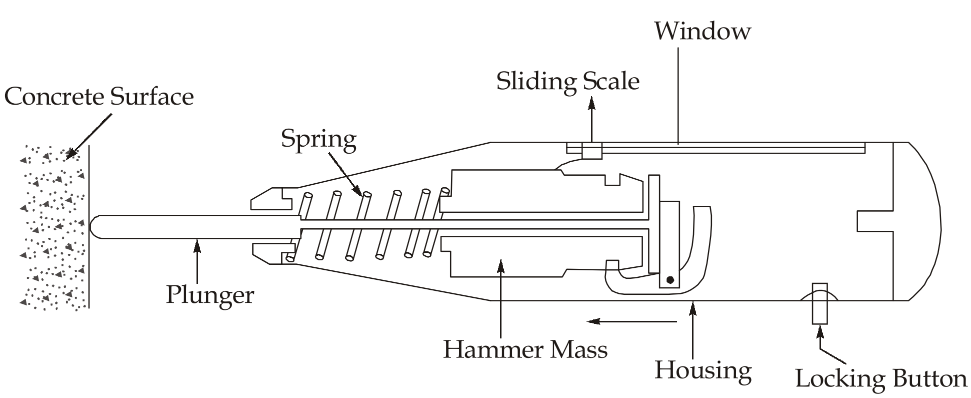
$$H_D = -\frac{+1.44 + 2.31}{5} = -0.75 \text{ kN } (\leftarrow)$$

$$V_A = -\frac{1.44 - 1.44}{3} = 0; \quad V_D = 0$$

Q.3 (b) Solution:

(i)

Non-destructive testing methods are quick and can be performed both in laboratory and in-situ with convenience.They help to assess physical properties like hardness, resistance to penetration by projectiles, rebound capacity, ultrasonic pulse transmission, and X-rays without damaging the concrete. NDT can be used on fresh as well as hardened concrete and provides information about strength and other properties indirectly.



Rebound hammer test

Rebound Hammer Test (Schmidt Hammer Test):

- It is done to find out the compressive strength of concrete by using rebound hammer.
- The principle of the test is that rebound of an elastic mass depends on the hardness of the surface against which it strikes.
- When the plunger of the rebound hammer is pressed against the surface of the concrete, the spring controlled mass rebounds and the extent of the rebound depends upon the surface hardness of the concrete.
- The surface hardness and therefore, the rebound are taken to be related to the compressive strength of the concrete.
- The rebound value is read from a graduated scale and is designated as the rebound number or rebound index. The compressive strength can be read directly from the graph provided on the body of the hammer.

Procedure:

- Apply light pressure on the plunger as it will release from the locked position and allow it to extend to the ready position for the test.
- Press the plunger against the surface of the concrete keeping the instrument perpendicular to the test surface. Apply a gradual increase in pressure until the hammer impacts.
- The spring controlled mass when rebounds, it takes with it a rider which slides along a graduated scale. It can be held in position on the scale by depressing the locking button.

Limitation and Advantages: The Schmidt hammer provides an inexpensive, simple and quick method of obtaining an indication of concrete strength, but accuracy of ± 15 to ± 20% is possible only for specimens cast cured and tested under conditions for which calibration curves have been established. The results are affected by factors such as smoothness of surface, size and shape of specimen, moisture condition of the concrete, type of cement and coarse aggregate and extent of carbonation of surface.

- The test provides useful information for surface layer up to 30 mm depth and is suitable for concrete having strength of 20-60 MPa.

(ii)

$$d' = 50 + \frac{35}{2} = 67.5 \text{ mm}$$

$$D = 450\text{mm}$$

$$\frac{d'}{D} = \frac{67.5}{450} = 0.15$$

$$\frac{P_u}{f_{ck}\, bD} = \frac{2500 \times 10^3}{25 \times 450 \times 450} = 0.494$$

$$\frac{M_u}{f_{ck}\, bD^2} = \frac{180 \times 10^6}{25 \times 450 \times 450^2} = 0.079$$

From chart given, $$\frac{P_t(\%)}{f_{ck}} = 0.10$$

∴ Percentage of reinforcement = 0.10 × 25 = 2.5%

∴ $$A_{sc} = \frac{2.5}{100} \times 450 \times 450 = 5062.5 \text{ mm}^2$$

Q.3 (c) Solution:

(i)

Shear lag effect:

One of the assumptions made in the simple theory of bending is that plane sections remain plane before and after the bending. However in reality, the shear strain induced influences the bending stresses in the flange and causes the sections to warp. This consequently modifies the bending stresses determined by the application of simple bending theory and results in higher stresses near the junction of web to flange element with the stress dropping as the distance from the beam web increases. The resultant stress distribution across the flange is therefore non-uniform and is shown below. This phenomenon is known as shear lag effect.

Shear lag effect depends upon

(i) Width-to-span ratio

(ii) Beam end constraints

(iii) Type of load

Point load causes more shear lag than uniform-load.

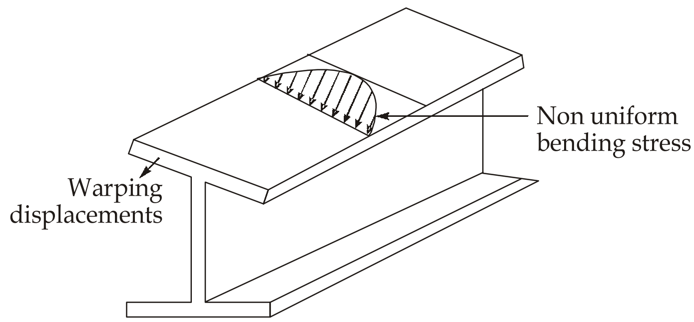
As per the provisions of IS 800 : 2007, the shear lag effects in the flanges may be disregarded provided,

- $b_0 \le \frac{L_o}{20}$ (For outstand elements)
- $b_0 \le \frac{L_o}{10}$ (For internal elements)

where L_o = Length between points of zero moment (inflection) in the span.

b_0 = Width of the flange outstand

b_i = Width of an internal element



Shear lag effect

(ii)

In cantilever, critical section for shear is at the support.

At critical section,

Shear force, $V_u = 80 \times 3 = 240$ kN

Bending moment, $M_4 = \frac{w_u L^2}{2} = \frac{80 \times (3)^2}{2} = 360$ kNm

Now, Nominal shear stress, $\tau_v = \frac{V_u - \frac{M_u}{d}\tan\beta}{bd} = \frac{240 - \frac{360}{0.55} \times \left(\frac{600 - 300}{3000}\right)}{300 \times 550} \times 10^3$

$= 1.06 \text{ N/mm}^2 < \tau_{c,max} = 0.625\sqrt{f_{ck}} = 3.1 \text{ N/mm}^2$ (OK)

Design shear strength of concrete:

Percentage of tensile steel at support $P_t(\%) = \frac{A_{st}}{bd} \times 100 = \frac{5 \times \frac{\pi}{4} \times (25)^2}{300 \times 550} \times 100 = 1.48\%$

From table given, $\tau_c = 0.70 + \frac{0.74 - 0.70}{1.50 - 1.25} \times (1.48 - 1.25)$

$= 0.7368 \text{ N/mm}^2$

Since, $\tau_v > \tau_c < \tau_{c,max}$

Hence, shear reinforcement has to be provided.

Design shear force, $V_{us} = (\tau_v - \tau_c)bd$

$= (1.06 - 0.7368) \times 300 \times 550 \times 10^{-3}$

$= 53.328$ kN

Provide 2-legged 8 mmϕ stirrups.

$$V_{us} = \frac{0.87 f_y \times A_{sv} \times d}{S_v}$$

$$\Rightarrow \quad S_v = \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 8^2 \times 550}{53.328 \times 10^3} = 374.35 \text{ mm}$$

But as per IS:456 : 2000,

$$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87 f_y}$$

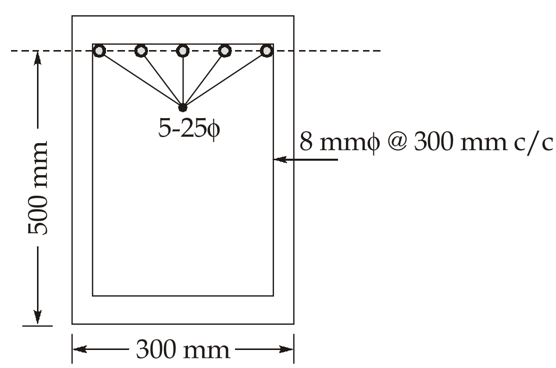
$$\Rightarrow \quad S_v \leq \frac{A_{sv} \times 0.87 f_y}{0.4b} \leq \frac{2 \times \frac{\pi}{4} \times 8^2 \times 0.87 \times 415}{0.4 \times 300}$$

$$\leq 302.47 \text{ mm}$$

$$\text{Maximum spacing} = 0.75d = 0.75 \times 550 = 412.5 \text{ mm}$$

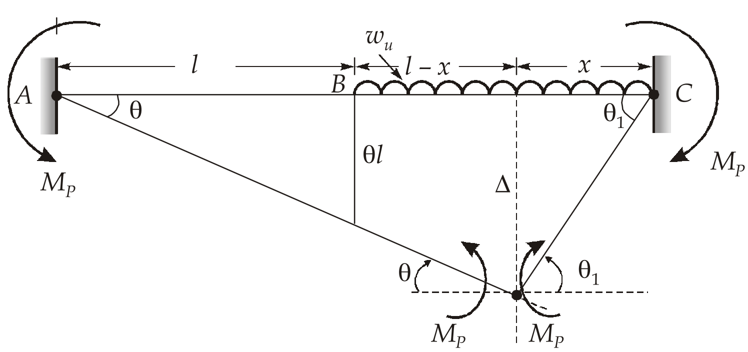
Hence, S_v = 300 mm, (limited to maximum 300 mm) should be adopted as the spacing.

Provide 2-legged 8 mmϕ @ 300 mm c/c.



Q.4 (a) Solution:

(i)



From the mechanism

$$\Delta = \theta(2l - x) = x\theta_1$$

$$\Rightarrow \quad \theta_1 = \frac{(2l-x)}{x}\theta$$

External work done = Intensity of load × Area of collapse mechanism diagram under the load.

$$\Rightarrow \quad W_e = w_u\left[\frac{1}{2}(\theta l+\Delta)(l-x)+\frac{1}{2}\Delta x\right]$$

$$\Rightarrow \quad W_e = w_u\left[\frac{1}{2}(\theta l+x\theta_1)(l-x)+\frac{1}{2}(x\theta_1)x\right]$$

$$\Rightarrow \quad W_e = w_u\left[\frac{1}{2}\left(\theta l+x\frac{(2l-x)}{x}\theta\right)(l-x)+\frac{x^2}{2}\left(\frac{2l-x}{x}\right)\theta\right]$$

$$\Rightarrow \quad W_e = \frac{w_u}{2}\left[(3\theta l-x\theta)(l-x)+x(2l-x)\theta\right]$$

$$\Rightarrow \quad W_e = \frac{w_u}{2}\left[3l^2-2lx\right]\theta$$

Now, internal workdone

$$W_i = M_p\theta + M_p(\theta + \theta_1) + M_p\theta_1$$

$$\Rightarrow \quad W_i = 2M_p(\theta + \theta_1)$$

$$\Rightarrow \quad W_i = 2M_P\left[\theta+\frac{(2l-x)}{x}\theta\right]$$

$$\Rightarrow \quad W_i = \frac{4M_Pl}{x}\theta$$

By principle of virtual work

$$W_e = W_i$$

$$\Rightarrow \quad \frac{w_u}{2}(3l^2-2lx) = \frac{4M_Pl}{x}$$

$$\Rightarrow \quad M_P = \frac{w_u(3l-2x)x}{8}$$

For the maximum value of M_P.

$$\frac{dM_P}{dx} = 0$$

$$\Rightarrow \quad \frac{w_u}{8}(3l - 4x) = 0$$

$$\Rightarrow \quad x = \frac{3l}{4} = 0.75l$$

Hence, $$M_P = \frac{w_u}{8}[3l - 2 \times 0.75l] \times 0.75l$$

$$\Rightarrow \quad M_P = 0.140625\, w_u l^2$$

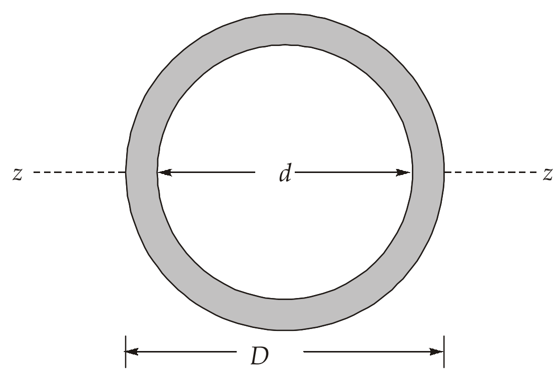
$$\Rightarrow \quad w_u = \frac{7.11 M_P}{l^2}$$

$\therefore$ Minimum collapse load $= w_u \times l$

$$= \frac{7.11 M_P}{l}$$

(ii)

Moment of inertia about zz-axis,



$$I_z = \frac{\pi}{64}(D^4 - d^4)$$

Elastic section modulus, $$Z_{ez} = \frac{\pi}{64} \times \frac{(D^4 - d^4)}{D/2} = \frac{\pi}{32} \times \frac{D^4 - d^4}{D}$$

Plastic section modulus, $$Z_{pz} = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$\bar{y}_1 = \bar{y}_2 = \frac{\frac{1}{2} \times \left[\frac{\pi D^2}{4}\left(\frac{2}{3} \times \frac{D}{\pi}\right) - \frac{\pi d^2}{4}\left(\frac{2}{3} \times \frac{d}{\pi}\right)\right]}{\frac{1}{2} \times \left[\frac{\pi}{4}(D^2 - d^2)\right]}$$

$$= \frac{2}{3\pi} \times \frac{(D^3 - d^3)}{(D^2 - d^2)}$$

$\therefore$

$$Z_{pz} = \frac{1}{2} \times \frac{\pi}{4}(D^2 - d^2)\left[2 \times \frac{2}{3\pi} \times \frac{(D^3 - d^3)}{(D^2 - d^2)}\right] = \frac{1}{6}(D^3 - d^3)$$

$$\text{Shape factor} = \frac{Z_{pz}}{Z_{ez}} = \frac{\frac{1}{6}(D^3 - d^3)}{\frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right)}$$

$$= \frac{32}{6\pi} \times \frac{D^3\left(1 - \frac{d^3}{D^3}\right)}{D^3\left(1 - \frac{d^4}{D^4}\right)} = \frac{32}{6\pi} \times \frac{\left(1 - \frac{d^3}{D^3}\right)}{\left(1 - \frac{d^4}{D^4}\right)}$$

Put,

$$k = \frac{d}{D}$$

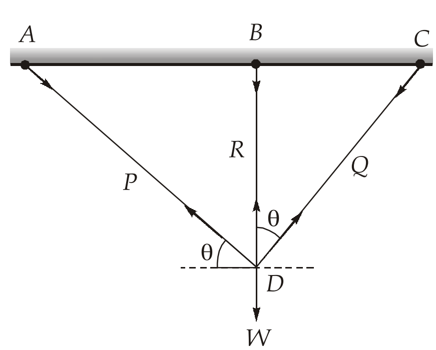
$$\text{Shape factor} = \frac{32}{6\pi} \times \frac{(1 - k^3)}{(1 - k^4)} = 1.7\frac{(1 - k^3)}{(1 - k^4)}$$

Q.4 (b) Solution:

It is easily seen that ADC is a right-angled triangle. Let the tensions in *DA*, *DC* and *DB* be *P*, *Q* and *R* respectively.

Let *DC* be inclined at θ with the vertical then *AD* will be inclined at θ with the horizontal.

For the equilibrium of the joint *D*, resolving the forces at *D* horizontally, we have $P\cos\theta = Q\sin\theta$, $P = Q\tan\theta$



$$\tan\theta = \frac{3}{4}$$

$$\therefore \quad \sin\theta = \frac{3}{5} \text{ and } \cos\theta = \frac{4}{5}$$

$$\Sigma H = 0$$

$$\Rightarrow \quad P\cos\theta = Q\sin\theta$$

$$\Rightarrow \quad P\left(\frac{4}{5}\right) = Q\left(\frac{3}{5}\right)$$

$$\Rightarrow \quad P = \frac{3}{4}Q$$

Resolving the forces at D vertically, we have,

$R + P\sin\theta + Q\cos\theta = W$

$$\Rightarrow \quad R + \frac{3}{4}Q \times \frac{3}{5} + Q\frac{4}{5} = W$$

$$\Rightarrow \quad R + \frac{5}{4}Q = W$$

$$\Rightarrow \quad Q = \frac{4}{5}(W - R)$$

But $$P = \frac{3}{4}Q$$

$$\therefore \quad P = \frac{3}{5}(W - R)$$

Strain energy stored, $$W_t = \frac{P^2(AD)}{2AE} + \frac{Q^2(DC)}{2AE} + \frac{R^2(BD)}{2AE}$$

$$\Rightarrow \quad W_t = \frac{1}{2AE}\left[5P^2 + 3.75Q^2 + 3R^2\right]$$

At equilibrium, strain energy is minimum.

$$\therefore \quad \frac{\partial W_i}{\partial R} = 0$$

$$\therefore \quad \frac{\partial W_i}{\partial R} = \frac{1}{2AE}\left[10P\frac{dP}{dR} + 7.5Q\frac{dQ}{dR} + 6R\right] = 0$$

$$P = \frac{3}{5}(W - R)$$

and $$Q = \frac{4}{5}(W - R)$$

$$\therefore \quad \frac{dP}{dR} = -\frac{3}{5} \text{ and } \frac{dQ}{dR} = -\frac{4}{5}$$

$$\therefore \quad 10\times\frac{3}{5}(W-R)\left(-\frac{3}{5}\right)+7.5\times\frac{4}{5}(W-R)\left(-\frac{4}{5}\right)+6R = 0$$

$$\Rightarrow \quad -\frac{18}{5}(W-R)-\frac{24}{5}(W-R)+6R = 0$$

$$\Rightarrow \quad \frac{72}{5}R = \frac{42}{5}W$$

$$\therefore \quad R = \frac{42}{72}W = \frac{7}{12}W$$

$$\therefore \quad P = \frac{3}{5}(W-R) = \frac{3}{5}\left(W-\frac{7}{12}W\right) = \frac{W}{4}$$

and
$$Q = \frac{4}{5}(W-R) = \frac{4}{5}\left(W-\frac{7}{12}W\right) = \frac{W}{3}$$

Horizontal component of the extension of $DA = \frac{P}{AE}\times 5\cos\theta = \frac{W}{4AE}\times 5\times\frac{4}{5} = \frac{W}{AE}$

Horizontal component of the extension of $DC = \frac{Q}{AE}\times 3.75\sin\theta$

$$= \frac{W}{3AE}\times 3.75\times\frac{3}{5} = \frac{0.75W}{AE}$$

$\therefore$ Horizontal movement of D,

= Difference between the horizontal components of the extensions of DA and DC

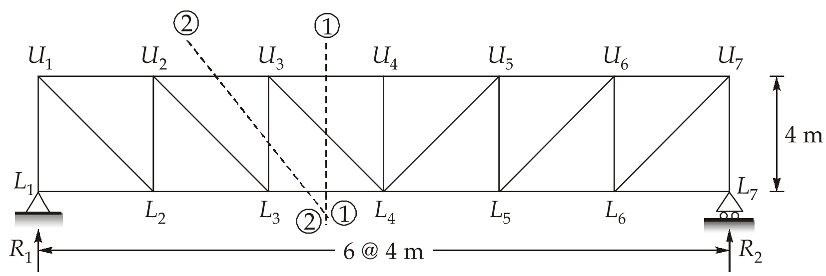
$$= \frac{W}{AE}-\frac{0.75W}{AE} = \frac{W}{4AE}$$

$$\text{Extension of } BD = \frac{R}{AE}\times 3 = \frac{7}{12}\frac{W}{AE}\times 3 = \frac{7W}{4AE}$$

$\therefore$ Horizontal movement of $D = \frac{1}{7}$ of the extension of BD.

Q.4 (c) Solution:

Let the reactions at the left support and right support be R_1 and R_2 respectively. Take the section 1-1 as shown below.



ILD for $F_{U_3L_4}$

When unit load is between L_1 and L_3,
Considering right hand side portion of truss.

$$\Sigma F_V = 0$$

$$\Rightarrow \quad F_{U_3L_4}\sin 45° + R_2 = 0$$

$$\Rightarrow \quad F_{U_3L_4}\times\sin 45° = -R_2$$

$$\Rightarrow \quad F_{U_3L_4} = -\frac{R_2}{\sin 45°} = \sqrt{2}R_2 \text{ (comp.) (Varies linearly)}$$

$$F_{U_3L_4} = 0, \text{ when load is at } L_1.$$

$$= \sqrt{2}\times\frac{8}{24} = \frac{\sqrt{2}}{3}, \text{ when load is at } L_3.$$

When unit load is in portion L_4 to L_7, considering the equilibrium of left hand side portion of truss.

$$\Sigma F_V = 0$$

$$\Rightarrow \quad -F_{U_3L_4}\times\sin 45° + R_1 = 0$$

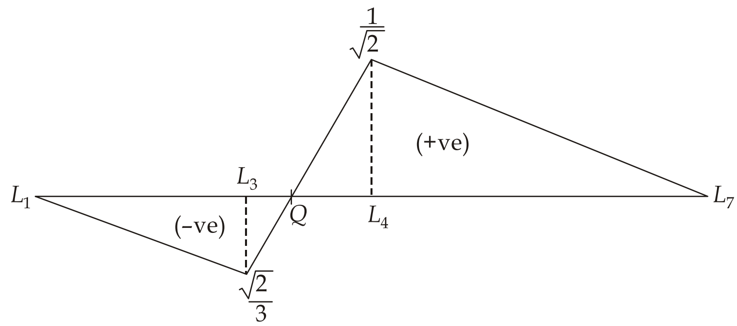
$$\Rightarrow \quad F_{U_3L_4}\times\sin 45° = R_1$$

$$\Rightarrow \quad F_{U_3L_4} = \frac{R_1}{\sin 45°} = \sqrt{2}R_1 \text{ (Tensile), which varies linearly}$$

$$= 0, \text{ when load is at } L_7.$$

$$= \sqrt{2}\times\frac{12}{24} = \frac{1}{\sqrt{2}}, \text{ when load is at } L_4.$$

Between L_3 and L_4, it varies linearly. Hence ILD for $F_{U_3L_4}$ is as shown below:



Let it intersect L_3L_4 at Q, then

$$\frac{L_3Q}{QL_4} = \frac{\sqrt{2}/3}{1/\sqrt{2}}$$

Also
$$L_3Q + QL_4 = 4$$

$$\Rightarrow \quad L_3Q + \frac{4/\sqrt{2}}{\sqrt{2}/3} L_3Q = 4$$

$$\Rightarrow \quad L_3Q = 1.6 \text{ m}$$

$\therefore$ Maximum compressive force occurs when udl covers the portion L_1Q

$$= \frac{1}{2} \times \frac{\sqrt{2}}{3} \times L_1Q \times 40$$

$$= \frac{1}{2} \times \frac{\sqrt{2}}{3} \times (8 + 1.6) \times 40 = 90.51 \text{ kN}$$

Maximum tensile force in the member U_3L_4 will occur when 40 kN/m *udl* covers the portion QL_7 and its value is:

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} \times (12 + 2.4) \times 40 = 203.65 \text{ kN}$$

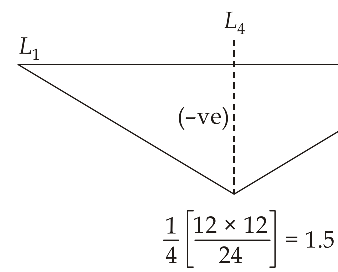
ILD for $F_{U_3U_4}$:

Considering section 1-1 and moment equilibrium about L_4, it can be observed that

$$F_{U_3U_4} \times 4 = M_{L_4}$$

$$\Rightarrow \quad F_{U_3U_4} = \frac{M_{L_4}}{4} \text{ (compressive)}$$

Therefore, ILD for $F_{U_3U_4}$ is similar to that of moment at L_4 as shown below:



$\therefore$ Force in U_3U_4 is maximum when load covers the entire span,

$$= \frac{1}{2} \times 24 \times 1.5 \times 40 = 720 \text{ kN (compressive)}$$

ILD for U_3L_3:

Considering section 2-2 and when the unit load is in portion L_1 to L_3, consider right side portion of truss:

$$\Sigma F_V = 0$$

$$\Rightarrow \quad F_{U_3L_3} = R_2 \text{ (tensile), varies linearly}$$

$$= 0, \text{ when load is at } L_1$$

$$= \frac{8}{24} = \frac{1}{3}, \text{ when load is at } L_3.$$

When the unit load is in portion L_4 to L_7, considering left hand side portion,

$$\Sigma F_V = 0$$

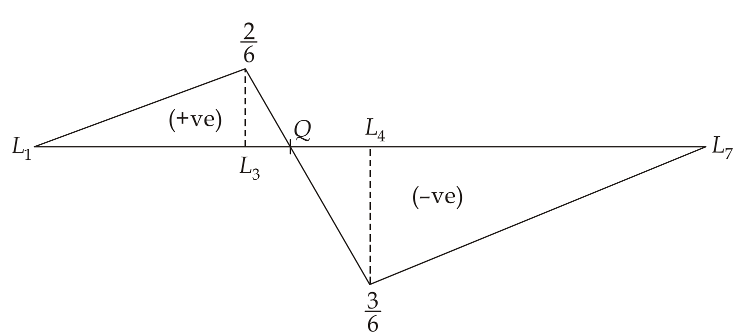
$$\Rightarrow \quad F_{U_3L_3} = R_1 \text{ (compressive), varies linearly}$$

$$= 0, \text{ when load is at } L_7.$$

$$= \frac{12}{24} = \frac{1}{2}, \text{ when load is at } L_4.$$

Between L_3 and L_4, it varies linearly,

ILD for $F_{U_3L_3}$ is as shown in figure.



$$\frac{L_3Q}{QL_4} = \frac{2/6}{3/6}$$

Also, $$L_3Q + QL_4 = 4$$

$\Rightarrow$ $$L_3Q + \frac{3}{2}L_3Q = 4$$

$\Rightarrow$ $$L_3Q = 1.6 \text{ m}$$

$\therefore$ Maximum tensile force in U_3L_3 due to 40 kN/m *udl* occurs when load covers the portion L_1Q only,

$$= \frac{1}{2}\times\frac{2}{6}\times L_1Q\times 40$$

$$= \frac{1}{2}\times\frac{2}{6}\times 9.6\times 40 = 64 \text{ kN}$$

Maximum compressive force occurs when the load is in portion QL_7 and is

$$= \frac{1}{2}\times\frac{3}{6}\times QL_7\times 40$$

$$= \frac{1}{2}\times\frac{3}{6}\times 14.4\times 40 = 144 \text{ kN}$$

Section - B

Q.5 (a) Solution:

Slump test: This is the most commonly used test due to the simplicity of the apparatus and also the test procedure. This test is very useful in detecting variations in the uniformity of a mix of given nominal proportions, from batch to batch. This test is not suitable for very wet mixes or very dry mixes. The test does not gauge the various factors influencing workability.

The test is made using a hollow metal frustum of a cone 300 mm high, with a bottom diameter of 200 mm and a top diameter of 100 mm and a metal thickness not less than 1.6 mm. The internal surface of the cone is thoroughly cleaned so as to be free from superfluous moisture and remains of set concrete. The cone is placed on a stable, non-absorbent horizontal surface, with the smaller opening at the top. The cone is filled with concrete in four layers each for a height of approximately one-fourth of the height of the cone. Each layer is tamped 25 times with a standard 16 mm diameter 600 mm long bullet ended steel rod. The tamping strokes should be distributed uniformly over the cross-section of the mould. After filling the mould with concrete, the concrete is struck off to level using a trowel. Any concrete leaked out between the mould and the base is wiped out. After this, the cone is slowly lifted upwards vertically leaving the concrete. The laterally unsupported concrete will now slump. The decrease in the height of the slumped concrete is called the slump and is measured.

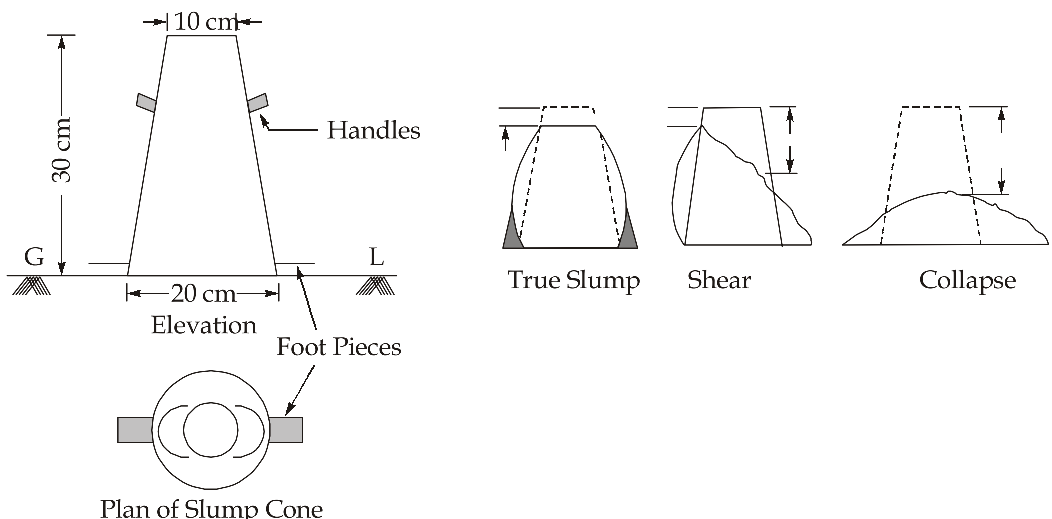
Precautions

(i) The test should be conducted at a place free from vibrations.

(ii) The test should be conducted within 2 minutes after sampling.

(iii) Any slump specimen which collapses or shears off laterally gives erroneous result and if this happens the test should be repeated with another sample.

As the mould is lifted off, the subsidence of concrete may take place in three possible types as shown in figure below.



Slump test

The results of the slump test can be interpreted as:

Slump Value	Degree of workability
0	Extremely low
0 - 25	Very low
25 - 50	Low
50 - 100	Medium
100-175	High

Q.5 (b) Solution:

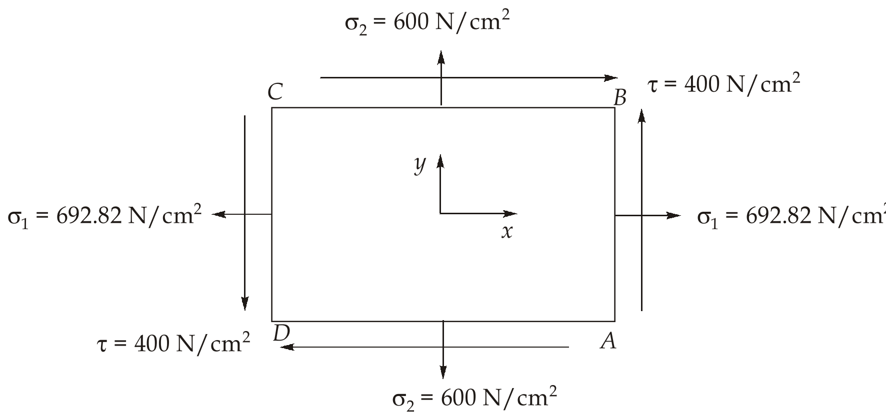
Given:

$$\text{Resultant stress on plane } AB = 800 \text{ N/cm}^2$$

$$\text{Angle of inclination of the above stress} = 30°$$

$$\text{Normal stress on plane } BC = 600 \text{ N/cm}^2$$

The resultant stress 800 N/cm^2 on plane AB is resolved into normal stress and tangential stress.



Normal stress on plane *AB*

$$= 800 \times \cos 30° = 692.82 \text{ N/cm}^2.$$

Tangential stress on plane *AB*

$$= 800 \times \sin 30° = 400 \text{ N/cm}^2.$$

The shear stress on plane *AB* is, i.e., τ_{AB} = 400 N/cm^2, then to maintain the equilibrium on the wedge *ABC*, another shear stress of same magnitude, i.e. τ_{BC} = 400 N/cm^2 must act on the plane *BC*. The free body diagram of the element *ABCD* is shown in figure above,, showing normal and shear stresses acting on different faces.

(i) Resultant stress on plane *BC*

On plane *BC*,

$$\sigma_2 = 600 \text{ N/cm}^2$$

Shear stress, $\tau = 400 \text{ N/cm}^2$

∴ Resultant stress on plane *BC*

$$= \sqrt{\sigma_2^2 + \tau^2} = \sqrt{600^2 + 400^2} = 721 \text{ N/cm}^2 \quad \textbf{Ans.}$$

The resultant will be inclined at an angle θ with the horizontal and is given by,

$$\tan\theta = \frac{\sigma_2}{\tau} = \frac{600}{400} = 1.5$$

$$\therefore \quad \theta = \tan^{-1}(1.5) = 56.3° \quad \textbf{Ans.}$$

(ii) Principal stresses and their directions

∴ Major principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right) + \tau^2}$$

$$= \frac{692.82 + 600}{2} + \sqrt{\left(\frac{692.82 - 600}{2}\right)^2 + 400^2}$$

$$= 646.41 - 402.68$$

$$= 1049.09 \text{ N/cm}^2 \text{ (Tensile).}$$ **Ans.**

$\therefore$ Minor principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{692.82 + 600}{2} - \sqrt{\left(\frac{692.82 - 600}{2}\right)^2 + 400^2}$$

$$= 646.41 - 402.68$$

$$= 243.73 \text{ N/cm}^2 \text{ (Tensile).}$$ **Ans.**

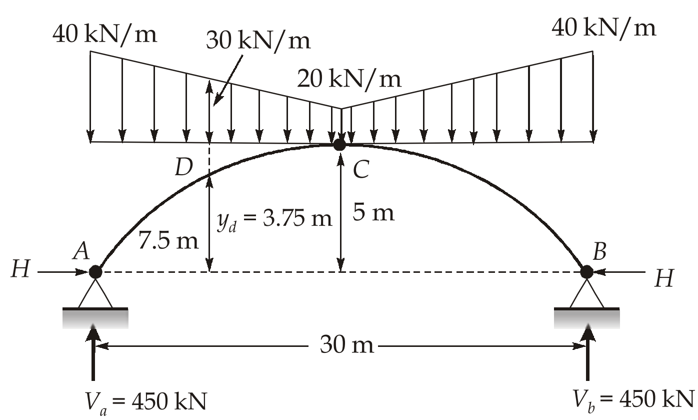
The directions of principal stresses are given by

$$\tan 2\theta = \frac{2\tau}{(\sigma_1 - \sigma_2)} = \frac{2 \times 400}{(692.82 - 600)} = \frac{800}{92.82} = 8.619$$

$\therefore$ $$2\theta = \tan^{-1} 8.619 = 83.38° \text{ or } 263.38°$$

$\therefore$ $$\theta = 41.69° \text{ or } 131.99°.$$ **Ans.**

Q.5 (c) Solution:



Total vertical load, $$V = (20 \times 30) + 2 \times \frac{2}{3} \times 15 \times 20 = 900 \text{ kN}$$

$\therefore$ $$V_a = V_b = \frac{900}{2} = 450 \text{ kN}$$

Taking moments about C from the left side:

$$5\,H + 20 \times 15 \times \frac{15}{2} + \frac{1}{2} \times 15 \times 20 \times \frac{2}{3} \times 15 = 450 \times 15$$

$\Rightarrow$ $H = 600$ kN

At the section D, 7.5 m from A

$$y_d = \frac{4 \times 5}{30 \times 30} \times 7.5(30-75) = 3.75 \text{ m}$$

Rate of loading at section $D = 20 + 10 = 30$ kN/m

B.M. at D, $$M_d = \left[450 \times 7.5 - 30 \times \frac{7.5^2}{2} - \frac{1}{2} \times 7.5 \times 10 \times \frac{2}{3}(7.5)\right]$$

$$- [600 \times 3.75] = 93.75 \text{ kNm}$$

Consider the forces acting on the part AD.

To keep this part in equilibrium,

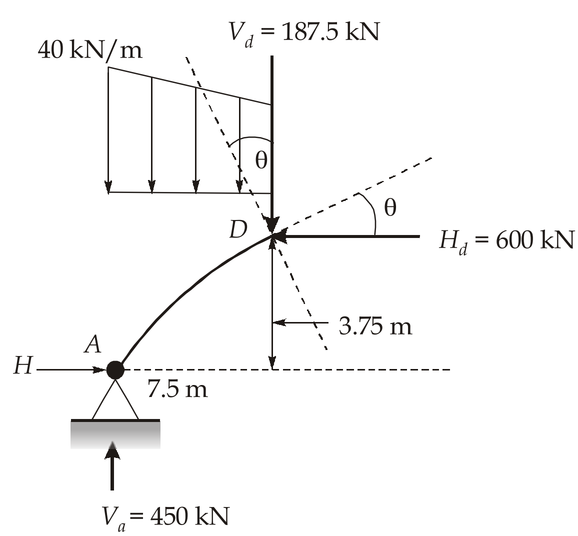
Vertical force required at D $V_d = 450 - 35 \times 7.5 = 187.5$ kN $\downarrow$

Horizontal force required at D,$H_d = 600$ kN $\leftarrow$

Equation of the arch is,

$$y = \frac{4 \times 5}{30 \times 30} x(30-x) \; i.e. \quad y = \frac{1}{45} x(30-x)$$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{45}(30-2x)$$



$\therefore$ At D, i.e., at $x = 7.5$ m, $\frac{dy}{dx} = \tan\theta = \frac{1}{45}(30 - 2\times 7.5) = \frac{1}{3}$

$\therefore$ $\sin\theta = \frac{1}{\sqrt{10}}, \cos\theta = \frac{3}{\sqrt{10}}$

Normal thrust at D, $P_n = H_d \cos\theta + V_d \sin\theta = 600 \times \frac{3}{\sqrt{10}} + 187.5 \times \frac{1}{\sqrt{10}} = 628.5$ kN

Radial shear at D, $S = H_d \sin\theta - V_d \cos\theta = 600 \times \frac{1}{\sqrt{10}} - 187.5 \times \frac{3}{\sqrt{10}} = 11.86$ m

Q.5 (d) Solution:

Given: $b = 200$ mm, $d = 400$ mm

$$A_{st} = 3 \times \frac{\pi}{4} \times 16^2 = 603.19 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N.mm}^2$$

$$L = 5 \text{ m}$$

- **Depth of neutral axis (x_u)**

$$x_u = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b} = \frac{0.87 \times 415 \times 603.19}{0.36 \times 20 \times 200}$$

$\Rightarrow$ $x_u = 151.2$ mm

- **Limiting depth of neutral axis ($x_{u\,\text{lim}}$)**

$$x_{u\,\text{lim}} = 0.48\,d = 0.48 \times 400$$

$$= 192 \text{ mm}$$

$x_{u\,\text{lim}} > x_u$ hence the section is under-reinforced.

- **Moment of resistance (M_u)**

$$M_u = 0.87 f_y \cdot A_{st} (d - 0.42\,x_u)$$

$\Rightarrow$ $M_u = 0.87 \times 415 \times 603.19 (400 - 0.42 \times 151.2)$

$\Rightarrow$ $M_u = 73.28 \times 10^6 \text{ Nmm} = 73.28$ kNm

- **Ultimate load**

Equating maximum factored bending moment and the ultimate moment of resistance.

Max. moment, $M = \frac{w_u \times l^2}{8}$

$\therefore$ $M = M_u$

$$\Rightarrow \qquad \frac{w_u \times l^2}{8} = 73.28$$

$$\Rightarrow \qquad w_u = 23.45 \text{ kN/m}$$

Q.5 (e) Solution:

In this connection packing plate of 8 mm thickness is to be used. Hence there shall be reduction in the shear strength of bolt. The reduction factor is given by

$$\beta_{pk} = (1 - 0.0125\, t_{pk})$$
$$= 1 - 0.0125 \times 8 = 0.9$$

∴ Nominal shear strength of one bolt in double shear

$$= \beta_{pk} \frac{f_{ub}}{\sqrt{3}}\left(1 \times \frac{\pi}{4} d^2 \times 0.78 + \frac{\pi}{4} d^2\right)$$

$$= 0.9 \times \frac{400}{\sqrt{3}}(1.78) \times \frac{\pi}{4} \times 20^2 = 116228 \text{ N}$$

Design shear strength of one bolt in shear

$$= \frac{116228}{1.25} = 92982.6 \text{ N}$$

∴ Design shear strength of 6 bolts in the joint

$$= 6 \times 92982.6 = 557895.6 \text{ N}$$
$$= 557.896 \text{ kN}$$

Strength of bolts in bearing:

$$K_b = \min.\begin{cases} \dfrac{e}{3d_o} = \dfrac{40}{3 \times 22} = 0.6061 \\ \dfrac{P}{3d_o} - 0.25 = \dfrac{60}{3 \times 22} - 0.25 = 0.659 \\ \dfrac{f_{ub}}{f_u} = \dfrac{400}{410} = 0.975 \\ 1 \end{cases}$$

$$\therefore \qquad K_b = 0.6061.$$

∴ Nominal strength of one bolt in bearing = $2.5\, K_b\, dt\, f_u$

$$= 2.5 \times 0.6061 \times 20 \times 10 \times 410$$
$$= 124250.5 \text{ N}$$

Thickness of thinner plate t = 10mm

$\therefore$ Design strength of a bolt $= \dfrac{124250.5}{1.25} = 99400$ N

Design strength of 6 bolts in bearing = 6 × 99400

= 596400 N

= 596.4 kN > 557.896 kN

$\therefore$ Strength of bolts in connection = 557.896 kN.

Strength of plates in the joint = Strength of thinner plate at weakest section.

$\therefore$ Design strength of plate

$$= \frac{0.9 A_n f_u}{\gamma_m} = \frac{0.9 \times (200 - 3 \times 22) \times 10 \times 410}{1.25}$$

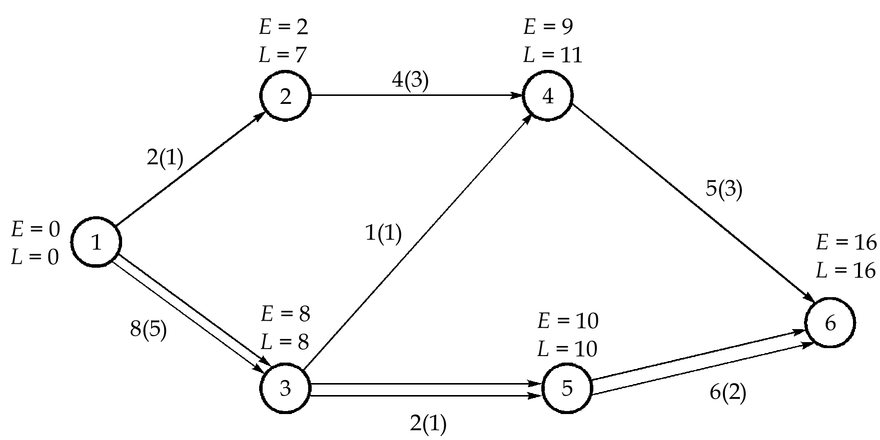
= 395568 N

= 395.568 kN < 597.896 kN

Design strength of the joint = 395.568kN **Ans.**

Q.6 (a) Solution:

Network diagram and cost slope for activities are given below



Activity (*i* - *j*)	1 - 2	1 - 3	2 - 4	3 - 4	3 - 5	4 - 6	5 - 6
Cost slope (thousand Rs/week)	5	2	4	0	7	3	6

Critical path is 1-3-5-6 and critical path duration is 16 weeks.

Direct cost of the project (in thousands) = Rs. (10 + 15 + 20 + 7 + 8 + 10 + 12) = Rs. 82

Total normal cost (in thousands) = 82 + 16 × 5 = 82 + 80 = Rs. 162

As activity 1-3 has least cost slope, among 1-3,3-5 and 5-6 activities which is Rs.2 thousands per week.

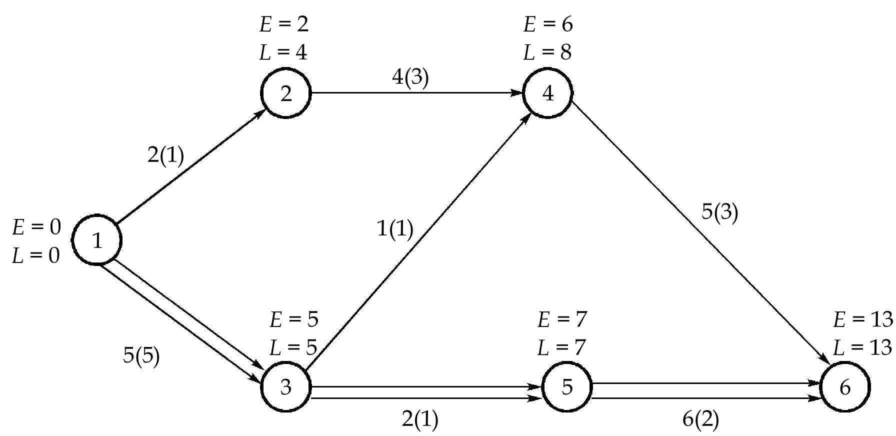
Hence, crashing activity 1-3 by 3 weeks,

$$\text{Total cost (in thousands)} = 82 + 13 \times 5 + 3 \times 2 = \text{Rs. } 153$$

$$\text{Crash cost} = 3 \times 2 = \text{Rs. 6 thousands}$$

$$\text{Total duration} = 13 \text{ weeks}$$

Modified network becomes as shown below.



Critical path is 1-3-5-6. We can't crash activity 1-3 further. Among activities 3-5 and 5-6, activity 5-6 has least cost slope.

Crash activity 5-6 by 2 weeks

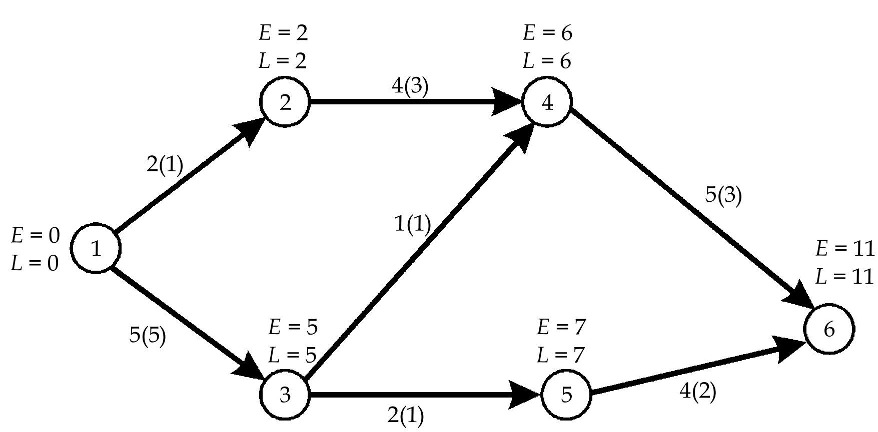
$$\text{Crash cost} = 2 \times 6 = \text{Rs. 12 thousands}$$

$$\text{Total duration} = 11 \text{ weeks}$$

$$\text{Total cost} = 82 + 11 \times 5 + 3 \times 2 + 2 \times 6$$

$$= \text{Rs. 155 (in thousand)}$$

Modified network is shown below.



Now, 1-2-4-6, 1-3-4-6 and 1-3-5-6 will become critical paths.

Options available are :

Activities 4 - 6, 5 - 6 $\rightarrow$ 3 + 6 = Rs. 9 thousands (minimum)

1-2, 3-5 $\rightarrow$ 5 + 7 = Rs. 12 thousands

2-4, 3-5 $\rightarrow$ 4 + 7 = Rs. 11 thousands

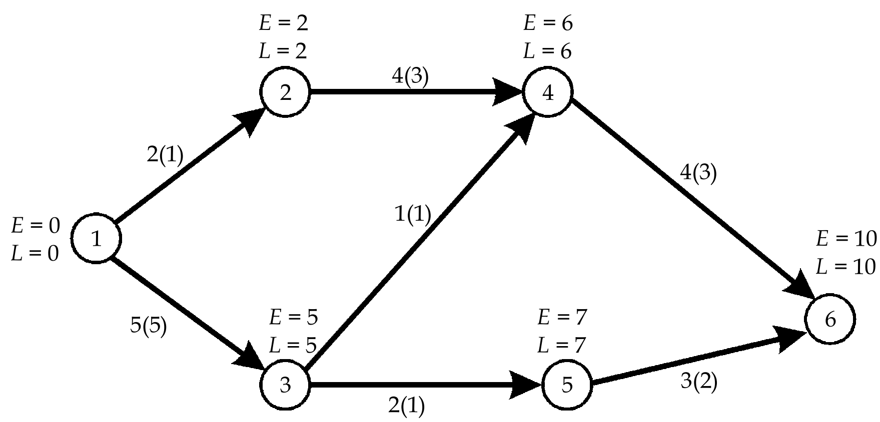
2-4, 5-6 $\rightarrow$ 4 + 6 = Rs.10 thousands

Total duration = 10 weeks

Crash cost = Rs. 9 thousands

Total cost (in thousands) = Rs. (82 + 10 × 5 + 9 + 6 + 12) = Rs. 159

Now, network becomes:



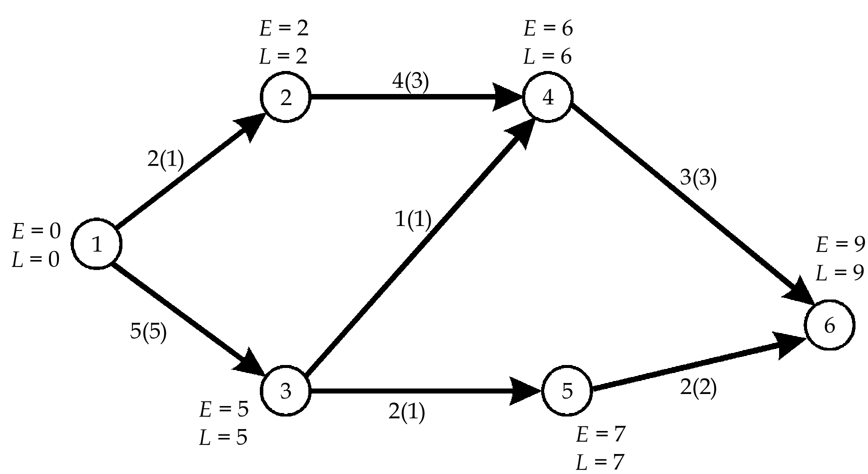
Crash activity 4-6 and 5-6 by 1 week again,

Crash cost = Rs. (3 + 6) = Rs. 9 thousands

Total cost (in thousands) = Rs. (82 + 9 × 5 + 9 + 9 + 6 + 12)

= Rs. 163

Network becomes,



Q.6 (b) Solution:

(i)

1. **Hardness Test :** A cylinder of diameter 25 mm and height 25 mm is taken out from the sample of stone and weighed. The sample is then placed in Dorry's testing machine and it is subjected to a pressure of 12.50 N/mm^2. Annular steel disc of the machine is then rotated at a speed of 28 rpm. Coarse sand of standard specification is sprinkled on the top of disc while rotating. The specimen is taken out and weighed after about a thousand revolutions. The coefficient of hardness for the stone specimen is found to judge the suitability for use.

$$\text{Coefficient of hardness} = \frac{20 - (\text{Loss of weight in g})}{3}$$

2. **Water Absorption Test:** The test procedure is as follows :

   (a) A stone sample, about 50 gm weight, is prepared. Its actual weight is recorded as W_1 gm.

   (b) Stone sample is then immersed in distilled water for a period of about 24 hours. After 24 hours the sample is taken out of water and its surface wiped off of water with a damp cloth.

   (c) It is weighed again. Let the weight be W_2 gm.

   (d) The sample is then suspended freely in water and weighed. Let this be W_3 gm.

   (e) The sample is then kept in a boiling water for about five hours.

   (f) Sample is removed and surface water is wiped off with a damp cloth. Its weight is recorded. Let it be W_4 gm.

   From the above observations, values of the following properties of stones are obtained.

$$\text{Percentage absorption by weight after 24 hours} = (W_2 - W_1) \times \frac{100}{W_1}$$

$$\text{Percentage absorption by volume after 24 hours} = (W_2 - W_1) \times \frac{100}{(W_2 - W_3)}$$

$$\text{Volume of displaced water} = W_2 - W_3$$

$$\text{Percentage porosity by volume} = (W_4 - W_1) \times \frac{100}{(W_2 - W_3)}$$

$$\text{Specific gravity} = \frac{W_1}{(W_2 - W_3)} kg/\text{m}^3$$

$$\text{Saturation coefficient} = \frac{\text{Water absorption}}{\text{Total porosity}} = \frac{(W_2 - W_1)}{(W_4 - W_1)}$$

3. **Acid Test:** A sample of stone weighing about 50 to 100 gm is placed in a solution of hydrochloric acid (HCl) (one percent strength) for seven days. The solution is agitated at regular intervals. A stone that maintains its sharp edges and keeps its surface free from powder at the end of this period is considered suitable for building material purposes. If the stone specimen contains calcium carbonate then its edges get broken and powder is formed on the surface. Such a stone possesses poor weathering resistance. This test is more commonly performed on sandstones.

(ii)

The lime is an important engineering material and its uses can be enumerated as follows:

- It is used as a chemical raw material in the purification of water and for sewage treatment.
- It is used as a flux in the metallurgical industry.
- It is used as a matrix for concrete and mortar.
- It is used as a refractory material for lining of open-hearth furnaces.
- It is used in the production of glass.
- It is used for making mortar for masonry work.
- It is used for plastering of walls and ceilings.
- It is used for soil stabilisation.
- It is used for whitewashing and for serving as a basic coat for distemper.
- Lime mortar can be used in place of the costly cement mortar.

Tests for lime:

Physical properties :

- Pure limestone is indicated by white colour.
- The hydraulic limestones are indicated by bluish grey, brown or some dark colour.
- The presence of lumps give an indication of quick lime or unburnt limestone.

Heat test:

- A piece of dry limestone is weighted and it is heated in an open fire for few hours. The sample is weighed again and loss of weight indicates the amount of carbon dioxide.

Acid test:

- A teaspoon of powdered lime is taken in a test tube and dilute hydrochloric acid is poured in it. The contents are now stirred and the test tube with its contents is then kept standing in its stand for 24 hours.
- If content of calcium carbonate is high, there will be vigorous efflorescence and less formation of residue.
- If a thick jet is formed, it indicates lime of class *A*.

Ball test:

- The balls of about 40 mm size of stiff lime formed by adding enough water are made and they are left undisturbed for six hours. The balls are then placed in a basin of water. If there are signs of slow expansion and slow disintegration within minutes after placing in water, it indicates class *C* lime.

Q.6 (c) Solution:

Let us consider the member BD as redundant.

Let us remove this member and find the forces in the members.

Joint *B* $P_{ba} = 0, P_{bc} = 0,\quad \tan\theta = \frac{5}{4}, \sin\theta = \frac{5}{\sqrt{41}}$ and $\cos\theta = \frac{4}{\sqrt{41}}$

Taking moments about, D, $V_c \times 4 = 60 \times 5$

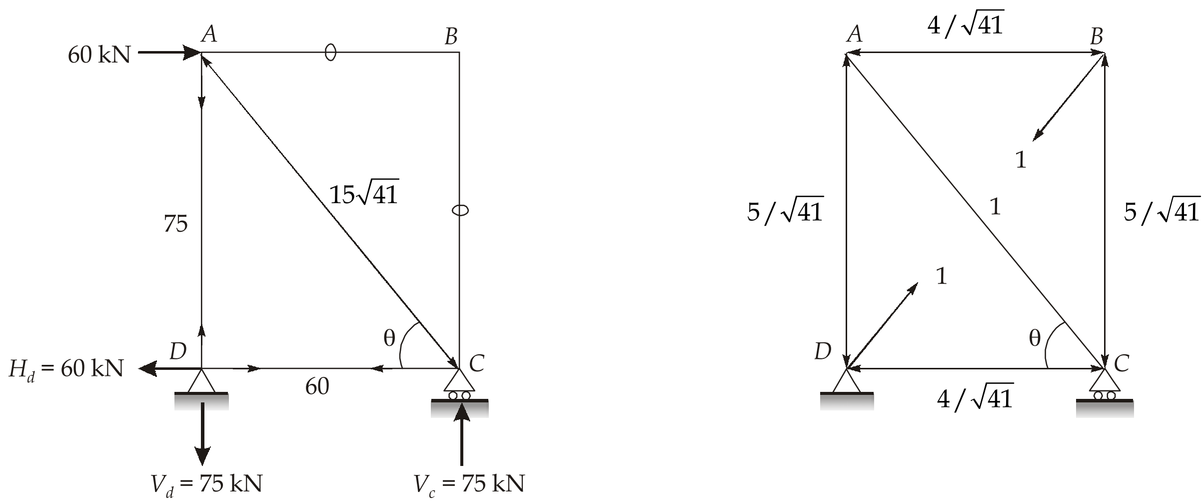
$\therefore \quad V_c = 75 \text{ kN} \uparrow$ and $V_d = 75 \text{ kN} \downarrow$ and $H_d = 60 \text{ kN} \leftarrow$

Joint C Resolving vertically, $P_{ca} \sin\theta = 75$

$$\therefore \quad P_{ca} = \frac{75\sqrt{41}}{5} = 15\sqrt{41} \text{ kN (compressive)}$$

Resolving horizontally,

$$P_{cd} = 15\sqrt{41}\cos\theta = 15\sqrt{41} \times \frac{4}{\sqrt{41}} = 60 \text{ kN(tensile)}$$



Joint D Resolving vertically, $P_{da} = 75$ kN (tensile)

Now remove the given loads and apply 1 kN loads at B and D in place of the member BD. Obviously there will be no reactions at the supports.

Joint *D* Resolving horizontally, $K_{dc} = 1 \cos\theta$

$$= \frac{4}{\sqrt{41}} \text{ kN (compressive)}$$

Resolving vertically, $K_{da} = 1 \sin\theta = \dfrac{5}{\sqrt{41}}$ kN (compressive)

Similarly, by considering the joint *B*,

$$K_{ba} = \frac{4}{\sqrt{41}} \text{ kN (compressive) and } K_{bc} = \frac{5}{\sqrt{41}} \text{ kN (compressive)}$$

Joint A Resolving vertically,

$$K_{ac} \sin\theta = \frac{5}{\sqrt{41}}$$

$$\therefore \quad K_{ac} = 1\ kN \text{ (tensile)}$$

The actual force in any member is given by

$$S = P + XK, \qquad \text{where } X = -\frac{\Sigma \frac{PKL}{A}}{\Sigma \frac{K^2L}{A}}$$

The summation $\Sigma \dfrac{PKL}{A}$ and $\Sigma \dfrac{K^2L}{A}$ are done below

Member	P	K	L(mm)	$A\left(mm^2\right)$	$\frac{PKL}{A}$	$\frac{K^2L}{A}$
AB	0	$+\frac{4}{\sqrt{41}}$	4000	900	0	1.7344
BC	0	$+\frac{5}{\sqrt{41}}$	5000	1200	0	2.5407
CD	–60	$+\frac{4}{\sqrt{41}}$	4000	900	–166.585	1.7344
DA	–75	$+\frac{5}{\sqrt{41}}$	5000	1200	–244.022	2.5407
AC	$15\sqrt{41}$	–1	$1000\sqrt{41}$	1500	–410	4.2687
BD	0	–1	$1000\sqrt{41}$	1500	0	4.2687
				Total	**–820.607**	**17.0876**

$\therefore$ $$X = -\left(\frac{-820.607}{17.0876}\right) = 48.0235$$

$\therefore$ The actual force in any member is given by,

$$S = P + 48.0235\, K$$

The actual forces in the members are calculated below:

$$S_{ab} = P_{ab} + X\,K_{ab} = 0 + 48.0235 \times \frac{4}{\sqrt{41}} = +\,30 \text{ kN (compressive)}$$

$$S_{bc} = P_{bc} + X\,K_{bc} = 0 + 48.0235 \times \frac{5}{\sqrt{41}} = +\,37.5 \text{ kN (compressive)}$$

$$S_{cd} = P_{cd} + X\,K_{cd} = -\,60 + 48.0235 \times \frac{4}{\sqrt{41}} = -\,30 \text{ kN (tensile)}$$

$$S_{da} = P_{da} + X\,K_{da} = -\,75 + 48.0235 \times \frac{5}{\sqrt{41}} = -\,37.5 \text{ kN (tensile)}$$

$$S_{ac} = P_{ac} + X\,K_{ac} = 15\sqrt{41} + 48.0235(-1) = +\,48.023 \text{ kN (compressive)}$$

$$S_{bd} = P_{bd} + X\,K_{bd} = 0 + 48.0235\,(-1) = -\,48.023 \text{ kN (tensile)}$$

Q.7 (a) Solution:

For Fe410 steel,

$$f_u = 410 \text{ N/mm}^2$$
$$f_y = 250 \text{ N/mm}^2$$
$$A_g = 2058 \text{ mm}^2$$

(i) Tensile strength due to gross-section yielding

$$T_{dg} = \frac{A_g f_y}{\gamma_{m0}} = \frac{2058 \times 250}{1.1} \text{N} = 467.73 \text{ kN}$$

(ii) Tensile strength due to rupture of critical section

Length of outstanding leg (w) = 115 mm

Weld length along the direction of load (L_c) = 150 mm

Width of shear lag (b_s) = w = 115 mm

$$\beta = 1.4 - 0.076\left(\frac{w}{t}\right)\left(\frac{f_y}{f_u}\right)\left(\frac{b_s}{L_c}\right) \le \frac{0.9 f_u \gamma_{mo}}{f_y \gamma_{m1}} \ge 0.7$$

$$= 1.4 - 0.076\left(\frac{115}{8}\right)\left(\frac{250}{410}\right)\left(\frac{115}{150}\right) = 0.8893$$

$$\frac{0.9 f_u \gamma_{m0}}{f_y \gamma_{m1}} = \frac{0.9 \times 410 \times 1.1}{250 \times 1.25} = 1.29888 > 0.7$$

$\therefore$ $$\beta < \frac{0.9 f_u \gamma_{mo}}{f_y \gamma_{m1}} > 0.7 \qquad \text{(OK)}$$

$\therefore$ $$\beta = 0.8893$$

Net area of connected leg (A_{nc}) = $\left(150 - \frac{8}{2}\right)8 = 1168 \text{ mm}^2$

Gross area of outstanding leg (A_{go}) = $\left(115 - \frac{8}{2}\right)8 = 888 \text{ mm}^2$

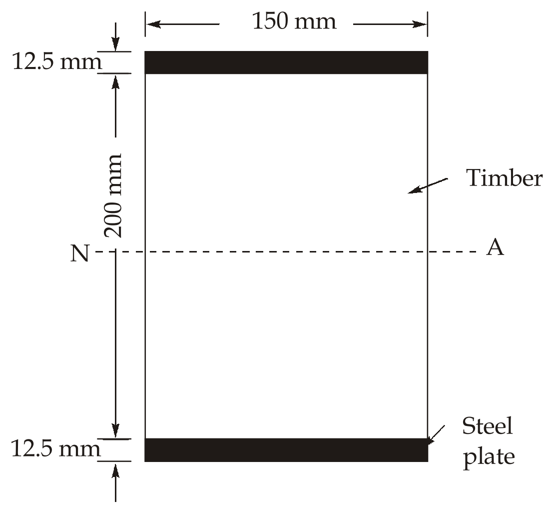
$\therefore$ $$T_{dn} = \frac{0.9 A_{nc} f_u}{\gamma_{m1}} + \beta \frac{A_{go} f_y}{\gamma_{m0}} = \frac{0.9(1168)410}{1.25} + \frac{0.8893(888)250}{1.1}$$

$$= (344.79 + 179.48) \text{ kN} = 524.27 \text{ kN}$$

Q.7 (b) Solution:

Given:

1st Case: Flitches attached symmetrically at the top and bottom.



Suffix 1 represents steel and suffix 2 represents timber.

Width of steel, $b_1 = 150$ mm

Depth of steel, $d_1 = 12.5$ mm

Width of timber, $b_2 = 150$ mm

Depth of timber, $d_2 = 200$ mm

Number of steel plates $= 2$

Max. stress in timber, $\sigma_2 = 6\text{N/mm}^2$

E for steel, $E_1 = E_s = 2 \times 10^5\ \text{N/mm}^2$

E for timber, $E_2 = E_t = 1 \times 10^4\ \text{N/mm}^2$

Distance of extreme fibre of timber from N.A.,

$$y_2 = 100 \text{ mm}$$

Distance of extreme fibre of steel from N.A.,

$$y_1 = 100 + 12.5 = 112.5 \text{ mm.}$$

Let σ_1 = Max. stress in steel

∴ σ_1 = Stress in steel at a distance of 100 mm from N.A.

Now we know that strain at the common surface is same. The strain at a common distance of 100 mm from N.A. is steel and timber would be same. Hence using equation, we get

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\Rightarrow \qquad \sigma_1 = \frac{E_1}{E_2} \times \sigma_2 = \frac{2 \times 10^5}{1 \times 10^4} \times 6 = 120 \text{ N/mm}^2.$$

But σ_1 is the stress in steel at a distance of 100 mm from N.A. Maximum stress in steel would be at a distance of 112.5 mm from N.A. As bending stresses are proportional to the distance from N.A.

Hence
$$\frac{\sigma_1}{100} = \frac{\sigma_1'}{112.5}$$

$$\Rightarrow \qquad \sigma_1' = \frac{112.5}{100} \times \sigma_1 = \frac{112.5}{100} \times 120 = 135 \text{ N/mm}^2.$$
Ans.

Now moment of resistance of steel is given by

$$M_1 = \frac{\sigma_1'}{y_1} \times I_1 \text{ (where } \sigma_1' \text{ is the maximum stress in steel)}$$

$$= \frac{135}{112.5} \times I_1$$

Where I_1 = M.O.I of two steel plates about N.A.

= 2 × [M.O.I. one steel plate about its C.G. + Area of one steel plate × (Distance between its C.G. and N.A.)^2]

$$= 2 \times \left[\frac{b_1 d_1^3}{12} + b_1 d_1 \times \left(100 + \frac{d_1}{2}\right)^2\right]$$

$$= 2 \times \left[\frac{150 \times 12.5^3}{12} + 150 \times 12.5 \times \left(100 + \frac{12.5}{2}\right)^2\right]$$

$$= 2 \times (24414.06 + 21166992.19)$$

$$= 42382812.5 \text{ mm}^4$$

$$\therefore \qquad M_1 = \frac{135}{112.5} \times 42382812.5$$

$$= 50859374.99 \text{ Nmm} = 50859.375 \text{ Nm}$$

Similarly,
$$M_2 = \frac{\sigma_2}{y_2} \times I_2 = \frac{6}{100} \times \frac{150 \times 200^3}{12}$$

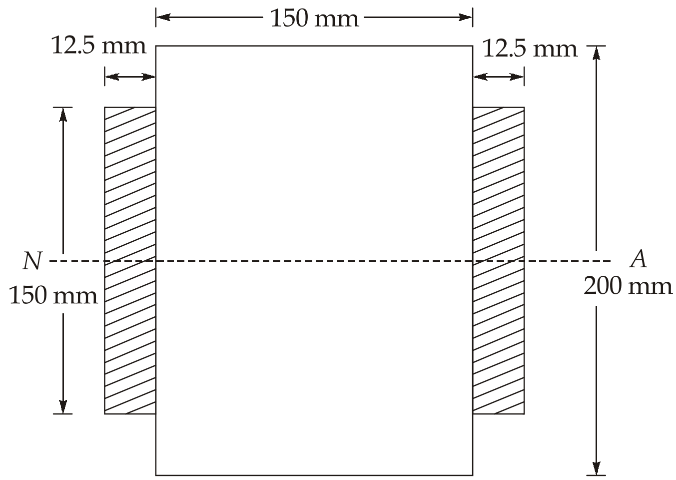
$$= 6000000 \text{ Nmm} = 6000 \text{ Nm}$$

$\therefore$ Total moment of resistance is given by,

$$M = M_1 + M_2$$

$$= 50859.375 + 6000 = 56859.375 \text{ Nm.}$$
Ans.

2nd Case: Flitches attached symmetrically on the sides of timber beam.



Here distance of the extreme fibre of steel from N.A.

$$= \frac{150}{2} = 75 \text{ mm.}$$

In the first case we have seen that stress in steel at a distance of 100 mm from N.A. is 120 N/mm^2.

Hence the stress in steel at a distance of 75 mm from N.A. is given by,

$$\sigma_1'' = \frac{120}{100} \times 75$$

($\because$ Stress are proportional to the distance from N.A.)

$$= 90 \text{ N/mm}^2$$

$\therefore$ Maximum stress in steel = $\sigma_1'' = 90 \text{ N/mm}^2$. **Ans.**

Total moment of resistance of two steel plates

$$= \frac{\sigma_1''}{y_{max}} \times I_1$$

(Here σ_1'' = Maximum stress in steel = 90 N/mm^2)

$$= \frac{90}{75} \times I_1 \qquad (y_{max} = 75 \text{ mm})$$

I_1 = M.O.I of two steel plates about N.A.

$$= 2 \times \frac{12.5 \times 150^3}{12} = 7031250 \text{ mm}^4$$

$\therefore$ $$M_1 = \frac{90}{75} \times 7031250 \text{ Nmm} = 8437500 \text{ Nmm} = 8437.5 \text{ Nm.}$$

Similarly, M_2 = Moment of resistance of timber section

$$= \frac{\sigma_2}{y_2} \times I_2$$

$$= \frac{6}{100} \times \frac{150 \times 200^3}{12} \quad \left(\because I_2 = \frac{150 \times 200^8}{12}\right)$$

$$= 6000000 \text{ Nmm} = 6000 \text{ Nm}$$

$\therefore$ Total moment of resistance,

$$M = M_1 + M_2$$

$$= 8437.5 + 6000 = 14437.5 \text{ Nm.}$$ **Ans.**

Q.7 (c) Solution:

(i) (a)

Silica fume: It is a product resulting from reduction of high purity quartz with coal in an electric arc furnace in the manufacture of silicon or ferrosilicon alloy.

- Micro silica is initially produced as an ultrafine undensified powder
- At least 85% SiO_2 content
- Mean particle size between 0.1 and 0.2 micron
- Minimum specific surface area is 15,000 m^2/kg
- Spherical shape of particles

Effect on fresh concrete:

- The increase in water demand of concrete containing microsilica will be about 1% for every 1% of cement substituted.
- Lead to lower slump but more cohesive mix.
- Make the fresh concrete sticky in nature and hard to handle.
- Large reduction in bleeding and concrete with microsilica could be handled and transported without segregation.
- Produces more heat of hydration at the initial stage of hydration.
- The total generation of heat will be less than that of standard concrete.

Effect on hardened concrete:

- Modulus of elasticity of microsilica concrete is less.

(b)

Rice husk ash: Rice husk ash is obtained by:

- Burning rice husk in a controlled manner without causing environmental pollution.
- Material of future as mineral additives.

Amount used:

- 10% by weight of cement.
- It greatly enhances the workability and impermeability of concrete.

Contains:

- Amorphous silica (90% SiO_2) in very high proportion when burnt in controlled manner.
- 5% carbon.
- 2% K_2O.

Effects:

- Reduces susceptibility to acid attack and improves resistance to chloride penetration.
- Reduces large pores and porosity thereby resulting concrete is of very low permeability.
- Reduces the free lime present in the cement paste.
- Improves overall resistance to CO_2 attack.
- Enhances resistance to corrosion of steel in concrete.
- Reduces micro cracking and improves freeze-thaw resistance.
- Improves capillary suction and accelerated chloride diffusivity.

(ii)

Analysis of rates: The determination of rate per unit of a particular item of work, from the cost of quantities of materials, the cost of labourers and other miscellaneous petty expenses required for its completion is known as analysis of rate.

In order to assess the rate of a particular item of work from the quantities of materials and labours required, hiring of tools and plants, water charges, contractor's profit etc., an analysis of rate is carried out. It is done in order to have an idea about the cost estimate of the work. Working out estimate of the work to be done is quite important before floating a tender. Contractor's profit is added on all items of work which have been arranged by him.

Analysis of rates comprises of the following components:

(a) Cost of material

(b) Cost of labour

(c) Tools and plant and sundries (miscellaneous items)

(d) Carriage or the transportation cost

(e) Contractor's profit (usually it is kept between 10-15%).

Purpose of rate analysis:

1. To determine the authenticity of rates quoted by contractor.
2. To assess the quantity of materials and labours required for work.
3. To reconsider and revise the schedule of rates due to cost revision in materials, labour etc.
4. To determine the prevalent rates for doing the work in the location under consideration.

Factors affecting rate analysis:

The following factors affect the rate of a particular item of work:

(i) Specifications of work and materials, quality of materials, proportion of mix, method of construction etc.

(ii) Quantities of materials and their rates.

(iii) Number of different types of labour and their rates.

(iv) Location of site of work and its distance from the sources of materials and rates of transport.

(v) Availability of water.

(vi) Miscellaneous and overhead expenses of contractor.

(vii) Site conditions.

Q.8 (a) Solution:

Distribution Factors.

Joint	Member	Stiffness	Total Stiffness	Distribution factors
B	*BA*	$\frac{4EI}{4} = EI$	2EI	$\frac{1}{2}$
	BC	$\frac{4EI}{4} = EI$		$\frac{1}{2}$

Fixed End Moments:

$$\overline{M}_{ab} = \frac{20\times1\times3^2}{4^2} = -11.25 \text{ kNm}$$

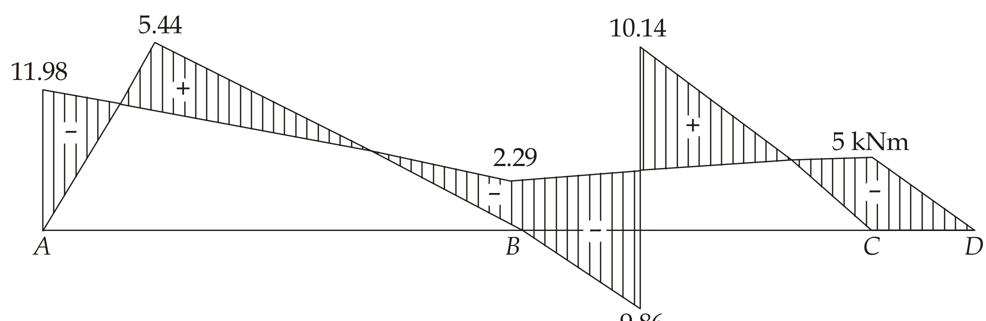
$$\overline{M}_{ba} = +\frac{20\times1^2\times3}{4^2} = +3.75 \text{ kNm}$$

$$\overline{M}_{bc} = +\frac{20\times2}{3^2}(3\times1-3) = 0$$

$$\overline{M}_{cb} = +\frac{20\times1}{3^2}(3\times2-3) = +6.67 \text{ kNm}$$

$$\overline{M}_{cd} = -5 \text{ kNm}$$

A		B		C	D
		$\frac{1}{2}$	$\frac{1}{2}$		
– 11.25		+ 3.75	+ 2.50 0 – 3.34	+ 5.00 + 6.57 – 6.67	– 5.00
– 11.25		+ 3.75 – 1.46	– 0.84 – 1.45	+ 5.00	– 5.00
– 0.73					
– 11.98		+ 2.29	– 2.29	– 5.00	– 5.00



BMD

Calculation of support reactions

$$\text{B.M. at } B = V_a \times 4 - 20\times3 - 11.98 = -2.29$$

$$\Rightarrow \quad V_a = 17.42 \text{ kN}$$

$$\text{B.M. at } B = V_c \times 3 - 50\times4 - 20 = -2.29$$

$$V_c = 12.57 \text{ kN}$$

$$V_b = [20+5] - [17.42+12.57] = -4.99 \text{ kN} \downarrow$$

Q.8 (b) Solution:

(i)

Strength of weld per mm length,

$$= 0.7 \times 6 \times 102.5 = 430.5 \text{ N}$$

Total effective length of weld required = L

$$= \frac{250000}{430.5}$$

$$\therefore \quad L = 580.72 \text{ mm}$$

Let 'L_1' and 'L_2' be the weld lengths on two sides of the angle

$$\therefore \quad 2(L_1 + L_2) = L = 580.72$$

$$\Rightarrow \quad L_1 + L_2 = 290.36 \text{ mm}$$

The lengths 'L_1' and 'L_2' are such that the resultant strength of the weld passes through the C.G. of the section.

$$\therefore \quad \frac{L_1}{90 - 28.7} = \frac{L_1 + L_2}{90}$$

$$\Rightarrow \quad L_1 = (90 - 28.70) \times \frac{290.36}{90} = 197.8 \text{ mm}$$

$$\therefore \quad L_2 = 290.36 - 197.8 = 92.56 \text{ mm}$$

$\therefore$ Adopt $L_1 = 200$ mm

$L_2 = 100$ mm

(ii)

Assessed value:

- It is the value of property recorded in the register of local authority and used for the purpose of determining the various taxes to be collected from the owner of the property.
- The rate of property taxes as government decides is applicable on assessed value of property.

Sinking fund:

- It is the fund which is built-up for the sole purpose of replacement or reconstruction of a property when it losses its utility either at the end of its useful life or becoming absolete.
- The fund is regularly deposited in a bank or with an insurance agency so that on the expiry of period of utility of the building, sufficient amount is available for its replacement.

- The calculation of sinking fund depends upon the life of building as well as the rate of interest.

(iii)

Given:

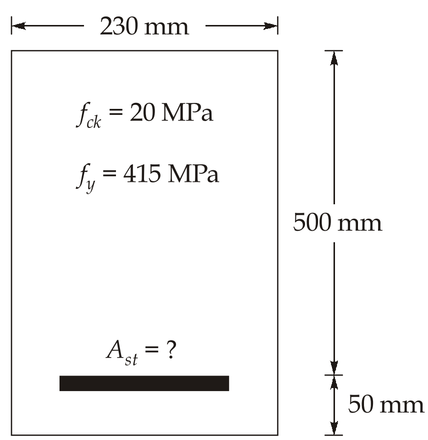
$$I_p = 0.08; \quad i = 8\%$$

$$I_c = \frac{i}{(1+i)^n - 1} = \frac{0.08}{(1+0.08)^{60} - 1} = 0.0008$$

Year's purchase, $$YP = \frac{1}{0.08+0.0008} = \frac{1}{0.0808} = 12.38$$

Value of the property = 16000 × 12.38 = Rs. 198080

Q.8 (c) Solution:



Given: $b = 230$ mm, $d = 500$ mm

$$M_u = 200 \text{ kNm} = 200 \times 10^6 \text{ Nmm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$\frac{d'}{d} = \frac{50}{500} = 0.1$$ [50 mm effective cover for compression steel]

∴ $f_{sc} = 353 \text{ N/mm}^2$ [From given table]

For Fe 415, $x_{u\,max} = 0.48\, d = 0.48 \times 500 = 240$ mm

- Limiting moment of resistance ($M_{u\,lim}$)

$$M_{u\,lim} = 0.36\, f_{ck}\, b.\, x_{u\,max}\,(d - 0.42\, x_{u\,max})$$

⇒ $$M_{u\,lim} = 0.36 \times 20 \times 230 \times 240\,(500 - 0.42 \times 240)$$

$\Rightarrow \quad M_{u\,lim} = 158658048 \text{ Nmm} = 158.65 \text{ kNm} > M_u \text{ (= 200 kNm)}$

$\therefore \quad M_{u_2} = M_u - M_{u\,lim}$

$\Rightarrow \quad M_{u_2} = 200 \times 10^6 - 158.65 \times 10^6 = 41.35 \text{ kNm}$

- Area of tension steel (A_{st})

$$A_{st} = A_{st_1} + A_{st_2}$$

$$A_{st_1} = \frac{M_{u\,lim}}{0.87 f_y (d - 0.42 x_{u\,max})}$$

$$\Rightarrow \quad A_{st_1} = \frac{158.65 \times 10^6}{0.87 \times 415 (500 - 0.42 \times 240)} = 1100.7 \text{ mm}^2$$

Now,

$$A_{st_2} = \frac{M_{u_2}}{0.87 f_y (d - d')} = \frac{41.35 \times 10^6}{0.87 \times 415 (500 - 50)}$$

$$\Rightarrow \quad A_{st_2} = 254.5 \text{ mm}^2$$

$$\text{Total area of tension steel} = A_{st_1} + A_{st_2}$$

$$\Rightarrow \quad A_{st} = 1100.7 + 254.5 = 1355.2 \text{ mm}^2$$

$$\text{Area of one 20 mm bar} = \frac{\pi}{4} \times 20^2 = 314 \text{ mm}^2$$

$$\text{No. of bars required} = \frac{1355.2}{314} = 4.3 \text{ say } 5$$

$\therefore$ Provide 5-20 mm diameter bars as tension steel.

Area of compression steel (A_{sc})

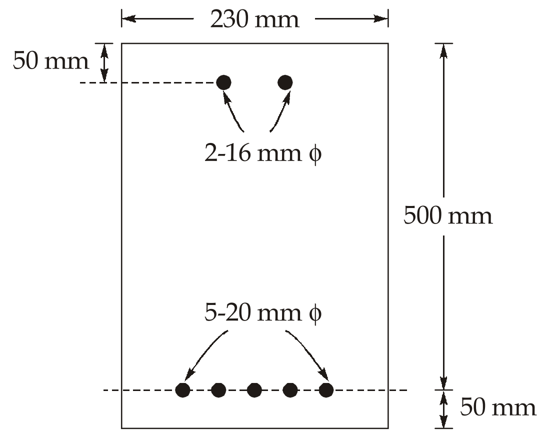
$$A_{sc} = \frac{M_{u2}}{f_{sc}(d - d')}$$

$$\Rightarrow \quad A_{sc} = \frac{41.34 \times 10^6}{353(500 - 50)} = 260.2 \text{ mm}^2$$

$$\therefore \quad \text{Area of one 16 mm dia bar} = \frac{\pi}{4} \times 16^2 = 201 \text{ mm}^2$$

$$\text{No. of 16 mm dia bars required} = \frac{260.2}{201} = 1.3 \text{ say } 2$$

$\therefore$ Provide 2 - 16 mm diameter bars as compression steel.


230 mm
50 mm
2-16 mm ϕ
500 mm
5-20 mm ϕ
50 mm

■■■■