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UPSC ESE 2020

Main Exam Detailed Solutions

Electrical Engineering

PAPER-II

EXAM DATE : 18-10-2020 | 2:00 PM to 5:00 PM

MADE EASY has taken due care in making solutions. If you find any discrepency/ error/typo or want to contest the solution given by us, kindly send your suggested answer with detailed explanations at info@madeeasy.in

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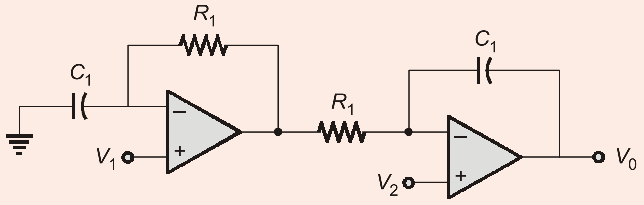
Electrical Engineering Paper Analysis
ESE 2019 Main Examination

Sl.	Subjects	Total Marks
1.	Analog and Digital Electronics	32
2.	Power Systems	104
2.	Systems & Signal Processing	72
4.	Control Systems	64
5.	Electrical Machines	104
6.	Power Electronics	84
7.	Communication Systems	20
	Total	**480**

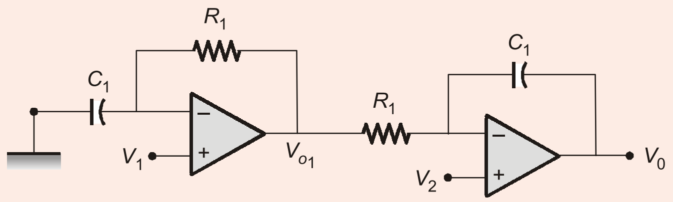
Scroll down for detailed solutions

1. (a) For the circuit shown in the figure below, derive the expression for output voltage and sketch the nature of the output when V_2 = 10 V and V_1 = 5 V.



[12 Marks]

Solution:



For op-Amp (1)

$$V_{o_1}(s) = \left[1+\frac{R_1}{\frac{1}{sC_1}}\right]\cdot\frac{V_1}{s} = [1+sR_1C_1]\cdot\frac{V_1}{s}$$

For op-Amp (2)

$$V_o(s) = -\frac{\frac{1}{sC_1}}{R_1}V_{o_1}(s)+\left[1+\frac{\frac{1}{sC_1}}{R_1}\right]\cdot\frac{V_2}{s}$$

$$V_o(s) = -\frac{1}{sC_1R_1}V_{o_1}(s)+\left[1+\frac{1}{sC_1R_1}\right]\cdot\frac{V_2}{s}$$

$$V_o(s) = -\frac{1}{sC_1R_1}[1+sC_1R_1]\cdot\frac{V_1}{s}+\left[1+\frac{1}{sC_1R_1}\right]\cdot\frac{V_2}{s}$$

$$= -\frac{V_1}{s^2R_1C_1}-\frac{V_1}{s}+\frac{V_2}{s}+\frac{V_2}{s^2R_1C_1}$$

On taking inverse Laplace transform.

$$V_o(t) = -V_1\frac{t}{\tau}-V_1\,u(t)+V_2\,u(t)+V_2\cdot\frac{t}{\tau} \quad [\because\ \tau = R_1C_1]$$

$$= (V_2-V_1)\cdot\frac{t}{\tau}+(V_2-V_1)\cdot u(t)$$

$$V_o(t) = (10-5)\frac{t}{\tau}+(10-5)\cdot u(t) = \frac{5t}{\tau}+5u(t)$$

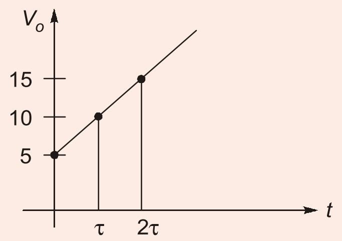
$\Rightarrow$ at $t = 0$

$V_o = 5V$

$\Rightarrow$ at $\quad t = \tau,\ V_o = 10$

$\Rightarrow$ at $\quad t = 2\tau,\ V_o = 15$

Output waveform:

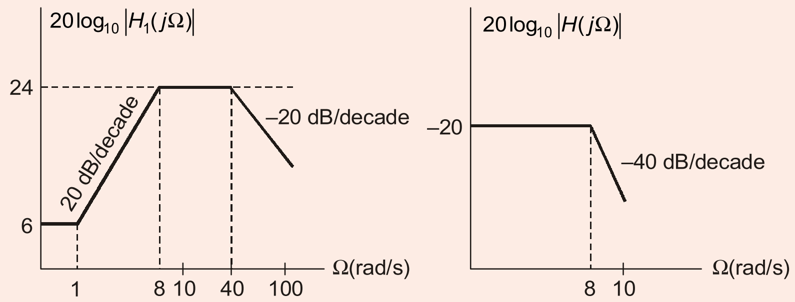


MADE EASY Source

- **MADE EASY Classnotes**

End of Solution

1. (b) A continuous LTIV system S with frequency response $H\,(j\Omega)$ is constructed by cascading two continuous-time LTIV systems with frequency response $H_1\,(j\Omega)$ and $H_2\,(j\Omega)$, respectively. Figures (a) and (b) show the straight-line approximations of Bode magnitude plots of $H_1\,(j\Omega)$ and $H(j\Omega)$, respectively. Find $H_2\,(j\Omega)$.



(a) (b)

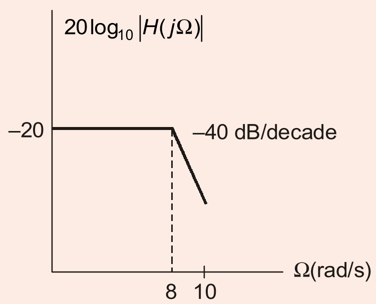
[12 Marks]

Solution:

$$H(j\Omega) = H_1(j\Omega) \cdot H_2(j\Omega)$$

$$H_2(j\Omega) = \frac{H(j\Omega)}{H_1(j\Omega)}$$

Forming $H(j\Omega)$:



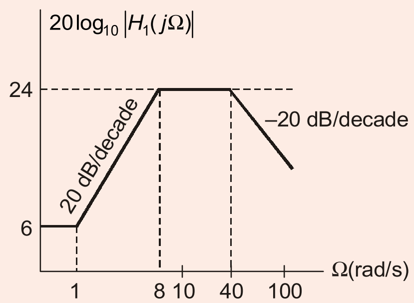
$$20\log_{10} K = -20$$
$$\log_{10} K = -1$$
$$K = 10^{-1} = 0.1$$

and it has 2 poles at $s = -8$

$$\therefore \quad H_1(j\Omega) = \frac{0.1}{\left(1+\frac{s}{8}\right)^2} = \frac{0.1\times 8^2}{(s+8)^2}$$

$$H_1(j\Omega) = \frac{6.4}{(s+8)^2}$$

Forming $H_1(j\Omega)$:



$$20\log_{10} K = 6$$
$$\log_{10} K = \frac{6}{20}$$
$$K = (10)^{6/20} = 2$$

it has a zero at $s = -1$ and a pole at $s = -8$ and one more pole at $s = -40$

$$H_1(j\Omega) = \frac{2\left(\frac{s}{1}+1\right)}{\left(\frac{s}{8}+1\right)\left(\frac{s}{40}+1\right)} = \frac{2(s+1)\times 8\times 40}{(s+8)(s+40)}$$

$$H_1(j\Omega) = \frac{640(s+1)}{(s+8)(s+40)}$$

$$H_2(j\Omega) = \frac{H(j\Omega)}{H_1(j\Omega)} = \frac{\frac{6.4}{(s+8)^2}}{\frac{640(s+1)}{(s+8)(s+40)}}$$

$$H_2(j\Omega) = \frac{0.01(s+40)}{(s+8)(s+1)}$$

End of Solution

1. (c) Consider a three-phase induction motor with the following parameters :

Number of poles	:	4
Supply frequency	:	50 Hz
Full load supply	:	1470 rpm
Rotor resistance	:	0.12 Ω
Standstill reactance	:	1.12 Ω

Find :

(i) Slip for maximum torque.

(ii) Ratio of maximum torque to full load torque.

[4 + 8 Marks]

Solution:

Given,

$$P = 4$$
$$f = 50 \text{ Hz}$$
$$N_r = 1470 \text{ rpm}$$
$$r_1' = 0.12\ \Omega$$
$$r_2' = 1.12\ \Omega$$

⇒ Synchronous speed of motor $= N_s = \dfrac{120f}{P} = \dfrac{120 \times 50}{4} = 1500$ rpm

Full load slip $S_{fl} = \dfrac{N_s - N_r}{N_s} = \dfrac{1500 - 1470}{1500} = 0.02$

(i) Slip at maximum torqueue is given by

$$s_{MT} = \frac{r_2'}{x_2'} \quad [\because \text{ When stator impedance is ignored}]$$

$$s_{MT} = \frac{0.12}{1.12} = 0.1071$$

(ii) Full load torque of induction motor is given as

$$T_{fL} = \frac{3}{\omega_s} \cdot \frac{V_{ph}^2 \cdot \dfrac{r_2'}{s_{fl}}}{\left(\dfrac{r_2'}{s_{fl}}\right)^2 + (x_2')^2} \quad \text{...(i)}$$

Maximum torque of induction motor is given as

$$T_{max} = \frac{3}{\omega_s} \cdot \frac{V_{ph}^2}{(2x_2')} \quad \text{...(ii)}$$

On dividing equation (ii) by equation (i)

$$\frac{T_{max}}{T_{fL}} = \frac{\dfrac{3}{\omega_s} \cdot \dfrac{V_{ph}^2}{(2x_2')}}{\dfrac{3}{\omega_s} \cdot \dfrac{V_{ph}^2 \cdot \dfrac{r_2'}{s_{fl}}}{\left(\dfrac{r_2'}{s_{fl}}\right)^2 + (x_2')^2}}$$

$$\frac{T_{max}}{T_{fL}} = \frac{\left(\frac{r_2'}{s_{fl}}\right)^2 + (x_2')^2}{2x_2' \cdot \frac{r_2'}{s_{fl}}}$$

On solving

$$\frac{T_{max}}{T_{fL}} = \frac{(s_{fl})^2 + (s_{MT})^2}{2 \cdot s_{fl} \cdot s_{MT}} = \frac{(0.02)^2 + (0.1071)^2}{2 \times 0.1071 \times 0.02}$$

$$\frac{T_{max}}{T_{fL}} = 2.77$$

MADE EASY Source

- **Mains Work Book:** (Q. 47 , page 123) **(Click Here for Reference)**
- **MADE EASY Classnotes**
- **Mains Classes Video Lecture**

End of Solution

1. (d) (i) What is Smart Grid?
(ii) Compared to Supervisory Control And Data Acquisition (SCADA) system, what are the advantages of Phasor Measurement Unit (PMU)?
(iii) Explain operation of PMU with a neat diagram.

[4+4+4 = 12 Marks]

Solution:

Smart grid:

A smart grid is an electrical grid which includes a variety of operational and energy measures including smart meters, smart appliances, renewable energy resources, and energy efficiency resources. Electronic power conditioning and control of the production and distribution of electricity are important aspects of the smart grid.

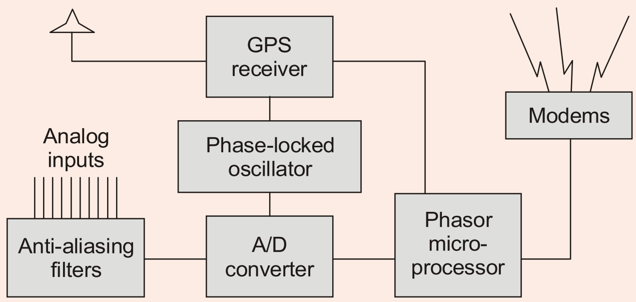
The first official definition of Smart Grid was provided by the Energy Independence and Security Act of 2007 (EISA-2007).

"It is the policy of the United States to support the modernization of the Nation's electricity transmission and distribution system to maintain a reliable and secure electricity infrastructure that can meet future demand growth and to achieve each of the following, which together characterize a Smart Grid:

- Increased use of digital information and controls technology to improve reliability, security, and efficiency of the electric grid.
- Dynamic optimization of grid operations and resources, with full cyber-security.
- Deployment and integration of distributed resources and generation, including renewable resources.
- Development and incorporation of demand response, demand-side resources, and energy- efficiency resources.
- Deployment of smart' technologies (real-time, automated, interactive technologies that optimize the physical operation of appliances and consumer devices) for metering, communications concerning grid operations and status, and distribution automation.

- Integration of smart' appliances and consumer devices.
- Deployment and integration of advanced electricity storage and peak-shaving technologies, including plug-in electric and hybrid electric vehicles, and thermal storage air conditioning.
- Provision to consumers of timely information and control options.
- Development of standards for communication and interoperability of appliances and equipment connected to the electric grid, including the infrastructure serving the grid.
- Identification and lowering of unreasonable or unnecessary barriers to adoption of smart grid technologies, practices, and services."

Phasor measurement units (PMUs) measure current and voltage by amplitude and phase at selected stations of the transmission system. The high precision time synchronization (via GPS) allows comparing measured values (synchrophasors) from different substations far apart and drawing conclusions as to the system state and dynamic events such as power swing conditions.



Block Diagram of PMU

Advantages of PMUs over SCADA system :

- Phasor measurement units (PMUs), produce accurate time stamped measurements of voltage and current magnitude as well as phasor angles.

  They also report the status of breakers with timestamps synchronized to those of the measurements. Because PMUs calculate synchrophasors with respect to a global angle reference, the number of critical measurements is less than when the state estimator uses SCADA measurements.
- Another advantage of using PMU measurements for state estimation, was that the angle measurements are made directly, thus reducing the errors introduced by inaccuracies in network parameters. The SCADA system measurements of voltage angles must be calculated using measurements of voltage, measurements of active power, reactive power measurements, network parameters, and a reference angle. Quality of results depends heavily on network quality parameters which are not always precise.

  Therefore, PMU measurements provides more direct and more precise than traditional SCADA measurements, because PMU measurements do not depend on network parameters.
- One of the major advantages of using synchrophasors is the ability to provide coherent data from different parts of the network. System operators and engineers require knowing the trends of voltage phase angle differences among coherent groups

of generators and major interconnections to monitor the stability of the system. The phase angle difference also provides knowledge to the system operators on the available power transfer margin. We cannot obtain precise high resolution voltage phase angle differences using SCADA based measurements.

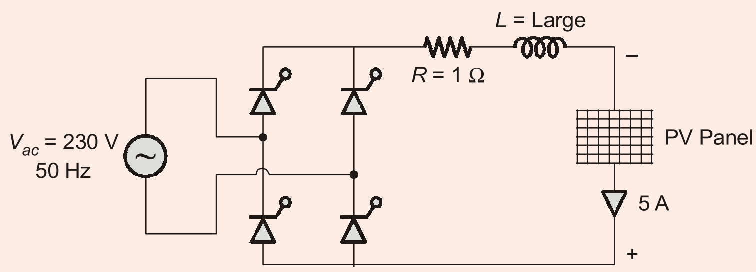
- Synchronized phasor measurements offer solutions to a number of vexing protection problems. These include the protection of series compensated lines, protection of multiterminal lines, and the inability to satisfactorily set out-of-step relays. In many situations the reliable measurement of a remote voltage or current on the same reference as local variables has made a substantial improvement in protection functions possible. In some examples communication of such measurements from one end of a protected line to the other is all that is required while in others communication across large distances is necessary.

MADE EASY Source

- **Theory Book:** Smart grid (Page No. 380) **(Click Here for Reference)**

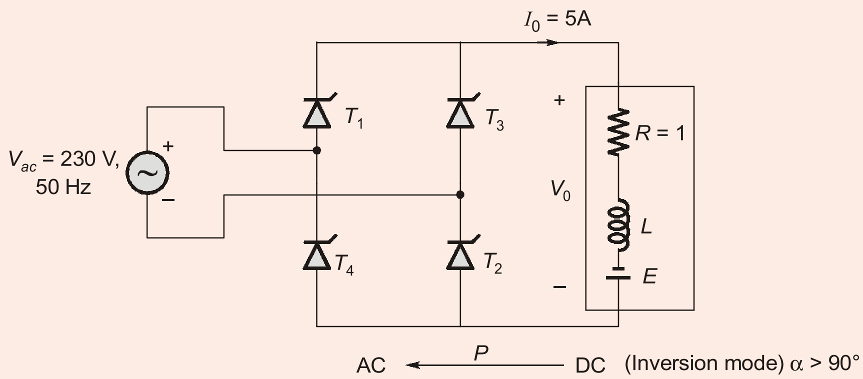
End of Solution

1. (e) A PV panel is connected with a single phase fully controlled converter as shown in the circuit below. The panel is supplying a current of 5 A and generated power is 1000 W. The series inductance in the circuit is large to make the current flat and continuous. Find (i) the triggering angle of the thyristor bridge, (ii) output voltage at rectifier terminal and (iii) input power factor.



[12 Marks]

Solution:



Generated power in the panel (solar cell)

$$= 1000\text{ W}$$

$$EI_o = 1000\text{ W}$$

$$E = \frac{1000}{I_o} = \frac{1000}{5} = 200\text{ V}$$

$$V_o = -E + I_oR$$

$$\frac{2V_M}{\pi}\cos\alpha = -200 + (5 \times 1) = -195\text{ V}$$

$$\frac{2.230\sqrt{2}}{\pi}\cdot\cos\alpha = -195$$

$$\cos\alpha = \frac{-195}{650.54} = -0.941698$$

(i) $\alpha = 160.338°$

(ii) $V_o = -195\text{ V}$

(iii) $PF = g.FDF$

$$PF = 0.9\cos\alpha$$

$$= 0.9\cos(160.3350)$$

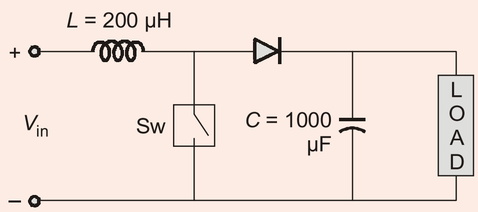
$$PF = -0.8475\text{ lag}$$

MADE EASY Source

- **ESE/Gate Classroom Workbook :** (Q.20, 25, Page 47) **(Click Here for Reference)**
- **MADE EASY Classnotes: Concept**

End of Solution

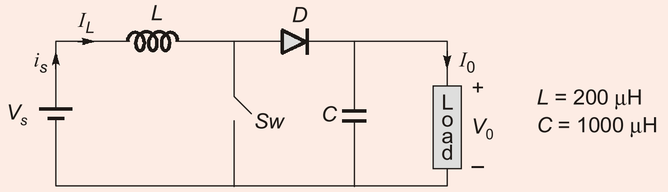
2. (a) The DC–DC converter given below is operating at 30 kHz and drawing an input current of 25 A at 48 V DC.
   (i) For a load current of 10 A, find
      I. the duty ratio of the switch,
      II. output voltage,
      III. peak inductor current,
      IV. output voltage ripple, and
      V. the load current where the inductor current just becomes discontinuous.
   (ii) Also find the critical value of L to keep the inductor current just continuous when the input voltage changes to 60 V with output remaining same. (Assume lossless operation of converter components).



[20 Marks]

Solution:

Boost Converter :



f = 30 kHz, I_s = 25 A, I_o = 10 A, V_s = 48 V

I.
$$P_o = P_{in}$$
$$V_o I_o = V_s . I_s$$
$$V_o = \frac{V_s \cdot I_s}{I_o} = \frac{48 \times 25}{10} = 120 \text{ V}$$

Let us assume it is continous conduction.

$$V_o = \frac{V_s}{1-\alpha}$$
$$120 = \frac{48}{1-\alpha}$$

II.
$$\alpha = 0.6$$
$$I_{mn} = I_L - \frac{\Delta I_L}{2}$$
$$= I_S - \frac{\alpha V_S}{2fL}$$
$$= 25 - \frac{0.6 \times 48}{2 \times 30.10^3 \times 200.10^{-6}}$$
$$I_{mn} = 22.6 \text{ A}$$

$I_{mn} > 0$ ∴ our assumption is true, i.e., continuous conduction.

III.
$$(i_L)_{peak} = I_{mx} = I_L + \frac{\Delta I_L}{2}$$
$$= 25 + \frac{\alpha V_S}{2fL}$$
$$= 25 + \frac{0.6 \times 48}{2 \times 30.10^3 \times 200.10^{-6}}$$
$$= 27.4 \text{ A}$$

IV.
$$\Delta V_o = \Delta V_c = \frac{\alpha I_o}{fC}$$
$$= \frac{0.6 \times 10}{30.10^3 \times 1000.10^{-6}} = 0.2 \text{ V}$$

V. At the Boundary :

$$I_{LB} = \frac{\Delta I_L}{2}$$
$$\frac{I_{OB}}{1-\alpha} = \frac{\alpha V_S}{2fL}$$

$$I_{OB} = \frac{\alpha(1-\alpha)V_S}{2fL}$$

$$= \frac{0.6\times(1-0.6)\times 48}{2\times 30.10^3\times 200.10^{-6}}$$

$$I_{OB} = 0.96 \text{ A}$$

(ii) Supply voltage changes to 60 V.
Output remaining same.

$$V_o = 120 \text{ V}, I_o = 10 \text{ A}$$

$$V_o I_o = V_s I_s$$

$$I_s = \frac{V_o I_o}{V_s} = \frac{120.10}{60}$$

$$I_s = 20 \text{ A}$$

$$V_o = \frac{V_s}{1-\alpha}$$

$$120 = \frac{60}{1-\alpha}$$

$$\therefore \quad 1-\alpha = \frac{60}{120} = 0.5$$

$$\alpha = 0.5$$

At the boundary :

$$I_{LB} = \frac{\Delta I_L}{2}$$

$$I_{SB} = \frac{\alpha V_S}{2fL_C}$$

$$\frac{I_{OB}}{1-\alpha} = \frac{\alpha V_S}{2fL_C}$$

$$\frac{10}{1-0.5} = \frac{0.5\times 60}{2\times 30\cdot 10^3\cdot L_C}$$

$$L_C = \frac{0.5\times 0.5\times 60}{10\times 2\times 30\times 10^3} = 0.25\times 10^{-4} \text{ H}$$

$$L_C = 25\ \mu\text{H}$$

MADE EASY Source

- **MADE EASY Classnotes: Concept (Click Here for Reference)**
- **Mains Classroom Video Lecture**

End of Solution

2. (b) A signal $m(t) = 2\cos(20\pi t) - \cos(40\pi t)$, where the unit of time is millisecond, is amplitude modulated using the carrier frequency (f_c) of 600 kHz. The AM signal is given by

$$s(t) = 5\cos 2\pi f_c t + m(t)\cos 2\pi f_c t$$

(i) Sketch the magnitude spectrum of $s(t)$. What is its bandwidth?

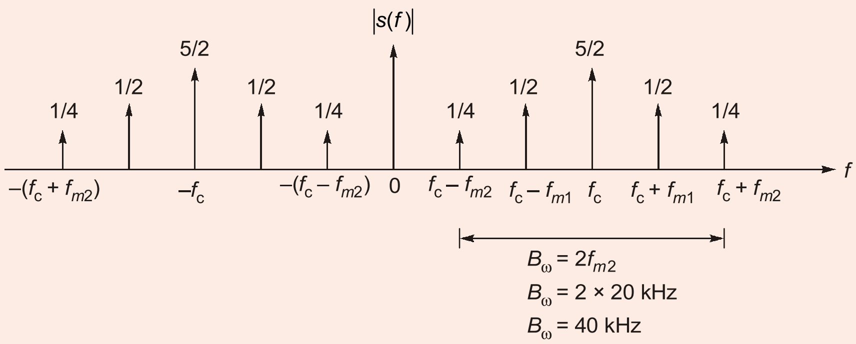
(ii) What is the modulation index?

(iii) The AM signal is passed through a high-pass filter with cut-off frequency 595 kHz (i.e., the filter passes all frequencies above 595 kHz, and cuts off all frequencies below 595 kHz). Find an explicit time-domain expression for the quadrature component of the filter output with respect to a 600 kHz frequency reference.

[20 Marks]

Solution:

(i)
$$m(t) = \cos(20\pi t) - \cos(40\pi t)$$
$$f_c = 600 \text{ kHz}$$
$$s(t) = 5\cos 2\pi f_c t + m(t)\cos 2\pi f_c t$$
$$= 5\left[1 + \frac{m(t)}{5}\right]\cos 2\pi f_c t$$
$$= 5\left[1 + \frac{2}{5}\cos(2\pi\times 10t) - \frac{1}{5}\cos(2\pi\times 20\times t)\right]\cos 2\pi f_c t$$
$$s(t) = A_c[1 + \mu_1\cos 2\pi f_{m1}t + \mu_2\cos(2\pi f_{m2}t)]\cos 2\pi f_c t$$
$$A_c = 5,\ \mu_1 = \frac{2}{5},\ f_{m1} = 10 \text{ kHz},\ \mu_2 = \frac{1}{5},\ f_{m2} = 20 \text{ kHz}$$



(ii) Modulation index of multitone AM signal is given as μ_T
The resultant modulation index/total modulation index is

$$\mu_T = \sqrt{\mu_1^2 + \mu_2^2 + ...\mu_n^2}, \qquad 0 < \mu_T < 1$$

Modulation index, $\mu_T = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{\left(\frac{2}{5}\right)^2 + \left(-\frac{1}{5}\right)^2} = \frac{1}{\sqrt{5}}$

(iii) The o/p of HPF will contains the SSB-USB components only

$$s(t) = \frac{5}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{2}[\delta(f - 610\text{ K}) + \delta(f + 610\text{ K})]$$

$$- \frac{1}{4}[\delta(f - 620\text{ K}) + \delta(f + 620\text{ K})]$$

Apply IFT

$$s(t) = 5\cos(2\pi\ 600\text{ Kt}) + \cos(2\pi\ (610\text{ Kt}) - \frac{1}{2}\cos(2\pi\ (620\text{ Kt})$$

$$= 5\cos(2\pi\ 600t) + \cos(2\pi\ 610t) - \frac{1}{2}\cos(2\pi\ 620t)$$

The quadrature component of the filter output is 0.

$$s(t) = 0$$

MADE EASY Source

- **ESE 2020 Mains Test Series:** Q.2(b) of Test-7 **(Click Here for Reference)**
- **Theory Book:** Communication system
- **MADE EASY Classnotes**

End of Solution

2. (c) A 400 V DC shunt motor has armature and field resistance of 0.2 Ω and 200 Ω respectively. It draws a current of 6 A on no-load and 70 A on full-load. If its no-load and full-load speeds are the same, determine the field weakening due to load current as percentage of no-load flux.

[20 Marks]

Solution:

Given that, V_t = 400 V DC

Armature resistance (r_a) = 0.2 Ω

Field resistance (r_{fL}) = 200 Ω

No load speed = Full load speed

$N_{NL} = N_{fL}$

Let field flux at no-load = ϕ_{NL} Wb

and field flux at full load = ϕ_{fL} Wb

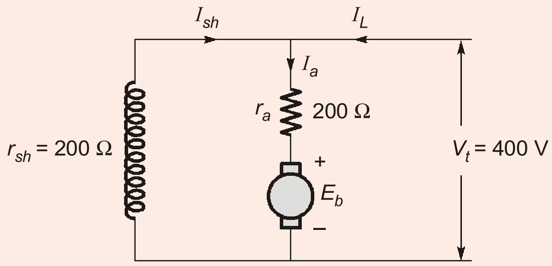
Back emf at no-load = $E_{b(NL)}$

Back emf at full load = $E_{b(FL)}$

Armature current at full load = $I_{a(FL)}$

Armature current at no-load = $I_{a(NL)}$

Electrical equivalent circuit of given shunt motor is



At no load armature current is $I_{a(NL)}$

$$I_{a(NL)} = I_{L(NL)} - I_{sh}$$

$$I_{a(NL)} = 6 - \frac{400}{200} = 4 \text{ amp}$$

Back emf of motor at no load = $E_{b(NL)}$

$$E_{b(NL)} = V_t - I_{a(NL)} \cdot r_a$$

$$E_{b(NL)} = 400 - 4 \times 0.2 = 399.2 \text{ volts}$$

At full load, armature current is = $I_{a(FL)}$

$$I_{a(FL)} = I_{L(fL)} - I_{sh}$$

$$I_{a(FL)} = 70 - \frac{400}{200} = 68 \text{ amp}$$

Back emf of motor at full load = $E_{b(FL)}$

$$E_{b(FL)} = V_t - I_{a(FL)} \cdot r_a$$

$$E_{b(FL)} = 400 - 68 \times 0.2 = 386.40 \text{ volts}$$

⇒ As we know that back emf of is directly proportional to speed and flux of motor $E_b \alpha\, N \cdot \phi$

$$\frac{E_{b(FL)}}{E_{b(NL)}} = \frac{N_{fL} \cdot \phi_{fL}}{N_{NL} \cdot \phi_{NL}}$$

$$\phi_{fL} = \frac{386.40}{399.20} \times \phi_{NL} \qquad [\because\ N_{fL} = N_{NL}]$$

$$\phi_{fL} = 0.9679\, \phi_{fL}$$

$$\text{Reduction in load flux} = \frac{\phi_{FL} - \phi_{NL}}{\phi_{NL}} = \frac{0.9679 - 1}{1}$$

Percentage reduction in flux = –0.0320 × 100% = –3.20%

So as the DC shunt motor loaded from no load to full load, by keeping the speed constant, it's field will be weak due to load current. For given motor, % reduction in field is 3.20%.

MADE EASY Source

- **Mains Classroom Video Lecture**

End of Solution

3. (a) A salient pole star connected alternator is connected to infinite bus operating at 1.0 p.u. voltage. The alternator has X_d = 0.75 p.u. and X_q = 0.5 p.u. on per phase basis. It is delivering 1.0 p.u. power to the infinite bus at 0.8 p.f. lag. Calculate (i) the load angle and excitation voltage under this condition, (ii) the maximum power that can be delivered by the alternator with same excitation and the corresponding load angle, (iii) the armature current and p.f. under maximum power condition, and (iv) the theoretical value of maximum power that the alternator can deliver when its field circuit is suddenly disconnected due to fault.

[20 Marks]

Solution:

Given, $V = 1$ pu, $P = 1$ pu

$\cos\phi = 0.8,\ \phi = 36.86$

$$I_a = \frac{P}{V\cos\phi} = \frac{1}{1\times 0.8} = 1.25 \text{ pu}$$

(i)
$$\tan\psi = \frac{V\sin\phi + I_a X_a}{V\cos\phi + I_a R_a} = \frac{1\times 0.6 + 1.25\times 0.5}{0.8}$$

$\psi = \tan^{-1}(1.53125) = 56.853°$

As we know

Load angle $(\delta) = 56.853 - 36.86 = 19.98°$ $[\because \delta = \psi - \phi]$

$I_q = I_a \cos\psi = 1.25 \times \cos(56.85) = 0.6835$

$I_d = I_a \sin\psi = 1.25 \sin(56.85) = 1.045$

$E = V\cos\delta + I_q R_a + I_d X_d$

$E = 1 \times \cos(19.98) + 1.0465 \times 0.75 = 1.724$ pu

(ii)
$$P = \frac{E\cdot V}{X_d}\sin\delta + \frac{V^2}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right]\cdot\sin 2\delta$$

$$P = \frac{1.724\times 1}{0.75}\sin\delta + \frac{1}{2}\times\frac{2}{3}\sin 2\delta$$

$P = 2.2986 \sin\delta + 0.33 \sin 2\delta$ ...(i)

For maximum power, $\dfrac{dP}{d\delta} = 0$

$$\frac{dP}{d\delta} = 2.2986\cos\delta + 0.66\cos 2\delta = 0$$

$\Rightarrow$ $\delta = 75.46°$

So maximum power delivered by salient pole generator at load angle $(\delta) = 75.46°$.

$P_{max} = 2.2986 \sin 75.46° + 0.33 \sin(2 \times 75.46°)$

$P_{max} = 2.3853$ pu

(iii)
$$Q = \frac{V\cdot E}{X_d}\cos\delta - \frac{V^2}{2X_d X_q}\left[(X_d + X_q) - (X_d + X_q)\cos 2\delta\right]$$

Relative power (Q) under maximum power delivered condition.

$$Q = \frac{1.724}{0.75}\cos(75.37) - \frac{1}{2\times0.5\times0.75}[1.25 - 0.25\cos(2\times75.37)]$$

$$Q = 0.5805 - 1.9574$$

$$Q = -1.3769 \text{ pu}$$

Power factor under maximum power delivering condition.

$$\cos\phi = \cos\left[\tan^{-1}\left(\frac{Q}{P}\right)\right] = \cos\left[\tan^{-1}\left(\frac{-1.3769}{2.387}\right)\right]$$

$$\cos\phi = 0.866 \text{ leading}$$

Armature current under maximum power delivering condition is

$$P = V_t \cdot I_a \cos\phi$$

$$I_a = \frac{2.387}{1\times0.866} = 2.7556 \text{ pu}$$

(iv) When excitation (E) is removed in generator, then generator will able to deliver reluctance power only and reluctance power will be maximum corresponds to $\delta = 45°$.

$$P_{max}\Big/_{E=0} = \frac{V_t^2}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right]$$

$$P_{max}\Big/_{E=0} = \frac{1}{2}\left[\frac{2}{3}\right] = \frac{1}{3} \text{ pu}$$

MADE EASY Source

- **ESE 2020 Mains Test Series:** Q.3(c) of Test-Mock-2 **(Click Here for Reference)**

End of Solution

3. (b) A closed loop system with unity feedback and having the forward loop transfer function as

$$G(s) = \frac{14.4}{s(1+0.1s)}$$

Modify the design using cascaded compensation to satisfy the optimum performance criterion, so that the transient response to unit step input reaches its final steady state value in minimum time without having any overshoot.

[20 Marks]

Solution:

Let the controller be PD controller.

Controller transfer function = $K_P + K_D s$

$\therefore$ Forward path transfer function = $\dfrac{14.4(K_P + K_D s)}{s(1+0.1s)}$

$\therefore$ Characteristic equation is

$$s(1 + 0.1s) + 14.4K_P + 14.4K_D s = 0$$

$$0.1s^2 + s + 14.4K_D s + 14.4K_P = 0$$
$$0.1s^2 + (14.4K_D + 1)s + 14.4K_P = 0$$
$$s^2 + (144K_D + 10)s + 144K_P = 0$$
$$\omega_n = \sqrt{144K_P}$$

Let $K_P = 2$

$$\omega_n = \sqrt{288} = 12\sqrt{2} = 16.97$$

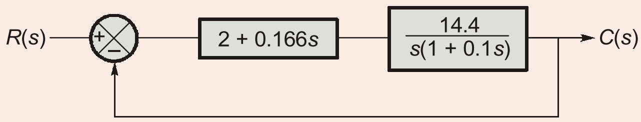
$\therefore$ $2\xi\omega_n = 144K_D + 10$

as $\omega_n = 12\sqrt{2}$ rad/sec

$$\frac{2\omega_n - 10}{144} = K_D$$

$\therefore$ $$K_D = \frac{24\sqrt{2} - 10}{144} = 0.166$$

$\therefore$ Controller transfer function is $2 + 0.166s$.



MADE EASY Source

- **MADE EASY Conventional Classnotes**

End of Solution

3. (c) Two 11 kV, 30 MVA, three-phase synchronous generators operate in parallel supplying a substation through a feeder having an impedance of (0.6 + j0.8) ohms to positive and negative sequence currents and (1.0 + j2.6) ohms to zero sequence currents. Each generator has X_1 = 0.8 ohms, X_2 = 0.5 ohms and X_0 = 0.2 ohms and has its neutral grounded through a reactance of 0.2 ohms. Evaluate the fault currents in each line and the potential above earth attained by the generator neutrals, consequent to simultaneous occurrence of earth fault on the Y and B phases at the sub-station. [20 Marks]

Solution:

Let the base MVA be 30 MVA and base kV, 11 kV.

For synchronous generators.

Per unit positive sequence reactance

$$X_1 = j0.8 \times \frac{30}{(11)^2} = j0.1983 \text{ p.u.}$$

Per unit negative sequence reactance

$$X_2 = j0.5 \times \frac{30}{(11)^2} = j0.1239 \text{ p.u.}$$

Per unit zero sequence reactance

$$X_0 = j0.2 \times \frac{30}{(11)^2} = j0.0495 \text{ p.u.}$$

For feeder,
$$X_1 = X_2 = (0.6 + j0.8) \times \frac{30}{(11)^2} = 0.2479(0.6 + j0.8)$$
$$= 0.14874 + 0.19832j \text{ p.u.}$$
$$X_0 = (1 + j2.6) \times \frac{30}{(11)^2} = 0.2479(1 + j2.6)$$
$$= 0.2479 + 0.646j \text{ p.u.}$$

Grounding reactance,
$$R_G = 0.2 \times \frac{30}{(11)^2} = j0.04958 \text{ p.u.}$$

As both generator (identical) operate in parallel
$$X_{1\text{eff}} = j\frac{0.1983}{2} + 0.14874 + j0.19832$$
$$= j0.09915 + 0.14874 + j0.19832$$
$$= 0.14874 + j0.29747 \text{ p.u.}$$
$$X_{2\text{eff}} = j\frac{0.1239}{2} + 0.14874 + j0.19832$$
$$= j0.06195 + 0.14874 + j0.19832 = 0.299\angle 60.25° \text{ p.u.}$$
$$X_{0\text{eff}} = j\frac{0.04958 + (3 \times j0.04958)}{2} + 0.2479 + j(0.6446)$$
$$= 0.2479 + j0.74376 \text{ p.u.} = 0.784\angle 71.56° \text{ p.u.}$$

For LLG fault on generator terminals

Assuming pre-fault voltage E_a be $1\angle 0°$ p.u.
$$I_{a1} = \frac{E_a}{X_{1\text{eff}} + \dfrac{X_{2\text{eff}} \times X_{0\text{eff}}}{X_{2\text{eff}} + X_{0\text{eff}}}} \text{ p.u.}$$
$$I_{a1} = \frac{1\angle 0°}{(0.14874 + j0.29747) + \dfrac{(0.14874 + j0.26027)(0.2479 + j0.74376)}{0.14874 + j0.26027 + 0.2479 + j0.74376}} \text{ p.u.}$$
$$= 1.81723\angle -63.4109 \text{ p.u.}$$
$$I_{a0} = -\left(\frac{E_a - I_{a1}X_{1\text{eff}}}{X_{0\text{eff}}}\right)$$
$$= -\left(\frac{1 - (0.84132 - j1.62609) \times (0.14874 + j0.19832)}{0.32227 + j0.66935}\right)$$
$$= -0.4135 + j0.62613 \text{ p.u.}$$
$$= -0.504\angle -71.598° \text{ p.u.}$$

I_{a0} for each generator
$$\frac{0.504\angle -71.598}{2} = -0.252\angle -71.598 \text{ p.u.}$$

Potential of alternator neutral w.r.t. ground
$$= 3 \times I_{a0} \text{ per generator} \times R_G$$
$$= 3 \times (-0.252\angle -71.598) \times j0.04958$$
$$= 0.03075 - j0.04656$$
$$= -3.7428 \times 10^{-3}\angle 18.4° \text{ p.u.}$$

Potential of alternator neutral w.r.t. ground

$$-3.7428 \times 10^{-2}\angle 18.4° \times \frac{11000}{\sqrt{3}} = -237.69\angle 18.4° \text{ V}$$

$$I_{R2} = I_{R1} - I_{R0}$$

$$I_{a2} = -1.320\angle -60.294° \text{ p.u.}$$

The sequence current in each generator will be

$$I_{g0} = \frac{I_{a0}}{2} = -0.252\angle -71.598°$$

$$I_{g1} = \frac{I_{a1}}{2} = 0.908615\angle -63.4109°$$

$$I_{g2} = \frac{I_{a2}}{2} = -0.66\angle -60.294°$$

$$\begin{bmatrix} I_{af=0} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_{af}^{(0)} \\ I_{af}^{(1)} \\ I_{af}^{(2)} \end{bmatrix}$$

$$I_{bf} = I_{af}^{(0)} + \alpha^2 I_{af}^{(1)} + \alpha I_{af}^{(2)}$$

$$= -0.252\angle -71.598° + 1\angle 240°(0.908615\angle -63.4109°) + 1\angle 120°(-0.66\angle -60.294°)$$

$$= -0.252\angle -71.598° + 0.90861\angle 176.5891 - 0.66\angle 59.706°)$$

$$= 1.3481\angle -168.156° \text{ p.u.}$$

$$I_{cf} = -0.252\angle -71.598° + (1\angle 120°) \times (0.908615\angle -63.4109°) + 1\angle 240°) \times (-0.66\angle -60.294°)$$

$$= -0.252\angle -71.598° + 0.908615\angle 56.589° - (0.66\angle -179.706°)$$

$$= 1.4684\angle 42.61° \text{ p.u.}$$

$$I_{bf\,(actual)} = 2122.707\angle -168.156° \text{ A},$$

$$I_{cf\,(actual)} = 2312.13\angle 42.61° \text{ A}$$

MADE EASY Source

- **Mains Work Book:** Power System (Q.37, Page 196) **(Click Here for Reference)**
- **MADE EASY Classnotes**

End of Solution

4. (a) A signal $g(t)$ band limited to B Hz is sampled by a periodic pulse train $p_T(t)$ made up of a rectangular pulse of width $\frac{1}{8B}$ sec (centered at origin) repeating at the Nyquist rate (2 B pulses per sec.) Show that the sampled signal $g_s(t)$ is given by

$$g_s(t) = \frac{1}{4}g(t) + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right) g(t) \cos(4n\pi Bt)$$

How will you recover $g(t)$ from the signal $g_s(t)$?

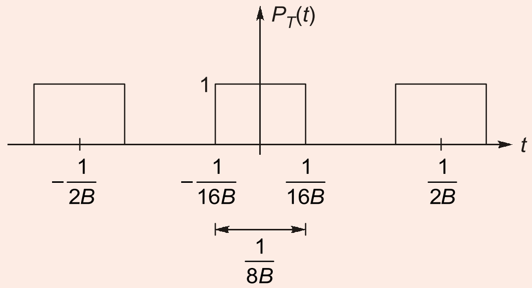
[15 + 5 Marks]

Solution:

$$g_s(t) = g(t)p_T(t)$$

Given that $p_T(t)$ is made up of rectangular pulse of width $\frac{1}{8B}$ sec and repeating at the Nyquist rate 2 B pulses/sec

$$T_s = \frac{1}{2B}$$



Fourier series of $P_T(t)$ can be written as

$$P_T(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

As the given signal is even signal,

$$b_n = 0$$

$$P_T(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t)$$

$$a_0 = \frac{1}{T}\int_0^T P_T(t)\cdot dt = \frac{1}{T}\int_{-\frac{1}{16}B}^{1/16B} 1\cdot dt$$

$$a_0 = \frac{1}{\left(\frac{1}{2B}\right)}\left[\frac{1}{16B} + \frac{1}{16B}\right] = \frac{\frac{1}{8B}}{\frac{1}{2B}} = \frac{1}{4}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} P_T(t)\cdot \cos(n\omega_0 t)\cdot dt$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\left(\frac{1}{2B}\right)} = 4B\pi$$

$$a_n = \frac{2}{\left(\frac{1}{2B}\right)}\int_{-\frac{1}{16B}}^{\frac{1}{16B}} 1\cdot \cos(4B\pi nt)\cdot dt$$

$$a_n = 4B\left[\frac{\sin(4B\pi nt)}{4B\pi n}\right]_{-1/16B}^{1/16B}$$

$$a_n = 4B\left[\frac{\sin\left(\frac{\pi n}{4}\right) - \sin\left(\frac{-\pi n}{4}\right)}{4B\pi n}\right]$$

$$a_n = \frac{2}{\pi n}\sin\left(\frac{\pi n}{4}\right)$$

$$\therefore \quad P_T(t) = \frac{1}{4} + \sum_{n=1}^{\infty}\frac{2}{\pi n}\sin\left(\frac{\pi n}{4}\right)\cos(4B\pi nt)$$

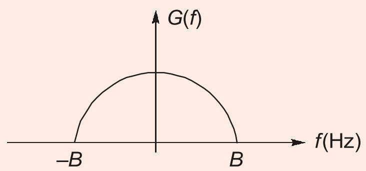
$$\therefore \quad g_s(t) = g(t)\cdot P_T(t)$$

$$\therefore \quad g_s(t) = \frac{g(t)}{4} + \sum_{n=1}^{\infty}\frac{2}{\pi n}\sin\left(\frac{\pi n}{4}\right)g(t)\cdot\cos(4B\pi nt)$$

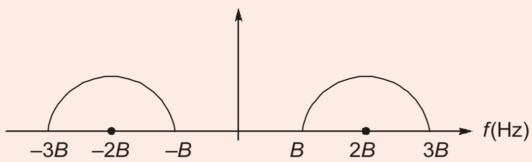
If we will expand $g_s(t)$,

then we can write $g_s(t) = \frac{g(t)}{4} + \frac{2}{\pi}\sin\left(\frac{\pi}{4}\right)g(t)\cos(4\pi Bt) + \frac{1}{\pi}\sin\left(\frac{\pi}{2}\right)g(t)\cos(8\pi Bt) + ...$

Suppose $g(t) \xleftrightarrow{F.T.} G(f)$



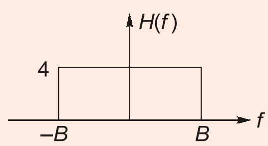
Then for $\frac{2}{\pi}\sin\frac{\pi}{4}g(t)\cdot\cos(4\pi Bt) \xleftrightarrow{F.T.}$



(By using modulation property of fourier transform)

It means to filter out $g(t)$ from $g_s(t)$ and to reject all other terms, we have to pass $g_s(t)$ through an LPF whose cut off frequency should be 'B' Hz and pass band gain should be '4'.

i.e.,



$$H(f) = 4\,\text{rect}\left(\frac{f}{2B}\right)$$

End of Solution

4. (b) A 3-phase half-controlled rectifier with free-wheeling diode is supplying a separately excited DC motor for speed control purpose. The AC input to the converter is 415 V, 3-phase, 50 Hz. The motor parameters are :

$$V = 220 \text{ V DC}, \quad P = 10.5 \text{ kW}$$
$$\text{Rated speed} = 1100 \text{ rpm},$$
$$\text{Armature resistance}, r_a = 0.4\ \Omega$$

The field current is kept constant at rated value. The motor is operated at rated speed delivering half rated torque.

(i) Find motor terminal voltage and triggering angle of thyristor bridge.

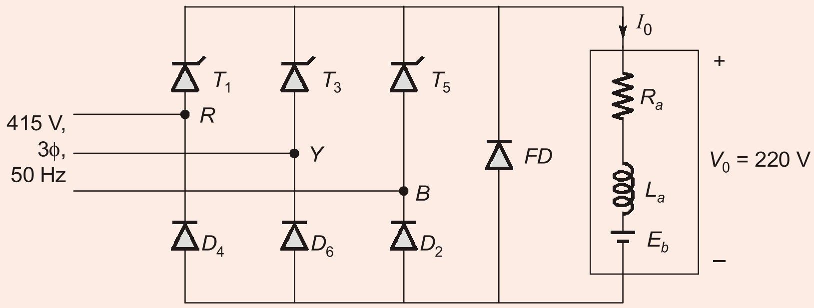
(ii) Find the speed of the motor if one of the input phases to the converter is out due to fault and the triggering angle is kept as before with same load torque.

(iii) Also find the new triggering angle if the motor speed is to be maintained at rated value with same load torque.

(Neglect losses in the machine)

[20 Marks]

Solution:



$$P_o = 10.5 \text{ kW}$$
$$N_r = 1100 \text{ rpm}$$
$$R_a = 0.4\ \Omega$$

Motor operated at rated speed and half the rated torque.

$$P_o = V_o I_o$$
$$10.5 \times 10^3 = 220 \times I_o$$
$$I_o = \frac{10.5 \times 10^3}{220} = 47.7 \text{ A}$$

Rated :

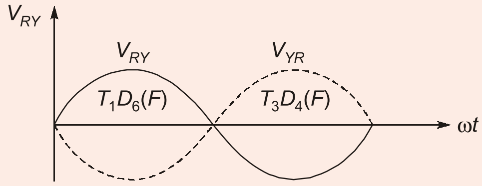
$$V_o = E_b + I_o R_a$$
$$220 = K_m \cdot \frac{2\pi N_r}{60} + I_o R_a$$
$$220 = K_m \cdot \frac{2\pi \times 1100}{60} + (47.7 \times 0.4)$$
$$K_m = \frac{201 \times 60}{2\pi \times 1100}$$
$$K_m = 1.745 \text{ V.s/rad}$$

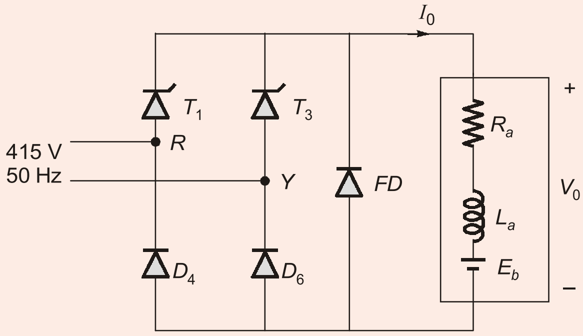
At half the rated torque, $\quad I_o = \frac{I_{oRated}}{2} = \frac{47.7}{2}$

$$I_o = 23.85 \text{ A}$$
$$V_o = E_b + I_o R_a$$
$$V_o = K_m \cdot \frac{2\pi N_r}{60} + I_o R_a$$
$$V_o = 1.745 \times \frac{2\pi \times 1100}{60} + (23.85 \times 0.4)$$
$$V_o = 201 + 9.54$$

(i)
$$V_o = 210.54 \text{ V}$$
$$\frac{3V_{mL}}{2\pi}(1+\cos\alpha) = 210.54$$
$$\frac{3\times 415\sqrt{2}}{2\pi}(1+\cos\alpha) = 210.54$$
$$1 + \cos\alpha = \frac{1322.86}{1760.695} = 0.751$$
$$\cos\alpha = -0.24867$$
$$\alpha = 104.41°$$

(ii)





Now it works like single phase half controlled rectifier.

$$V_o = \frac{V_{mL}}{\pi}(1+\cos\alpha)$$
$$V_o = \frac{415\sqrt{2}}{\pi}(1+\cos 104.41)$$
$$V_o = 186.815(1+\cos 104.41)$$
$$V_o = 140.324 \text{ volts}$$
$$V_o = E_b + I_o R_a$$
$$V_o = K_m \frac{2\pi N}{60} + (23.85 \times 0.4)$$
$$140.324 = 1.745 \times \frac{2\pi N}{60} + 9.54$$

$$130.778 = 1.745 \times \frac{2\pi}{60} N$$

$$\therefore \quad N = \frac{130.778 \times 60}{1.745 \times 2\pi}$$

$$N = 716.061 \text{ rpm}$$

(iii) α = ?

$$N = N_r = 1100 \text{ rpm}$$

$$V_o = E_b + I_o R_a$$

$$\frac{V_{mL}}{\pi}(1 + \cos\alpha) = K_m \cdot \frac{2\pi N_r}{60} + I_o R_a$$

$$\frac{415\sqrt{2}}{\pi}(1 + \cos\alpha) = 1.745 \times \frac{2\pi \cdot 1100}{60} + (23.85 \times 0.4)$$

$$186.815(1 + \cos\alpha) = 201 + 9.54$$

$$1 + \cos\alpha = \frac{210.549}{186.815} = 1.127$$

$$\cos\alpha = 0.127$$

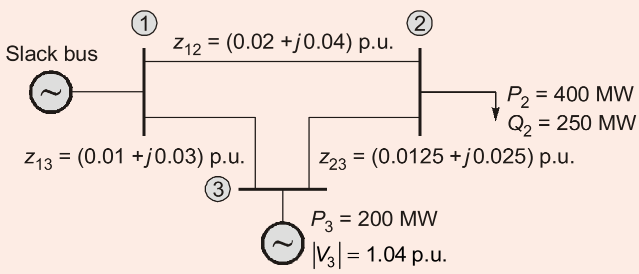
$$\alpha = 82.7°$$

MADE EASY Source

- **MADE EASY Classnotes: Concept**

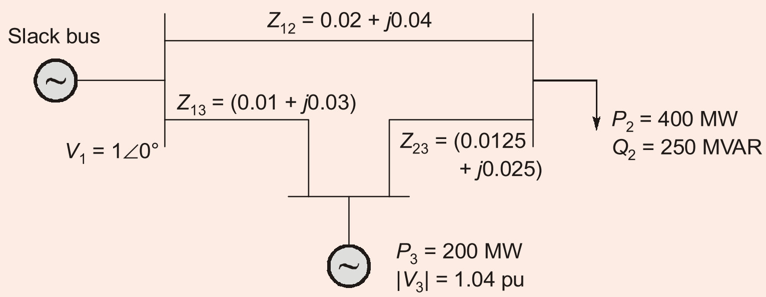
End of Solution

4. (c) The figure below shows single line diagram of a power system with generators at bus-1 and bus-3. The voltage at bus-1 is 1.05∠0° p.u. and at bus-3, $|V|$ = 1.04 p.u. Line impedances are in p.u. and line charging suseptances are neglected. Obtain state vector using Fast Decoupled Load Flow (FDLF) for one iteration.



[20 Marks]

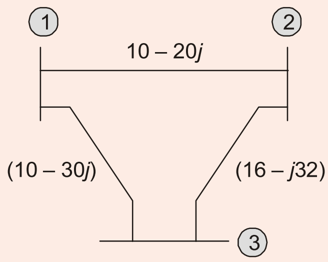
Solution:



$$Y_{12} = \frac{1}{Z_{12}} = 10 - 20j$$

$$Y_{13} = \frac{1}{(0.01 + j0.013)} = 10 - 30j$$

$$Y_{23} = \frac{1}{(0.0125 + j0.025)} = 16 - j32$$



$$Y_{Bus} = \begin{bmatrix} 20 - j50 & (-10 + j20) & (-10 + j30) \\ -10 + j20 & (26 - j52) & (-16 + j32) \\ (-10 + j30) & (-16 + j32) & (26 - j62) \end{bmatrix}$$

$$= \begin{bmatrix} 53.85\angle -68.195 & 22.36\angle 116.566^\circ & 31.622\angle 108.44 \\ 22.36\angle 116.566^\circ & 58.37\angle -63.434^\circ & 35.777\angle 116.566 \\ 31.622\angle 108.44 & 35.777\angle 116.566 & 67.230\angle -67.246 \end{bmatrix}$$

Bus 2 is *PQ* Bus required to calculate $|V_2|\angle\delta_2$
Bus 3 is *PV* Bus required to calculate δ_3, Q_3

$$B' = \begin{bmatrix} -52 & 32 \\ 32 & -62 \end{bmatrix}$$

The inverse of the above matrix is

$$[B']^{-1} = \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix}$$

From the expressions for real power at bus 2 and 3 and the reactive power at bus 2 are
$P_2 = |V_2||V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2^2||Y_{22}| \cos\theta_{22} + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$
$P_3 = |V_3||V_1||Y_{31}| \cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \cos(\theta_{32} - \delta_3 + \delta_2) + |V_3^2|||Y_{33}| \cos\theta_{33}$
$Q_2 = -|V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - |V_2^2||Y_{22}| \sin\theta_{22} - |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$
The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \text{ p.u.}$$

$$P_3^{sch} = \frac{200}{100} = 2.0 \text{ p.u.}$$

The slack bus voltage is $V_1 = 1.05\angle 0$ p.u. and the bus 3 voltage magnitude is $|V_3| = 1.04$ pu. Starting with an initial estimate of $|V_2^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are computed from the equation.

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -4.0 - (-1.14) = -2.86$$

$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = 2.0 - (0.5616) = 1.4384$$

$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -2.5 - (-2.28) = -0.22$$

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix} = -\begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix} \begin{bmatrix} \frac{-2.8600}{1.0} \\ \frac{1.4384}{1.04} \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix} = \begin{bmatrix} -0.060483 \\ -0.008909 \end{bmatrix}$$

Since bus 3 is a regulated bus, the corresponding row and column of B' are eliminated and we get

$$B'' = [-52]$$

$$\Delta|V_2| = \left[\frac{-1}{52}\right]\left[\frac{-0.22}{1.0}\right] = -0.0042308$$

The new bus voltages in the first iteration are

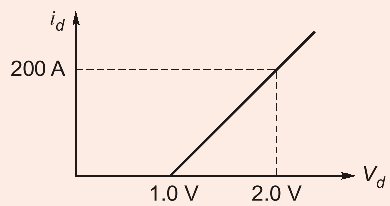
$$\delta_2^{(1)} = 0 + (-0.060483) = -0.060483 \text{ rad}$$
$$\delta_3^{(1)} = 0 + (-0.008909) = -0.008909 \text{ rad}$$
$$|V_2^{(1)}| = 1 + (-0.0042308) = 0.995769 \text{ p.u.}$$

MADE EASY Source

- **ESE 2020 Mains Test Series:** Q.7(a) of Mock-2 **(Click Here for Reference)**
- **MADE EASY Classnotes**
- **Mains Classroom Video Lecture**

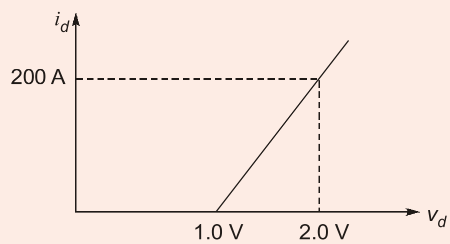
End of Solution

5. (a) A thyristor is having the I-V characteristic as given in the figure below. It is used in a half wave rectifier circuit with resistive load operating at α = 30° and carrying a peak load current of 100 A. Determine the average conduction loss in the thyristor.

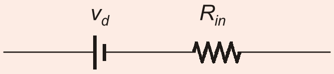


[12 Marks]

Solution:



Equivalent conduction circuit for thyristor is



Now $$R_{in} = \frac{2-1}{200} = \frac{1}{200}\Omega = 5\ m\Omega$$

$$V_f = 1 \text{ volt}$$

Now, $$I_{peak} = 100\ A,\ \alpha = 30°$$

For half wave rectifier and resistive load

$$I_{dc} = \frac{I_{peak}}{2\pi}(1+\cos\alpha) = 29.698 \text{ Amp}$$

$$I_{rms} = \frac{I_{peak}}{2\sqrt{\pi}}\left[(\pi-\alpha)+\frac{1}{2}\sin 2\alpha\right]^{\frac{1}{2}}$$

$$= \frac{100}{2\sqrt{\pi}}\left[\left(\pi-\frac{\pi}{6}\right)+\frac{1}{2}\sin 60°\right]^{\frac{1}{2}} = 49.273 \text{ Amp}$$

$$\text{Average power loss} = I_{dc}\times V_f + I^2_{rms}R_{in}$$

$$= 29.698\times 1+\frac{(49.273)^2}{200}$$

$$= 41.8371 \text{ watts}$$

MADE EASY Source

- **ESE/Gate Classroom Workbook :** (Q.1, Page 40) **(Click Here for Reference)**
- **MADE EASY Classnotes: Concept**

End of Solution

5. (b) A three-phase equilateral transmission line has a total corona loss of 55 kW at 110 kV and 100 kW at 114 kV. What is the disruptive critical voltage between lines ? What is the corona loss at 120 kV ?

[12 Marks]

Solution:

Three phase corona power loss

$$P_c = 3\times\frac{244}{\delta}(f+25)\sqrt{\frac{r}{d}}(V_{Ph}-V_{do})^2\times 10^{-5}\ \text{kW/h}$$

As air density factor, δ, supply frequency f, radius of conductor r and conductor spacing same.

$$P_c \propto (V_{Ph}-V_{do})^2$$

So, $$\frac{P_{c1}}{P_{c2}} = \frac{(V_{ph1}-V_{do})^2}{(V_{ph2}-V_{do})^2}$$

$$\frac{55}{100} = \frac{\left(\frac{110}{\sqrt{3}}-V_{do}\right)^2}{\left(\frac{114}{\sqrt{3}}-V_{do}\right)^2} = \frac{(63.50-V_{do})^2}{(65.81-V_{do})^2}$$

$$0.7416 = \frac{63.50 - V_{do}}{65.81 - V_{do}}$$

$$(0.7416)(65.81 - V_{do}) = (63.5 - V_{do})$$

$$48.806 - 0.7416V_{do} = 63.5 - V_{do}$$

$$0.2584V_{do} = 14.693$$

$$V_{do} = 56.861 \text{ kV}$$

For corona loss at 120 kV

As $$P_c \propto \left(\frac{120}{\sqrt{3}} - V_{do}\right)^2 \propto (69.282 - V_{do})^2$$

$$\frac{P_c}{55} = \frac{(69.282 - V_{do})^2}{(63.50 - V_{do})^2}$$

Using $$V_{do} = 56.861 \text{ kV}$$

$$\frac{P_c}{55} = \left[\frac{69.282 - 56.861}{63.50 - 56.861}\right]^2 = \left(\frac{12.421}{6.639}\right)^2$$

$$P_c = 192.5176 \text{ kW}$$

Disruptive critical voltage between lines

$$= \sqrt{3}V_{do} = \sqrt{3} \times 56.861 \text{ kV}$$

$$= 98.486 \text{ kV}$$

End of Solution

5. (c) A Gaussian pulse is specified by

$$g(t) = Ae^{-\alpha^2 t^2},$$

where α is an arbitrary attenuation coefficient and A is constant. Show that the Fourier transform of $g(t)$ is also Gaussian.

[12 Marks]

Solution:

$$G(\omega) = A\int_{-\infty}^{\infty} e^{-\alpha^2 t^2} e^{-j\omega t} \cdot dt$$

$$= A\int_{-\infty}^{\infty} e^{-(\alpha^2 t^2 + j\omega t)} dt$$

Substituting, $$\alpha^2 t^2 + j\omega t = \left(\alpha t + \frac{j\omega}{2\alpha}\right)^2 + \frac{\omega^2}{4\alpha^2}$$

$$G(\omega) = A\int_{-\infty}^{\infty} e^{-\left(\alpha t + \frac{j\omega}{2\alpha}\right)^2} \cdot e^{-\frac{\omega^2}{4\alpha^2}} dt$$

$$G(\omega) = Ae^{-\frac{\omega^2}{4\alpha^2}} \int_{-\infty}^{\infty} e^{-\left(\alpha t + j\frac{\omega}{2\alpha}\right)^2} dt$$

Let $u = \alpha t + j\dfrac{\omega}{2\alpha}$

$$du = \alpha\ dt$$

$$dt = \frac{du}{\alpha}$$

As $t \to \infty$, $u \to \infty$

As $t \to -\infty$, $u \to -\infty$

$$G(\omega) = Ae^{-\frac{\omega^2}{4\alpha^2}} \int_{-\infty}^{\infty} e^{-u^2} \cdot \frac{du}{\alpha}$$

$$= Ae^{-\frac{\omega^2}{4\alpha^2}} \cdot \frac{2}{\alpha} \int_0^{\infty} e^{-u^2} \cdot du$$

Since $\displaystyle\int_0^{\infty} e^{-u^2} \cdot du = \frac{\sqrt{\pi}}{2}$

$$G(\omega) = A\frac{\sqrt{\pi}}{2} \cdot e^{-\frac{\omega^2}{4\alpha^2}}$$

$G(\omega)$ is a Gaussian pulse

∴ Hence proved.

MADE EASY Source

- **MADE EASY Classnotes Click Here for Reference)**

End of Solution

5. (d) What are the advantages and limitations of Lead and Lag networks in a practical control system ?

[12 Marks]

Solution:

Advantages of Lead :

- Gain crossover frequency (ω_{gc}) increases hence bandwidth increases due to which speed increases.
- Phase becomes less lag hence phase margin increases due to which stability increases.
- Effect of resonance decreases due to which oscillations reduces.

Limitations :

- Due to increase in bandwidth, noise also increases.
- Steady state error cannot become zero.
- Ineffective if the type of a system is very high, in such case multistage compensator is preferred.

Advantages of Lag :

- Effect of noise decreases.
- Useful to reduce damping in heavily damped system.
- Steady state error can be reduced.

Limitations :

- Lag action causes less speed and stability hence recommended to be used in a system with good speeds and stability margins.
- If desired steady state error is zero then it cannot be achieved.

MADE EASY Source

- **MADE EASY Classnotes**

End of Solution

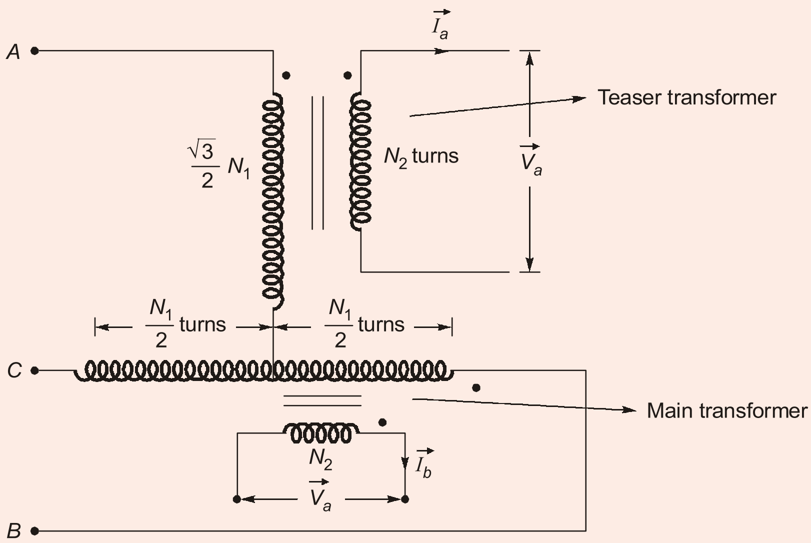
5. (e) For a Scott connected transformer, prove that the number of turns on primary of the teaser transformer is $\frac{\sqrt{3}}{2}$ times the number of turns in primary of main transformer.

[12 Marks]

Solution:

Scott connection is most common method of connecting two single phase transformer to perform a 3-ϕ to 2-ϕ conversion and vice-versa. Two transform are connected electrically but not magnetically, one transformer is main transformer and other is known as teaser transformer.

Scott connection of transformers:



Frequently identical interchangeable transformers are used for the scott connection, in which each transformer has primary winding of N_1 turns and is provided with tapping at 0.289 N_1, 0.5 N_1, 0.866 N_1.

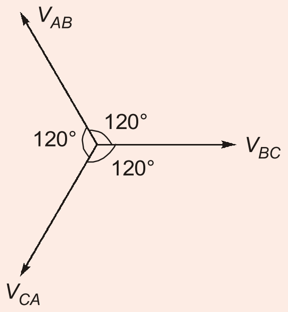
Phasor diagram: The line voltages of the 3-ϕ system V_{AB}, V_{BC} and V_{CA}, which are balanced the same voltages are shown as a closed equilateral triangle in figure below.

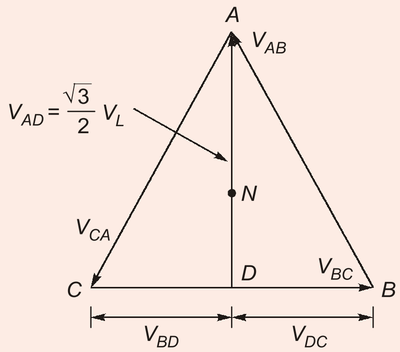
$$|V_{AB}| = |V_{BC}| = |V_{CA}| = V_L \text{ (say)}$$

Let V_{BC} be taken as reference.

$$V_{BC} = V_L \angle 0°$$

$$V_{CA} = V_L \angle -120^\circ$$
$$V_{AB} = V_L \angle +120^\circ$$





Voltages on transformer primary winding.

⇒ Since, D divides the primary BC of main transformer in two equal halves.

Number of turns in portion BD = Number of turns in portion DC = $\frac{N_1}{2}$.

Voltages V_{BD} and V_{DC} are equal. They are in phase with V_{BC},

$$V_{BC} = V_{DC} = \frac{1}{2} V_{BC} = \frac{1}{2} V_L \angle 0^\circ \text{ volts}$$

The voltage between A and D is,

$$V_{AD} = V_{AB} + V_{BD}$$
$$= V_L \angle 120^\circ + \frac{V_L}{2} \angle 0^\circ$$
$$V_{AD} = 0.866\ V_L \angle 90^\circ = \frac{\sqrt{3}}{2} \angle 90^\circ$$

So teaser transformer has primary voltage rating $\frac{\sqrt{3}}{2}$ of the voltage rating of main transformer.

For the same flux in each transformer, voltage per turn should be the same. In order to keep voltage per turn same in primary of the main transformer and primary of teaser transformer, the number of turns in primary of the teaser, that is in portion AD should be equal to $\frac{\sqrt{3}}{2} N_1$.

So, $$\frac{N_{teaser}}{N_{main}} = \frac{\sqrt{3}}{2}$$

N = Number of turns

MADE EASY Source

- **MADE EASY Classnotes (Click Here for Reference)**
- **Mains Classroom Video Lecture**

End of Solution

6. (a) A 15 kW, 400 V, 3-phase, star connected synchronous motor has synchronous impedance of 0.4 + j4 Ω. Find the motor excitation voltage for full load output at 0.866 leading power factor. Take the armature efficiency of 95%.

[20 Marks]

Solution:

Given that,

$$P_o = 15 \text{ kW}$$

$$\vec{Z}_s = (0.4 + j4)\Omega$$

$$\vec{V}_t = 400 \text{ V } (L - L)$$

$$\text{Power factor} = \cos\phi = 0.866 \text{ lead}$$

$$\phi = 30°$$

$$\eta = 0.95$$

$$P_{in} = \frac{P_{out}}{\eta} = \frac{15}{0.95} = 15.7894 \text{ kW}$$

As we know,

$$P_{in} = \sqrt{3}\cdot V_L \cdot I_L \cos\phi$$

$$\vec{I}_L = \frac{15789.40}{\sqrt{3}\times 400\times 0.866} = \angle\cos^{-1}(0.866) = 26.3165\angle 30° \text{ Amp}$$

Excitation voltage for given motor is = $\vec{E}_f$

$$\vec{E}_f = \vec{V}_t - \bar{I}_a\bar{Z}_s$$

$$= \frac{400}{\sqrt{3}}\angle 0 - (26.31\angle 30°)\,(0.4 + j4)$$

$$\vec{E}_f = 290.90 \angle -19.35 \text{ Volts } [L - N]$$

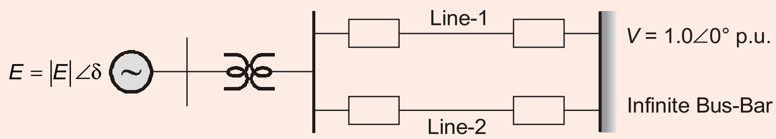
$$\vec{E}_f = 503.85 \angle -19.35 \text{ Volts } [L - L]$$

MADE EASY Source

- **Mains Work Book:** (Q.39, Page 122) **(Click Here for Reference)**
- **MADE EASY Classnotes**
- **Mains Classroom Video Lecture**

End of Solution

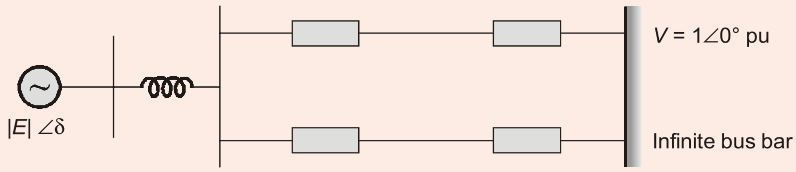
6. (b) A synchronous machine is connected to an infinite bus through a transformer and a double circuit line shown in the figure. The infinite bus voltage is $V = 1.0\angle 0°$ p.u. The direct axis transient reactance of the machine is 0.20 p.u., the transformer reactance is 0.10 p.u. and the reactance of each of the transmission lines is 0.4 p.u., all to a base of the rating of the synchronous machine. Initially the machine is delivering 0.8 p.u. power with a terminal voltage of 1.05 p.u. The inertia constant H = 5 MJ/MVA. All resistances are neglected. Determine the equation of motion of the machine rotor.

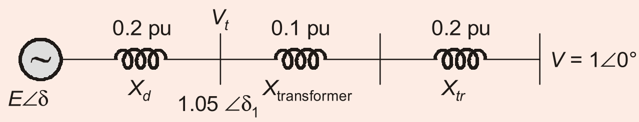


[20 Marks]

Solution:

For given power system :





Using power relation

$$\frac{V_t V \sin\delta_1}{X} = 0.8$$

$$\frac{1(1.05)\sin\delta_1}{0.3} = 0.8 \text{ or } \delta_1 = \sin^{-1} 0.2285 = 13.21°$$

Now using voltage equation

$$I_a = \frac{1.05\angle 13.21° - 1\angle 0°}{0.3j} = 0.803\angle -5.289° \text{ p.u.}$$

$$E = 1.05\angle 13.21 + 0.803\angle{-5.23} \times 0.2\angle 90°$$

$$E = 1.111\angle 21.086° \text{ p.u.}$$

$$\delta = 21.086°$$

$$M\frac{d^2\delta}{dt^2} = P_m - P_e \qquad \text{...(swing equation)}$$

where

$$P_e = \frac{EV\sin\delta}{X_{eq}} = \frac{1.11 \times 1\sin\delta}{0.5} = 2.222 \sin\delta$$

$$M = \frac{SH}{\pi f} = \frac{1 \times 5}{\pi \times 50} = 0.0318$$

$$0.0318\frac{d^2\delta}{dt^2} = 0.8 - 2.222 \sin\delta$$

MADE EASY Source

- **ESE 2020 Mains Test Series:** Q.6(c) (i) of Test-5
- **Mains Work Book:** Power System (Q.24-a, Page 157) **(Click Here for Reference)**
- **MADE EASY Classnotes**

End of Solution

6. (c) The open loop transfer function of unity feedback control system is given by

$$G(s) = \frac{K}{s(s+a)(s+b)},\quad 0 < a \le b$$

(i) Find the range of the gain constant K (> 0) for stability using Routh-Hurwitz criterion.

(ii) What type of control do you use if the system is required to have zero steady-state error for ramp input ? Let 'A' be the parameter that can be varied in the introduced control. Find the range of 'K' for stability in terms of parameters a, b and A using Routh-Hurwitz criterion.

[20 Marks]

Solution:

(i) Characteristic equation

$$1 + G(s).H(s) = 0$$
$$s(s^2 + (a+b)s + ab) + K = 0$$
$$s^3 + (a+b)s^2 + abs + K = 0$$

$$\begin{array}{c|cc} s^3 & 1 & ab \\ s^2 & a+b & K \\ s^1 & \dfrac{(a+b)ab-K}{a+b} & \\ s^0 & K & \end{array}$$

For stability $K > 0$

and $(a+b)ab - K > 0$

$$(a+b)ab > K$$
$$K < (a+b)ab$$

$\therefore$ $0 < K < (a+b)ab$

(ii) We need to use PI Controller to eliminate steady state error.

Let controller transfer function is $1+\dfrac{A}{s}$.

Characteristic equation is

$$1+\frac{(A+s)K}{s^2(s+a)(s+b)} = 0$$

$$s^4 + (a+b)s^3 + abs^2 + Ks + AK = 0$$

$$\begin{array}{c|ccc} s^4 & 1 & ab & AK \\ s^3 & (a+b) & K & \\ s^2 & \dfrac{(a+b)ab-K}{a+b} & AK & \\ s^1 & \dfrac{\left[\dfrac{(a+b)ab-K}{a+b}\right]K-(a+b)AK}{\dfrac{(a+b)ab-K}{a+b}} & & \\ s^0 & AK & & \end{array}$$

For the stability

$$\left[\frac{(a+b)ab-K}{a+b}-(a+b)A\right]K > 0$$

$$(a + b)ab - K - (a + b)^2A > 0$$

$$(a + b)ab - (a + b)^2A > K$$

$$K < (a + b)ab - (a + b)^2A$$

and
$$\frac{(a+b)ab-K}{a+b} > 0$$

$$(a + b)ab > K$$

$$K < (a + b)ab$$

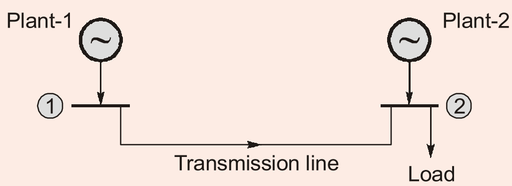
$\therefore$
$$K < (a + b)ab - (a + b)^2A$$

End of Solution

7. (a) A system consists of two plants connected by a transmission line and a load is at power plant-2 as shown in the figure. Data for the loss equation consists of the information that 200 MW transmitted from Plant-1 to the load results in transmission loss of 20 MW. Find the optimum generation schedule considering transmission losses to supply a load of 204.41 MW. Also evaluate the amount of financial loss that may be incurred if at the time of scheduling transmission losses are not coordinated. The incremental fuel cost characteristics of plant-1 and plant-2 are given by

$$\frac{df_1}{dP_1} = 0.025\, P_1 + 14 \quad ₹/\text{MW-hr}$$

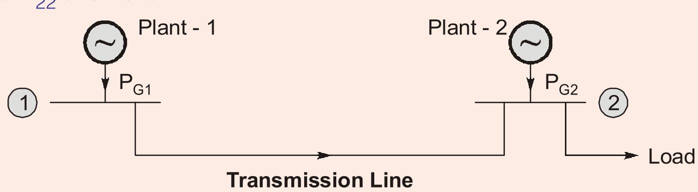
$$\frac{df_2}{dP_2} = 0.05\, P_2 + 16 \quad ₹/\text{MW-hr}$$



[20 Marks]

Solution:

As load is not present at bus-1 so transmission loss is independent of plant 2 generation. So B_{12} and B_{22} are zero.



Power loss due to flow of 200 MW from bus (1) to bus (2) is 20 MW.

$$20 = B_{11}(200)^2$$

$$B_{11} = 5 \times 10^{-4}\ \text{MW}^{-1}$$

$$P_L = 5\times10^{-4}P_{G1}^2$$

and $$\frac{\partial P_L}{\delta P_{G2}} = 0$$

Penalty factor for plant 1,

$$L_1 = \frac{1}{1-\frac{\delta P_L}{\delta P_{G1}}} = \frac{1}{1-10\times 10^{-4} P_{G1}}$$

Penalty factor for plant 2,

$$L_2 = \frac{1}{1-0} = 1$$

Now for optimum scheduling

$$IC_1 L_1 = IC_2 L_2 = \lambda$$

$$\frac{(0.025P_{G1}+14)}{1-10^{-3}P_{G1}} = (0.05P_{G2} + 16) \times 1$$

$$P_D = P_{G1} + P_{G2} - P_L$$

$$204.41 = P_{G1} + P_{G2} - 5 \times 10^{-4} P_{G1}^2$$

$$P_{G2} = 204.41 + 5 \times 10^{-4} P_{G1}^2 - P_{G1}$$

$$\frac{(0.025P_{G1}+14)}{(1-10^{-3}P_{G1})} = 0.05[204.41 + 5 \times 10^{-4}P_{G1}^2 - P_{G1}] + 16$$

$$\frac{(0.025P_{G1}+14)}{1-0.001P_{G1}} = [10.2205 + 2.5 \times 10^{-5}P_{G1}^2 - 0.05P_{G1} + 16]$$

$$0.025P_{G1} + 14 = [26.2205 + 2.5 \times 10^{-5}P_{G1}^2 - 0.05P_{G1}] \times [1 - 0.001P_{G1}]$$

$$2.5 \times 10^{-8} P_{G1}^3 + 7.5 \times 10^{-5}P_{G1}^2 - 0.10122P_{G1} + 12.2205 = 0$$

$$P_{G1} = 134.801 \text{ MW}$$

$$P_L = 9.08552 \text{ MW}$$

$$P_{G2} = 78.6945 \text{ MW}$$

Cost function of each generation plant can be given as :

$$C_1 = 0.025\frac{P_{G1}^2}{2} + 14P_{G1} + K_1$$

$$C_2 = 0.05\frac{P_{G2}^2}{2} + 16P_{G2} + K_2$$

Using values of P_{G1} and P_{G2}, we get

$$C_1 = 2114.355 + K_1$$

$$C_2 = 1413.932 + K_2$$

$$\text{Total investment cost} = 2114.355 + K_1 + 1413.932 + K_2$$

$$= 3528.287 + K_1 + K_2$$

Without considering the losses

$$P_{G1} + P_{G2} = 204.41 \quad \text{...(1)}$$

$$\frac{dF_1}{dP_{G1}} = \frac{dF_2}{dP_{G2}}$$

$$0.025P_{G1} + 14 = 0.05P_{G2} + 16$$

$$0.025P_{G1} - 0.05P_{G2} = 2$$

$$P_{G1} = 162.94 \text{ MW}$$
$$P_{G2} = 41.47 \text{ MW}$$

Also,
$$C_1 = \frac{0.025}{2}P_{G1}^2 + 14P_{G1} + K_1 = 2613.028 + K_1$$
$$C_2 = \frac{0.05}{2}P_{G2}^2 + 16P_{G2} + K_2 = 706.514 + K_2$$
$$\text{Total Cost} = 3319.542 + K_1 + K_2$$
$$\text{Financial Loss} = (3528.287 + K_1 + K_2) - (3319.542 + K_1 + K_2)$$
$$= 208.745 \text{ Rs./hr.}$$

MADE EASY Source

- **Mains Work Book:** Power Systems (Q.4, Page 153) **(Click Here for Reference)**
- **MADE EASY Classnotes**

End of Solution

7. (b) A continuous-time integrator has a system function $H_a(s) = \frac{1}{s}$.

(i) Design a discrete-time integrator using bilinear transformation and find the difference equation relating the input $x[n]$ to the output $y[n]$ of the discrete-time system.

(ii) Find the frequency response of the discrete-time integrator found in part (i) and determine whether or not this system is a good approximation of the continuous time system.

(For $\theta << 1$, $\sin\theta \approx \theta$ and $\cos\theta \approx 1-\frac{\theta^2}{2}$)

[10 + 10 Marks]

Solution:

Given that, $H_a(s) = \frac{1}{s}$

Where, $H_a(s)$ = System function of continous time integrator

(i) Let $H(z)$ be the system function of discrete time integrator using Bilinear transformation. Hence, we can write

$$H(z) = Ha(s)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

Where T = sampling interval

$$= \frac{1}{s}\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$\Rightarrow \quad \frac{Y(z)}{X(z)} = \frac{1}{\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$\Rightarrow \quad \frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{\frac{2}{T}\left[1-z^{-1}\right]}$$

$$\Rightarrow \frac{2}{T}Y(z) - \frac{2}{T}Z^{-1}Y(z) = X(z) + z^{-1}X(z)$$

By applying inverse Z-T.

$$\frac{2}{T}y(n) - \frac{2}{T}y(n-1) = x(n) + x(n-1)$$

The above difference equation is for the desired discrete time integrator.

(ii) As we have seen that in part (i),

$$H(z) = \frac{1+z^{-1}}{\frac{2}{T}[1-z^{-1}]}$$

Put $z = e^{j\omega}$ in the above function

$$H(e^{j\omega}) = \frac{1+e^{-j\omega}}{\frac{2}{T}[1-e^{-j\omega}]} = \frac{1+\cos\omega - j\sin\omega}{\frac{2}{T}[1-\cos\omega + j\sin\omega]}$$

$$= \frac{(1+\cos\omega) - j\sin\omega}{\frac{2}{T}[(1-\cos\omega)+j\sin\omega]} \times \frac{1-\cos\omega - j\sin\omega}{1-\cos\omega - j\sin\omega}$$

$$\Rightarrow \quad H(e^{j\omega}) = \frac{[(1-j\sin\omega)+\cos\omega][(1-j\sin\omega)-\cos\omega]}{\frac{2}{T}[(1-\cos\omega)^2 + \sin^2\omega]}$$

$$= \frac{T}{2}\cdot\frac{(1-j\sin\omega)^2 - \cos^2\omega}{1+\cos^2\omega - 2\cos\omega + \sin^2\omega}$$

$$= \frac{T}{2}\cdot\frac{1-\sin^2\omega - 2j\sin\omega - \cos^2\omega}{2-2\cos\omega}$$

$$\left[\because \sin^2\omega + \cos^2\omega = 1\right]$$

$$= \frac{T}{2}\cdot\frac{-2j\sin\omega}{2(1-\cos\omega)} = \frac{-T}{2}j\cdot\frac{\sin\omega}{1-\cos\omega}$$

$$= -\frac{Tj}{2}\frac{2\sin\frac{\omega}{2}\cos\frac{\omega}{2}}{2\sin^2\frac{\omega}{2}}$$

$$= -\frac{Tj}{2}\cdot\frac{\cos\frac{\omega}{2}}{\sin\frac{\omega}{2}} \qquad \text{...(i)}$$

Since it is given that if $\theta << 1$ then $\sin\theta \approx \theta$ and $\cos\theta = 1-\frac{\theta^2}{2}$

So, by using the above approximation,

$$\cos\frac{\omega}{2} = \frac{1-(\omega/2)^2}{2} = 1-\frac{\omega^2}{8}$$

$$\sin\frac{\omega}{2} = \frac{\omega}{2}$$

By using equation (i),

$$H(e^{j\omega}) = -\frac{Tj}{2}\cdot\frac{\left(1-\frac{\omega^2}{8}\right)}{\frac{\omega}{2}} = -Tj\cdot\left[\frac{1-\frac{\omega^2}{8}}{\omega}\right]$$

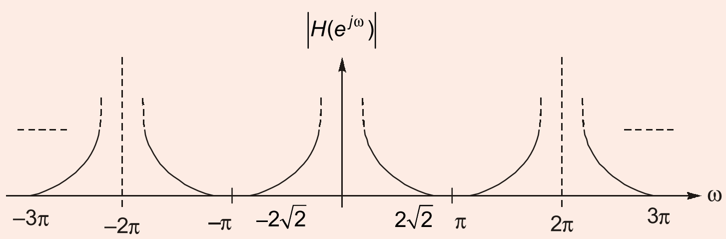
Assuming, $T = 1$ sec,

$$H(e^{j\omega}) = -j\left[\frac{1-\frac{\omega^2}{8}}{\omega}\right]$$

Desired system function of discrete time integrator.

Now,

ω	$\lvert H(e^{j\omega})\rvert = \left\lvert\frac{1-\omega^2/8}{\omega}\right\rvert$
0	∞
1	7 / 8
2	1 / 4
$2\sqrt{2}$	0



Since, the above magnitude characteristics of discrete time integrator is well approximating the magnitude characteristics of continuous time integrals. Therefore, the obtained discrete time integrator is good approximation of continous time integrators.

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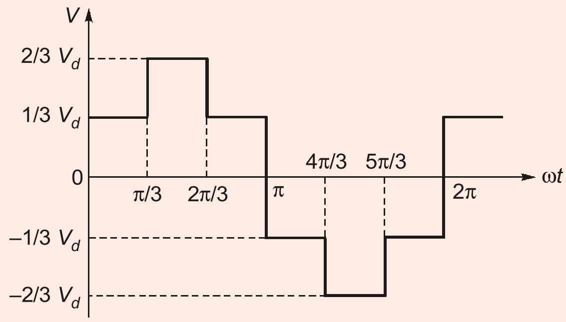
End of Solution

7. (c) For a 3-phase, 50 Hz, 415 V, 4-pole induction motor, the standstill resistance and reactance are 3.0 Ω and 5.0 Ω at 50 Hz respectively. The machine has magnetizing inductance of 350 mH and stator resistance of 1.2 Ω. The machine is supplied from a 3-phase voltage source inverter with qausi square wave output voltage waveform per phase as shown in the figure below. The DC bus voltage is 500 V.

If the machine is operating at 4% slip, find (i) the fundamental input current, (ii) harmonic copper losses in the machine up to 13 harmonics, and (iii) input power factor.

Assume negligible core losses, equal distribution of stator and rotor leakage reactances and linear magnetic circuit.



[20 Marks]

Solution:

The per phase equivalent parameters of induction motor is

$$r_1 = 1.2\ \Omega$$
$$r'_2 = 3\ \Omega$$
$$x_1 = j5\ \Omega$$
$$x_2 = j5\ \Omega$$
$$L_m = 350\text{ mH}$$
$$X_m = 2\pi \times 50 \times 350 \times 10^{-3} = j109.95\ \Omega$$

Input supply phase waveform is six step waveform.

$$V_{on} = \sum_{6K\pm 1}^{\infty} \frac{2V_d}{n\pi} \sin n\omega t$$

Fundamental output voltage of inverter

$$V_{01} = \frac{2V_d}{\sqrt{2}\pi} = 225.079\text{ V}$$

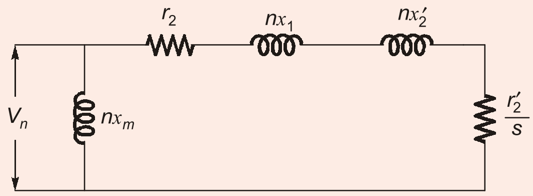
$$V_{05} = \frac{2V_d}{\sqrt{2} \times 5 \times \pi} = 45.0158\text{ V}$$

$$V_{07} = \frac{2V_d}{\sqrt{2} \times 7 \times \pi} = 32.154\text{ V}$$

$$V_{011} = \frac{2V_d}{\sqrt{2} \times 11 \times \pi} = 20.461\text{ V}$$

$$V_{013} = \frac{2V_d}{\sqrt{2} \times 13 \times \pi} = 17.3137\text{ V}$$

Using approximate equivalent circuit of induction motor



Equivalent impedance at fundamental frequency

$$Z_{eq} = jX_m \times \left[\frac{r_1 + \frac{r_2'}{s} + j(x_1 + x_2')}{jX_m + r_1 + \frac{r_2'}{s} + j(x_1 + x_2')}\right]$$

$$Z_{eq} = 59.46\angle 39.90\Omega$$

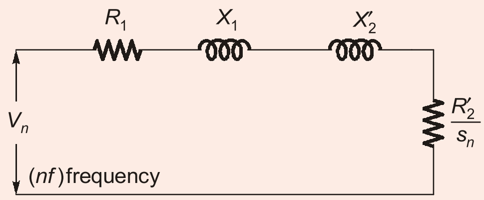
(i) Fundamental input current

$$= \frac{V_{01}}{Z_{eq1}} = 3.78\angle -39.90 \text{ A}$$

(iii) Fundamental input power factor

$$\cos\phi = \cos 39.90 = 0.76714 \text{ lag}$$

(ii) **Induction Motor Equivalent Circuit w.r.t. Harmonics**



n : Order of harmonic
s : Fundamental slip

Harmonic slip, $$s_n = \frac{N_{sn} \pm N}{N_{sn}}$$

$$N_{sn} = n.N_s$$

$$N = N_s(1 - s)$$

$$s_n = \frac{nN_s \pm N_s(1-s)}{n \cdot N_s}$$

$$s_n = \frac{n \pm (1-s)}{n} \qquad (+ \Rightarrow 6n - 1; - \Rightarrow 6n + 1)$$

$n = 1$, Fundamental $$s_n = \frac{1-(1-s)}{1} = s = 0.04$$

For $n = 5$, $$s_5 = \frac{5+(1-s)}{5}$$

$\therefore$ $$s_5 = \frac{6-s}{5} = \frac{6-0.04}{5} = 1.192$$

For $n = 7$, $$s_7 = \frac{7-(1-s)}{7}$$

$$s_7 = \frac{6+s}{7} = \frac{6+0.04}{7} = 0.8628$$

For $n = 11$,
$$s_{11} = \frac{11+(1-s)}{11} = \frac{12-s}{11}$$

$$s_{11} = \frac{12-0.04}{11} = 1.08$$

For $n = 13$,
$$s_{13} = \frac{13-(1-s)}{13} = \frac{12+s}{13}$$

$$s_{13} = \frac{12+0.04}{13} = 0.926$$

Now,
$$I_{05} = \frac{V_{05}}{R_1 + jnX_1 + jnX_2' + \frac{R_2}{s_5}} = 0.897\angle{-85.74°}\text{ A}$$

$$I_{07} = \frac{V_{07}}{R_1 + jnX_1 + jnX_2 + \frac{R_2}{s_7}} = 0.4583\angle{-86.177°}\text{ A}$$

$$I_{011} = \frac{V_{011}}{R_1 + jnX_1 + jnX_2 + \frac{R_2}{s_{11}}} = 0.18588\angle{-87.928}\text{ A}$$

$$I_{013} = \frac{V_{013}}{R_1 + jnX_1 + jnX_2 + \frac{R_2}{s_{13}}} = 0.1331\angle{-88.044}\text{ A}$$

$$P_L = I_{05}^2\left(r_1 + \frac{r_2'}{s_5}\right) + I_{07}^2\left(r_1 + \frac{r_2'}{s_7}\right) + I_{011}^2\left(r_1 + \frac{r_2'}{s_{11}}\right) + I_{013}\left(r_1 + \frac{r_2'}{s_{13}}\right)$$

$$P_L = 2.990 + 0.9823 + 0.1374 + 0.078 \text{ watts}$$
$$P_L = 4.1877 \text{ watts}$$

End of Solution

8. (a) A 50 Hz, 3-phase induction motor has a slip of 0.2 for maximum torque, when operated on rated frequency and rated voltage. If the motor is run on 60 Hz supply with application of rated voltage, find the ratio of

(i) Starting currents
(ii) Starting torques
(iii) Maximum torques

with respective values at 50 Hz.
Neglect the stator impedance.

[7 + 7 + 6 Marks]

Solution:

Given, $f = 50$ Hz, 3-ϕ

$S_{mT} = 0.2$

Now machine made to operate on 60 Hz.

(i) Starting current of induction motor is given as

$$I_{st} = \frac{V_{ph}}{\sqrt{(r_2')^2 + (x_2')^2}} \quad [\because \text{ stator impedance is ignored}]$$

$$\frac{I_{st}(60\text{ Hz})}{I_{st}(50\text{ Hz})} = \frac{\sqrt{(r_2')^2 + (x_2')^2}}{\sqrt{(r_2')^2 + (x_2'')^2}} \quad [x_2'' = \text{impedance at 60 Hz}]$$

$[x_2'' = 1.2x_2']$

$$= \frac{\sqrt{1+\left(\frac{x_2'}{r_2'}\right)^2}}{\sqrt{1+\left(\frac{1.2x_2'}{r_2'}\right)^2}}$$

$$\frac{I_{st}(60\text{ Hz})}{I_{st}(50\text{ Hz})} = \frac{\sqrt{26}}{\sqrt{1+36}} = \sqrt{\frac{26}{37}} \quad \left[S_{mT} = \frac{r_2'}{x_2'} = 0.2, \Rightarrow \frac{x_2'}{r_2'} = 5\right]$$

$$\frac{I_{st}(60\text{ Hz})}{I_{st}(50\text{ Hz})} = 0.8382 \quad \text{...(i)}$$

(ii) Starting torque of induction motor at 60 Hz w.r.t. starting torque of induction motor at 50 Hz

$$T_{st} = \frac{3I_{st}^2 r_2'}{\omega_s} = \frac{3}{\omega_2}\frac{V_{PL}^2}{\left(r_2'^2 + x_2'^2\right)} \cdot r_2'$$

$$\frac{T_{st}(60\text{ Hz})}{T_{st}(50\text{ Hz})} = \frac{\frac{3}{\omega_s'}\frac{V_{ph}^2}{\left(r_2''^2 + x_2''^2\right)} \cdot r_2'}{\frac{3}{\omega_s}\frac{V_{ph}^2}{\left(r_2'^2 + x_2'^2\right)} \cdot r_2'}$$

Where, $\omega_s \rightarrow$ angular synchronous speed at 50 Hz

$\omega_s' \rightarrow$ angular synchronous speed at 60 Hz

$$\frac{T_{st}(60\text{ Hz})}{T_{st}(50\text{ Hz})} = \frac{\omega_s\left(r_2'^2 + x_2'^2\right)}{\omega_s'\left(r_2'^2 + x_2''^2\right)}$$

$$= \frac{50(1+5^2)}{60\left[1+(1.2\times 5)^2\right]} = \frac{50\times 26}{60\times 37} = 0.5856$$

(ii) Maximum torque at 60 Hz w.r.t 50 Hz

$$T_{max} = \frac{3V_{ph}^2}{2\omega_s x_2'}$$

$$\frac{T_{max}(60\text{ Hz})}{T_{max}(50\text{ Hz})} = \frac{\dfrac{3V_{ph}^2}{2\omega_s' x_2''}}{\dfrac{3V_{ph}^2}{2\omega_s x_2'}} = \frac{\omega_s x_2'}{\omega_s' x_2''}$$

$$= \frac{50 \times 50}{60 \times 60} = \frac{25}{36} = 0.695$$

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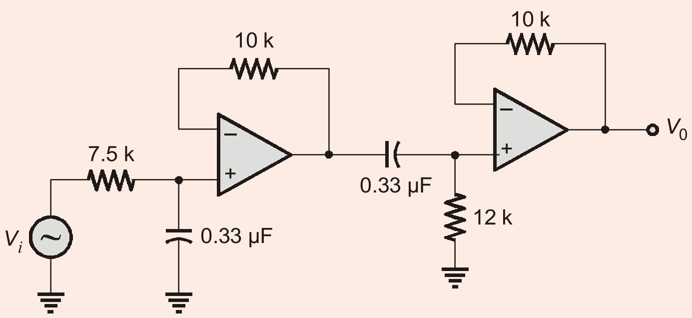
- **Mains Classroom Video Lecture**

End of Solution

8. (b) The current of an induction motor is sensed through a suitable arrangement and converted to equivalent voltage. The current contains fundamental and higher order 5th and 7th harmonics. In order to separate the fundamental, the equivalent voltage waveform is passed through the following circuit as given in figure. Find the (i) cut-off frequencies of each section, (ii) overall gain attenuation in dB for fundamental, 5th and 7th harmonics, and (iii) overall phase shift of the measured fundamental current.



[20 Marks]

Solution:

Current sensed by arrangement is $i_s(t)$

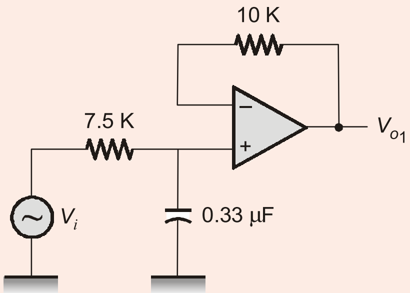
$$i_s(t) = I_1 \cdot \sin \omega t + I_5 \cdot \sin 5\omega t + I_7 \cdot \sin 7\omega t$$

This current is converted into equivalent voltages $V_s(t)$

$$V_s(t) = V_1 \sin \omega t + V_5 \sin 5\omega t + V_7 \cdot \sin 7\omega t$$

$$V_i = V_1 \cdot \sin \omega t + V_5 \cdot \sin 5\omega t + V_7 \cdot \sin 7\omega t$$

(i) Now cut-off frequency of 1st section: [Low pass filter]



Cut-off frequency for given LPF is f_{c1}

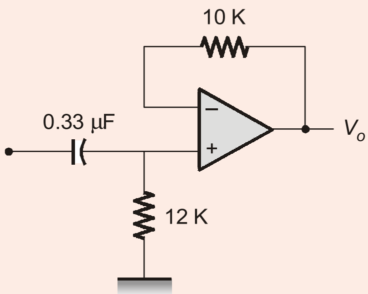
$$f_{c1} = \frac{1}{2\pi\tau} \qquad \tau = R_{eq} \cdot c$$

where R_{eq} is the resistance across c

So, $$f_{c1} = \frac{1}{2\pi \times 7.5 \times 10^3 \times 0.33 \times 10^{-6}} \qquad R_{eq} = 7.5\text{ K}$$

$$f_{c1} = 64.30\text{ Hz}$$

Cut-off frequency of 2nd section: [High pass filter]



$$\tau = R_{eq} \cdot c \qquad R_{eq} = 12\text{ K}$$
$$\tau = 12 \times 10^3 \times 0.33 \times 10^{-6} \qquad c = 0.33\ \mu\text{F}$$
$$\tau = 3.96 \times 10^3\text{ sec}$$

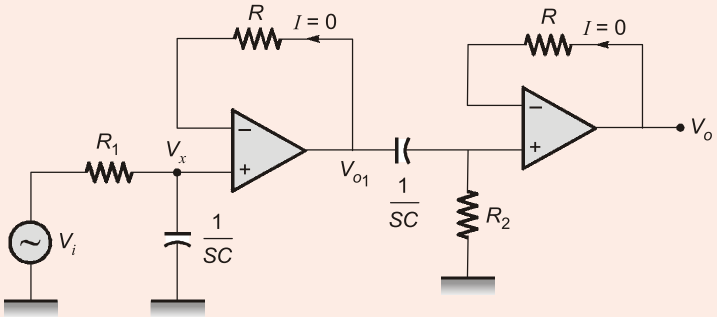
Cut-off of 2nd stage is f_{c2}

$$f_{c2} = \frac{1}{2\pi\tau} = \frac{1}{2 \times 3.96 \times 10^3} = 40.19\text{ Hz}$$

So cut-off frequencies are

$$f_{c1} = 64.30\text{ Hz},\ f_{c2} = 40.19\text{ Hz}$$

(ii)



$$V_x = \frac{\frac{1}{sC}}{R_1 + \frac{1}{sC}} \times V_i$$

Due to virtually ground

$$V_x = V_o \quad [\because \; I = 0 \text{ to } \infty \text{ input impedance of op-amp device}]$$

$$V_{o1} = \frac{V_i}{1 + sR_1C} \qquad \text{[for LPF]}$$

Also,
$$V_o = \frac{R_2}{R_2 + \frac{1}{sC}} V_{o1}$$

$$\frac{V_o}{V_{o1}} = \frac{sR_2C}{1 + sR_2C} \qquad \text{[for HPF]}$$

Overall gain of given band pass filter is

$$\frac{V_o}{V_i} = \frac{V_{o1}}{V_i} \times \frac{V_o}{V_{o1}} = \frac{sR_2C}{(1+sR_1C)(1+sR_2C)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{sR_2C}{(1+sR_1C)(1+sR_2C)}$$

On putting values of C, R_2 and R_1

$$\frac{V_o(s)}{V_i(s)} = \frac{3.96 \times 10^{-3} s}{(1 + 2.475 \times 10^{-3} s)(1 + 3.96 \times 10^{-3} s)}$$

Put,
$$s = j\omega$$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{j3.96 \times 10^{-3}\omega}{(1 + j2.475 \times 10^{-3}\omega)(1 + j3.96 \times 10^{-3}\omega)} \qquad \text{...(i)}$$

⇒ Overall gain (β_1) for fundamental frequency (f = 50Hz) is

$$\beta_1 = \left|\frac{V_o(j314)}{V_i(j314)}\right| = \left|\frac{j1.24344}{(1+j0.777)(1+j1.24344)}\right| = 0.6153$$

Attenuation for fundamental frequency = α_1

$$\alpha_1 = \frac{1}{\beta_1} = 1.625$$

Attenuation (α_1) in (dB) = 20 $\log_{10}(\alpha_1)$ = 4.21 dB

$\Rightarrow$ Overall gain for 5th harmonic = β_5

$$\beta_5 = \left|\frac{V_o(j1570)}{V_i(j1570)}\right| = \left|\frac{j6.2172}{25.2662\angle 156.43}\right| = 0.2460$$

$$\text{Attenuation } \alpha_5 = \frac{1}{\beta_5} = 4.06$$

$$\alpha_5 \text{ in (dB)} = 20 \log 4.06 = 12.17 \text{ dB}$$

$\Rightarrow$ Overall gain for 7th harmonic = β_7

$$\beta_7 = \left|\frac{V_o(j2198)}{V_i(j2198)}\right| = \left|\frac{j8.704}{48.46\angle 163.39}\right| = 0.1269$$

$$\text{Attenuation in (dB)} = 20\log\left(\frac{1}{\beta_7}\right) = 37.92 \text{ dB}$$

So overall attenuation for fundamental frequency = 4.21 dB

5th harmonic = 12.17 dB

7th harmonic = 37.92 dB

(iii) Since filter is passing fundamental component of frequency only.

Output of filter = I_{o1} [only fundamental component of frequency]

$$I_{o1} = I_1 \times \left|\frac{V_o(j314)}{V_i(j314)}\right| \times \sin\left(314t + \angle\left(\frac{V_o(j314)}{V_i(j314)}\right)^\circ\right)$$

$$I_{o1} = 0.6153\, I_1 \sin(314t + 0.9597^\circ) \text{ Amp}$$

So phase shift provided by band pass filter in fundamental component of current is 0.9597° only.

End of Solution

8. (c) Given the following facts about a real signal $x(t)$ with Laplace transform $X(s)$:

A : $X(s)$ has exactly two poles

B : $X(s)$ has no zeros in the finite s-plane

C : $X(s)$has a pole at $s = -1 + j$

D : $e^{2t} x(t)$ is not absolutely integrable

E : $X(0) = 8$

Determine $X(s)$ and specify its region of convergence.

[10 + 10 Marks]

Solution:

Since $X(s)$ has exactly two poles in the finite s-plane, $X(s)$ can be written as

$$X(s) = \frac{A}{(s-a)(s-b)}$$

Since $x(t)$ is real, the poles of $X(s)$ must occur in conjugate reciprocal pairs.

Therefore $\quad b = a^*$

$$X(s) = \frac{A}{(s-a)(s-a^*)}$$

$$X(s) = \frac{A}{(s+1-j)(s+1+j)} = \frac{A}{(s+1)^2+1}$$

Given that, $\quad X(0) = 8$

$$\frac{A}{1+1} = 8$$

$$A = 16$$

$$\therefore \quad X(s) = \frac{16}{s^2+2s+2}$$

From the pole locations we know that there are two possible choices of ROC, either $\sigma > -1$ (or) $\sigma < -1$.

Given that $e^{2t}\,x(t)$ is not absolutely integrable,

Let $\quad y(t) = e^{2t}x(t) \xleftrightarrow{L.T.} Y(s) = X(s-2)$

The ROC of $Y(s)$ is the ROC of $X(s)$ shifted by 2 to the right. The poles of $Y(s)$ are located at $-1 + j + 2 = 1 + j$ and $-1 - j + 2 = 1 - j$.

Since it is given that $y(t)$ is not absolutely integrable, the ROC of $Y(s)$ should not include the $j\omega$-axis. This is possible only if ROC of $X(s)$ is $\sigma > -1$.

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- **MADE EASY Conventional Classnotes Click Here for Reference)**

End of Solution

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Exam Year	Total Vacancies	Total Selections	Selection %	All India Rank-1 (Stream-wise)	Selections in Top 10 (out of 40)	Selections in Top 20 (out of 80)
ESE-2019	494	465	94%	All 4 Streams	40	78
ESE-2018	511	477	94%	All 4 Streams	38	78
ESE-2017	500	455	91%	All 4 Streams	40	78
ESE-2016	604	505	84%	All 4 Streams	39	76
ESE-2015	434	352	82%	All 4 Streams	38	73
ESE-2014	589	445	75%	All 4 Streams	32	64
ESE-2013	702	482	69%	All 4 Streams	34	62
ESE-2012	635	395	62%	All 4 Streams	32	60
ESE-2011	693	401	60%	CE, ME, EE	29	55
ESE-2010	584	295	51%	ME, EE, ET	26	51

Last 10 Years Results of GATE

Exam Year	Total AIR-1	All India Rank-1 (Stream-wise)	Ranks in Top 10	Ranks in Top 20	Ranks in Top 100
GATE-2020	9	CE, ME, EC, CS, IN, PI	61	109	441
GATE-2019	7	CE, ME, EE, EC, CS, IN, PI	60	118	426
GATE-2018	5	CE, ME, CS, IN, PI	57	103	406
GATE-2017	6	CE, ME, EE, CS, IN, PI	60	101	351
GATE-2016	6	ME, EE, EC, CS, IN, PI	53	96	368
GATE-2015	6	ME, EE, EC, CS, IN, PI	48	80	314
GATE-2014	5	CE, ME, EE, EC, IN	34	58	214
GATE-2013	3	CE, ME, PI	26	42	178
GATE-2012	3	CE, IN, PI	18	22	89
GATE-2011	2	ME, PI	06	11	57

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