

ESE

GATE

State Engg. Exams

MADE EASY
WORKBOOK 2026



**Detailed Explanations of
Try Yourself *Questions***

Mechanical Engineering
Heat Transfer



1

Conduction

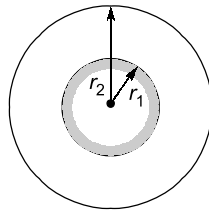


Detailed Explanation of Try Yourself Questions

T1 : Solution

$$r_1 = 1.2 \text{ cm} = 0.012 \text{ m}, r_2 = 1.8 \text{ cm} = 0.018 \text{ m}, T_1 = 500^\circ\text{C}$$

$$\left(\frac{dT}{dr}\right)_{r=r_1} = 0 \quad (\text{Insulated at inner surface})$$



Temperature profile in cylinder is given by

$$T = \frac{-\dot{q}r^2}{4k} + C_1 \ln r + C_2 \quad \dots(i)$$

At $r = r_1, T = 500^\circ\text{C}$

$$500 = \frac{-500 \times 10^3 \times (0.012)^2}{4 \times 0.55} + C_1 \ln(0.012) + C_2$$

$$C_2 - 4.42 C_1 = 532.73 \quad \dots(ii)$$

At $r = r_1, \frac{dT}{dr} = 0$

$$0 = \frac{-\dot{q}r}{2k} + \frac{C_1}{r} + 0$$

$$0 = \frac{-500 \times 10^3 \times 0.012}{2 \times 0.55} + \frac{C_1}{0.012}$$

$$\begin{aligned}
 C_1 &= 65.45 \\
 C_2 &= 822.019 \\
 \text{From equation (ii)} \quad T &= \frac{-500 \times 10^3 r^2}{4 \times 0.55} + 65.45 \ln r + 822.019 \\
 \text{From equation (i)} \quad T &= -227272.72 \times r^2 + 65.45 \ln r + 822.019 \\
 \text{at} \quad r &= r_2 \\
 T &= -227272.72 \times (0.018)^2 + 65.45 \ln 0.018 + 822.019 = 485.44^\circ\text{C}
 \end{aligned}$$

T2 : Solution

$$T = 120 - 100x + 24x^2 + 40x^3 - 30x^4$$

$$\frac{\partial^2 T}{\partial x^2} + 0 + 0 + 0 = \frac{1}{\alpha} \frac{\partial T}{\partial \tau}$$

Rate of heating or cooling,

$$\frac{\partial T}{\partial \tau} = \alpha \frac{\partial^2 T}{\partial x^2}$$

Location for maxima.

$$\frac{\partial}{\partial x} \left(\frac{\partial T}{\partial \tau} \right) = 0$$

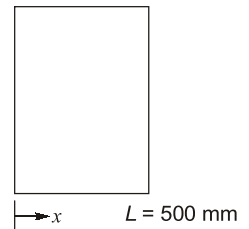
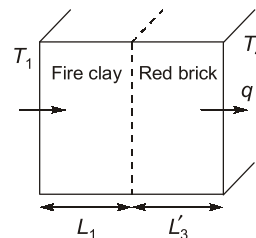
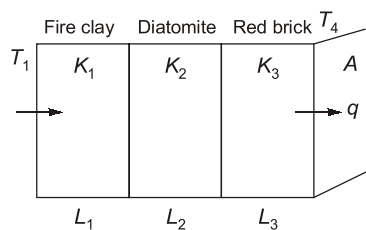
$$\frac{\partial}{\partial x} \left(\frac{\partial T}{\partial \tau} \right) = \alpha \frac{\partial^3 T}{\partial x^3}$$

$$\alpha \frac{\partial^3 T}{\partial x^3} = 0$$

$$\frac{\partial^3 T}{\partial x^3} = 0 - 0 + 0 + 240 - 720x$$

$$240 - 720x = 0$$

$$x = \frac{240}{720} = 0.333 \text{ m}$$


T3 : Solution


$$q = \frac{T_1 - T_4}{\frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A}}; \quad q = \frac{T_1 - T_4}{\frac{L_1}{K_1 A} + \frac{L'_3}{K_2 A}}$$

$$\frac{T_1 - T_4}{\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A}} = \frac{T_1 - T_4}{\frac{L_1}{k_1 A} + \frac{L'_3}{k_2 A}}$$

$$\frac{1}{\frac{0.11}{0.94} + \frac{0.06}{0.13} + \frac{0.25}{0.7}} = \frac{1}{\frac{0.11}{0.94} + \frac{L'_3}{0.7}}$$

$$L'_3 = 0.573 = 57.3 \text{ cm}$$

T4 : Solution

$t = 150 \text{ mm}$, $k = 15 \text{ W/mK}$

$$(i) \quad \dot{q} = \frac{h(T - T_\infty)}{0.150} = \frac{500 \times (100 - 20)}{0.150}$$

$$= 0.267 \times 10^6 \text{ W/m}^2$$

$$T(X) = a + bx + cx^2$$

$$T(0) = T_0 = 100^\circ\text{C}, T_\infty = 20^\circ\text{C}, h = 500 \text{ W/m}^2\text{-K}$$

$X = 0$
at $X = 0$,

$$T_0 = 100^\circ\text{C}$$

$$100 = a + 0 + 0, \quad (a = 100)$$

$$\left. \frac{dT}{dX} \right|_{x=L} = 0$$

$$b + 2cx = 0$$

$$b + 2c(0.15) = 0$$

$$b + 0.3c = 0$$

(ii) For steady state,

$$-\left[-k \frac{dT}{dX} \right]_{x=0} = h(T - T_\infty)$$

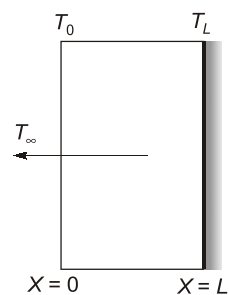
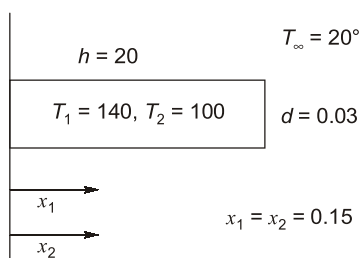
$$15 \times b = 500 \times (100 - 20)$$

$$b = 266.67 \times 10$$

$$b = 2.67 \times 10^3 \text{ k/m}$$

$$(iii) \quad 2.67 \times 10^3 + 0.3c = 0$$

$$c = -8.9 \times 10^3 \text{ k/m}$$

**T5 : Solution**

Since it is mentioned long rod, i.e., $L \rightarrow \infty$

$$\frac{T_1 - T_\infty}{T_s - T_\infty} = e^{-mx_1} \quad \dots(i)$$

$$\frac{T_2 - T_\infty}{T_s - T_\infty} = e^{-mx_2} \quad \dots(ii)$$

$$\frac{T_1 - T_\infty}{T_2 - T_\infty} = \frac{e^{-mx_1}}{e^{-mx_2}}$$

$$\frac{140 - 30}{100 - 30} = e^{m(x_2 - x_1)}$$

$$m \times 0.15 = \ln \left[\frac{110}{70} \right]$$

$$m = 3.01$$

$$\sqrt{\frac{h \times 4}{K \cdot d}} = 3.01$$

$$\frac{20 \times 4}{K \times 0.03} = 3.01^2$$

$$K = 294.3308 \text{ W/mK}$$

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2

Heat Exchanger



Detailed Explanation of Try Yourself Questions

T1 : Solution

(a) LMTD method: The rate of heat transfer in the heat exchanger is found as

$$Q = [\dot{m}c_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}}$$

$$= 1.2(4.18)(90 - 30) = 301 \text{ kW}$$

The outlet temperature of geothermal fluid is determined as

$$T_{\text{out}} = 160 - \frac{301}{2(4.31)} = 125.10^\circ\text{C}$$

Therefore,

$$\Delta T_1 = 160 - 90 = 70^\circ\text{C}$$

$$\Delta T_2 = 125.10 - 30 = 95.10^\circ\text{C}$$

and

$$LMTD = \frac{70 - 95.10}{\ln(70/95.10)} = 81.91^\circ\text{C}$$

Hence

$$A = \frac{Q}{U(LMTD)} = \frac{301 \times 10^3}{(600)(81.91)} = 6.12 \text{ m}^2$$

To provide this surface area, the length of the tube required is found as

$$L = \frac{A}{\pi D} = \frac{6.12}{\pi(0.015)} = 129.87 \text{ m}$$

(b) *NTU Method*: We first determine the heat capacity rates of the hot and cold fluids to identify the smaller value of the two.

$$C_h = \dot{m}_h c_h = 2(4.31) = 8.62 \text{ kW/}^\circ\text{C}$$

$$C_c = \dot{m}_c c_c = 1.2(4.18) = 5.02 \text{ kW/}^\circ\text{C}$$

Therefore,

$$C_{\min} = C_c = 5.02 \text{ kW/}^\circ\text{C}$$

and

$$C = C_{\min}/C_{\max} = 5.02/8.62 = 0.583$$

$$\varepsilon = \frac{Q}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{301.00}{5.02(160 - 30)} = 0.461$$

Now we determine the value of NTU by making use of the expression of NTU for a counterflow heat exchanger from equation.

$$NTU = \frac{1}{C-1} \ln \left(\frac{\epsilon-1}{\epsilon C-1} \right) = \frac{1}{0.582-1} \ln \left(\frac{0.4615-1}{0.4615 \times 0.582-1} \right) = 0.7325$$

We know

$$NTU = \frac{UA}{C_{\min}}$$

or

$$A = \frac{NTU \cdot C_{\min}}{U} = \frac{0.7325(5.016 \times 10^3)}{600} = 6.1237 \text{ m}^2$$

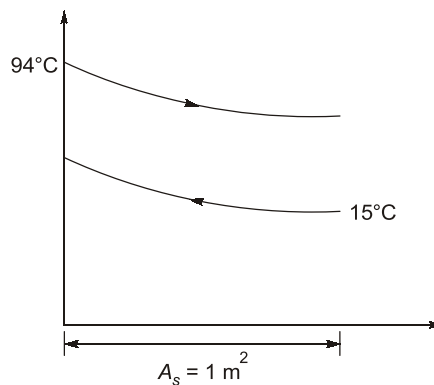
$$= \pi \times 0.015 \times L$$

Hence,

$$L = 129.95 \text{ m}$$

Therefore, we find that almost same result is obtained in both the methods.

T2 : Solution



$$NTU = \frac{UA_s}{C_{\min}} = \frac{1075 \times 1}{305.4} = 3.52$$

$$U = 1075 \text{ W/m}^2\text{-K}$$

$$C_h = \dot{m}_h c_h = 0.1527 \times 2000 = 305.4$$

$$C_{\min} = 305.4$$

$$C_c = \dot{m}_c c_c = 0.361 \times 480 = 1508.98$$

$$C_{\max} = 1508.98$$

$$C_h < C_c$$

$$R = \frac{C_{\min}}{C_{\max}} = 0.20$$

$$\dot{m}_h = \frac{550}{3600} = 0.1527 \text{ kg/sec}$$

$$\dot{m}_c = \frac{1300}{3600} = 0.361 \text{ kg/sec}$$

$$\frac{C_h \Delta T_h}{C_{\min} \Delta T_{\max}} = \epsilon = \frac{1 - \exp[-NTU(1-R)]}{1 - R \exp[-NTU(1-R)]}$$

$$\frac{[T_{hi} - T_{ho}]}{[T_{hi} - T_{ci}]} = \frac{1 - \exp[-3.52(1 - 0.2)]}{1 - 0.2 \exp[-3.52(1 - 0.2)]}$$

$$\frac{94 - T_{ho}}{94 - 15} = \frac{1 - 0.0598}{1 - 0.2 \times 0.598} = 0.932$$

$$T_{ho} = 20.372^\circ\text{C}$$

$$\begin{aligned} q &= \dot{m}_h c_h [T_{hi} - T_{ho}] = 305.4 [94 - 20.372] \\ &= 22.486 \text{ kW} \end{aligned}$$

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Detailed Explanation of Try Yourself Questions

T1 : Solution

We know that,

$$\frac{Q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{R}$$

Here $A_1 = A_2 = A$, and R is the equivalent resistance of the thermal network

$$\epsilon_1 = 0.75, \epsilon_2 = 0.70$$

By summation rule of view factors

$$F_{33} + F_{31} + F_{32} = 1$$

$$F_{33} = 0 \text{ (in consideration of furnace surfaces to be plane)}$$

From symmetry,

$$F_{31} = F_{32}$$

Hence,

$$F_{31} = F_{32} = 0.5$$

Again from the reciprocity relation, $F_{13} = F_{31} = 0.5$

$$F_{23} = F_{32} = 0.5 \text{ (since } A_1 = A_2 = A_3 = A)$$

Again

$$F_{11} + F_{12} + F_{13} = 1$$

$$F_{11} = 0 \text{ and } F_{13} = 0.5$$

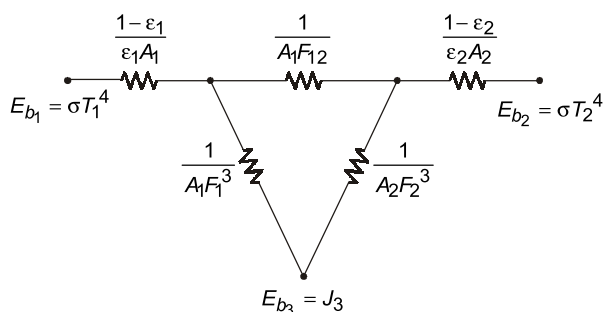
Hence,

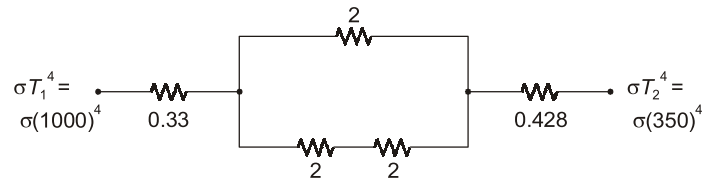
$$F_{12} = 0.5$$

As we know,

$$F_{12} = F_{13} = F_{23} = 0.5$$

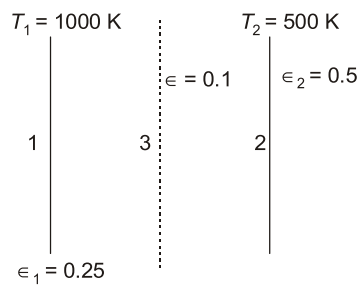
Therefore





$$R = \frac{1-0.75}{0.75} + \frac{1}{0.5 + (2+2)^{-1}} + \frac{0.3}{0.7} = 2.09$$

$$\begin{aligned} \frac{Q}{A} &= \frac{5.67 \times 10^8 (1000^4 - 350^4)}{2.09} \\ &= 26.72 \times 10^3 \text{ W/m}^2 = 26.72 \text{ kW/m}^2 \end{aligned}$$

T2 : Solution

$$(a) \quad \frac{\dot{q}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{12}} + \frac{1-\epsilon_2}{\epsilon_2}} = \frac{5.67 \times 10^{-8} \times (1000^4 - 500^4)}{3 + 1 + 1}$$

$$\frac{q}{A} = 10631.25 \text{ W/m}^2 = 10.631 \frac{\text{kW}}{\text{m}^2}$$

$$(b) \quad \frac{\dot{q}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{13}} + \left(\frac{1-\epsilon}{\epsilon}\right) + \left(\frac{1-\epsilon}{\epsilon}\right) + \frac{1}{F_{23}} + \frac{1-\epsilon_2}{\epsilon_2}}$$

$$\left(\frac{\dot{q}}{A}\right) = \frac{5.67 \times (1000^4 - 500^4) \times 10^{-8}}{3 + 1 + 9 + 9 + 1 + 1}$$

$$\frac{q}{A} = 2214.843 \text{ W/m}^2$$

or

$$\frac{q}{A} = 2.214 \frac{\text{kW}}{\text{m}^2}$$

$$(c) \quad \frac{\dot{q}}{A} = \frac{\sigma(T_1^4 - T^4)}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{23}} + \frac{1-\epsilon}{\epsilon}}$$

$$2214.84 = \frac{5.67 \times 10^{-8} \times (1000^4 - T^4)}{3 + 1 + 9}$$

$$T = 837.59 \text{ K}$$



4

Convection



Detailed Explanation of Try Yourself Questions

T1 : Solution

$$Pr = \frac{\mu C_p}{k} = \frac{2.131 \times 10^{-5} \times 1.01 \times 10^3}{0.031} = 0.694$$

$$Re = \frac{\rho VL}{\mu} = \frac{0.962 \times 12 \times 2}{2.131 \times 10^{-5}} = 1.0834 \times 10^6 > 5 \times 10^5$$

So, flow is turbulent at the end of the plate. Distance upto which flow is laminar (x_{cr}):

$$Re_{cr} = 5 \times 10^5 = \frac{\rho V x_{cr}}{\mu}$$

$$5 \times 10^5 = \frac{0.9620 \times 12 \times x_{cr}}{2.131 \times 10^{-5}}$$

$$x_{cr} = 0.923 \text{ m}$$

For laminar region, $Nu = 0.332 Re_x^{1/2} Pr^{1/3}$

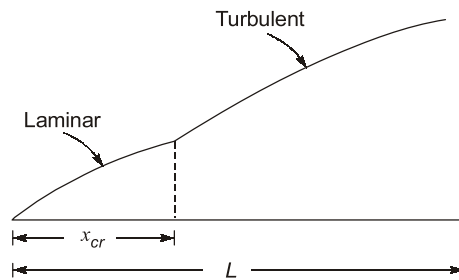
$$\frac{h_x \times x}{0.031} = 0.332 \left(\frac{0.962 \times 12 \times x}{2.131 \times 10^{-5}} \right)^{1/2} (0.694)^{1/3}$$

$$h_{x,L} = \frac{6.7067}{\sqrt{x}}$$

For turbulent region, $Nu = 0.0296 Re_x^{4/5} Pr^{1/3}$

$$\frac{h_{x,T} x}{0.031} = 0.0296 \left(\frac{0.962 \times 12 x}{2.131 \times 10^{-5}} \right)^{4/5} (0.694)^{1/3}$$

$$h_{x,T} = \frac{31.39}{x^{1/5}}$$



Average heat transfer coefficient,

$$\begin{aligned}\bar{h} &= \frac{1}{L} \left[\int_0^{x_{cr}} h_{x,L} dx + \int_{x_{cr}}^L h_{x,T} dx \right] = \frac{1}{2} \left[\int_0^{0.923} \frac{6.7067}{\sqrt{x}} dx + \int_{0.923}^2 \frac{31.39}{x^{1/5}} dx \right] \\ &= \frac{1}{2} \left[6.7067 \left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_0^{0.923} + 31.39 \left[\frac{x^{-\frac{1}{5}+1}}{-\frac{1}{5}+1} \right]_{0.923}^2 \right] \\ &= \frac{1}{2} \left[2 \times 6.7067 \times 0.923^{1/2} + 31.39 \times \frac{5}{4} \times (2^{4/5} - 0.923^{4/5}) \right] \\ &= 22.20 \text{ W/m}^2\text{-K}\end{aligned}$$

T2 : Solution

Given: Height, $L = 1.5$, Width, $W = 1$ m, Plate temp, $t_s = 150^\circ\text{C}$, Surrounding temp, $t_\infty = 30^\circ\text{C}$,

Average temp $t_{avg} = 90^\circ\text{C}$

for

$$\rho = 0.946 \text{ kg/m}^3, k_a = 0.0313 \text{ W/mK}$$

$$\nu = 22.10 \times 10^{-6} \text{ m}^2/\text{s}, C_p = 1.009 \text{ kJ/kg-K}$$

So,

$$Gr = \frac{\rho \beta \Delta t L^3}{\nu^2} = \frac{9.81 \times \frac{1}{273+90} \times 120 \times 1.5^3}{(22.10 \times 10^{-6})^2} \quad [\text{As } \beta = \frac{1}{273+t_{avg}}]$$

$$Gr = 2.24095 \times 10^{10}$$

Prandtl number,

$$Pr = \frac{\rho \gamma C_p}{k} = 0.67395$$

$$Ra_L = Gr \times Pr = 1.51 \times 10^{10}$$

As given

$$Nu_L = 0.59 (Ra_L)^{0.25}$$

$$\frac{hL}{k_a} = 50.562 \times 0.59 = 206.832$$

$$h = 4.316 \text{ W/m}^2\text{K}$$

So rate of heat transfer from both the surfaces,

$$'Q' = 2 \times h \times A \times (t_s - t_\infty) = 1553.76 \text{ W}$$

