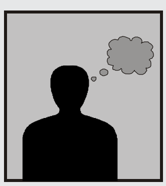
ESE GATE

State Engg. Exams

MADE EASY Workbook 2026



Detailed Explanations of Try Yourself *Questions*

Mechanical Engineering

Industrial Engineering

MADE EASY PUBLICATIONS

7 LPP, Transportation and Assignment

Detailed Explanation of Try Yourself Questions

T1 : Solution

By introducing slack variable s_1, s_2, s_3 and s_4 the problem may be written as follows

Maximum $Z = 3x_1 + 2x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4$

$$x_1 + x_2 + s_1 = 6$$
$$2x_1 + x_2 + s_2 = 8$$
$$-x_1 + x_2 + s_3 = 1$$
$$x_2 + s_4 = 2$$
$$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$$

The above information is tabulated

Table - 1

Basis \ C_j	3	2	0	0	0	0		$\theta = \frac{b}{C_i}$
	x_1	x_2	s_1	s_2	s_3	s_4	b	
0 s_1	1	1	1	0	0	0	6	6
← 0 s_2	[2]*	1	0	1	0	0	8	4 ←
0 s_3	–1	1	0	0	1	0	1	–1
0 s_4	0	1	0	0	0	1	2	∞
E_j	0	0	0	0	0	0		
$E_j - C_j$	–3 ↑	–2	0	0	0	0		

E_j and $E_j - C_j$ are calculated

The entering column is identified and the θ values are calculated, then the leaving row are identified

The pivotal element/value is identified from the table

s_2 leaves the basis, x_1 enters the basis

The following row operations are performed

$$R_{2new} = R_{2old}/2$$
$$R_{1new} = R_{1old} - R_{2new}$$
$$R_{3new} = R_{3old} + R_{2new}$$

Then the table 1 becomes

E_j and $E_j - C_j$ are calculated

The entering column is identified and the θ values are calculated, then the leaving row are identified

The pivotal element/value is identified from the table

s_4 leaves the basis, x_2 enters the basis

Table - 2

Basis \ C_j	3 x_1	2 x_2	0 s_1	0 s_2	0 s_3	0 s_4	b	$\theta = \frac{b}{C_i}$
0 s_1	1	1/2	1	–1/2	0	0	2	4
← 0 s_2	1	1/2	0	1/2	0	0	4	8
0 s_3	0	3/2	0	1/2	1	0	5	10/3
0 s_4	0	[1]*	0	1	0	1	2	2
E_j	3	3/2	0	3/2	0	0		
$E_j - C_j$	0	–1/2 ↑	0	3/2	0	0		

The following row operations are performed

$$R_{1new} = R_{1old} - (R_{4new})/2$$
$$R_{2new} = R_{2old} - (R_{4new})/2$$
$$R_{3new} = 2R_{3old} - 3(R_{4new})/2$$

Then the table 2 becomes table 3

E_j and $E_j - C_j$ are calculated

Table - 3

Basis \ C_j	3 X_1	2 X_2	0 S_1	0 S_2	0 S_3	0 S_4	b
0 S_1	0	0	1	–1/2	0	–1/2	1
← 0 X_1	1	0	0	1/2	0	–1/2	3
0 S_3	0	0	0	1/2	1	–3/2	2
0 X_2	0	1	0	0	0	1	2
E_j	3	2	0	3/2	0	1/2	
$E_j - C_j$	0	0	0	3/2	0	1/2	

It is an optimum matrix since all the values obtained in '$E_j - C_j$' row are positive

Thus the optimum solution will change as

$$\Rightarrow \left.\begin{matrix} x_1 = 3 \\ x_2 = 2 \\ S_1 = 1 \\ S_3 = 2 \end{matrix}\right\} \text{Basic variables} \quad \left.\begin{matrix} S_2 = 0 \\ S_4 = 0 \end{matrix}\right\} \text{Non basic variables}$$

Hence, $Z = 3 \times 3 + 2 \times 2 = 13$

T2 : Solution

The simplex problem can be expressed in the form of matrices as follows

$$\text{Minimum } Z = \overset{A}{(5 \;\; 2 \;\; 2)} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}^{X}$$

$$\begin{pmatrix} 2 & 3 & 1 \\ 6 & 8 & 5 \\ 7 & 1 & 3 \\ 1 & 2 & 4 \end{pmatrix}^{B} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}^{X} \geq \begin{pmatrix} 20 \\ 30 \\ 40 \\ 50 \end{pmatrix}^{C}$$

Note: If primal problem is $\begin{pmatrix} \min Z = AX \\ BX \geq C \end{pmatrix}$ then Dual problem is $\begin{pmatrix} \min Z = C^T Y \\ B^T Y \leq A^T \end{pmatrix}$

So the dual for the given problem becomes

$$\text{Maximum } Z = (20 \; 30 \; 40 \; 50) \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 6 & 7 & 1 \\ 3 & 8 & 1 & 2 \\ 1 & 5 & 3 & 4 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \leq \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$$

T3 : Solution

Let s_1, s_2, s_3 be slack variables then the constraints become

$$x + y + s_1 = 90$$
$$x + 2y + s_2 = 80$$
$$x + y + s_3 = 50$$
$$x, y, s_1, s_2, s_3 \geq 90$$

$\Rightarrow$ $$Z = 45x + 40y + 0s_1 + 0s_2 + 0s_3$$

The above information is tabulated, E_j and $E_j - C_j$ are calculated.

The entering column is identified and the θ values are calculated, then the leaving row are identified

Table - 1

Basis ↓ / C_j →	45 x	40 y	0 s_1	0 s_2	0 s_3	b	$\theta = \frac{b}{C_i}$
0 s_1	1	1	1	0	0	90	90
0 s_2	1	2	0	1	0	80	80
0 s_3	[1]*	1	0	0	1	50	50 ←
E_j	0	0	0	0	0		
$E_j - C_j$	–45 ↑	–40	0	0	0		

s_3 leaves the basis and x enters into the basis, then the table 1 becomes table 2, E_j and $E_j - C_j$ are calculated.

Table - 2

Basis ↓ / C_j →	45 x	40 y	0 s_1	0 s_2	0 s_3	b	$\theta = \frac{b}{C_i}$
0 s_1	0	0	1	0	–1	40	
0 s_2	0	1	0	1	–1	30	
45 x	1	1	0	0	1	50	
E_j	45	45	0	0	45		
$E_j - C_j$	0	5	0	0	46		

As '$E_j - C_j$' values present in the matrix are positive. So it is an optimum matrix.
So solution can be read out from matrix i.e.,
$x = 50,\ y = 0,$
$\therefore$ by putting $(x, y) = (50, 0)$ in the objective function the maximum value of $Z = 45x + 40y = 2250$.

T4 : Solution

6	5	8	11	16
1	13	16	1	10
16	11	8	8	8
9	14	12	10	16
10	13	11	8	16

Steps I - If the number of lines is equal to the order of matrix then it is the optimal table. If the number of lines is less than the order of matrix, then select the minimum number which is not covered by the lines. Subtract this value from each and every element which is uncovered and add this value at the intersection point of lines. Here adding "3" minimum uncovered value at intersection and subtracting at non covered values.

1	[0]	3	6	11
~~0~~	12	15	~~0~~	9
8	3	[0]	~~0~~	~~0~~
[0]	5	3	1	7
2	5	3	[0]	8

We will continue to repeat step I, till number of lines are equal to the order of the matrix. Now number of lines are equal to the order of the matrix, so select zeros such that every column and row consists of one zero i.e., one allocation for one machine.

1	[0]	~~0~~	6	8
~~0~~	12	12	[0]	6
11	6	~~0~~	3	[0]
[0]	5	~~0~~	1	4
2	5	[0]	~~0~~	5

A–II, B–IV, C–V, D–I, E–III

The total processing time is 34.

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