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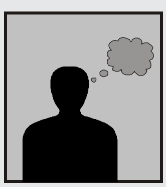

Detailed Explanations of Try Yourself *Questions*

Mechanical Engineering

Theory of Machines

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1 Mechanism

Detailed Explanation of Try Yourself Questions

T1 : Solution

Pair Symbol	Constrained motion	Relative Motion	Degrees of Freedom
Revolute pair	1	Circular	5
Cylindrical pair	2	Cylindrical	4
Screw pair	1	Helical	5
Spherical pair	3	Spherical	3

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3 Gears and Gear Trains

Detailed Explanation of Try Yourself Questions

T1 : Solution

Given data : $T_p = 36$, $T_g = 96$, $\phi = 20°$, $m = 10$ mm, $a_m = 10$ mm

Pitch circle radius,

$$R = \frac{mT_g}{2} = \frac{10 \times 96}{2} = 480 \text{ mm}$$

Gear Addendum radius,

$$R_a = R + 10 = 490 \text{ mm}$$

$$r = \frac{mT_p}{2} = \frac{10 \times 36}{2} = 180 \text{ mm, pinion}$$

$$r_a = r + 10 = 190 \text{ mm}$$

Path of contact $= \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi + \sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi$

or $= \sqrt{490^2 - (480\cos 20°)^2} - 480\sin 20° + \sqrt{190^2 - (180\cos 20°)^2} - 180\sin 20°$

$= 191.446 - 164.17 + 86.54 - 61.56 =$ **52.256 mm**

Arc of contact $= \frac{\text{Path of contact}}{\cos 20°} = \frac{52.256}{\cos 20°} =$ **55.6 mm**

Contact ratio $= \frac{\text{Arc of contact}}{\text{Circular pitch}} = \frac{55.6}{\pi \times 10} =$ **1.77**

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6 Governor

Detailed Explanation *of* Try Yourself Questions

T1 : Solution

As per given information, r_1 = 120 mm, r_2 = 80 mm,
ball arm = sleeve arm ($a = b$), m = 2 kg

N_2 = 400 rpm, $\omega_1 = \frac{2\pi \times 400}{60}$, N_1 = 420 rpm, $\omega_2 = \frac{2\pi \times 420}{60}$

Sprint constant?

$$F_1 = mr_1\omega_1^2 = 2 \times 0.120 \times \left(\frac{2\pi \times 420}{60}\right)^2 = 464.266 \text{ N}$$

$$F_2 = mr_2\omega_2^2 = 2 \times 0.80 \times \left(\frac{2\pi \times 400}{60}\right)^2 = 280.735 \text{ N}$$

$$\text{Spring constant, } K = 2\left(\frac{a}{b}\right)^2\left(\frac{F_1 - F_2}{r_1 - r_2}\right)$$

$$= 2(1)^2\left(\frac{464.266 - 280.735}{0.040}\right) = 9.176 \times 10^3 \text{ N/m}$$

(ii) Spring constant, K = 9.176 N/mm

$$F_2 \times a = 0 + \frac{F_{s1}}{2} \cdot b$$

$$F_{s1} = 2F_2 = 2 \times 280.735 \text{ N}$$

(i) Initial compression $= \frac{F_{s1}}{K} = \frac{2 \times 280.735}{9.176}$ N = 61.1889 mm

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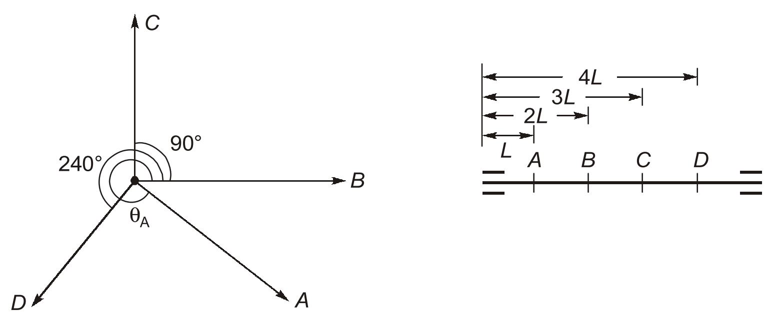
7 Balancing

Detailed Explanation of Try Yourself Questions

T1 : Solution



$\Sigma F_x = 0$, $m_A\, r \cos\theta + m_B\, r\cos 0° + m_C\, r\cos 90° + m_D\, r\cos 240° = 0$

$\Sigma F_y = 0$, $m_A\, r\sin\theta + m_B\, r\sin 0° + m_C\, r\sin 90° + m_D\, r\sin 240° = 0$

$$\Sigma F_x = mr\cos\theta + m_B r - \frac{m_D r}{2} = 0$$

$$m_A\cos\theta + m_B = \frac{m_D}{2}$$

$$m_A\cos\theta + 7 = \frac{m_D}{2}$$

$$\Sigma F_y = 0$$

$$m_A\sin\theta + m_C - \frac{\sqrt{3}}{2}m_D = 0$$

$$m_A\sin\theta + m_C = \frac{\sqrt{3}}{2}m_D$$

Dynamic

$\Sigma M_x = 0$

$m_A\, r\, l\cos\theta + m_B\, r\, 2\, l\cos 0° + m_C\, r\, 3\, l\cos 90° + m_D\, r\, 4\, l\cos 240° = 0$

$$m_A\cos\theta + 2\, m_B = 2\, m_D$$

$\Sigma M_y = 0$

$m_A\, r\, l \sin\theta + m_B\, r\, 2\, l \sin 0° + m_C\, r\, 3\, l \sin 90° + m_D\, r\, 4\, l \sin 240° = 0$

$$m_A \sin\theta + 3\, m_C = 2\sqrt{3}\, m_D$$

$$m_A \cos\theta + 7 = \frac{m_D}{2} \quad \text{... (i)}$$

$$m_A \sin\theta + m_C = \frac{\sqrt{3}}{2} m_D \quad \text{... (ii)}$$

$$m_A \cos\theta + 14 = 2\, m_D \quad \text{... (iii)}$$

$$m_A \sin\theta + 3\, m_C = 2\sqrt{3}\, m_D \quad \text{... (iv)}$$

From equation (iii) – (i)

$$m_D = 4.667 \text{ kg}$$

$$m_A \cos\theta + 7 = 2.33$$

$$m_A \sin\theta + m_C = 4.04$$

$$m_A \cos\theta + 14 = 9.332$$

$$m_A \sin\theta + 3\, m_C = 16.14$$

$$m_A \sin 0° + m_C = 4.04$$

$$m_C = 6.0667 \text{ kg}$$

$$m_A \sin\theta = -2$$

$$m_A \cos\theta = -4.66$$

$$m_A = 5.087 \text{ kg}$$

$$\theta = 203.456°$$

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10 Vibration

Detailed Explanation of Try Yourself Questions

T1 : Solution

Given; d = 50 mm = 0.05 m; l = 300 mm = 0.03 m; m = 100 kg; E = 200 GN/m^2 = 200 × 109 N/m^2

We knowt hat cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(0.05)^2 = 1.96 \times 10^{-3} \text{m}^2$$

and moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.3 \times 10^{-6} \text{m}^4$$

Frequency of longitudinal vibration

We know that static deflection of the shaft,

$$\delta = \frac{W.l.}{A.E} = \frac{100 \times 9.81 \times 0.3}{1.96 \times 10^{-3} \times 200 \times 10^9} = 0.751 \times 10^{-6} \text{ m}; \quad \omega_n = \sqrt{g/\delta}$$

∴ Frequency of longitudinal vibration,

$$f_n = \frac{\omega_n}{2\pi} = \left(\frac{\sqrt{g}}{2\pi}\right) \times \frac{1}{\sqrt{\delta}}$$

$$\Rightarrow \quad f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.751 \times 10^{-6}}} = 575 \text{ Hz}$$

Frequency of transverse vibration

We know that static deflection of the shaft,

$$\delta = \frac{W.l^3}{3E.I} = \frac{100 \times 9.81 \times (0.3)^3}{3 \times 200 \times 10^9 \times 0.3 \times 10^{-6}} = 0.147 \times 10^{-3} \text{ m}$$

∴ Frequency of transverse vibration,

$$f_n = \frac{\omega_n}{2\pi} = \left(\sqrt{\frac{g}{2\pi}}\right) \times \frac{1}{\sqrt{\delta}}$$

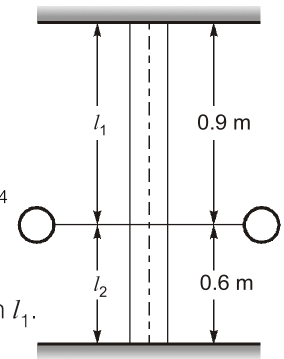
$$\Rightarrow \quad f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.147 \times 10^{-3}}} = 41 \text{Hz}$$

T2 : Solution

Given: d = 50 mm = 0.05 m; m = 500 kg; E = 200 GN/m^2 = 200 × 10^9 N/m^2

We know that cross-sectional area of shaft,

$$A = \frac{\pi}{4}\times d^2 = \frac{\pi}{4}(0.05)^2 = 1.96\times10^{-3}\text{m}^2$$



and moment of inertia of shaft,

$$I = \frac{\pi}{64}\times d^4 = \frac{\pi}{64}(0.05)^4 = 0.307\times10^{-6}\text{m}^4$$

Natural frequency of longitudinal vibration

Let m_1 = Mass of flywheel carried by the length l_1.

∴ $m - m_1$ = Mass of flywheel carried by length l_2.

We know that extension of length l_1

$$= \frac{W_1\cdot l_1}{A.E} = \frac{m_1.g.l_1}{A.E} \quad ...(i)$$

Similarly, compression of length l_2

$$= \frac{(W-W_1)l_2}{A.E} = \frac{(m-m_1)g.l_2}{A.E} \quad ...(ii)$$

Since extenson of length l_1 must be equal to compression of length l_2, therefore equating equations (i) and (ii),

$$m_1.l_1 = (m-m_1)l_2$$

$$m_1\times0.9 = (500 - m_1)0.6 = 300 - 0.6\, m_1 \quad \text{or} \quad m_1 = 200 \text{ kg}$$

∴ Extension of length l_1,

$$\delta = \frac{m_1.g.l_1}{A.E} = \frac{200\times9.81\times0.9}{1.96\times10^{-3}\times200\times10^9} = 4.5\times10^{-6}\text{m}$$

We know that natural frequency of longitudinal vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{4.5\times10^{-6}}} = 235 \text{ Hz}$$

Alternate: Natural freuency of longitudinal vibration

$$\text{Axial stiffness} = \frac{AF}{l} \Rightarrow s_1 = \frac{AE}{l_1}; s_2 = \frac{AE}{l_2}$$

The 2-stiffness are in parallel

⇒ $$s = s_1 + s_2 = 1.0908\times10^9$$

$$w_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{1.0908\times10^9}{500}} = 1477.04$$

$$f = \frac{w_n}{2\pi} = 235 \text{ Hz}$$

Natural frequency of tranverse vibration

We know that the static deflection for a shaft fixed at both ends and carrying a point load is given by

$$\delta = \frac{Wa^3b^3}{3EIl^3} = \frac{500\times9.81(0.9)^3(0.6)^3}{3\times200\times10^9\times0.307\times10^{-6}(1.5)^3} = 1.24\times10^{-3}\text{m}$$

...(Substituting $W = m.g$; $a = l_1$, and $b = l_2$)

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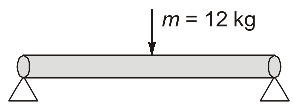
We know that natural frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{1.24 \times 10^{-3}}} = 14.15 \text{ Hz}$$

T3 : Solution

Given:



Since it is a short bearing
∴ It is simply supported.

(i) Deflection at mid span,

$$\Delta = \frac{(mg)L^3}{48EI}$$

$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{48EI}{L^3 \times m}} = \sqrt{\frac{48 \times 200 \times 10^9 \times \pi \times (.01)^4}{64 \times 0.4^3 \times 12}}$$

$$\omega_n = 78.332 \text{ rad/s} \Rightarrow 748 \text{ rpm}$$

(ii)

$$\Delta = \frac{Fl^3}{48EI} + \frac{5\omega l^4}{384EI}$$

Where

$$\omega = \frac{W}{L} = \frac{\rho \times V_g}{L} = \rho \times \frac{\pi}{4} d^2 \times g$$

∴

$$\Delta = 1.5987 \times 10^{-3} + 1.962 \times 10^{-5}$$

$$\omega_n = \sqrt{\frac{g}{\Delta}} = 77.85 \text{ rad/s}$$

$$N = \frac{\omega_n \times 60}{2\pi} = 743.48 \text{ rpm} \approx 744 \text{ rpm}$$

T4 : Solution

Given: $f_d = 90/\text{min} = 90/60 = 1.5$ Hz

We know that time period,

$$t_p = 1/f_d = 1/1.5 = 0.67 \text{ s}$$

Let

x_1 = Initial amplitude, and

x_2 = Final amplitude after one complete vibration

$= 20\%\ x_1 = 0.2\, x_1$ ...(Given)

We know that $\log_e\left(\frac{x_1}{x_2}\right) = a.t_p$ or $\log_e\left(\frac{x_1}{0.2x_1}\right) = a \times 0.67$

∴ $\log_e 5 = 0.67\, a$ or $1.61 = 0.67\, a$ or $a = 2.4$ ...(∵ $\log_e 5 = 1.61$)

We also know that frequency of free damped vibration

$$f_d = \frac{1}{2\pi}\sqrt{(\omega_n)^2 - a^2}$$

or

$$(\omega_n)^2 = (2\pi \times f_d)^2 + a^2 \quad \text{...(By squaring and arranging)}$$

$$= (2\pi \times 1.5)^2 + (2.4)^2 = 94.6$$

$$\omega_n = 9.726 \text{ rad/s}$$

We know that frequency of undamped vibration,

$$f_n = \frac{\omega_n}{2\pi} = \frac{9.726}{2\pi} = 1.55 \text{ Hz}$$

Alternate

Damped frquency, $f_d = 1.5$ /s

Given $n_1 = 0.2\, x_0$

$$\Rightarrow \quad \frac{x_0}{x_1} = \frac{x_1}{x_2} = \ldots \frac{x_{n-1}}{x_n} = 5 = e^{\delta}$$ where δ = logarithmic decrement

$$\Rightarrow \quad d = \mathit{ln}\, 5 = 1.609$$

$$\Rightarrow \quad \frac{2\pi_{\xi}}{\sqrt{1-\xi^2}} = 1.609$$

$$\Rightarrow \quad \frac{\xi^2}{1-\xi^2} = 0.066$$

$$\Rightarrow \quad \frac{1}{\xi^2} - 1 = 15.241$$

$$\Rightarrow \quad \xi = 0.248$$

$$\omega_d = \sqrt{1-\xi^2}\,\omega_n$$

$$\Rightarrow \quad \frac{\omega_d}{2\pi} = \sqrt{1-\xi^2}\,\frac{\omega_n}{2\pi}$$

$$\Rightarrow \quad f_d = \sqrt{1-\xi^2}\, f$$

$$\Rightarrow \quad f_n = \frac{f_d}{\sqrt{1-\xi^2}} = \frac{1.5}{\sqrt{1-0.248^2}} = 1.55 \text{ Hz}$$

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