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**PTQ**

**Prelims  
Through  
Questions**

*for*

**ESE 2021**

**Civil Engineering**

**Day 1 of 11**

**Q.1 - Q.50**

(Out of 500 Questions)

Strength of Materials + Structural Analysis

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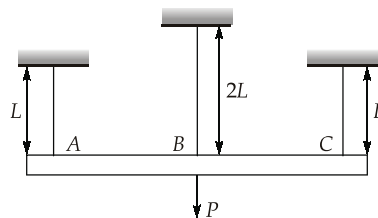
## Strength of Materials + Structural Analysis

**Q.1** The property of metal which allows it to deform continuously at slow rate without any further increase in stress is known as

- (a) fatigue (b) creep  
(c) plasticity (d) resilience

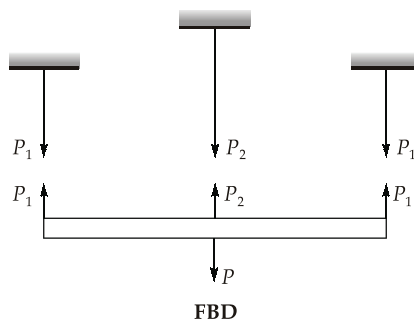
1. (b)

**Q.2** A rigid bar  $ABC$  is supported by three rods of the same materials as shown below. If the bar remains horizontal after applying a load  $P$ , then the forces in the outer bars due to the applied load is



- (a)  $0.6P$  (b)  $0.8P$   
(c)  $0.4P$  (d)  $0.2P$

2. (c)



From statical equilibrium

$$2P_1 + P_2 = P$$

From compatibility equation,  $\Delta_1 = \Delta_2$

$$\frac{P_1 L}{AE} = \frac{P_2 (2L)}{AE}$$

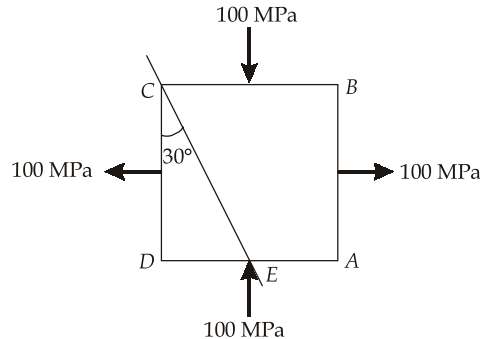
$$P_1 = 2P_2$$

$\therefore 2(2P_2) + P_2 = P$

$$P_2 = 0.2 P$$

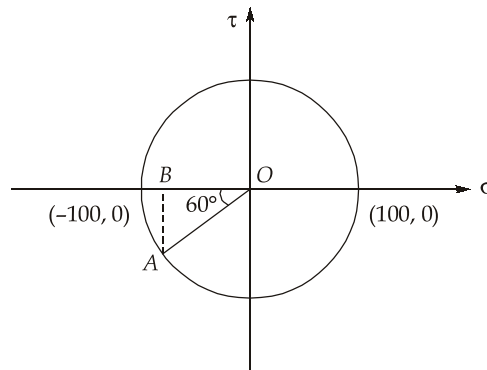
$$P_1 = 0.4 P$$

**Q.3** For the state of stress as shown in the figure below, magnitude of normal stress acting on the plane *CE* shown in figure is



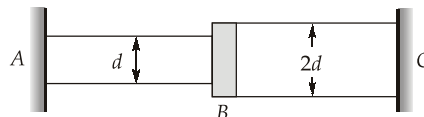
- (a) 100 MPa  
(b) 50 MPa  
(c)  $50\sqrt{3}$   
(d) zero

**3. (b)**  
In Mohr's circle of stress elements,



$$\begin{aligned}
 OA &= \text{Radius of Mohr's circle} = 100 \text{ MPa} \\
 \text{Normal stress on plane } CE, \quad OB &= -OA \cos 60^\circ \\
 &= -100 \times \frac{1}{2} = -50 \text{ MPa} \\
 \text{Magnitude of normal stress} &= 50 \text{ MPa}
 \end{aligned}$$

**Q.4** Two shafts *AB* and *BC* of equal length and diameter *d* and *2d* are made of the same material. They are joined at *B* through a shaft coupling. A twisting moment (*T*) is applied at *B*. If  $T_A$  and  $T_C$  represent the twisting moments at the ends *A* and *C*, respectively then



- (a)  $T_C = T_A$   
(b)  $T_C = 8T_A$   
(c)  $T_C = 16 T_A$   
(d)  $T_A = 16T_C$

4. (c)

$$\theta_{BA} = \theta_{BC}$$

Now  $\frac{T}{I_P} = \frac{G\theta}{L} \quad \left( \because \theta = \frac{TL}{GI_P} \right)$

$\therefore \theta_{BA} = \frac{T_A \cdot L}{GI_{PAB}}$

and  $\theta_{BC} = \frac{T_C \cdot L}{GI_{PBC}}$

Also,  $I_{PAB} = \frac{\pi d^4}{32}$

$$I_{PBC} = \frac{\pi(2d)^4}{32} = \frac{16\pi d^4}{32}$$

$\therefore \frac{32T_A \cdot L}{G\pi d^4} = \frac{32T_C \cdot L}{G16\pi d^4}$

$\Rightarrow \frac{T_A}{T_C} = \frac{1}{16}$

**Q.5** Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

**List-I**

- A. Partial derivative of total strain energy with respect to load
- B. Second order derivative of deflection
- C. Fourth order derivative of deflection
- D. Derivative of bending moment

**List-II**

- 1. Expression for shear force
- 2. Expression for load
- 3. Expression for bending moment
- 4. Deflection under the load

**Codes:**

	A	B	C	D
(a)	4	3	2	1
(b)	2	1	3	4
(c)	4	2	3	1
(d)	2	3	1	4

5. (a)





$$= \frac{5.60 \times 10^8 - 18000 \times (120.7)^2}{120.7}$$

$$= 2467 \times 10^3 \text{ mm}^3$$

$$\therefore M_r = 30 \times 2467 \times 10^3$$

$$= 74.01 \times 10^6 \text{ N-mm} \approx 74 \times 10^6 \text{ Nmm} = 74 \text{ kNm}$$

Let  $w$  be the permissible load in kN/m

$$\therefore M = \frac{wL^2}{8} = \frac{w \times 4^2}{8} = 2w \text{ kNm}$$

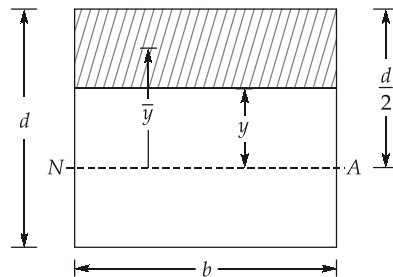
$$\text{So } 2w = 74$$

$$\Rightarrow w = 37 \text{ kN/m}$$

**Q.7** If a beam of rectangular cross-section is subjected to a vertical shear force  $V$ , the shear force carried by the upper one-third of the cross-sections is

- (a) zero (b)  $\frac{7V}{27}$   
 (c)  $\frac{8V}{27}$  (d)  $\frac{V}{3}$

7. (b)



$$\bar{y} = \left( \frac{\frac{d}{2} - y}{2} \right) + y = \frac{d}{4} + \frac{y}{2} = \frac{1}{2} \left( \frac{d}{2} + y \right)$$

$$\tau = \frac{SA\bar{y}}{Ib}$$

$$= \frac{V \times \left( \frac{d}{2} - y \right) \times b \times \left[ \frac{\left( \frac{d}{2} + y \right)}{2} \right]}{Ib}$$

$$\Rightarrow \tau = \frac{V \times \left( \frac{d^2}{4} - y^2 \right)}{2I}$$

$$dF = \tau \times bdy$$

$$= \frac{V \times \left( \frac{d^2}{4} - y^2 \right)}{2I} \times bdy$$

$$\begin{aligned} \therefore F &= \frac{Vb}{2I} \int_{d/6}^{d/2} \left( \frac{d^2}{4} - y^2 \right) dy \\ &= \frac{Vb}{2I} \left[ \frac{d^2}{4} y - \frac{y^3}{3} \right]_{d/6}^{d/2} \\ &= \frac{Vb}{2bd^3} \left[ \frac{d^2}{4} \left( \frac{d}{2} \right) - \frac{d^3}{24} - \frac{d^2}{4} \times \frac{d}{6} + \frac{d^3}{648} \right] \\ &= 6V \left[ \frac{1}{8} - \frac{1}{24} - \frac{1}{24} + \frac{1}{648} \right] \\ &= 6V \left[ \frac{7}{162} \right] = \frac{7V}{27} \end{aligned}$$

**Q.8** The bending moment, 'M' applied to a solid shaft produces the maximum direct stress ' $f_y$ ' at elastic failure. Now when only twisting moment 'T' is applied and produces the same maximum direct stress  $f_y$  then the relationship between 'M' and twisting moment 'T' as per maximum principal strain theory for elastic failure will be [Take  $\mu = 0.3$ ]

- (a)  $2M = T$  (b)  $M = 0.65 T$   
(c)  $M = T$  (d)  $M = 0.81 T$

**8. (b)**

The stress,  $f_y$  caused by bending moment, 'M' is,

$$f_y = \frac{32M}{\pi d^3}$$

Similarly, shear stress, ' $\tau$ ' produced by torque, 'T' is,

$$\tau = \frac{16T}{\pi d^3}$$

The induced principal stress,  $\sigma_1 = -\sigma_2 = \frac{16T}{\pi d^3}$

So, principal stresses acting on the shaft are,  $\frac{16T}{\pi d^3}, 0, \frac{-16T}{\pi d^3}$

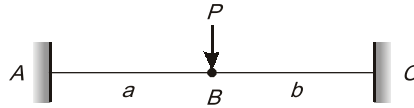
As per Maximum principal strain theory,  $\sigma_1 - \mu\sigma_2 = f_y$

$$\Rightarrow \frac{16T}{\pi d^3} + \frac{0.3 \times 16T}{\pi d^3} = \frac{32M}{\pi d^3}$$

$$\Rightarrow \frac{M}{T} = 0.65$$

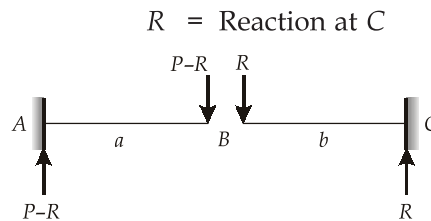
$$\Rightarrow M = 0.65T$$

**Q.9** The reaction at support C in the beam as shown below is



- (a)  $\frac{Pa^3}{a^3 + b^3}$                       (b)  $\frac{Pa^2}{a^2 + b^2}$   
 (c)  $\frac{Pb^3}{a^3 + b^3}$                       (d)  $\frac{Pb^2}{a^2 + b^2}$

**9. (a)**  
Let,



For left span,  $\Delta_B = \frac{(P-R)a^3}{3EI}$

For right span,  $\Delta_B = \frac{Rb^3}{3EI}$

From compatibility equation,  $(\Delta_B)_{\text{left}} = (\Delta_B)_{\text{right}}$

$$\therefore Pa^3 - Ra^3 = Rb^3$$

$$\Rightarrow R = \frac{Pa^3}{a^3 + b^3}$$

$$\therefore \text{Reaction at C} = \frac{Pa^3}{a^3 + b^3}$$

**Q.10** A thin cylindrical shell of internal diameter 'D' and thickness 't' is subjected to internal pressure 'P'. The change in diameter is given by [E and  $\mu$  are elastic constants]

- (a)  $\frac{PD^2}{4tE}(2-\mu)$                       (b)  $\frac{PD^2}{4tE}(1-2\mu)$   
 (c)  $\frac{PD^2}{2tE}(1-2\mu)$                       (d)  $\frac{PD^2}{2tE}(2-\mu)$

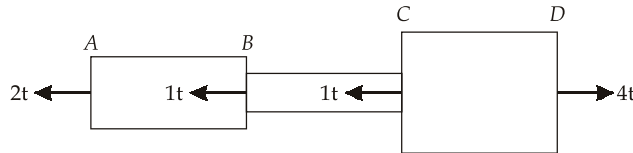
**10. (a)**  
Let  $\sigma_c$  and  $\sigma_l$  be two principal stresses at any point on the cylinder

$$\epsilon_c = \text{Hoop strain} = \frac{\sigma_c}{E} - \frac{\mu\sigma_l}{E}$$

$$\frac{\Delta D}{D} = \frac{1}{E}(\sigma_c - \mu\sigma_l) = \frac{1}{E}\left(\frac{PD}{2t} - \frac{\mu PD}{4t}\right)$$

$$\Delta D = \frac{PD^2}{2tE}\left(1 - \frac{\mu}{2}\right) = \frac{PD^2}{4tE}(2 - \mu)$$

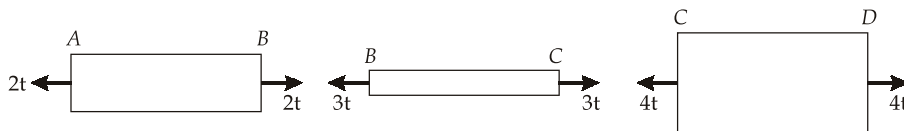
**Q.11** A mild steel bar is in three parts each 20 cm long. The diameter of parts AB, BC and CD are 2 cm, 1 cm and 3 cm respectively. The bar is subjected to axial forces as shown in figure.



$E = 2 \times 10^6 \text{ kg/cm}^2$ . The elongation in bars are  $\Delta_1, \Delta_2, \Delta_3$  respectively for AB, BC and CD. The ratio of greatest to the least of these elongation will be

- (a) 1.12 (b) 6.0  
(c) 6.7 (d) 9.8

11. (c)



$$\Delta_1 = \frac{PL}{AE} = \frac{2000 \times 20}{\frac{\pi}{4}(2)^2 \times 2 \times 10^6} = \frac{0.02}{\pi} \text{ cm}$$

$$\Delta_2 = \frac{3000 \times 20}{\frac{\pi}{4} \times (1)^2 \times 2 \times 10^6} = \frac{0.12}{\pi} \text{ cm}$$

$$\Delta_3 = \frac{4000 \times 20}{\frac{\pi}{4}(3)^2 \times 2 \times 10^6} = \frac{0.0178}{\pi} \text{ cm}$$

$$\therefore \frac{\Delta_2}{\Delta_3} = \frac{0.12}{0.0178} = 6.74 \approx 6.7$$

**Q.12** An aluminium bar ( $E = 70 \text{ GPa}$ ,  $\mu = 0.33$ ) of diameter 20 mm is stretched by an axial force  $P$  causing its diameter to decrease by 0.02 mm. The value of  $P$  is approximately

- (a) 67 kN (b) 77 kN  
(c) 87 kN (d) 97 kN

12. (a)

$$\epsilon_D = -\frac{0.02}{20}, \epsilon_L = -\frac{\epsilon_D}{\mu} = \frac{0.02}{20 \times 0.33} = \frac{1}{330}$$

But,

$$\epsilon_L = \frac{P}{A \times E}$$

$$\Rightarrow P = \epsilon_L \times A \times E$$

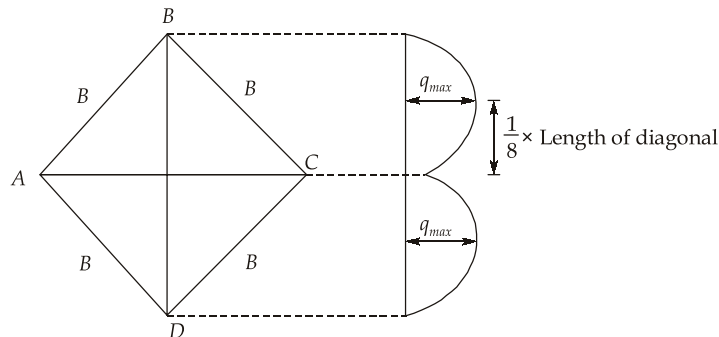
$$= \frac{\pi}{4} \times 20^2 \times 70 \times 10^3$$

$$= \frac{330}{330} \text{ N} = 66.6 \text{ kN} \simeq 67 \text{ kN (say)}$$

**Q.13** A beam of square cross-section ( $B \times B$ ) is used as a beam with one diagonal horizontal. The location of the maximum shear stress from neutral axis will be at a distance of

- (a) zero  
(b)  $\frac{B}{4}$   
(c)  $\frac{B}{4\sqrt{2}}$   
(d)  $\frac{B}{8\sqrt{2}}$

13. (c)



The length of diagonal =  $B\sqrt{2}$

The location of maximum shear stress from neutral axis is  $\frac{1}{8} \times$  length of diagonal i.e.

$$B\sqrt{2} \times \frac{1}{8} = \frac{B}{4\sqrt{2}}$$

**Q.14** A 10 mm diameter steel wire is coiled to form a close coiled helical spring having 8 nos. of coils of 75 mm mean diameter and the spring has a stiffness 'K'. If the same length of wire is coiled to 10 coils of 60 mm mean diameter, then the spring stiffness will be:

- (a) 1.00 K  
(b) 1.25 K  
(c) 1.56 K  
(d) 1.95 K

14. (c)

$$P = K\Delta$$

$$K = \frac{Gd^4}{64R^3n} = \frac{G \times 10^4}{64 \times 8 \times \left(\frac{75}{2}\right)^3}$$

$$K' = \frac{G \times 10^4}{64 \times 10 \times \left(\frac{60}{2}\right)^3}$$

$$\frac{K'}{K} = \frac{64 \times 8 \times 75^3}{64 \times 10 \times 60^3} = \frac{8}{10} \times \left(\frac{75}{60}\right)^3 = \frac{8}{10} \times \left(\frac{5}{4}\right)^3$$

$$K' = 1.56 K$$

**Q.15** A cantilever beam 'A' with rectangular cross section is subjected to a concentrated load at its free end. If width and depth of another cantilever beam 'B' are twice those of beam A, then the deflection at free end of the beam 'B' as compared to that of 'A' will be

- (a) 6.25% (b) 12.5%  
(c) 36% (d) 25%

15. (a)

$$\text{Deflection} \propto \frac{1}{I}$$

So, for rectangular cross section

$$\text{Deflection} \propto \frac{1}{bd^3}$$

$$\text{So } \frac{\delta_B}{\delta_A} = \frac{\frac{1}{2b.(2d)^3}}{\frac{1}{bd^3}} = \frac{1}{16}$$

$$\Rightarrow \delta_B = 6.25 \% \text{ of } \delta_A$$

**Q.16** Which one of the following pairs is not correctly matched?

Boundary conditions of column	Euler's buckling load
(a) Pin-pin	$\frac{\pi^2 EI}{l^2}$
(b) Fixed-fixed	$\frac{4\pi^2 EI}{l^2}$
(c) Fixed-free	$\frac{0.25\pi^2 EI}{l^2}$
(d) Fixed-pin	$\frac{\sqrt{2}\pi^2 EI}{l^2}$

16. (d)

For fixed-pin condition,

$$P_{cr} = \frac{2\pi^2 EI}{l^2}$$

**Q.17** The modulus of rigidity and bulk modulus of material are found as 70 GPa and 150 GPa respectively. Then

1. modulus of elasticity of material is 200 GPa.
2. Poisson's ratio of material is 0.22.
3. modulus of elasticity of material is 182 GPa.
4. Poisson's ratio of material is 0.30.



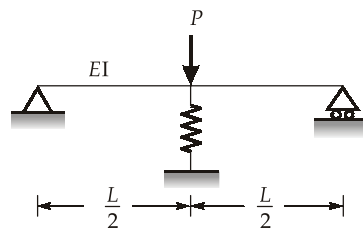


$$T_D = \frac{425}{12} = 35.42 \text{ kNm}$$

From eq. (i)

$$T_A = 25 - T_D = 25 - 35.42 = -10.42 \text{ kNm}$$

- Q.19** A simply supported beam of span length  $L$  and flexural stiffness  $EI$  has another spring support at the centre span of stiffness  $K$  as shown in figure. The central deflection of the beam due to a central concentrated load of  $P$  will be



- (a)  $\frac{PL^3}{48EI} + \frac{P}{k}$                       (b)  $\frac{PL^3}{48EI + kL^3}$   
 (c)  $\left(\frac{PL^3}{48EI}\right)\left(\frac{P}{k}\right)$                       (d)  $\frac{PL^3}{48EI} + k$

**19. (b)**

Compatibility condition is applied at the centre of span

Deflection due to point load - deflection due to spring force = Actual deflection

$$\Rightarrow \frac{PL^3}{48EI} - \frac{(k\Delta)L^3}{48EI} = \Delta$$

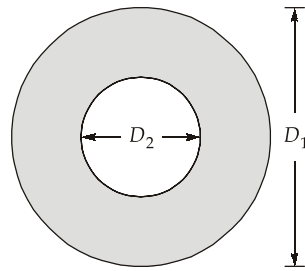
$$\Rightarrow \Delta = \frac{PL^3 \times 48EI}{48EI(48EI + kL^3)}$$

$$= \frac{PL^3}{(48EI + kL^3)}$$

- Q.20** A hollow circular column of internal diameter ' $d$ ' and external diameter  $1.5d$  is subjected to a compressive load. The maximum distance of the point of application of load from the centre for no tension is

- (a)  $\frac{d}{8}$     (b)  $\frac{13d}{48}$   
 (c)  $\frac{d}{4}$     (d)  $\frac{13d}{96}$

20. (b)



$$e = \frac{D_1^2 + D_2^2}{8D_1} = \frac{(1.5d)^2 + d^2}{8 \times 1.5d} = \frac{3.35}{12}d = \frac{13}{48}d$$

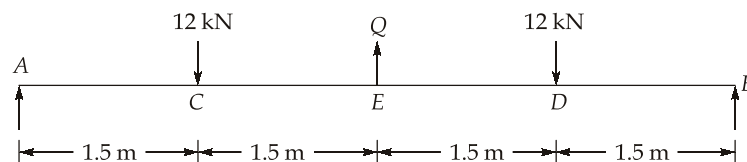
**Q.21** At a point in beam, the principal stresses are  $100 \text{ N/mm}^2$  and  $50 \text{ N/mm}^2$ . The stress in  $x$ -direction is  $65 \text{ N/mm}^2$ . The stress in  $y$ -direction will be

- (a)  $35 \text{ N/mm}^2$  (b)  $50 \text{ N/mm}^2$   
(c)  $85 \text{ N/mm}^2$  (d) Cannot be determined

21. (c)

$$\begin{aligned} \because \quad \sigma_1 + \sigma_2 &= \sigma_x + \sigma_y \\ \therefore \quad 100 + 50 &= 65 + \sigma_y \\ \Rightarrow \quad \sigma_y &= 150 - 65 = 85 \text{ N/mm}^2 \end{aligned}$$

**Q.22** For the loaded beam shown in figure. The magnitude of counter weight  $Q$  for which the absolute value of bending moment is as small as possible is



- (a) 20 kN (b) 16 kN  
(c) 32 kN (d) 48 kN

22. (b)

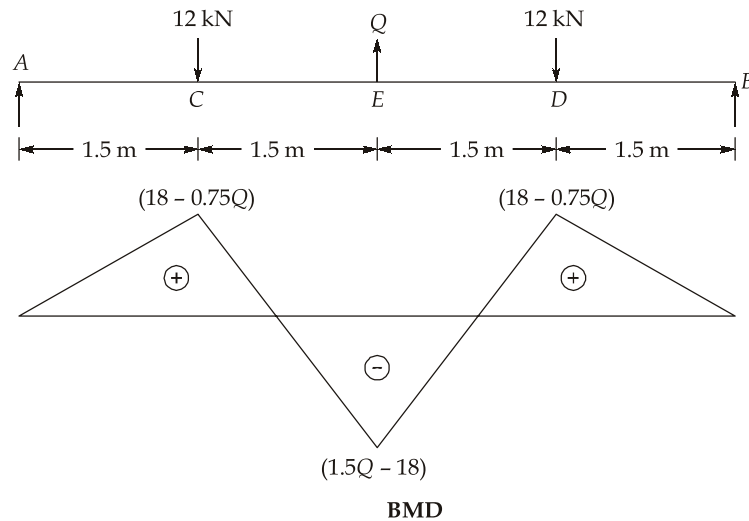
Due to counter weight an upward force  $Q$  is transmitted to beam at  $E$ .  
Reaction at each support,

$$V_A = V_B = \frac{24 - Q}{2}$$

$$\text{BM at } C = \text{BM at } D = \left(\frac{24 - Q}{2}\right) \times 1.5 = (18 - 0.75Q) \text{ kNm}$$

$$\text{BM at } E = \left(\frac{24 - Q}{2}\right) \times (3) - 12(1.5) = -(1.5Q - 18) \text{ kNm}$$

For the condition that maximum BM should be small as possible, the maximum sagging and hogging BM should be numerically equal.

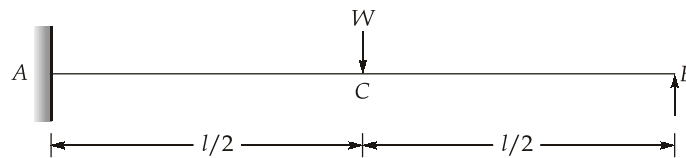


$$1.5Q - 18 = (18 - 0.75Q)$$

$$2.25Q = 36$$

$$\therefore Q = 16 \text{ kN}$$

**Q.23** A cantilever of length  $l$  carries a concentrated load  $W$  at its mid-span section. If free end of the cantilever be supported on rigid prop. The contraflexure point from the propped end will be at a distance of



- |                    |                     |
|--------------------|---------------------|
| (a) $\frac{5}{8}l$ | (b) $\frac{3}{11}l$ |
| (c) $\frac{3}{8}l$ | (d) $\frac{8}{11}l$ |

**23. (d)**

Since deflection at  $B$  is zero, downward deflection of  $B$  due to load  $W$  = upward deflection of  $B$  due to  $R_B$

$$\frac{W\left(\frac{l}{2}\right)^3}{3EI} + \frac{W\left(\frac{l}{2}\right)^2}{2EI} \left(\frac{l}{2}\right) = \frac{R_B l^3}{3EI}$$

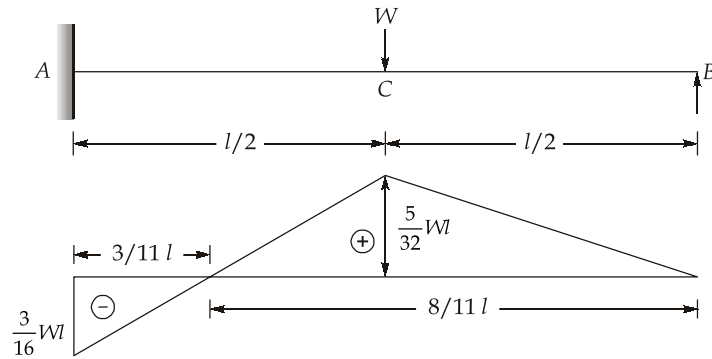
$$\therefore R_B = \frac{5W}{16}$$

$$\text{Reaction at A} = W - \frac{5}{16}W = \frac{11}{16}W$$

$$\text{Now, BM at C} = \frac{5}{16}W \left(\frac{l}{2}\right)$$

$$\text{BM at A} = \frac{5}{16}W\left(\frac{l}{2}\right) - \frac{Wl}{2} = -\frac{3}{16}Wl$$

There will be a point of contraflexure between A and C.

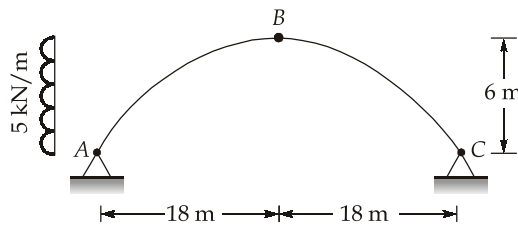


Let BM to be zero at  $x$  distance from B.

$$\frac{5}{16}W(x) - W\left(x - \frac{l}{2}\right) = 0$$

$$\therefore x = \frac{8}{11}l$$

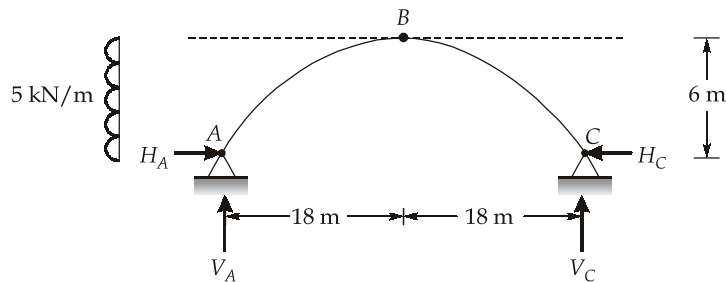
**Q.24** Consider the following three hinged arch



If this arch is carrying uniformly distributed horizontal load of 5 kN/m on left side, then magnitude of horizontal thrust at A is

- (a) 12.5 kN
- (b) 22.5 kN
- (c) 15 kN
- (d) 30 kN

**24. (b)**



Taking  $\Sigma M_B = 0$  (From left)

$$V_A \times 18 - H_A \times 6 - \frac{5 \times 6^2}{2} = 0$$

$$V_A \times 18 = 90 + 6H_A$$

$$V_A = 5 + \frac{H_A}{3} \quad \dots(i)$$

Taking,  $\Sigma M_C = 0$

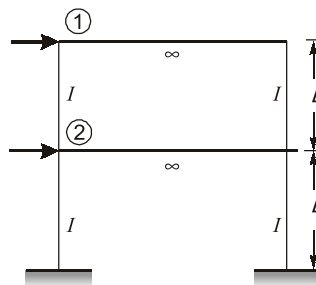
$$V_A \times 36 + 5 \times 6 \times 3 = 0$$

$$\therefore V_A = \frac{-5 \times 6 \times 3}{36} = -2.5 \text{ kN}$$

Now, from eq. (i)

$$\therefore H_A = (-2.5 - 5) \times 3 = -7.5 \times 3 = -22.5 \text{ kN} = 22.5 \text{ kN} (\rightarrow)$$

- Q.25** The beams in the two storey frame shown in the figure below have a cross section such that the flexural rigidity may be considered infinite. Which among the following is the stiffness matrix for the structure in respect of the global coordinates 1 and 2?



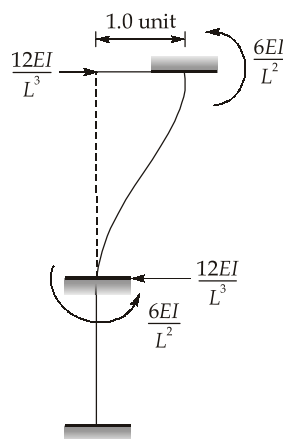
(a)  $\frac{24EI}{L^3} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

(b)  $\frac{24EI}{L^3} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

(c)  $\frac{24EI}{L^3} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

(d)  $\frac{24EI}{L^3} \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$

25. (a) For first column of matrix  $D_1 = 1.0$ ;  $D_2 = 0$   
The shape of columns will be



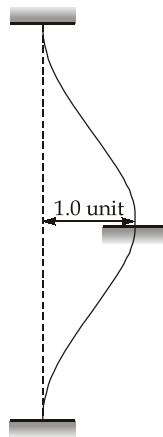
Since there are two column.

$$\therefore f_{11} = \frac{12EI}{L^3} + \frac{12EI}{L^3} = \frac{24EI}{L^3}$$

$$f_{21} = -\frac{24EI}{L^3}$$

For second column of matrix

$$D_1 = 0; D_2 = 1.0$$



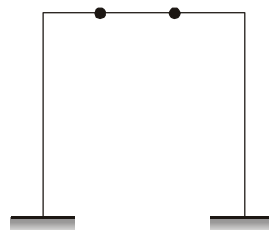
The shape of the columns will be

$$\therefore f_{12} = -\frac{24EI}{L^3}$$

$$f_{22} = \frac{24EI}{L^3} + \frac{24EI}{L^3} = \frac{48EI}{L^3}$$

$$\text{Stiffness Matrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} = \frac{24EI}{L^3} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

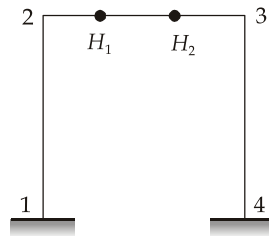
**Q.26** Assuming the member to be inextensible, the degree of kinematic indeterminacy of the structure shown below is



- (a) 12  
(c) 9

- (b) 7  
(d) 16

26. (c)



**Method-1:**

$$\begin{array}{c|c|c|c} \theta_2 & \theta_{H_{11}} & \theta_{H_{21}} & \theta_3 \\ \Delta x_2 & \theta_{H_{12}} & \theta_{H_{22}} & \Delta x_3 \\ \Delta y_2 & \Delta x_{H_1} & \Delta x_{H_2} & \Delta y_3 \\ \hline & \Delta y_{H_1} & \Delta y_{H_2} & \end{array}$$

$$D_k = 14$$

If member is inextensible

$$\Delta y_2 = \Delta y_3 = 0 \sim (-2)$$

$$\Delta x_2 = \Delta x_{H_1} = \Delta x_{H_2} = \Delta x_3 \sim (-3)$$

$\therefore$

$$D_k = 14 - 5 = 9$$

**Method-2:**

By use of formula

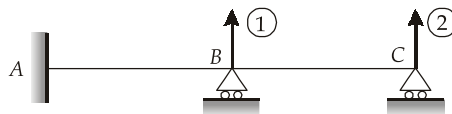
$$D_k = 3(j + j') - r_e + r_r - m$$

$$j = 4; j' = 2, r_e = 6, r_r = 2, m = 5$$

$$D_k = 3(4 + 2) - 6 + 2 - 5 = 9$$

**Q.27** Flexibility matrix of the beam shown below is :

$$\delta = \frac{1}{3EI} \begin{bmatrix} 1 & 2 \\ 2 & 8 \end{bmatrix}$$



If support B settles by  $\frac{\Delta}{EI}$  units, what is the reaction at B ?

(a) 0.75  $\Delta$

(b) 3.0  $\Delta$

(c) 6.0  $\Delta$

(d) 24.0  $\Delta$

27. (c)

We know that

$$D_Q = D_{QS} + FQ$$

$D_Q$  is the final displacement matrix corresponding to the redundants in the actual structure.

$$D_{QS} = D_{QL} + D_{QT} + D_{QP} + D_{QR}$$

$D_{QS}$  includes the effects of external loads ( $D_{QL}$ ), temperature ( $D_{QT}$ ), prestrain effects ( $D_{QP}$ ), support settlements, etc.

$F$  is the Flexibility matrix and  $Q$  is the unknown reactions matrix.

Assuming upward displacements as positive.

$$D_{QL} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; D_{QP} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$D_{QT} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; D_{QR} = \begin{bmatrix} -\frac{\Delta}{EI} \\ 0 \end{bmatrix}$$

$$\therefore D_{QS} = \begin{bmatrix} -\frac{\Delta}{EI} \\ 0 \end{bmatrix} \text{ and } Q = \begin{bmatrix} V_B \\ V_C \end{bmatrix}$$

Final displacement,  $D_Q = 0$

$$\therefore D_{QS} + FQ = 0$$

$$\Rightarrow FQ = -D_{QS}$$

$$\Rightarrow Q = -F^{-1} D_{QS}$$

$$\Rightarrow \begin{bmatrix} V_B \\ V_C \end{bmatrix} = -\frac{3EI}{4} \begin{bmatrix} 8 & -2 \\ -2 & 1 \end{bmatrix}_{2 \times 2} \begin{bmatrix} -\frac{\Delta}{EI} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_B \\ V_C \end{bmatrix} = -\frac{3EI}{4} \times \left(-\frac{\Delta}{EI}\right) \begin{bmatrix} 8 & -2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

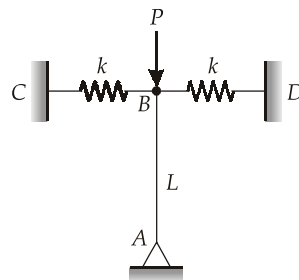
$$\Rightarrow \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \frac{3\Delta}{4} \begin{bmatrix} 8 + (-2) \times 0 \\ (-2) \times 1 + 1 \times 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \frac{3\Delta}{4} \begin{bmatrix} 8 \\ -2 \end{bmatrix}$$

$$\therefore V_B = \frac{3\Delta}{4} \times 8 = 6\Delta$$

$$\text{and } V_C = \frac{3\Delta}{4} \times (-2) = -1.5\Delta$$

**Q.28** Consider the following statements regarding the structure shown below  
[Member AB is rigid]





1. Degree of kinematic indeterminacy = 3.
2. The bar AB is stable if  $2kL > P$ .

Which of the above statement(s) is(are) INCORRECT?

- (a) 1 only (b) 2 only  
(c) Both 1 and 2 (d) Neither 1 nor 2

28. (a)

Give small horizontal displacement  $\Delta$  at B  
Overturning moment due to P about A

$$M_0 = P \times \Delta$$

The horizontal force which is trying to restore the bar back to its horizontal position =  $2k\Delta$ .

$\therefore$  Stabilizing moment =  $2k\Delta L$

$\therefore$  Bar is in stable equilibrium if,

$$M_s > M_0$$

$$\Rightarrow 2kL > P$$

Degree of kinematic indeterminacy = 1

Q.29 Consider the following statements about two hinged arches subjected to downward UDL:

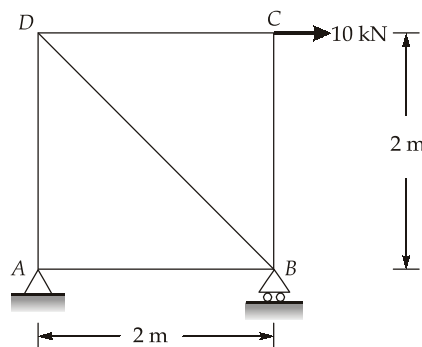
1. If one of the support settles down vertically, then the horizontal thrust remains unchanged.
2. If one of the support yields horizontally outward then the horizontal thrust is decreased.
3. If ambient temperature increases, then the horizontal thrust at support will increase.

Which of the above statements are CORRECT?

- (a) 1 and 2 (b) 2 and 3  
(c) 1 and 3 (d) All of these

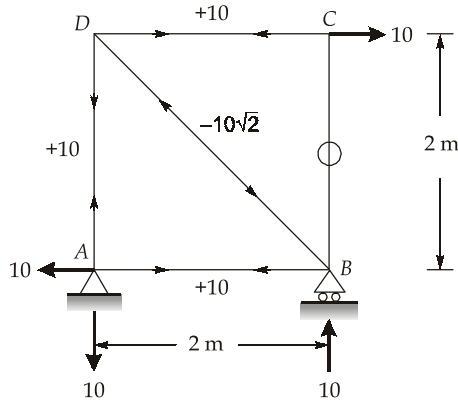
29. (b)

Q.30 The horizontal deflection for joint C in the given truss is [Take axial rigidity as 10000 kN]

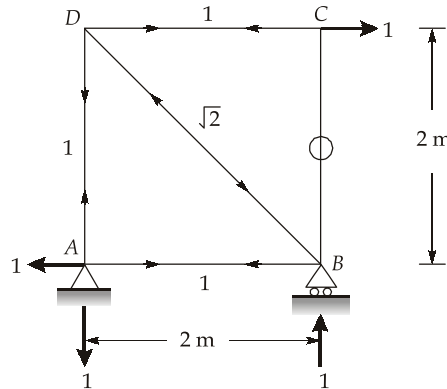


- (a)  $\frac{60 - 40\sqrt{2}}{10000}$  m (b)  $\frac{60 + 40\sqrt{2}}{10000}$  m  
(c)  $\frac{60(1 + \sqrt{2})}{10000}$  m (d)  $\frac{60(1 - \sqrt{2})}{10000}$  m

30. (b)  
P-force system (in kN)



k-force system (in kN)



	$P$	$k$	$L$	$PkL$
AB	10	1	2	20
BC	0	0	2	0
CD	10	1	2	20
DA	10	1	2	20
BD	$-10\sqrt{2}$	$-\sqrt{2}$	$2\sqrt{2}$	$40\sqrt{2}$
				$\sum PkL = (60 + 40\sqrt{2})$

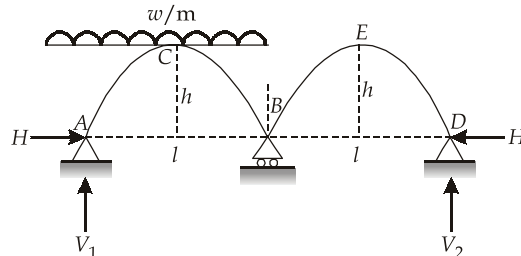
Using virtual work method,

$$\Delta_C = \frac{\sum PkL}{AE}$$

$$\therefore \Delta_C = \frac{(10 \times 1 \times 2) \times 3 + (-10\sqrt{2} \times (-\sqrt{2}) \times 2\sqrt{2})}{AE}$$

$$= \frac{(60 + 40\sqrt{2})}{10000} \text{ m}$$

- Q.31** Two identical parabolic arches have their outer ends hinged with their inner ends resting on a common roller. The horizontal thrust, when the arch *AB* carries a uniform distributed load *w* over its whole span will be



- (a)  $\frac{wl^2}{8h}$  (b)  $\frac{wl^2}{32h}$   
(c)  $\frac{wl^2}{16h}$  (d)  $\frac{wl^2}{4h}$

31. (c)

In this case, the roller support *B* will move towards the right, say by  $\delta$ .

If instead of loading the arch *AB*, we apply the loading on the arch *BD*, the horizontal thrust at *A* and *D* will again be equal to *H*. But, in this case the roller support *B* will move towards left by  $\delta$ .

Hence if both the arches are loaded, the horizontal thrust at *A* and *D* will be  $2H$ .

Hence for this case horizontal thrust is given as:

$$\Rightarrow 2H = \frac{wl^2}{8h}$$

$$\Rightarrow H = \frac{wl^2}{16h}$$

This is the horizontal thrust when any one of the arch is loaded.

- Q.32** A SDOF system consists of mass of 1000 kg and spring with stiffness 36 N/mm. The system has damping force of 2 kN with velocity 250 mm/s. The value of damping ratio is  
(a) 1.5 (b) 0.67  
(c) 3 (d) 0.33

32. (b)

$$m = 1000 \text{ kg; } k = 36 \text{ N/mm} = 36000 \text{ N/m}$$

$$\text{Natural frequency, } \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{36000}{1000}} = 6 \text{ rad/s}$$

Critical damping coefficient,

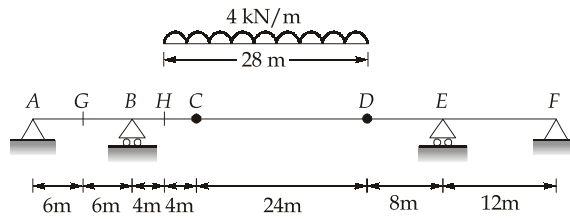
$$c_{cr} = 2m\omega_n = 2 \times 1000 \times 6 = 12000 \text{ Ns/m}$$

Damping force;  $c\dot{x} = 2000 \text{ N}$

$$\Rightarrow c = \frac{c\dot{x}}{\dot{x}} = \frac{2000}{0.25} = 8000 \text{ Ns/m}$$

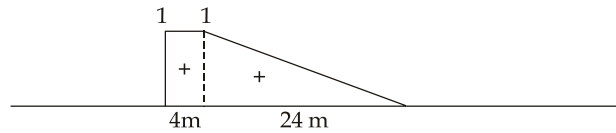
$$\therefore \text{Damping ratio} = \frac{c}{c_{cr}} = \frac{8000}{12000} = 0.67$$

**Q.33** For the given beam, the maximum shear force at section  $H$ , when a UDL of  $4 \text{ kN/m}$  of length  $28 \text{ m}$  moves from  $A$  to  $F$ , will be



- (a) 48 kN  
(b) 64 kN  
(c) 52 kN  
(d) 58 kN

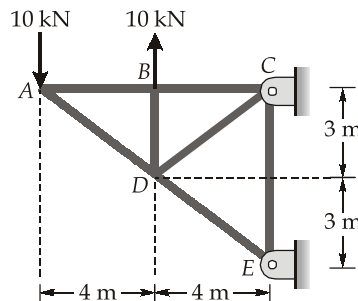
**33. (b)**  
ILD for shear force at  $H$  is given as,



So maximum shear force at section  $H$  is given as

$$= \left[ 1 \times 4 + \frac{1}{2} \times 24 \times 1 \right] \times 4 = 64 \text{ kN}$$

**Q.34** Consider the following truss

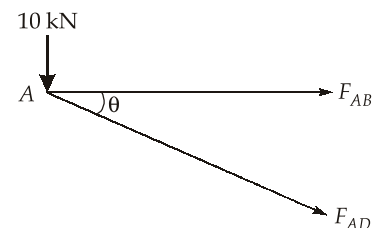


The force in member  $BC$  is

- (a)  $\frac{50}{3} \text{ kN (T)}$   
(b)  $\frac{40}{3} \text{ kN (C)}$   
(c)  $\frac{40}{3} \text{ kN(T)}$   
(d)  $\frac{50}{3} \text{ kN(C)}$

**34. (c)**  
Applying the method of joints, consider joint  $A$ ,

$$\begin{aligned} \Rightarrow \sum F_y &= 0 \\ F_{AD} \sin \theta &= -10 \\ \Rightarrow F_{AD} &= \frac{-10}{\sin \theta} \\ \Rightarrow F_{AD} &= -\frac{50}{3} \text{ kN} = \frac{50}{3} \text{ kN (C)} \end{aligned}$$

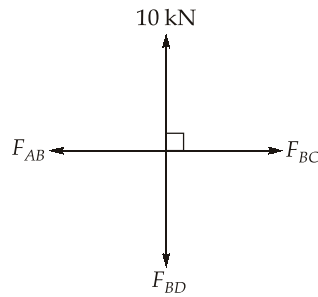


$$\sum F_x = 0$$

$$\Rightarrow F_{AD} \cos \theta + F_{AB} = 0$$

$$\Rightarrow F_{AB} = -F_{AD} \cos \theta = -\frac{50}{3} \times \frac{4}{5} = -\frac{40}{3} \text{ kN (T)}$$

\(\therefore\) Now consider joint B



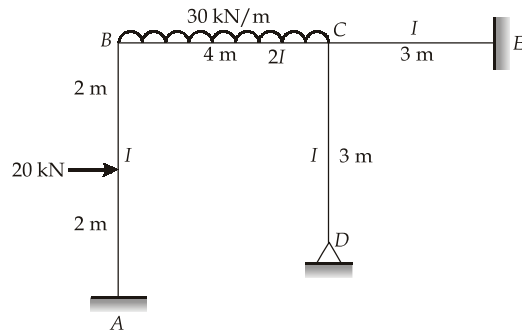
$$\therefore \text{ Take } \sum F_x = 0$$

$$\Rightarrow F_{AB} = F_{BC}$$

$$\Rightarrow F_{BC} = \frac{40}{3} \text{ kN (T)}$$

So, correct option is (c).

**Q.35** The distribution factor of member CE for the following frame is



- (a)  $\frac{4}{13}$
- (b)  $\frac{6}{13}$
- (c)  $\frac{3}{13}$
- (d) None of these

35. (a)

Joint	Member	K	\(\sum K\)	D.F
C	CB	$\frac{8EI}{4}$	$\frac{13EI}{3}$	$\frac{6}{13}$
	CD	$\frac{3EI}{3}$		$\frac{3}{13}$
	CE	$\frac{4EI}{3}$		$\frac{4}{13}$

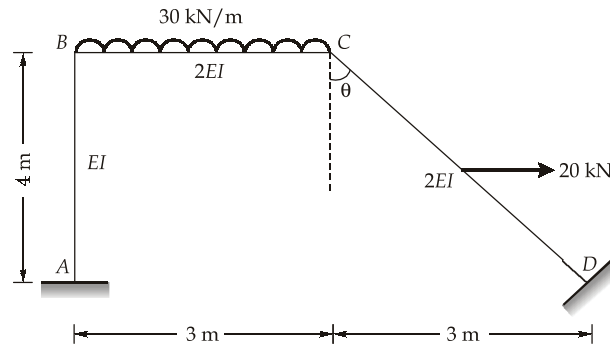
- Q.36** The horizontal thrust developed in a two hinged semicircular arch of radius  $R$ , subjected to some concentrated load at crown is 20 kN. Now, if another semicircular arch is used having same boundary conditions and same loading but cross-sectional area twice that of first arch, then horizontal thrust developed in the second case will be
- (a) 14.14 kN (b) 28.28 kN  
(c) 6.36 kN (d) 40 kN

36. (c)  
For two hinged semicircular arch subjected to concentrated load  $P$  at crown,

$$H = \frac{P}{\pi} \text{ which is independent of area of arch}$$

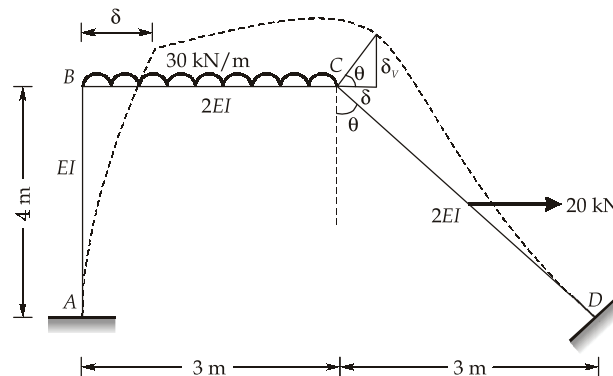
$$= \frac{20}{\pi} = 6.36 \text{ kN}$$

- Q.37** The slope deflection equation for member  $BC$  at join  $B$  in the given frame is [ $\delta$  is the horizontal displacement of joint  $B$  and  $C$ ]



- (a)  $M_{BC} = +22.5 + \frac{4EI}{3} \left( 2\theta_B + \theta_C + \frac{3\delta}{4} \right)$   
 (b)  $M_{BC} = -22.5 + \frac{4EI}{3} \left( 2\theta_B + \theta_C + \frac{3\delta}{4} \right)$   
 (c)  $M_{BC} = +22.5 + \frac{4EI}{3} \left( 2\theta_B + \theta_C + \frac{4\delta \tan \theta}{3} \right)$   
 (d)  $M_{BC} = -22.5 + \frac{4EI}{3} \left( 2\theta_B + \theta_C - \frac{4\delta \tan \theta}{3} \right)$

37. (b)



$$\therefore M_{BC} = \bar{M}_{BC} + \frac{2(2EI)}{3} \left[ 2\theta_B + \theta_C - \frac{3\delta_v}{3} \right]$$

Now,

$$\frac{\delta_v}{\delta} = \tan \theta = \frac{3}{4}$$

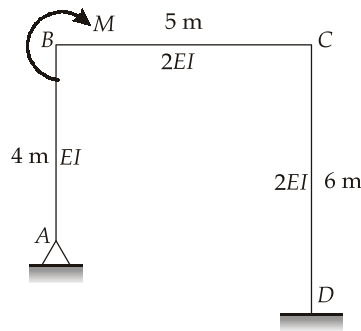
$$\therefore \delta_v = \frac{3\delta}{4}$$

$$\bar{M}_{BC} = -\frac{wl^2}{12} = -\frac{30 \times (3)^2}{12} = -22.5 \text{ kNm}$$

$$\therefore M_{BC} = -22.5 + \frac{4EI}{3} \left( 2\theta_B + \theta_C - \left( -\frac{3\delta}{4} \right) \right)$$

$$= -22.5 + \frac{4EI}{3} \left( 2\theta_B + \theta_C + \frac{3\delta}{4} \right)$$

**Q.38** Consider the following frame :



Which one of the following options is correct regarding above frame?

- (a)  $M_{BA} = EI (\theta_B - \theta_A)$                       (b)  $M_{BA} = \frac{EI}{2} (\theta_B - \theta_A)$
- (c)  $M_{BA} = \frac{EI}{4} (\theta_B - \theta_A)$                       (d)  $M_{BA} = \frac{3EI}{4} (\theta_B - \theta_A)$

**38. (b)**

Applying slope deflection method,

Fixed end moments

$$M_{FAB} = M_{FBA} = M_{FBC} = M_{FCB} = M_{FCD} = M_{FDC} = 0$$

$$M_{AB} = M_{FAB} + \frac{2EI}{L} \left( 2\theta_A + \theta_B - \frac{3\delta}{L} \right)$$

$$\Rightarrow 0 = \frac{4EI\theta_A}{4} + \frac{2EI\theta_B}{4} - \frac{6EI\delta}{16}$$

$$\Rightarrow 0 = EI\theta_A + \frac{EI\theta_B}{2} - \frac{3EI\delta}{8}$$

$$\Rightarrow \frac{3}{8} EI\delta = EI\theta_A + \frac{EI\theta_B}{2} \quad \dots(i)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L} \left( 2\theta_B + \theta_A - \frac{3\delta}{L} \right)$$

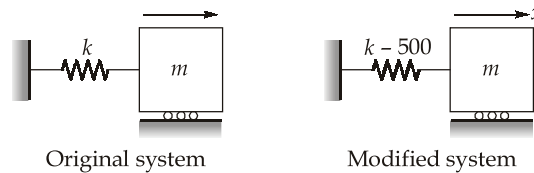
$$\begin{aligned}
 &= \frac{2EI}{4} \left( 2\theta_B + \theta_A - \frac{3\delta}{4} \right) \\
 &= EI\theta_B + \frac{EI\theta_A}{2} - \frac{3}{8}EI\delta \\
 &= EI\theta_B + \frac{EI\theta_A}{2} - EI\theta_A - \frac{EI\theta_B}{2} \qquad \text{[Using eq. (i)]} \\
 &= \frac{EI\theta_B}{2} - \frac{EI\theta_A}{2} = \frac{EI}{2}(\theta_B - \theta_A)
 \end{aligned}$$

**Q.39** A SDOF system of mass  $m$  and stiffness  $k$  is found to vibrate with a natural frequency of 15 Hz. If the stiffness is decreased by 500 N/m and the natural frequency is reduced by 50% then stiffness ( $k$ ) of the original system will be

- (a) 6000 N/m                      (b) 600 N/m  
(c) 333.33 N/m                 (d) 666.67 N/m

39. (d)

Consider given two systems



Frequency of original system

$$= 2\pi \times 15 = 30\pi$$

Now since  $w_n = \sqrt{\frac{k}{m}}$

$\therefore w_n^2 \propto k$

$\therefore \left( \frac{w_{n_2}}{w_{n_1}} \right)^2 = \frac{k_2}{k_1}$

$\Rightarrow \left( \frac{0.5w_{n_1}}{w_{n_1}} \right)^2 = \frac{k - 500}{k}$

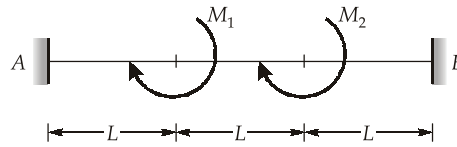
$\Rightarrow \frac{1}{4} = \frac{k - 500}{k}$

$\Rightarrow k = 4k - 2000$

$\Rightarrow k = \frac{2000}{3} = 666.67 \text{ N/m}$



**Q.40** The magnitude of fixed end moment at support 'A' of the fixed beam loaded as shown below is



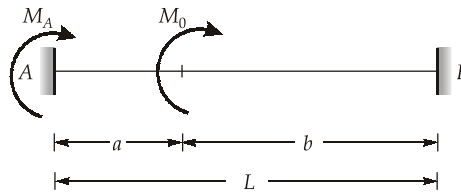
(a)  $\frac{M_1 + M_2}{3}$

(b)  $\frac{M_1 + M_2}{2}$

(c)  $\frac{M_1}{3}$

(d)  $\frac{M_2}{3}$

40. (d)

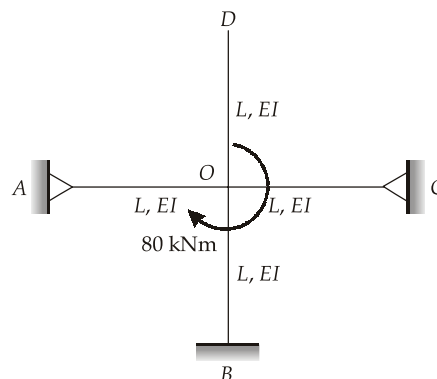


$$M_A = \frac{M_0 b(3a - L)}{L^2}$$

$$\therefore M_A = \frac{M_1 \times (2L)(3L - 3L)}{(3L)^2} + \frac{M_2 \times (L)(6L - 3L)}{9L^2}$$

$$= 0 + M_2 \times \frac{3L^2}{9L^2} = \frac{M_2}{3}$$

**Q.41** Consider the frame as shown in the figure. A moment of 80 kN-m acts at the joint O, given that the support at B fails at any moment greater than 18 kN-m. Then support B is



(a) Unsafe

(b) Safe

(c) Just Safe

(d) Cannot be determined

41. (b)

$$\text{Stiffness of } DO = \frac{3EI}{L}$$

$$\text{Stiffness of } OC = \frac{3EI}{L}$$

$$\text{Stiffness of } OB = \frac{4EI}{L}$$

$$\therefore OA : OC : OB = 3 : 3 : 4$$

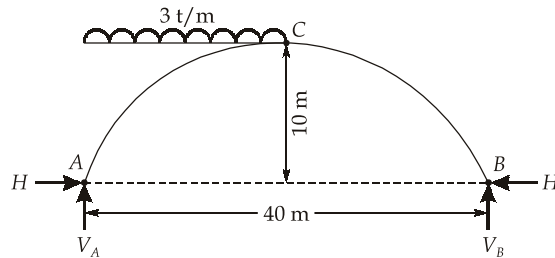
$$\therefore M_{OB} = 80 \times \frac{4}{10} = 32 \text{ kN-m}$$

$$\therefore M_{BO} = \frac{1}{2} \times M_{OB} = 16 \text{ kN-m} < 18 \text{ kN-m. Hence safe}$$

**Q.42** A three-hinged symmetric parabolic arch is hinged at the springings and at the crown. The span and rise are 40 m and 10 m respectively. The left half of the arch is loaded with UDL of 3t/m. The horizontal thrust at the springings will be

- (a) 15 t (b) 20 t  
(c) 30 t (d) 40 t

42. (c)



$$\Rightarrow \Sigma M_B = 0$$

$$V_A \times 40 = 3 \times 20 \times 30$$

$$\Rightarrow V_A = \frac{90}{2} = 45 \text{ t}$$

$$\therefore V_B = 3 \times 20 - 45 = 15 \text{ t}$$

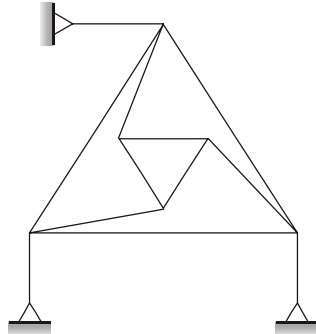
Now,  $\Sigma M_C = 0$

$$\Rightarrow H \times 10 = 15 \times 20$$

[Taking moment about central hinge from right side]

$$\Rightarrow H = 30 \text{ t}$$

**Q.43** The following two statements are made with reference to the plane truss shown below



I. The truss is statically determinate

II. The truss is kinematically determinate

With reference to the above statements, which of the following applies?

(a) Both statements are true.

(b) Both statements are false.

(c) II is true but I is false.

(d) I is true but II is false.

**43. (d)**

Degree of static indeterminacy

$$D_s = m + r_e - 2j$$

Here,  $m = 12$ ,  $j = 9$  and  $r_e = 6$

$$\therefore D_s = 12 + 6 - 2 \times 9 = 0$$

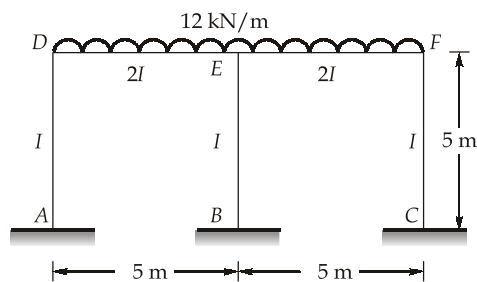
Degree of kinematic indeterminacy

$$D_k = 2j - r_e - m$$

$$= 2 \times 9 - 6 - 0 = 12$$

$\therefore$  Thus truss is statically determinate and kinematically indeterminate.

**Q.44** In the frame shown in figure below, the value of  $M_{BE}$  will be (in kNm)



(a) 12.5

(b) 25

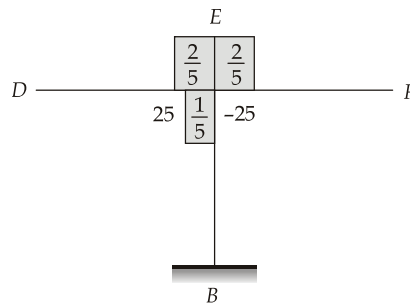
(c) 50

(d) None of these

44. (d)  
Distribution factor,

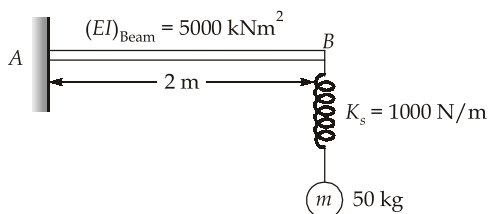
Joints	Member	Stiffness	Total stiffness	DF
D	DE	2I/5	3I/5	2/3
	DA	I/5		1/3
E	ED	2I/5	I	2/5
	EB	I/5		1/5
	EF	2I/5		2/5
F	FE	2I/5	3I/5	2/3
	FC	I/5		1/3

FEM,  $M_{DE} = -\frac{wl^2}{12} = -25 \text{ kNm}$   
 $M_{ED} = 25 \text{ kNm}$   
 $M_{EF} = -25 \text{ kNm}, M_{FE} = 25 \text{ kNm}$   
 $\therefore M_{ED} + M_{EF} = 0$   
 $\Rightarrow$  No unbalanced moment is there at joint E.  
 $\Rightarrow M_{EB} = 0$



Alternate solution:  
 Since the frame is symmetrical about the member BE,  
 Rotation of joint EB will be 0  
 Therefore,  $M_{BE} = 0$   
 $\therefore M_{EB} = 0$

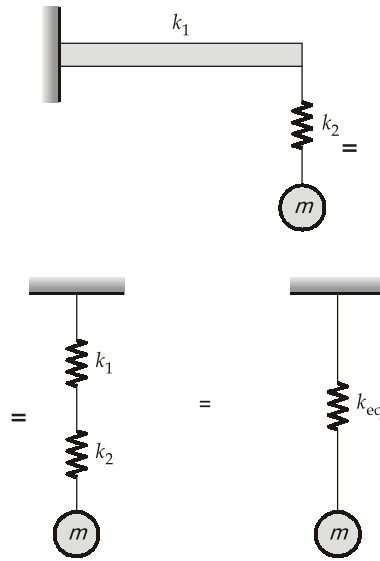
Q.45



The fundamental time period of vibration of the system shown in the figure, by neglecting the self-weight of the beam, is nearly

- (a) 0.2 sec (b) 0.8 sec  
 (c) 1.4 sec (d) 2.8 sec

45. (c)



$$k_1 = \frac{3EI}{L^3} = \frac{3 \times 5000 \times 10^3}{2^3}$$

$$= 1875000 \text{ N/m}$$

$$k_2 = 1000 \text{ N/m}$$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{1875000 \times 100}{1875000 + 100}$$

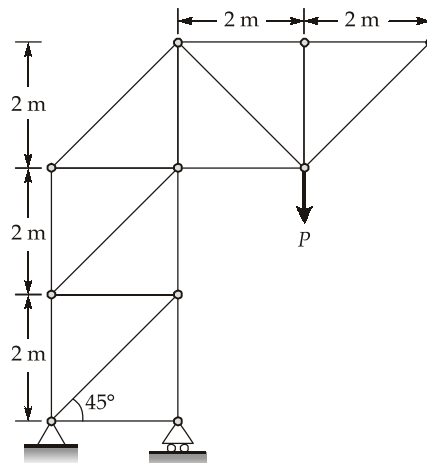
$$\approx 1000 \text{ N/m}$$

Now,

$$T = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{k_{eq}}{m}}} = 2\pi \sqrt{\frac{50}{1000}}$$

$$= 1.4 \text{ sec}$$

**Q.46** For the simple truss as shown below, what is number of members with zero force?



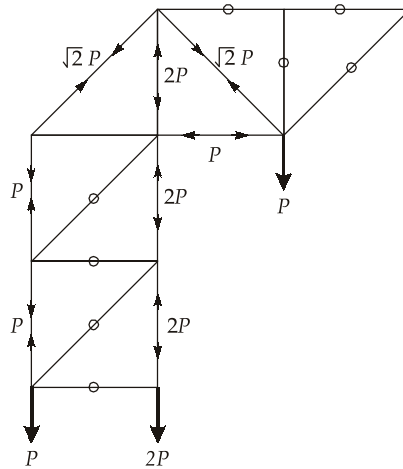
(a) 4

(b) 5

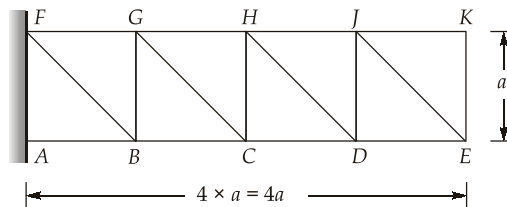
(c) 8

(d) 9

46. (c)

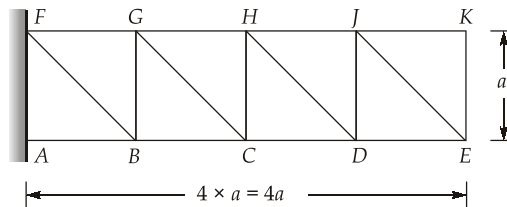


Q.47 In the pin-end cantilever truss shown below, member  $FG$  had been fabricated 10 mm longer than required. How much will point  $E$  deflect vertically?



- (a) 10 mm
- (b) 20 mm
- (c) 30 mm
- (d) 40 mm

47. (c)



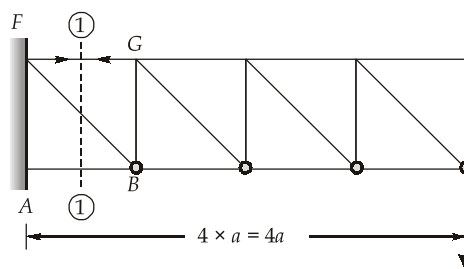
$\delta V_E$  = Vertical deflection of joint  $E$

$$= \sum \frac{Pkl}{AE} = \sum k \cdot \delta$$

$\delta_{FG}$  = Elongation member  $FG$  = +10 mm

$\delta = 0$  for all other member

$k$  for member  $FG$



Using method of sections

Cut F-G by (1)-(1)

Equilibrium of RHS of (1)-(1)

$$\begin{aligned} \therefore \quad \Sigma M_B &= 0 \\ &= -F_{FG} \times a + 1 \times 3a = 0 \end{aligned}$$

$$\Rightarrow \quad F_{FG} = 3$$

$$\text{So,} \quad \delta V_E = \Sigma k\delta = +3 \times (10) = 30 \text{ mm}$$

**Direction:** The following items consists of two statements, one labelled as **Statement (I)** and the other labelled as **Statement (II)**. You have to examine these two statements carefully and select your answers to these items using the codes given below:

**Codes:**

- (a) Both Statement (I) and Statement (II) are true and Statement (II) is the correct explanation of Statement (I).
- (b) Both Statement (I) and Statement (II) are true but Statement (II) is not a correct explanation of Statement (I).
- (c) Statement (I) is true but Statement (II) is false.
- (d) Statement (I) is false but Statement (II) is true.

**Q.48 Statement (I):** A beam of circular cross-section is stronger in shear as compared to one of rectangular section having same cross-section area.

**Statement (II):** For equal cross-sectional areas and shearing force, the maximum intensity of shear stress in the section of a beam of circular cross-section is smaller than that for the rectangular section.

48. (a)

For same area of cross-section A, the average shear stress will be same in both circular and rectangular cross-section.

$$\text{But } \tau_{\max} \text{ for rectangular section} = 1.5q_{av}$$

$$\text{And } \tau_{\max} \text{ for circular-section} = \frac{4}{3}q_{av}$$

$$T = \tau Z_p$$

$$\therefore \quad \tau \propto \frac{1}{Z_p}$$

$$\therefore \quad Z_p \Big|_{\text{circular section}} > Z_p \Big|_{\text{rectangular section}}$$

For more strongest section, polar section modulus will be large.

**Q.49 Statement (I):** The failure surface of an axially loaded mild steel tension specimen of circular cross-section is along a plane at 45° to the axis of the specimen.

**Statement (II):** The failure occurs on a plane of the specimen subjected to maximum shear stress and mild steel is relatively weak in shear.

49. (a)

**Q.50 Statement (I):** The slope-deflection method is a stiffness method in which the joint displacements are found by applying the equilibrium conditions at each joint.

**Statement (II):** The displacements at a joint of a member are independent of the displacements of the member at the far end of the joint.

50. (b)

The degree of kinematic indeterminacy or the degree of freedom of a structure is equal to the number of independent displacement components at all the joints. Thus displacement at member ends are independent but moment or reaction is dependent on displacement components at both ends.

■■■■