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Detailed Solutions

**ESE-2021  
Mains Test Series**

**Civil Engineering  
Test No : 10**

**Q.1 (a) Solution:**

**Composition of cement clinker:**

The various constituents combine during burning and form cement clinker. The compounds formed in the burning process have the properties of setting and hardening in the presence of water. They are known as Bogue's compounds after the name of Bogue who identified them. Le-Chatelier and Tornebohm have referred these compounds as Alite ( $C_3S$ ), Belite ( $C_2S$ ), Celite ( $C_3A$ ) and Felite ( $C_4AF$ ). The following Bogue's compounds are formed during clinkering process:

Principal mineral compounds in Portland cement	Formula	Name	Symbol
1. Tricalcium silicate	$3CaO.SiO_2$	Alite	$C_3S$
2. Dicalcium silicate	$2CaO.SiO_2$	Belite	$C_2S$
3. Tricalcium aluminate	$3CaO.Al_2O_3$	Celite	$C_3A$
4. Tetracalcium alumino ferrite	$4CaO.Al_2O_3.Fe_2O_3$	Felite	$C_4AF$

The properties of Portland cement varies markedly with the proportions of the above four compounds, reflecting substantial difference between their individual behaviour.

**Tricalcium silicate:** It is supposed to be the best cementing material and is well burnt cement. It is about 25-50% (normally about 40 per cent) of cement. It renders the clinker easier to grind, increases resistance to freezing and thawing, hydrates rapidly generating high heat and develops an early hardness and strength. However, raising of  $C_3S$  content beyond the specified limits increases the heat of hydration and solubility of cement in

water. The hydrolysis of  $C_3S$  is mainly responsible for 7 days strength and hardness. The rate of hydrolysis of  $C_3S$  and the character of gel developed are the main causes of the hardness and early strength of cement paste. The heat of hydration is about 500 J/g.

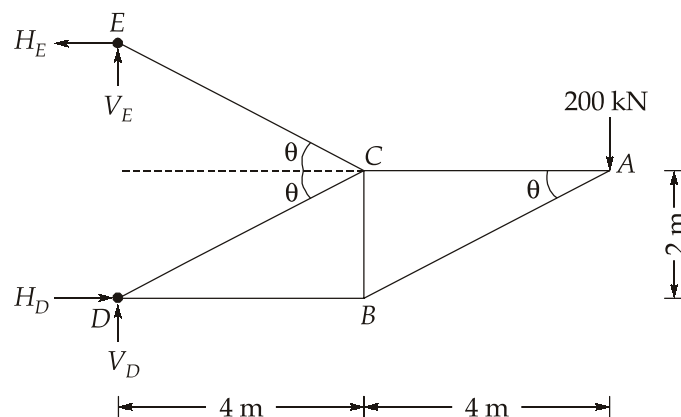
**Dicalcium silicate:** It is about 25-40% (normally about 32 per cent) of cement. It hydrates and hardens slowly and takes long time to add to the strength (after a year or more). It imparts resistance to chemical attack. Raising of  $C_2S$  content renders clinker harder to grind, reduces early strength, decreases resistance to freezing and thawing at early ages and decreases heat of hydration. The hydrolysis of  $C_2S$  proceeds slowly. At early ages, less than a month,  $C_2S$  has little influence on strength and hardness. While after one year, its contribution to the strength and hardness is proportionately almost equal to  $C_3S$ . The heat of hydration is about 260 J/g.

**Tricalcium aluminate:** It is about 5-11% (normally about 10.5 per cent) of cement. It rapidly reacts with water and is responsible for flash set of finely grounded clinker. The rapidity of action is regulated by the addition of 2-3% of gypsum at the time of grinding cement. Tricalcium aluminate is responsible for the initial set, high heat of hydration and has greater tendency to volume changes causing cracking. Raising the  $C_3A$  content reduces the setting time, weakness resistance to sulphate attack and lowers the ultimate strength, heat of hydration and contraction during air hardening. The heat of hydration is about 865 J/g.

**Tetracalcium aluminoferrite:** It is about 8-14% (normally about 9 percent) of cement. It is responsible for flash set but generates less heat. It has poorest cementing value. Raising the  $C_4AF$  content reduces the strength slightly. The heat of hydration is about 420 J/g.

### Q.1 (b) Solution:

**FBD of truss:**



From equilibrium equation:

$$\Sigma M_D = 0$$

$$\Rightarrow H_E \times 4 = 200 \times 8$$

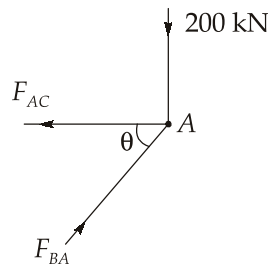
$$\Rightarrow H_E = 400 \text{ kN}$$

Also,  $\Sigma H = 0$

$$\Rightarrow H_E = H_D = 400 \text{ kN}$$

To determine the forces in members of the truss, consider equilibrium of each joint.

**Joint A:**



$$\tan \theta = \frac{2}{4} = \frac{1}{2}$$

$$\cos \theta = \frac{2}{\sqrt{5}}$$

$$\sin \theta = \frac{1}{\sqrt{5}}$$

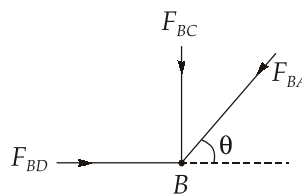
$$\Sigma V = 0$$

$$\Rightarrow F_{BA} \sin \theta = 200$$

$$\Rightarrow F_{BA} = 200\sqrt{5} = 447.21 \text{ kN (comp.)}$$

$$F_{AC} = F_{BA} \cos \theta = 447.21 \times \frac{2}{\sqrt{5}} = 400 \text{ kN (tensile)}$$

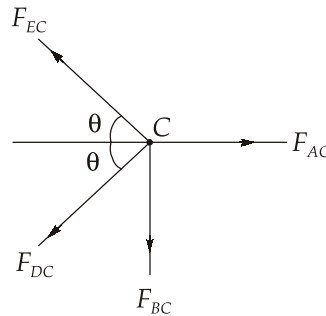
**Joint B:**



$$F_{BC} = F_{BA} \sin \theta = 447.21 \times \frac{1}{\sqrt{5}} = 200 \text{ kN (tension)}$$

$$F_{BD} = F_{BA} \cos \theta = 447.21 \times \frac{2}{\sqrt{5}} = 400 \text{ kN (comp.)}$$

**Joint C:**



$$\Sigma H = 0$$

$$\Rightarrow F_{EC} \cos \theta + F_{DC} \cos \theta = F_{AC}$$

$$\Rightarrow \frac{2}{\sqrt{5}}(F_{EC} + F_{DC}) = 400$$

$$\Rightarrow F_{EC} + F_{DC} = 200\sqrt{5} \quad \dots(i)$$

$$\Rightarrow F_{EC} \sin \theta = F_{DC} \sin \theta + F_{BC}$$

$$\Sigma V = 0$$

$$\Rightarrow (F_{EC} - F_{DC}) \frac{1}{\sqrt{5}} = 200$$

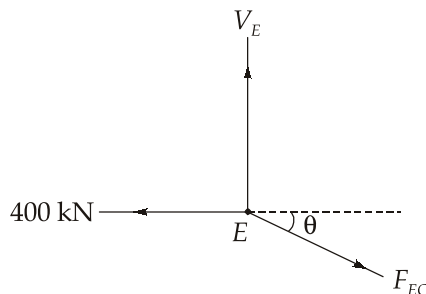
$$\Rightarrow F_{EC} - F_{DC} = 200\sqrt{5} \quad \dots(ii)$$

From eq. (i) and (ii)

$$F_{EC} = 200\sqrt{5} = 447.21 \text{ kN (tension)}$$

**Joint E:**

$$F_{DC} = 0$$



$$V_E = F_{EC} \sin \theta = 200 \text{ kN } (\uparrow)$$

$\therefore$

$$V_D = 200 - 200 = 0 \text{ kN}$$

**Q.1 (c) Solution:**

Let  $\sigma_1$  be the stress in each outer pillar,

and  $\sigma_2$  be the stress in the middle pillar, and let the decrease in length of the outer pillar be  $\delta$ .

$\therefore$  Decrease in length of the middle pillar =  $\delta - 0.35$

$$\text{For the outer pillar,} \quad \delta = \frac{\sigma_1}{E} \times 1750 \quad \dots(i)$$

$$\text{For the inner pillar,} \quad \delta - 0.35 = \frac{\sigma_2}{E} \times 1749.65 \quad \dots(ii)$$

From eq. (i) and (ii)

$$\frac{\sigma_1}{E} \times 1750 - 0.35 = \frac{\sigma_2}{E} \times 1749.65$$

$$\Rightarrow 1750\sigma_1 - 0.35E = 1749.65\sigma_2$$

$$\Rightarrow \sigma_1 - \frac{0.35E}{1750} = \frac{1749.65}{1750}\sigma_2$$

$$\Rightarrow \sigma_1 - \frac{0.35 \times 12500}{1750} = \frac{1749.65}{1750}\sigma_2$$

$$\Rightarrow \sigma_1 - 2.50 = 0.9998\sigma_2$$

Given  $\sigma_1 = 3.5 \text{ N/mm}^2$

$$\therefore \sigma_2 = \frac{3.5 - 2.5}{0.9998} = 1 \text{ N/mm}^2$$

$$\begin{aligned} \therefore \text{Safe load} &= \sigma_1 (\text{area of outer pillars}) + \sigma_2 (\text{area of middle pillar}) \\ &= 3.5 (2 \times 250^2) + 1 (250^2) \\ &= 500,000 \text{ N} = 500 \text{ kN} \end{aligned}$$

**Q.1 (d) Solution:**

(a) In this arrangement, the tension in the rope is uniform since pulley is smooth. Let the tension in the rope be  $P$ . Hence for the equilibrium of the system,

$$2P = W$$

$$\Rightarrow P = \frac{W}{2} = 2500 \text{ N}$$

Intensity of stress in the rope section,

$$\sigma = \frac{P}{A} = \frac{2500}{800} = 3.125 \text{ N/mm}^2$$

Length of the rope,  $L = 4.56 + 4.56 + 1.52 = 10.64 \text{ m}$

$\therefore$  Increase in length of the rope

$$= \frac{\sigma \times L}{E} = \frac{3.125 \times 10.64 \times 10^3}{980} = 33.93 \text{ mm}$$

Let downward movement of the pulley be  $\delta$  mm

$$\therefore 2\delta = 33.93$$

$$\Rightarrow \delta = 16.965 \text{ mm}$$

- (b) In this arrangement, the loads shared by the two ropes  $AB$  and  $CB$  should be such that they extend by the same amount. Let the stress intensity on the sections of the ropes  $AB$  and  $CB$  be  $\sigma_1$  and  $\sigma_2$  respectively.

$$\therefore \frac{\sigma_1 L_1}{E} = \frac{\sigma_2 L_2}{E}$$

$$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{L_2}{L_1} = \frac{6.08}{4.56} = \frac{4}{3}$$

$$\Rightarrow \sigma_1 = \frac{4}{3} \sigma_2 \quad \dots(i)$$

But tension in  $AB$  + tension in  $CB = W$

$$\Rightarrow \sigma_1 A + \sigma_2 A = 5000$$

$$\Rightarrow \sigma_1 + \sigma_2 = \frac{5000}{800} \quad \dots(ii)$$

From eq. (i) and (ii)

$$\frac{4}{3} \sigma_2 + \sigma_2 = \frac{5000}{800}$$

$$\Rightarrow \frac{7}{3} \sigma_2 = \frac{5000}{800}$$

$$\Rightarrow \sigma_2 = 2.68 \text{ N/mm}^2$$

$$\therefore \sigma_1 = \frac{4}{3} \times 2.68 = 3.57 \text{ N/mm}^2$$

∴ Downward movement of the pulley

= Extension of the rope  $AB$  or  $CB$

$$= \frac{\sigma_1 L_1}{E} = \frac{3.57}{980} \times 4.56 \times 10^3 = 16.61 \text{ mm}$$

**Q.1 (e) Solution:**

- (i) (a) **Effect of moisture on mechanical properties of wood:** Variations in the moisture content of the cell walls are accompanied by large changes in the strength and stiffness of wood. After years of seasoning, large timbers may lose enough water to effect an increase in tensile and compressive strength and in stiffness, but defects arising from shrinkage stresses often decrease the resistance to horizontal shear stresses. In kiln seasoning, the normal increase in strength due to loss of moisture is often nullified by case hardening, a condition which prevents complete drying of the piece and produces internal stresses.

The mechanical properties of wood are not materially affected by a reduction in the moisture content until the point of fibre saturation is reached. Further drying causes a large, proportionate increase in strength and stiffness.

- (b) **Effect of temperature on strength of wood:** The effect of temperature on wood is dependent upon the moisture content. Dry wood expands slightly when heated, while wet wood shrinks owing to evaporation of moisture. When the temperature of wood is raised above room temperature it becomes weaker in most strength properties. Very high temperatures, such as those used in vulcanizing, slightly weaken dry wood. Freezing somewhat increases both the strength and stiffness of wood. If wood is kept moist during the heating process, it is referred very pliable and is weakened.

- (c) **Stress-strain relationship in the wood:** Wood has three principal axes—longitudinal, radial and tangential—along which properties are fairly constant. Since wood is a nonsteroid material, it has three values of modulus of elasticity varying by as much as 150 to 1, three shear moduli varying by 20 to 1, and six Poisson's ratios varying by 40 to 1. There is no sharply defined elastic limit in wood but there is a proportional limit. However, the stress-strain diagram in any direction is fairly straight over a considerable range before it gradually curves off. It is a ductile material.

The relative stress-strain curves for direct tension, direct compression and bending stress intensities parallel to the grain in both, direct compression and bending, the proportional limit is in the vicinity of 65 to 75 per cent of the

ultimate strength. For all practical purposes, there is no proportional limit in direct tension.

Modulus of elasticity of the grain is practically the same in direct tension, direct compression and bending, if shear deformation in bending is eliminated.

## (ii) Preparation of Brick Earth

It consists of the following operations:

**Unsoiling:** The soil used for making building bricks should be processed so as to be free of gravel, coarse sand (particle size more than 2 mm), lime and kankar particles, organic matter, etc. About 20 cm of the top layer of the earth, normally containing stones, pebbles, gravel, roots, etc., is removed after clearing the trees and vegetation.

**Digging:** After removing the top layer of the earth, proportions of additives such as fly ash, sandy loam, rice husk ash, stone dust, etc. should be spread over the plane ground surface on volume basis. The soil mass is then manually excavated, puddled, watered and left over for weathering and subsequent processing. The digging operation should be done before rains.

**Weathering:** Stones, gravels, pebbles, roots, etc. are removed from the dug earth and the soil is heaped on level ground in layers of 60-120 cm. The soil is left in heaps and exposed to weather for at least one month in cases where such weathering is considered necessary for the soil. This is done to develop homogeneity in the mass of soil, particularly if they are from different sources, and also to eliminate the impurities which get oxidized. Soluble salts in the clay would also be eroded by rain to some extent, which otherwise could have caused scumming at the time of the brick in the kiln. The soil should be turned over at least twice and it should be ensured that the entire soil is wet throughout the period of weathering. In order to keep it wet, water may be sprayed as often as necessary. The plasticity and strength of the clay are improved by exposing the clay to weather.

**Blending:** The earth is then mixed with sandy-earth and calcareous - earth in suitable proportions to modify the composition of soil. Moderate amount of water is mixed so as to obtain the right consistency for moulding. The mass is then mixed uniformly with spades. Addition of water to the soil at the dumps is necessary for the easy mixing and workability, but the addition of water should be controlled in such a way that it may not create a problem in moulding and drying. Excessive moisture content may effect the size and shape of the finished brick.

**Tempering:** Tempering consists of kneading the earth with feet so as to make the mass stiff and plastic (by plasticity, we mean the property wet clay has to being



permanently deformed without cracking). It should preferably be carried out by storing the soil in a cool place in layers of about 30 cm thickness for not less than 36 hours. This will ensure homogeneity in the mass of clay for subsequent processing. For manufacturing good bricks, tempering is done in pug mills and the operation is called pugging.

**Q.2 (a) Solution:**

- (i) Rheology may be defined as the science of the deformation and flow of materials, and is concerned with relationships between stress, strain, rate of strain and time.

Following factors affect rheological properties:

**1. Mix proportions**

- (a) A concrete mix having an excess amount of coarse aggregate will lack sufficient mortar to fill the void system, resulting in a loss of cohesion and mobility.
- (b) A high fine aggregate content increases the surface area of particles, which increases the amount of paste required to coat these surfaces to have the same amount of mobility. This, in turn, can result in increased drying shrinkage and cracking.

**2. Consistency**

The consistency of concrete, as measured by the slump test, is an indicator of the relative water content in the concrete mix. An increase in the water content or slump above that required to achieve a workable mix produces greater fluidity and decreased internal friction. The reduction in the cohesion within the mixture increases the potential for segregation and excessive bleeding.

**3. Hardening and stiffening**

Elevated temperature, use of rapid-hardening cement, cement deficient in gypsum, and use of accelerating admixtures, increase the rate of hardening which reduces the mobility of concrete. The dry and porous aggregate will rapidly reduce workability by absorbing water from the mixture.

**4. Aggregate shape and texture**

The shape of the aggregate particles and aggregate texture influence the rheology of concrete appreciably. The rough and highly angular aggregate particles will result in higher percentage of voids being filled by mortar, requiring higher fine aggregate contents and correspondingly higher water content. Similarly, an angular fine aggregate will increase internal friction in

the concrete mixture and require higher water content than well-rounded natural sand.

#### 5. Aggregate grading

A well-graded aggregate gives good workability. The absence of a particular size of aggregate (gap-graded) or a change in the size distribution may have appreciable effect on the void system and workability. These effects are more in the fine aggregate than in coarse aggregate.

#### 6. Maximum aggregate size

An increase in the maximum size of aggregate will reduce the fine aggregate content required to maintain a given workability and will thereby reduce the surface area to be wetted and hence the cement content necessary for a constant water-cement ratio.

#### 7. Admixture

Out of the large number of admixtures used in concrete to obtain improved performance characteristics, the admixtures which have significant effect on the rheology of concrete are plasticizers and super-plasticizers, air-entraining agents, accelerators and retarders.

- (ii) Pozzonalas are finely ground siliceous materials which as such, do not possess cementing property in themselves, but react chemically with calcium hydroxide ( $\text{Ca}(\text{OH})_2$ ) released from the hydration of portland cement at normal temperature to form compounds of low solubility having cementing properties. The action is termed as pozzolanic action.

The pozzolanic materials can be divided into two groups namely, natural pozzolanas and artificial pozzolanas.

#### Examples:

**Natural pozzonalas:** Clay, shales, opaline charts, diatomaceous earth and volcanic tuffs and pumice.

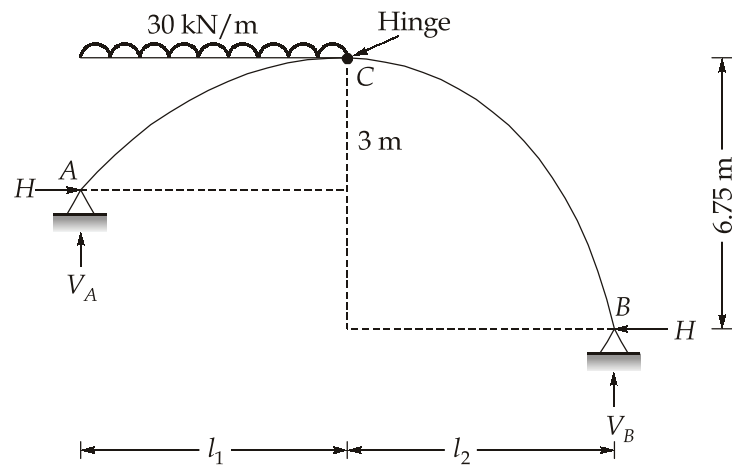
**Artificial pozzolanas:** Flash, blast-furnace-slag, silica fume, rice husk ash, metakaolin, and surkhi.

Various implications seen on application of pozzolana in cement concrete:

- (a) Improved workability with lesser amount of water.
- (b) Lower heat of hydration and thermal shrinkage.
- (c) Improved resistance to attack from salts and surfaces from soils and sea water.
- (d) Reduced susceptibility to dissolution and leaching of calcium hydroxide.

- (e) Reduced permeability
- (f) Lower costs (economical)
- (g) Reduction in the rate of development of strength
- (h) An increase in drying shrinkage.
- (i) Reduction in durability at times
- (j) Longer curing periods in case of use of flyash concrete.

**Q.2 (b) Solution:**



Let  $l_1$  = horizontal distance between A and C  
and  $l_2$  = horizontal distance between B and C

$$l_1 + l_2 = 22.5$$

Since cable profile is parabolic

$$\therefore \frac{l_1}{l_2} = \sqrt{\frac{y_1}{y_2}}$$

$$\Rightarrow \frac{l_1}{22.5 - l_1} = \sqrt{\frac{3}{6.75}}$$

$$\Rightarrow l_1 = 9 \text{ m}$$

$$l_2 = 22.5 - 9 = 13.5 \text{ m}$$

Let  $V_A$  and  $V_B$  be the vertical reactions at the support A and B respectively. Let  $H$  be the horizontal reaction at each support.

Taking moment about C from the left,

$$V_A \times 9 = 3H + 30 \times 9 \times 4.5$$

$$\Rightarrow V_A = \frac{H}{3} + 135 \quad \dots(i)$$

Taking moments about C from the right,

$$\Rightarrow V_B \times 13.5 = 6.75 H$$

$$V_B = \frac{H}{2} \quad \dots(\text{ii})$$

Also,  $\Sigma F_V = 0$

$$\Rightarrow V_A + V_B = 30 \times 9 = 270$$

$$\Rightarrow \frac{H}{3} + 135 + \frac{H}{2} = 270$$

$$\Rightarrow \frac{5H}{6} = 135$$

$$\Rightarrow H = 162 \text{ kN}$$

Substituting for  $H$ ,

$$V_A = \frac{162}{3} + 135 = 189 \text{ kN}$$

$$V_B = \frac{162}{2} = 81 \text{ kN}$$

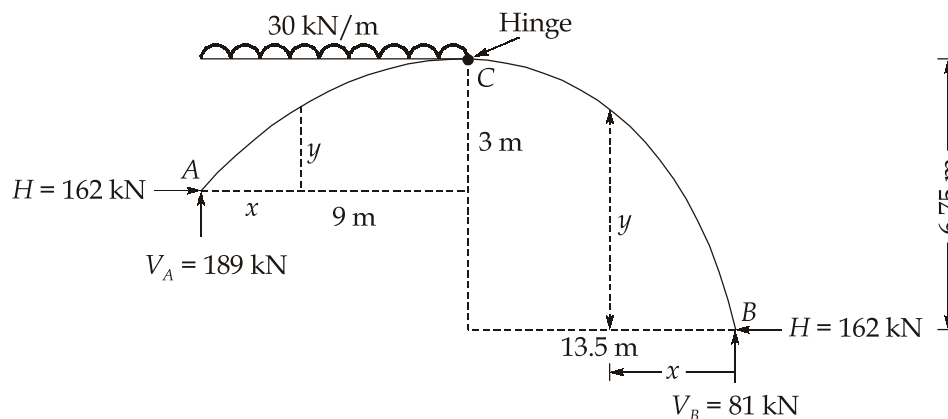
### Maximum positive BM:

The maximum positive BM will occur in portion AC.

The equation of arch from A to C with A as origin is

$$y = \frac{(4 \times 3)x}{(18)^2} (18 - x)$$

$$\Rightarrow y = \frac{12x}{(18)^2} (18 - x)$$



The BM at a section in AC distant  $x$  from A is given by

$$M_x = 189x - 15x^2 - \frac{162 \times 12x}{18^2}(18 - x)$$

$$\Rightarrow M_x = 189x - 15x^2 - 108x + 6x^2$$

$$\Rightarrow M_x = 81x - 9x^2$$

For the condition of maximum BM,

$$\frac{dM_x}{dx} = 0$$

$$\Rightarrow \frac{dM_x}{dx} = 81 - 18x = 0$$

$$\Rightarrow x = 4.5 \text{ m}$$

$$\therefore M_{\max} = 81 \times 4.5 - 9 \times 4.5^2 = 182.25 \text{ kNm}$$

**Maximum negative BM:**

The maximum negative BM will occur in portion BC.

The equation to the arch from B to C with B as origin is

$$y = \frac{(4 \times 6.75)x(27 - x)}{(27)^2} = \frac{x}{27}(27 - x)$$

The BM at a section in BC distant  $x$  from B is given by

$$M_x = 81x - 162 \times \frac{x}{27}(27 - x)$$

$$\Rightarrow M_x = 6x^2 - 81x$$

For the condition of maximum BM,

$$\frac{dM_x}{dx} = 0$$

$$\Rightarrow 12x - 81 = 0$$

$$\Rightarrow x = 6.75 \text{ m}$$

$$\therefore M_{\max} = 6 \times 6.75^2 - 81 \times 6.75 = -273.375 \text{ kNm}$$

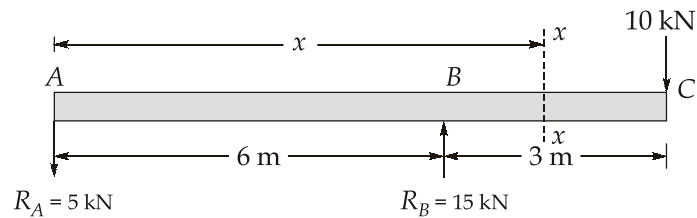
**Q.2 (c) Solution:**

$$\Sigma M_A = 0$$

$$\Rightarrow R_B \times 6 = 10 \times 9$$

$$\Rightarrow R_B = 15 \text{ kN } (\uparrow)$$

$$\therefore R_A = \text{Total load} - R_B = 10 - 15 = -5 \text{ kN} = 5 \text{ kN } (\downarrow)$$



The BM at any section at a distance  $x$  from the support A is given by

$$EI \frac{d^2y}{dx^2} = -R_A x + R_B (x - 6)$$

$$\Rightarrow EI \frac{d^2y}{dx^2} = -5x + 15(x - 6)$$

Integrating the above equation, we get

$$EI \frac{dy}{dx} = \left( -\frac{5x^2}{2} + C_1 \right) + \frac{15(x-6)^2}{2} \quad \dots(i)$$

Integrating again, we get

$$EIy = \left( -\frac{5x^3}{6} + C_1x + C_2 \right) + \frac{15(x-6)^3}{6}$$

$$\Rightarrow EIy = \left( -\frac{5}{6}x^3 + C_1x + C_2 \right) + \frac{5}{2}(x-6)^3 \quad \dots(ii)$$

where  $C_1, C_2$  are constants of integration.

Values of constant  $C_1$  and  $C_2$  are obtained from boundary conditions which are:

(i) At  $x = 0, y = 0$  and thus  $C_2 = 0$

(ii) At  $x = 6 \text{ m}, y = 0$

$$\therefore -\frac{5}{6} \times (6)^3 + C_1 \times 6 = 0$$

$$\Rightarrow C_1 = 30$$

Substituting the values of  $C_1$  and  $C_2$  in equation (i) and (ii), we get

$$EI \frac{dy}{dx} = -\frac{5}{2}x^2 + 30x + \frac{15}{2}(x-6)^2 \quad \dots(iii)$$

$$EIy = -\frac{5}{6}x^3 + 30x + \frac{5}{2}(x-6)^3 \quad \dots(iv)$$

(a) Slope at support A

By substituting  $x = 0$  in equation (iii) upto dotted line, we get the slope at support A (since the point  $x = 0$  lies in the first part AB of the beam).

$$\therefore EI\theta_A = -\frac{5}{2} \times 0 + 30 = 30 \text{ kNm}^2 = 30 \times 10^9 \text{ Nmm}^2$$

$$\Rightarrow \theta_A = \frac{30 \times 10^9}{2 \times 10^5 \times 5 \times 10^8} = 0.0003 \text{ radian}$$

(b) Slope at support B

By substituting  $x = 6$  m in eq. (iii) upto dotted line, we get the slope at support B (since the point  $x = 6$  m lies in the first part AB of the beam)

$$\therefore EI\theta_B = -\frac{5}{2} \times 6^2 + 30 = -90 + 30 = -60 \text{ kNm}^2$$

$$\Rightarrow EI\theta_B = 60 \times 10^9 \text{ Nmm}^2$$

$$\Rightarrow \theta_B = -\frac{60 \times 10^9}{2 \times 10^5 \times 5 \times 10^8} = -0.0006 \text{ radians}$$

(c) Slope at right end i.e., at C

By substituting  $x = 9$  m in eq. (iii), we get the slope at C. In this case, complete equation is to be taken as  $x = 9$  m lies in the last part of the beam.

$$\therefore EI\theta_C = -\frac{5}{2} \times 9^2 + 30 + \frac{15}{2} (9-6)^2$$

$$\Rightarrow EI\theta_C = -202.5 + 30 + 67.5 = -105 \text{ kNm}^2 = -105 \times 10^9 \text{ Nmm}^2$$

$$\Rightarrow \theta_C = -\frac{105 \times 10^9}{2 \times 10^5 \times 5 \times 10^8} = -0.00105 \text{ radians}$$

(d) Maximum upward deflection between the supports.

For maximum deflection between the supports,  $\frac{dy}{dx}$  should be zero. Hence equating the slope given by equation (iii) to be zero upto dotted line, we get

$$0 = -\frac{5x^2}{2} + 30$$

$$\Rightarrow x = \sqrt{\frac{60}{5}} = 3.464 \text{ m}$$

Now, substituting  $x = 3.464$  m in eq. (i) upto dotted line, we get the maximum deflection as

$$\begin{aligned} EIy_{\max} &= -\frac{5}{6} \times 3.464^3 + 30 \times 3.464 \\ &= -34.638 + 103.92 = 69.282 \text{ kNm}^3 \\ &= 69.282 \times 10^{12} \text{ Nmm}^3 \end{aligned}$$

$$\therefore y_{\max} = \frac{69.282 \times 10^{12}}{2 \times 10^5 \times 5 \times 10^8} = 0.6928 \text{ mm (upward)}$$

(e) Deflection at the right end i.e., at point C

By substituting  $x = 9$  m in eq. (iv), we get deflection at point C. Here complete equation is to be taken as point  $x = 9$  m lies in the last part of the beam

$$\therefore EIy_c = -\frac{5}{6} \times 9^3 + 30 \times 9 + \frac{5}{2} (9-6)^3$$

$$\Rightarrow EIy_c = -607.5 + 270 + 67.5 = -270 \text{ kNm}^3 \\ = -270 \times 10^{12} \text{ Nmm}^3$$

$$\therefore y_c = -\frac{270 \times 10^{12}}{2 \times 10^5 \times 5 \times 10^8} = -2.7 \text{ mm (downward)}$$

**Q.3 (a) Solution:**

**Distribution factors:**

Joint	Member	Relative stiffness	Total stiffness	DF
B	BC	$\frac{3}{4} \times \frac{2I}{3} = \frac{I}{2}$	$\frac{3I}{4}$	$\frac{2}{3}$
	BC	$\frac{I}{4}$		$\frac{1}{3}$
C	CB	$\frac{I}{4}$	$\frac{I}{2}$	$\frac{1}{2}$
	CD	$\frac{I}{4}$		$\frac{1}{2}$

**Fixed end moment:**

$$M_{FAE} = +\frac{2 \times 1.5^2}{2} = +2.25 \text{ kNm}$$

$$M_{FAB} = -\frac{36 \times 2 \times 1^2}{3^2} = -8 \text{ kNm}$$



$$M_{FBA} = +\frac{36 \times 1 \times 2^2}{3^2} = +16 \text{ kNm}$$

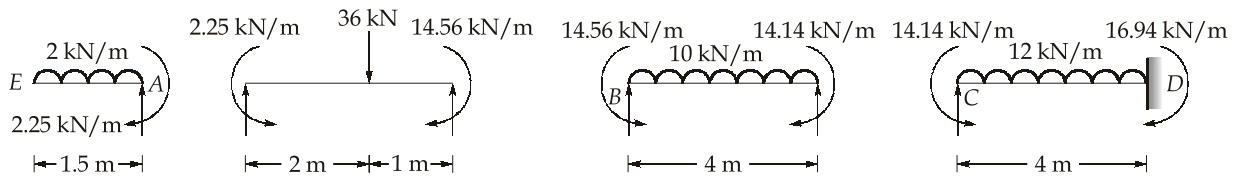
$$M_{FBC} = -\frac{10 \times 4^2}{12} = -13.33 \text{ kNm}$$

$$M_{FCB} = +\frac{10 \times 4^2}{12} = 13.33 \text{ kNm}$$

$$M_{FCD} = -\frac{12 \times 4^2}{12} = -16 \text{ kNm}$$

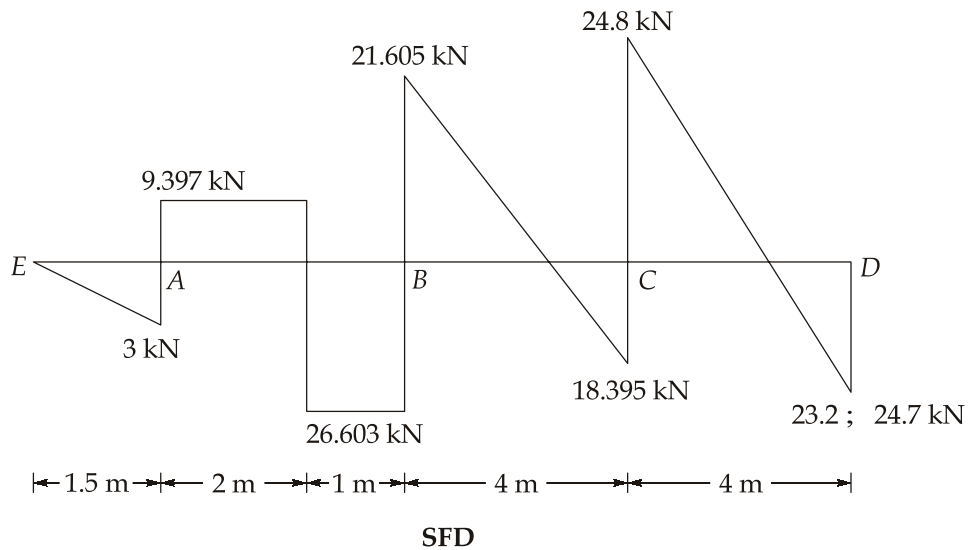
$$M_{FDC} = +\frac{12 \times 4^2}{12} = +16 \text{ kNm}$$

	E	A	B	C	D			
DF	-	0	1	2/3	1/3	1/2	1/2	0
FEM		+2.25	-8	+16	-13.33	+13.33	-16	+16
Balance			+5.75	-1.78	-0.89	+1.335	+1.335	
COM				2.875	0.6675	-0.445		0.6675
FEM		2.25	-2.25	17.095	-13.55	14.22	-14.665	16.6675
Balance				-2.363	-1.182	+0.223	+0.223	
COM					0.1115	-0.591		0.1115
FEM		2.25	-2.25	14.732	-14.621	13.852	-14.44	16.78
Balance				-0.074	-0.037	+0.294	+0.294	
COM					0.147	-0.0185		0.147
FEM		2.25	-2.25	+14.66	-14.51	+14.13	-14.15	16.93
Balance				-0.1	-0.05	0.01	0.01	
COM					0.005	-0.025		0.05
FEM		2.25	-2.25	14.56	-14.56	14.12; 14.14	-14.14	16.94



	E (kN)	A (kN)	A (kN)	B (kN)	B (kN)	C (kN)	C (kN)	D (kN)
Reaction due to applied load	0	$2 \times 1.5 = 3 (\uparrow)$	$\frac{36 \times 1}{3} = 12 (\uparrow)$	$\frac{36 \times 2}{3} = 24 (\uparrow)$	$\frac{10 \times 4}{2} = 20 (\uparrow)$	$\frac{10 \times 4}{2} = 20 (\uparrow)$	$\frac{12 \times 4}{2} = 24 (\uparrow)$	$\frac{12 \times 4}{2} = 24 (\uparrow)$
Reaction due to end moment	0	$\frac{2.25}{1.5} = 1.5 (\uparrow)$	$\frac{14.56 - 2.25}{3} = 4.103 (\downarrow)$	$\frac{14.56 - 2.25}{3} = 4.103 (\uparrow)$	$\frac{14.56 - 14.14}{4} = 0.105 (\uparrow)$	$\frac{14.56 - 14.14}{4} = 0.105 (\downarrow)$	$\frac{16.94 - 14.14}{4} = 0.7 (\downarrow)$	$\frac{16.94 - 14.14}{4} = 0.7 (\uparrow)$

$R_E = 0,$                        $R_A = 12.397 \text{ kN } (\uparrow),$                        $R_B = 48.208 \text{ kN } (\uparrow)$   
 $R_C = 43.195 \text{ kN } (\uparrow),$                        $R_D = 24.7 \text{ kN } (\uparrow)$



**Q.3 (b) Solution:**

Let us do it by using slope deflection method:

Fixed end moment:

$$M_{FAB} = \frac{-2 \times 1 \times 3^2}{4^2} = -1.125 \text{ kNm}$$

$$M_{FBA} = \frac{2 \times 3 \times 1^2}{4^2} = 0.375 \text{ kNm}$$

$$M_{FBC} = -\frac{16 \times 0.5 \times 1.5^2}{2^2} = -4.5 \text{ kNm}$$

$$M_{FCB} = \frac{16 \times 1.5 \times 0.5^2}{2^2} = 1.5 \text{ kNm}$$

$$M_{FCD} = -\frac{10 \times 2^2}{12} = -3.33 \text{ kNm}$$

$$M_{FDC} = \frac{10 \times 2^2}{12} = 3.33 \text{ kNm}$$

Slope deflection equations:

**For span AB:**

$$M_{AB} = M_{FAB} + \frac{2EI}{L_{AB}} \left[ 2\theta_A + \theta_B - \frac{3\delta}{L_{AB}} \right]$$

$$\Rightarrow M_{AB} = -1.125 + \frac{EI\theta_B}{2} - \frac{3}{8}EI\delta \quad (\because \theta_A = 0)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L_{BA}} \left[ 2\theta_B + \theta_A - \frac{3\delta}{L_{BA}} \right]$$

$$\Rightarrow M_{BA} = +0.375 + EI\theta_B - \frac{3}{8}EI\delta$$

**For span BC:**

$$\begin{aligned} M_{BC} &= M_{FBC} + \frac{2EI \times 2}{L_{BC}} \left( 2\theta_B + \theta_C - \frac{3\delta}{L_{BC}} \right) \\ &= -4.5 + 4EI\theta_B + 2EI\theta_C \end{aligned}$$

$$\begin{aligned} M_{CB} &= M_{FCB} + \frac{2EI \times 2}{L_{BC}} \left( 2\theta_C + \theta_B - \frac{3\delta}{L_{BC}} \right) \\ &= +1.50 + 2EI\theta_B + 4EI\theta_C \end{aligned}$$

**For span CD:**

$$\begin{aligned} M_{CD} &= M_{FCD} + \frac{2EI \times 2}{L_{CD}} \left( 2\theta_C + \theta_D - \frac{3\delta}{L_{CD}} \right) \\ &= -3.33 + 4EI\theta_C - 3EI\delta \quad (\because \theta_D = 0) \end{aligned}$$

$$M_{DC} = M_{FDC} + \frac{2EI \times 2}{L_{CD}} \left( 2\theta_D + \theta_C - \frac{3\delta}{L_{CD}} \right)$$

$$= +3.33 + 2EI\theta_C - 3EI\delta$$

Equilibrium condition at B,

$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow 0.375 + EI\theta_B - \frac{3}{8}EI\delta - 4.5 + 4EI\theta_B + 2EI\theta_C = 0$$

$$\Rightarrow 5EI\theta_B + 2EI\theta_C - \frac{3}{8}EI\delta = 4.125$$

$$\Rightarrow 40EI\theta_B + 16EI\theta_C - 3EI\delta = 33 \quad \dots(i)$$

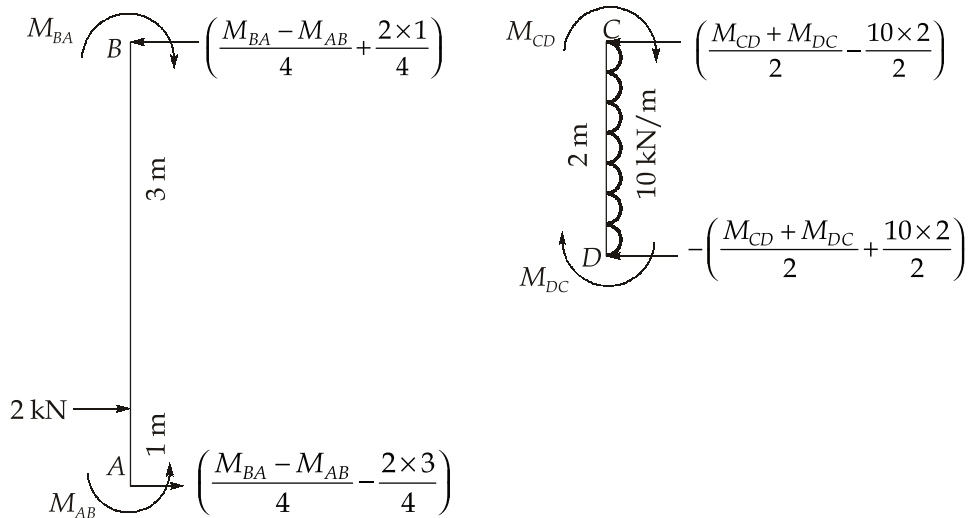
Equilibrium condition at C,

$$M_{CB} + M_{CD} = 0$$

$$\Rightarrow 1.50 + 2EI\theta_B + 4EI\theta_C - 3.33 + 4EI\theta_C - 3EI\delta = 0$$

$$\Rightarrow 2EI\theta_B + 8EI\theta_C - 3EI\delta = 1.83 \quad \dots(ii)$$

For horizontal equilibrium,



$$R_A + 2 = R_D + (10 \times 2) \quad \dots(i)$$

$$\Rightarrow \frac{M_{BA} - M_{AB}}{4} - \frac{2 \times 3}{4} + 2 = -\left(\frac{M_{DC} + M_{CD}}{2} + \frac{10 \times 2}{2}\right) + 20$$

$$\Rightarrow M_{BA} - M_{AB} - 6 + 8 = -(M_{DC} + M_{CD} + 10)2 + 80$$

$$\Rightarrow M_{BA} - M_{AB} + 2(M_{CD} + M_{DC}) = 58$$

$$\Rightarrow 0.375 + EI\theta_B - \frac{3}{8}EI\delta + 1.125 - \frac{EI\theta_B}{2} + \frac{3}{8}EI\delta$$

$$+ 2(-3.33 + 4EI\theta_C - 3EI\delta + 3.33 + 2EI\theta_C - 3EI\delta) = 58$$

$$\Rightarrow \frac{EI\theta_B}{2} + 1.5 + 12EI\theta_C - 12EI\delta = 58$$

$$\Rightarrow EI\theta_B + 24EI\theta_C - 24EI\delta = 113 \quad \dots(\text{iii})$$

Solving eq. (i), (ii) and (iii)

$$EI\theta_B = 1.4526$$

$$EI\theta_C = -3.0037$$

$$EI\delta = -7.6515$$

$$\therefore M_{AB} = -1.125 + \frac{1.4526}{2} - \frac{3}{8}(-7.6515) = 2.47 \text{ kNm}$$

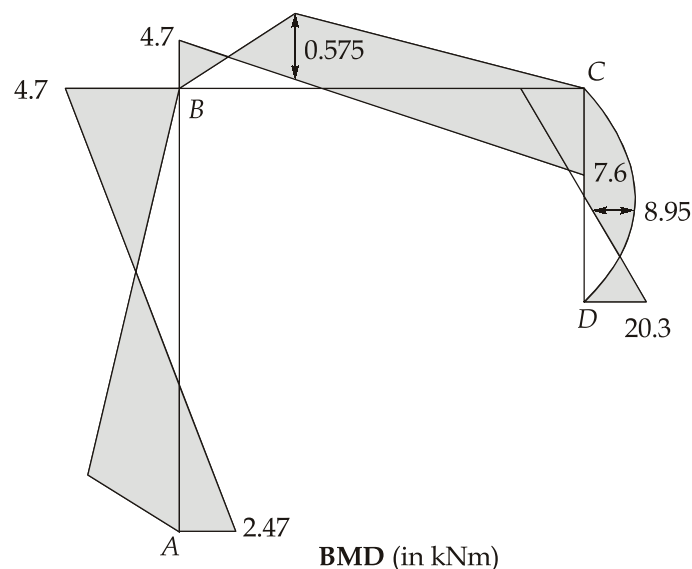
$$M_{BA} = 0.375 + 1.4526 - \frac{3}{8}(-7.6515) = 4.7 \text{ kNm}$$

$$M_{BC} = -4.5 + 4(1.4526) + 2(-3.0037) = -4.7 \text{ kNm}$$

$$M_{CB} = 1.5 + 2(1.4526) + 4(-3.0037) = -7.6 \text{ kNm}$$

$$M_{CD} = -3.33 + 4(-3.0037) - 3(-7.6515) = 7.6 \text{ kNm}$$

$$M_{DC} = 3.33 + 2(-3.0037) - 3(-7.6515) = 20.3 \text{ kNm}$$



**Q.3 (c) Solution:**

Given, Number of bolts,  $n = 12$   
 Pitch circle diameter  $= 25 \text{ cm} = 250 \text{ mm}$   
 Maximum shear stress in shaft,  
 $\tau = 55 \text{ N/mm}^2$

Maximum shear stress in bolts,  
 $q = 20 \text{ N/mm}^2$

Diameter of solid shaft,  $D = 5 \text{ cm} = 50 \text{ mm}$

External diameter of hollow shaft,  
 $D_0 = 10 \text{ cm} = 100 \text{ mm}$

Let,  $D_i =$  Internal diameter of hollow shaft,  
 $d =$  Diameter of the bolt in mm

In case of coupling, the torque from the shaft to other shaft is transmitted through bolts. As the shaft and coupling are all equally strong in torsion, the torque transmitted by the bolts must be equal to the torque transmitted by hollow shaft.

The torque transmitted by solid shaft is given by

$$T = \frac{\pi}{16} \tau D^3 = \frac{\pi}{16} \times 55 \times 50^3 = 1349903.093 \text{ Nmm} \quad \dots(i)$$

$$\text{Area of one bolt} = \frac{\pi}{4} d^2$$

$$\therefore \text{Area of 12 bolts} = 12 \times \frac{\pi}{4} d^2$$

$$\text{Shear force in 12 bolts} = 20 \times 12 \times \frac{\pi}{4} \times d^2 = 60\pi d^2 \text{ N}$$

$$\begin{aligned} \text{Torque transmitted by the bolts} &= \text{Shear force in bolts} \times \frac{\text{Pitch circle diameter}}{2} \\ &= 60\pi d^2 \times \frac{250}{2} \\ &= 7500 \pi d^2 \text{ Nmm} \end{aligned} \quad \dots(ii)$$

Equating eq.(i) and (ii), we get

$$1349903.093 = 7500 \pi d^2$$

$$\Rightarrow d = \sqrt{\frac{1349903.093}{7500\pi}} = \sqrt{57.3} = 7.57 \text{ mm}$$

Torque transmitted by hollow shaft,

$$T = \frac{\pi}{16} \times \tau \times \left[ \frac{D_o^4 - D_i^4}{D_o} \right] = \frac{\pi}{16} \times 55 \left[ \frac{100^4 - D_i^4}{100} \right]$$

$$= 0.10799(10^8 - D_i^4) \quad \text{..(iii)}$$

Equating eq. (i) and (iii)

$$1349903.093 = 0.10799(10^8 - D_i^4)$$

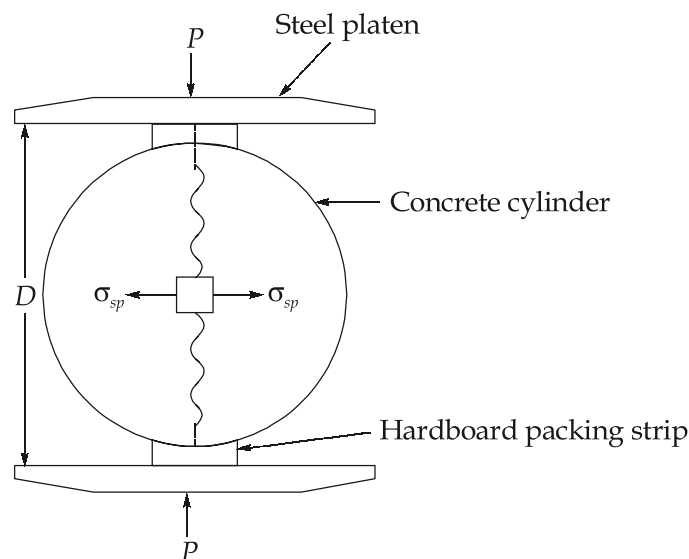
$$\Rightarrow D_i^4 = 10^8 - \frac{1349900}{0.10799}$$

$$\Rightarrow D_i^4 = 10^8 - 1250 \times 10^4$$

$$\Rightarrow D_i = (8750 \times 10^4)^{1/4} = 96.72 \text{ mm}$$

#### Q.4 (a) Solution:

- (i) The cylinder splitting test is a well known indirect test used for determining the tensile strength of concrete, sometimes referred to as the splitting tensile strength of concrete. The test is carried out by placing a cylindrical specimen (300 mm long and 150 mm in diameter) horizontally between the loading surfaces of a compression testing machine and the load is applied until failure of the cylinder, along the vertical diameter. When the load is applied, a fairly uniform tensile stress is induced over two-third of the loaded diameter as obtained from an elastic analysis.



The magnitude of this tensile stress is given by

$$\sigma_{sp} = \frac{2P}{\pi DL}$$

where  $P$  is the applied load,  $D$  is the diameter and  $L$  is the length of cylinder.

**(ii) Precautions in using mortar**

Following precautions are to be taken while making use of mortar:

- (a) Consumption of mortar:** After preparation, the mortar should be consumed as early as possible. The lime mortar should be consumed within 36 hours after its preparation and it should be kept wet or damp. The cement mortar should be consumed within 30 minutes after adding water and for this reason, it is advisable to prepare cement mortar of one bag of cement at a time. The gauged mortar or composite mortar should be used within 2 hours of the addition of cement.
- (b) Frost action:** The setting action of mortar is affected by the presence of frost. It is therefore advisable to stop the work in frosty weather or to execute it with cement mortar which will set before it tries to freeze.
- (c) Sea water:** In absence of pure water, the sea water may be used with hydraulic lime or cement. It helps in preventing too quick drying of the mortar. However it is not advisable to use sea water in making pure lime mortar or surkhi mortar because it will lead to efflorescence.
- (d) Soaking of building units:** The presence of water in mortar is essential to cause its setting action. Hence the building units should be soaked in water before mortar is applied. If this precaution is not taken, the water of mortar will be absorbed by the building units and the mortar will become weak.
- (e) Sprinkling of water:** The construction work carried out by mortar should be kept damp or wet by sprinkling water to avoid rapid drying of mortar. The water may be sprinkled for about 7 to 10 days. The exposed surfaces are sometimes covered to give protection against sun and wind.
- (f) Workability:** The mortar should not contain excess water and it should be as stiff as can be conveniently used. The joints should be well formed and the excess mortar from joints should be neatly taken off by a trowel. The surfaces formed by mortar for building units should be even.

**Q.4 (b) Solution:**

Considering the stiffening girder as a simply supported beam supporting the given external load system, we will first calculate the reactions. Taking moments about  $A$ , we have



$$V_B \times 120 = 240 \times 25 + 300 \times 80$$

$$\Rightarrow V_B = 250 \text{ kN}$$

$$\therefore V_A = 240 + 300 - 250 = 290 \text{ kN}$$

Beam moment at C, the middle point of the girder,

$$M_C = 290 \times 60 - 240 \times 35 = 9000 \text{ kNm}$$

Beam moment under the 240 kN load =  $290 \times 25 = 7250 \text{ kNm}$

Beam moment under the 300 kN load =  $250 \times 40 = 10000 \text{ kNm}$

Horizontal reaction at each end of the cable,

$$H = \frac{M_c}{h} = \frac{9000}{12} = 750 \text{ kN}$$

Let uniformly distributed load transferred to cable be  $w_e$  per unit run

$$\therefore H = \frac{w_e l^2}{8h} = \frac{w_e \times 120^2}{8 \times 12} = 750$$

$$\Rightarrow w_e = 5 \text{ kN/m}$$

Each vertical reaction for the cable,

$$V = \frac{w_e l}{2} = \frac{5 \times 120}{2} = 300 \text{ kN}$$

$\therefore$  Maximum tension in the cable,

$$T_{\max} = \sqrt{V^2 + H^2} = \sqrt{300^2 + 750^2} = 807.8 \text{ kN}$$

For girder, the SF at any section is given by  $S$ ,

$$S = \text{Beam shear} - H \tan \theta$$

$$\text{For cable, } \tan \theta = \frac{4h}{l^2}(l - 2x)$$

$$\therefore \text{At 40 m for left end, } \tan \theta = \frac{4 \times 12}{120 \times 120} \times (120 - 20 \times 40) = \frac{2}{15}$$

Beam shear at 40 m from left end,

$$= 290 - 240 = 50 \text{ kN}$$

$\therefore$  Actual shear force at 40 m from left end

$$= 50 - 750 \times \frac{2}{15} = -50 \text{ kN}$$

For the girder, the BM at any section is given by

$$M = \text{Beam moment} - \text{Moment due to } H$$

$$\Rightarrow M = \text{Beam moment} - Hy$$

Beam moment at 40 m from left end

$$= 290 \times 40 - 240 \times 15 = 8000 \text{ kNm}$$

At 40 m from the left end, for the cable

$$y = \frac{4hx}{l^2}(l-x) = \frac{4 \times 12}{(120)^2} \times (40 \times 80) = \frac{32}{3} \text{ m}$$

$\therefore$  Actual BM at 40 m from left end

$$= 8000 - 750 \times \frac{32}{3} = 0$$

#### Q.4 (c) Solution:

Length of column,  $l = 4 \text{ m} = 4000 \text{ mm}$

End conditions : Both ends fixed

$\therefore$  Effective length,  $L_e = \frac{l}{2} = \frac{4000}{2} = 2000 \text{ mm}$

$$\text{FOS} = \frac{\text{Crippling load}}{\text{Safe load}}$$

$$\Rightarrow 5 = \frac{\text{Crippling load}}{250}$$

$\Rightarrow$  Crippling load,  $P = 5 \times 250 = 1250 \text{ kN}$

Area of column  $A = \frac{\pi}{4} [D^2 - (0.8D)^2] = \frac{\pi}{4} [D^2 - 0.64D^2] = \pi \times 0.09D^2$

Moment of inertia,  $I = \frac{\pi}{64} [D^4 - (0.8)^4 D^4] = 0.009225\pi D^4$

But,  $I = Ak^2$

where  $k$  is the radius of gyration of column section

$$\Rightarrow k = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.009225 \times \pi D^4}{\pi \times 0.09 D^2}} = 0.32D$$

From Rankine's formula:

$$\text{Crippling load} \quad P_{cr} = \frac{\sigma_c A}{1 + \alpha \left( \frac{L_e}{k} \right)^2}$$

$$\Rightarrow 1250 \times 10^3 = \frac{550 \times \pi \times 0.09 D^2}{1 + \frac{1}{1600} \left( \frac{2000}{0.32 D} \right)^2}$$

$$\Rightarrow \frac{1250 \times 10^3}{550 \times \pi \times 0.09} = \frac{D^2}{1 + \frac{24414.0625}{D^2}}$$

$$\Rightarrow 8038.128 = \frac{D^4}{D^2 + 24414.0625}$$

$$\Rightarrow D^4 - 8038.128 D^2 - 196243359.4 = 0$$

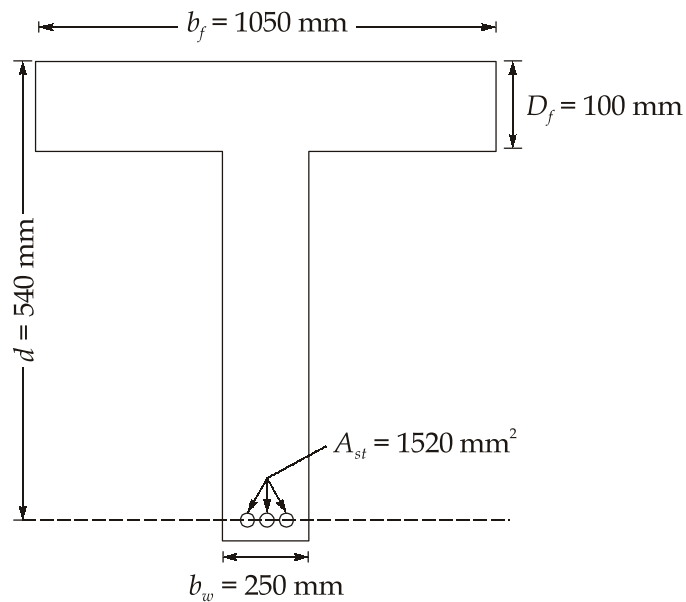
Solving for  $D$ , we get

$$D = 136.4 \text{ mm}$$

$$d = 0.8D = 0.8 \times 136.4 = 109.12 \text{ mm}$$

### Q.5 (a) Solution:

As per data given: T-beam is drawn below:



Determination of limiting moment of resistance:

For Fe415, limiting depth of NA,

$$x_{u', \text{lim}} = 0.48 \times d = 0.48 \times 540$$

$$x_{u, \text{lim}} = 259.2 \text{ mm} > 100 \text{ mm}$$

Hence, NA lies in the web.

$$\frac{3}{7} x_{u, \text{lim}} = \frac{3}{7} \times 259.2 = 111.09 \text{ mm}$$

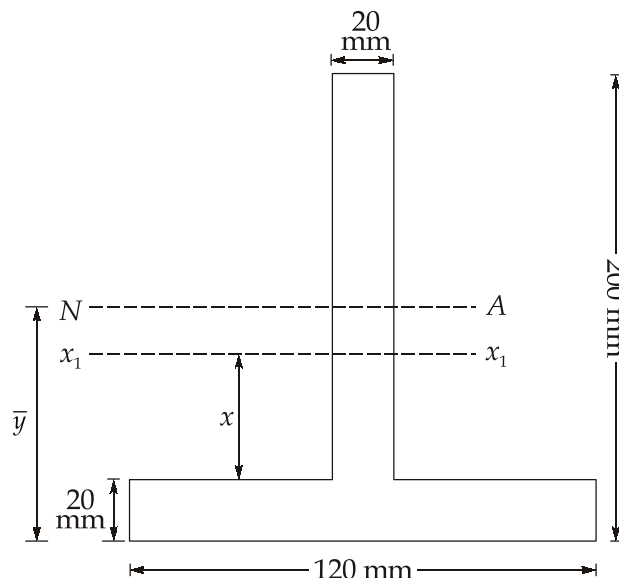
$$\therefore D_f < \frac{3}{7} (x_{u, \text{lim}})$$

Hence,

$$\begin{aligned} \therefore M_{u, \text{lim}} &= 0.36 f_{ck} x_{u, \text{lim}} b_w (d - 0.42 x_{u, \text{lim}}) + 0.45 f_{ck} (b_f - b_w) D_f \left( d - \frac{D_f}{2} \right) \\ &= 0.36 \times 20 \times 259.2 \times 250 (540 - 0.42 \times 259.2) \\ &\quad + 0.45 \times 20 \times (1050 - 250) \times 100 \left( 540 - \frac{100}{2} \right) \text{ Nmm} \\ &= 553.95 \text{ kNm} \end{aligned}$$

**Q.5 (b) Solution:**

(i)



Let  $\bar{y}$  be the distance of NA from bottom,

$$\therefore \bar{y} = \frac{120 \times 20 \times 10 + 180 \times 20 \times 110}{120 \times 20 + 180 \times 20} = 70 \text{ mm}$$

Moment of inertia about NA

$$\begin{aligned}
 I_{NA} &= \sum I_{\text{self}} + \sum A_i (\bar{y}_i - \bar{y})^2 \\
 &= \frac{120 \times (20)^3}{12} + 120 \times 20(10 - 70)^2 \\
 &\quad + \frac{20 \times (180)^3}{12} + 20 \times 180 \times (110 - 70)^2 \\
 &= 242 \times 10^5 \text{ mm}^4
 \end{aligned}$$

Section Modulus,

$$Z_{ez} = \frac{I_{NA}}{y_{\text{max}}} = \frac{242 \times 10^5}{130} = 186153.85 \text{ mm}^3$$

Let  $x_1 - x_1$  be the equal area axis at a distance  $x$  from the bottom of the flange as shown in figure.

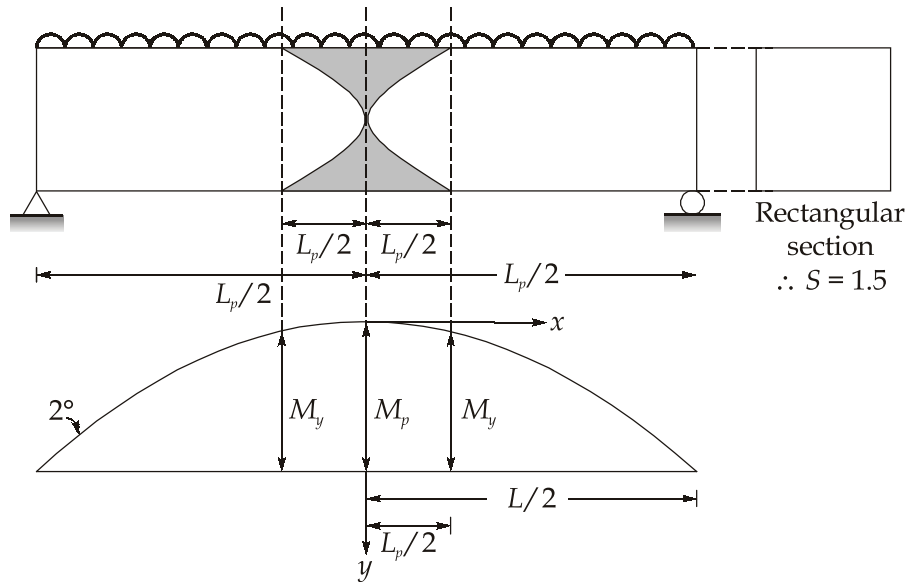
$$\begin{aligned}
 \therefore \quad &20 \times 120 + 20x = 20 \times (180 - x) \\
 \Rightarrow \quad &2400 + 20x = 3600 - 20x \\
 \Rightarrow \quad &40x = 1200 \\
 \Rightarrow \quad &x = 30 \text{ mm}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \quad \bar{y}_1 &= \text{CG of area above } x_1-x_1 = \frac{180-30}{2} = 75 \text{ mm} \\
 \bar{y}_2 &= \text{CG of area below } x_1-x_1 \\
 &= \frac{20 \times 120 \times 40 + 20 \times 30 \times 15}{20 \times 120 + 30 \times 20} = 35 \text{ mm}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \quad \text{Plastic section modulus, } Z_p &= \frac{A}{2} \cdot [\bar{y}_1 + \bar{y}_2] \\
 &= \frac{(20 \times 180 + 20 \times 120)}{2} [75 + 35] = 33 \times 10^4 \text{ mm}^3
 \end{aligned}$$

$$\therefore \quad \text{Shape factor, } S = \frac{Z_p}{Z_e} = \frac{33 \times 10^4}{186153.85} = 1.773$$

(ii) At collapse condition, plastic hinge is developed at the mid section.



At  $x = \frac{L}{2}, y = M_p$

$\therefore M_p = a \times \left(\frac{L}{2}\right)^2$

$\Rightarrow a = \frac{4 M_p}{L^2}$

Also, at  $x = \frac{L}{2}; y = M_p - M_y$

$\therefore (M_p - M_y) = \frac{4 M_p}{L^2} \cdot \left(\frac{L_p}{2}\right)^2$

$\Rightarrow \left(1 - \frac{1}{S}\right) = \left(\frac{L_p}{L}\right)^2 \quad \left(\because \frac{M_p}{M_y} = S\right)$

$\Rightarrow L_p = L \times \sqrt{1 - \frac{1}{S}}$

Since, cross-section is rectangular,

$\therefore S = 1.5$

$\therefore L_p = L \times \sqrt{1 - \frac{1}{1.5}} = L \times \sqrt{1 - \frac{2}{3}} = \frac{L}{\sqrt{3}}$

**Q.5 (c) Solution:**

Explosive is a chemical compound which under favorable conditions detonates quickly and produces tremendous high pressure.

It is a powerful source of energy. Without explosives, it would not be possible to get large quantities of coal, limestone, iron ore and other minerals. Furthermore explosives are extensively used in all forms of rock excavation (either open cut or tunnelling), for making roadways through hilly areas, for demolishing of structures etc.

Upon initiation of the explosives, tremendous amount of heat is generated which expands the gases and causes them to exert enormous pressure, thereby breaking the rock.

The following properties of an explosive are of much concern to users:

- Strength (energy content to be released)
- Velocity of detonation
- Density
- Water resistance
- Sensitivity
- Oxygen balance

**Types of explosives**

The explosives are classified on the following basis:

Based on the chemical nature, the explosives fall into three main classes:

1. **Low explosive:** On initiation of low explosives, the explosive composition burns over a relatively sustained period of time, without the production of an intense shock wave and thereby the gases are released at lower pressures. It is used where a slow heaving action is required. Black blasting powder (gun powder) belongs to this class. It is used extensively in quarries particularly in the production of building and monumental stone. The main constituents are sulphur 10% approximately, charcoal 15% approximately, sodium/potassium nitrate 75% approximately.
2. **High explosive:** High explosives usually contain either nitroglycerine (NGL) or trinitrotoulene (TNT) as the main explosive ingredient and are initiated by detonator. An intense shock wave is followed by the production of large volume of gases, at exceptionally high pressure. Hence, the action of high explosive is extremely fast and violent.  
High explosives generally fall into two distinct groups viz. Nitroglycerine and Non-nitroglycerine explosive.
3. **Initiating explosive:** Initiating explosives, as the name suggests, are used to initiate the explosion. When ignited, they produce an intense local blow or shock which starts the reaction in less sensitive high explosives.

### Other High Explosives

Several special type high explosives are being manufactured these days with the following names but these should be used under the guidance of the expert crew:

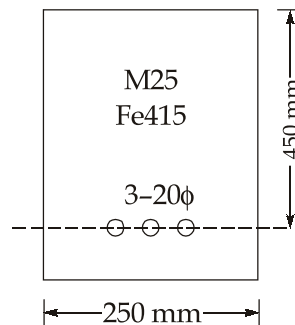
TNT = Tri-Nitro-Toluene

RDX = Rapid Detonating Explosive

PENT = Penta-Enythrital

### Q.5 (d) Solution:

Determination of Moment of Resistance of beam section.



For Fe415,

$$x_{u,lim} = 0.48d = 0.48 \times 450 = 216 \text{ mm}$$

$$C = T$$

⇒

$$0.36 f_{ck} x_u b = 0.87 f_y A_{st}$$

⇒

$$x_u = \frac{0.87 \times 415 \times 3 \times \frac{\pi}{4} (20)^2}{0.36 \times 25 \times 250} = 151.24 \text{ mm}$$

∴

$x_u < x_{u,lim}$  and thus section is under-reinforced

∴ Moment of resistance,

$$\begin{aligned} M_u &= 0.36 \cdot f_{ck} x_u b (d - 0.42x_u) \\ &= 0.36 \times 25 \times 151.24 \times 250 (450 - 0.42 \times 151.24) \text{ Nmm} \\ &= 131.5 \text{ kNm} \end{aligned}$$

Factored shear force at the centre of support

$$V_u = 250 \text{ kN}$$

Anchorage length of bar at simply supported end of beam can be determined by

$$L_d \leq 1.3 \frac{M_u}{V_u} + l_0$$

⇒

$$l_0 \geq L_d - 1.3 \frac{M_u}{V_u}$$



But,

$$L_d = \frac{\phi \cdot (0.87 f_y)}{4 \tau_{bd}} = \frac{20 \times 0.87 \times 415}{4 \times 2.24} = 805.92 \text{ mm}$$

$$\therefore l_o \geq 805.92 - 1.3 \times \frac{131.5 \times 10^3}{250}$$

$$\geq 122.12 \text{ mm}$$

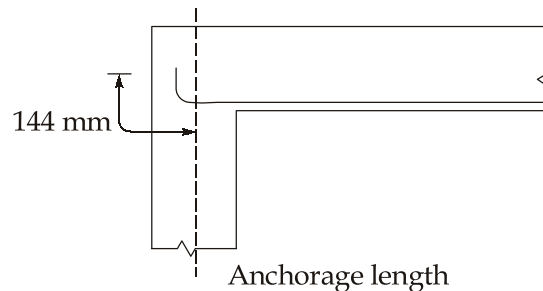
The anchorage length of bar shall not be less than the minimum length of bar to be extended beyond the centre of support.

The minimum length of bar to be extended beyond the centre of support

$$= \frac{L_d}{3} - \frac{\text{Width of support}}{2} = \frac{805.92}{3} - \frac{250}{2}$$

$$= 143.64 \text{ mm} > (L_o = 122.12 \approx 123 \text{ mm})$$

$\therefore$  The anchorage length of 144 mm is provided as shown in figure.



### Q.5 (e) Solution:

Given: Fe 410 grade steel

$$f_y = 250 \text{ N/mm}^2$$

$$\gamma_{m0} = 1.1$$

$$f_{ck} = 25 \text{ N/mm}^2$$

$\therefore$  Permissible bearing pressure of concrete

$$= 0.45 f_{ck} = 0.45 \times 25 = 11.25 \text{ N/mm}^2$$

Area of bearing plate,

$$A = \frac{220 \times 10^3}{11.25} = 19555.56 \text{ mm}^2$$

Width of bearing plate = Width of concrete pedestal  
= Thickness of masonry wall = 240 mm

Length of bearing plate,

$$b_l = \frac{19555.56}{240} = 81.48 \text{ mm} \neq b_l = 210 \text{ mm}$$

∴  $b_l = 210 \text{ mm}$

Provide 240 mm × 210 mm bearing plate,

∴ Area of plate provided = 240 × 210 = 50400 mm<sup>2</sup>.

Thickness of bearing plate,

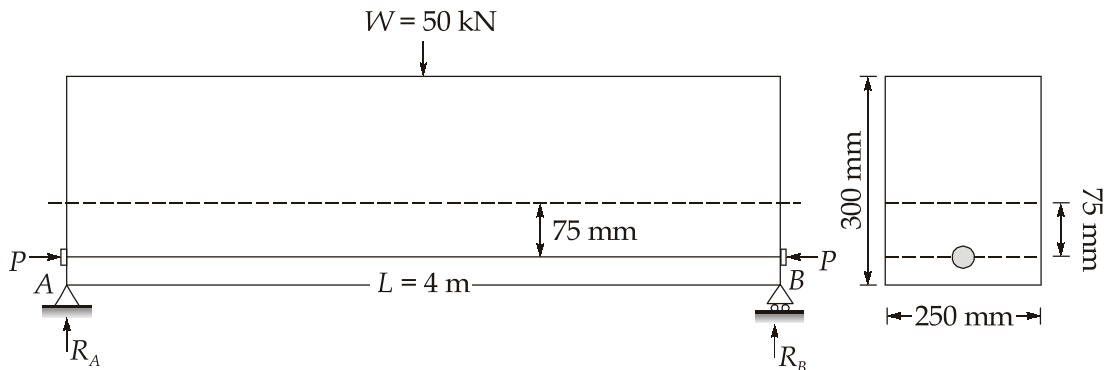
$$t = \sqrt{\frac{2.75 R n^2}{A \cdot f_y}}$$

$$n = \frac{b_l}{2} - \frac{t_w}{2} - R_1 = \frac{210}{2} - \frac{12}{2} - 20 = 79 \text{ mm}$$

∴  $t = \sqrt{\frac{2.75 \times 220 \times 10^3 \times (79)^2}{50400 \times 250}} = 17.31 \text{ mm} \approx 18 \text{ mm}$

∴ Provide bearing plate of size 240 mm × 210 mm × 18 mm

**Q.6 (a) Solution:**



Given:  $P = 500 \text{ kN}$

Reaction at each support,  $R_A = R_B = \frac{50}{2} = 25 \text{ kN}$

At support, bending moment is zero, therefore pressure line is at,  $e = 75 \text{ mm}$

At quarter span,

Bending moment,  $M = R_A \times 1 = 25 \times 1 = 25 \text{ kNm}$

∴ Shift of  $p$ -line,  $a = \frac{M}{P} = \frac{25 \times 10^6}{500 \times 10^3} = 50 \text{ mm}$

Eccentricity of the  $p$ -line,  $e = 50 - 75 = -25 \text{ mm}$

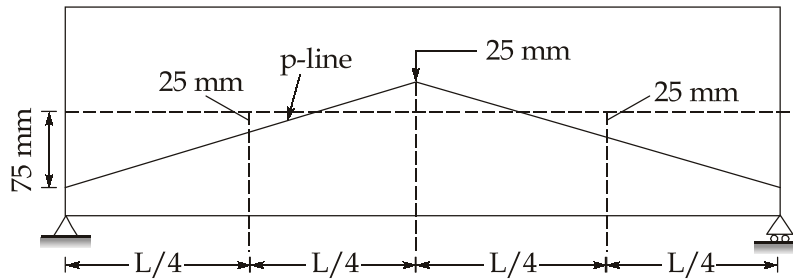
(i.e. below 25 mm from mid depth)

At mid span,  $M = R_A \times 2 = 25 \times 2 = 50 \text{ kNm}$

∴ Shift of p-line, 
$$a = \frac{M}{P} = \frac{50 \times 10^6}{500 \times 10^3} = 100 \text{ mm}$$

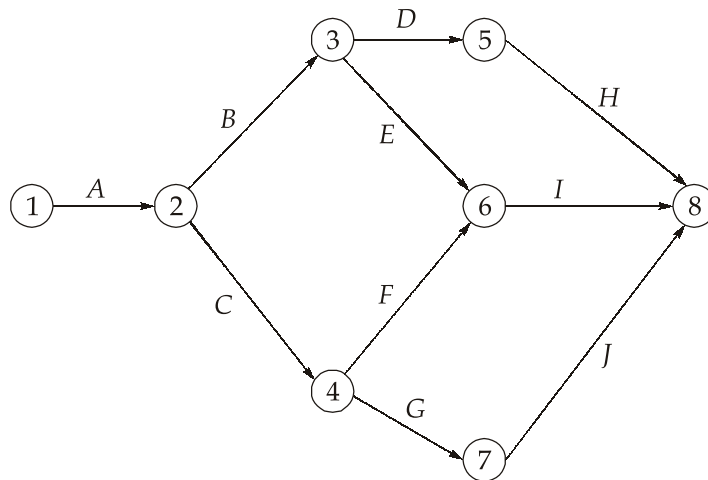
∴ Eccentricity of the p-line,  $e' = 100 - 75 = 25 \text{ mm}$  (i.e. above the mid depth)

The pressure line is shown in figure below:

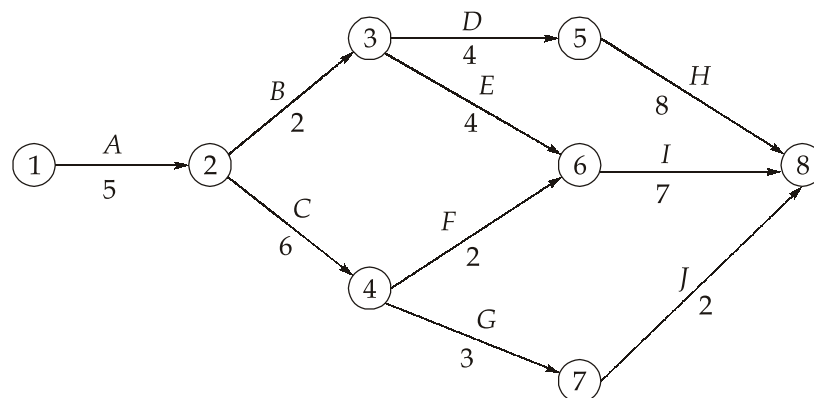


**Q.6 (b) Solution:**

(i) Network diagram:



Network diagram with expected duration of activities:



S.No.	Paths	Duration (days)
1.	1-2-3-5-8	$5+2+4+8=19$
2.	1-2-3-6-8	$5+2+4+7=18$
3.	1-2-4-6-8	$5+6+2+7=20$
4.	1-2-4-7-8	$5+6+3+2=16$

Therefore the critical path is 1-2-4-6-8 with project completion duration equal to 20 days.

- (ii) The process of replanning and rescheduling based on the results which serve a guidance for decision by performing calculations made by taking into consideration the new knowledge and latest information at an intermediate stage of the project thus modifying the original network, is known as the process of updating.

**Data required for updating:**

The following information is necessary to update the plan at an intermediate stage of execution of a project:

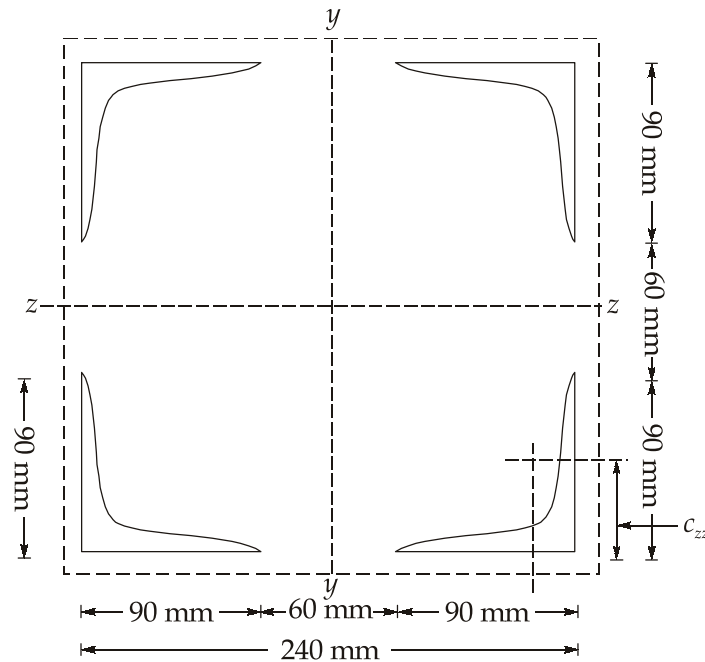
- Original network
- Original network calculation chart
- Stage at which updating is being done i.e., point in time of updating
- Execution position of the project at that stage and
- New information and knowledge which will affect the duration of time of the activities to be performed.

**When to update:**

The following points must be kept in view while deciding the time of updating:

- For shorter duration projects, the updating must be done frequently by taking into account the latest position of the execution of the project.
- For large duration projects, the process of updating must be increased as the project is progressing toward completion. Duration of project goes on decreasing as project progresses and start behaving more or less like a small duration project.
- Whenever there is major change in the duration of any of the activity then the updating is to be done.
- Updating is essential if there is change in the estimated duration of any activity falling on the critical path. If the duration of a critical activity increases, remedial measures are necessary and if the activity duration decreases, this may allow changes in the project plan which were not possible previously.

Q.6 (c) Solution:



Gross area of all angle sections,

$$A_e = 4 \times A = 4 \times 1376 = 5504 \text{ mm}^2.$$

$$\begin{aligned} I_{zz, \text{combi}} = I_{yy, \text{combi}} &= 4 \cdot [I_{zz, \text{one}} + A_{\text{one}} (120 - c_{zz})^2] \\ &= 4 [104.2 \times 10^4 + 1376 (120 - 25.1)^2] \\ &= 53737079.04 \text{ mm}^4 = 5373.7 \times 10^4 \text{ mm}^4 \end{aligned}$$

$$\therefore I_{\min} = I_{zz, \text{combi}} = I_{yy, \text{combi}}$$

$$\text{Minimum radius of gyration, } r_{\min} = \sqrt{\frac{I_{zz, \text{combi}}}{A}} = \sqrt{\frac{5373.7 \times 10^4}{5504}} = 98.8 \text{ mm}$$

Since, column is laced, hence effective length of column is increased by 5%

$$\therefore \text{Slenderness ratio, } \lambda = \frac{KL \times 1.05}{r_{\min}}$$

$$\text{Here, } K = 0.65; \quad L = 10\text{m.}$$

Since both the ends of the column are fixed

$$\therefore \lambda = \frac{0.65 \times 10 \times 10^3 \times 1.05}{98.8} = 69.08$$

From table given,  $f_{cd} = 150 + \frac{133 - 150}{70 - 60}(69.08 - 60) = 134.564 \text{ N/mm}^2$

∴ Factored load on the column,

$$P_u = f_{cd} A_e = 134.564 \times 5504 \times 10^{-3} \text{ kN} = 740.64 \text{ kN}$$

### Design of lacing:

Since lacing system is provided with double lacing system with the lacing flats inclined at  $45^\circ$ . Bolts are provided at the centre of the leg of the angle

Spacing of lacing bars,  $C = s + 2g = 60 + 2 \times 45 = 150 \text{ mm}$

$$\therefore \frac{c}{r_{\min}^{\text{one}}} \not\geq 50 \not\geq 0.7 \lambda_{\text{whole}}$$

$$r_{\min}^{\text{one}} = \sqrt{\frac{104.2 \times 10^4}{1376}} = 27.5 \text{ mm}$$

$$\therefore \frac{C}{l_{\min}^{\text{one}}} = \frac{150}{27.5} = 5.45 \not\geq 50$$

$$\not\geq 0.7 \times 69.08 = 48.356 \quad (\text{OK})$$

Transverse shear to be resisted,

$$V_t = \frac{2.5}{100} \times 740.64 \times 10^3 \text{ N} = 18.516 \times 10^3 \text{ N}$$

Transverse shear in each plane,

$$\frac{V_t}{N} = \frac{18.516 \times 10^3}{2} = 9.258 \times 10^3 \text{ N}$$

As double lacing is provided,

Compressive force in each lacing bar

$$\begin{aligned} &= \frac{1}{2} \times \frac{V_t}{N} \times \text{cosec } \theta \\ &= \frac{1}{2} \times 9.258 \times 10^3 \times \text{cosec } 45^\circ = 6.55 \times 10^3 \text{ N} \end{aligned}$$

### Section of lacing flats:

Since M20 bolts of grade 4.6 are used

$$\therefore \text{Minimum width of flat} = 3 \times 20 = 60 \text{ mm}$$

$$\text{Thickness of lacing flat} = \frac{1}{60} \times (150) \times \text{cosec } 45^\circ = 3.54 \text{ mm.}$$

Provide thickness of lacing flat as 6 mm

Provide 60 ISF 6 mm flat section,

$$\lambda_{\text{lacing}} = \frac{l_e}{(t/\sqrt{12})} = \frac{0.7 \times 150\sqrt{2}}{(6/\sqrt{12})} = 85.73 < 145 \quad (\text{OK})$$

Also, from table given, for  $\lambda = 85.73$

$$f_{cd} = 118 + \frac{105 - 118}{90 - 80}(85.73 - 80) = 110.551 \text{ N/mm}^2$$

Design compressive strength of lacing flat,

$$\begin{aligned} P_d &= f_{cd} A_e = 110.551 \times 60 \times 6 \times 10^{-3} \text{ kN} \\ &= 39.8 \text{ kN} > 6.55 \text{ kN} \end{aligned} \quad (\text{OK})$$

Tensile strength of lacing flat is minimum of

$$\begin{aligned} \text{(i)} \quad 0.9 \times (B - d_0)t \frac{f_u}{\gamma_{m1}} &= 0.9 \times (60 - 22) \times 6 \times \frac{410}{1.25} \times 10^{-3} \text{ kN} \\ &= 67.3 \text{ kN} \end{aligned}$$

$$\text{(ii)} \quad A_g \times \frac{f_y}{\gamma_{m0}} = (60 \times 6) \times \frac{250}{1.1} \times 10^{-3} \text{ kN} = 81.82 \text{ kN.}$$

Hence, tensile strength of lacing flat is

$$67.3 \text{ kN} > 6.55 \text{ kN} \quad (\text{OK})$$

**Connection:** Here bolt is in double shear

Assume both the shear planes intercepting the thread

$$\begin{aligned} \therefore V_{dsb} &= 2 \times A_{nb} \times \frac{f_{ub}}{\sqrt{3} \gamma_{mb}} \\ &= 2 \times \frac{\pi}{4} \times (20)^2 \times 0.78 \times \frac{400}{\sqrt{3} \times 1.25} \times 10^{-3} \text{ kN} \\ &= 90.54 \text{ kN} \end{aligned}$$

Strength of 20 mm bolt in bearing,

$$V_{dpb} = 2.5 K_b dt \frac{f_u}{1.25}$$

$$\text{Let } K_b = 1.0$$

$$\therefore V_{dpb} = 2.5 \times 1.0 \times 20 \times 6 \times \frac{410}{1.25} \times 10^{-3} \text{ kN} = 98.4 \text{ kN}$$

∴  $V_{db} = \text{minimum of } (V_{dsb}, V_{dpb}) = 90.54 \text{ kN}$

∴  $\text{Number of bolts} = \frac{2 \times 6.55 \times \cot 45^\circ}{90.54} = 0.145 \approx 1$

Provide 1 bolt of 20 mm dia, at the ends of flats.

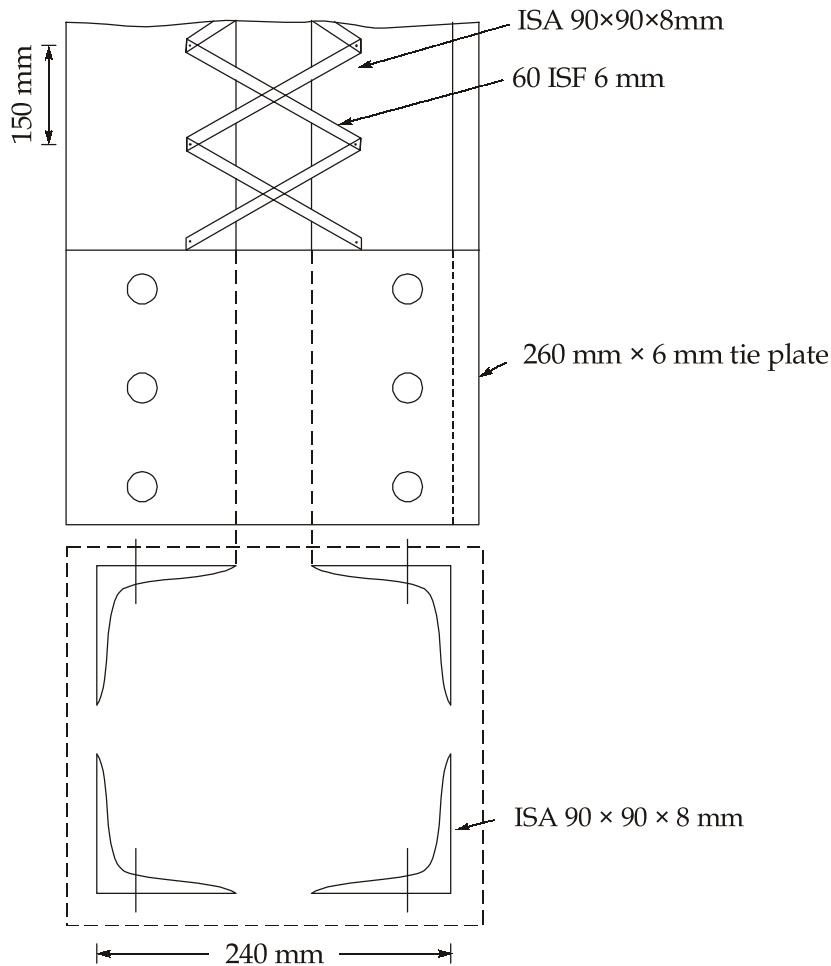
**Tie plate:**

Tie plates are provided at each end of the built up column.

Effective length of tie plate =  $240 - 2 \times 25.1 = 189.8 \text{ mm} > 2 \times 90 \text{ mm}$

Overall depth of the tie plate =  $189.8 + 2 \times 1.5 \times 22 = 255.8 \text{ mm} \approx 260 \text{ mm}$

Length of tie plate = 240 mm





## Q.7 (a) Solution:

$$C_i = \text{Rs. } 12000$$

$$C_s = \text{Rs. } 1500$$

$$n = 5 \text{ years}$$

$$i = 8\%$$

$$D_m = \text{Depreciation for } m^{\text{th}} \text{ year}$$

$$B_m = \text{Book value at the end of } m^{\text{th}} \text{ year}$$

$$1. \text{ Sinking fund method, } D = (C_i - C_s) \left[ \frac{i}{(1+i)^n - 1} \right]$$

$$\Rightarrow D = (12000 - 1500) \left[ \frac{0.08}{(1+0.08)^5 - 1} \right]$$

$$\Rightarrow D = \text{Rs. } 1789.793$$

$$\text{Using, } D_m = D(1+i)^{m-1}$$

$$D_1 = 1789.793 (1 + 0.08)^0 = \text{Rs. } 1789.793$$

$$B_1 = 12000 - 1789.793 = \text{Rs. } 10210.207$$

$$D_2 = 1789.793 (1 + 0.08)^{2-1} = \text{Rs. } 1932.96$$

$$B_2 = 10210.207 - 1932.96 = \text{Rs. } 8277.231$$

$$D_3 = 1789.793 (1 + 0.08)^{3-1} = \text{Rs. } 2087.61$$

$$B_3 = 8277.231 - 2087.61 = \text{Rs. } 6189.621$$

$$D_4 = 1789.793 (1 + 0.08)^{4-1} = \text{Rs. } 2254.62$$

$$B_4 = 6189.621 - 2254.62 = \text{Rs. } 3935.001$$

$$D_5 = 1789.793 (1 + 0.08)^{5-1} = \text{Rs. } 2434.994$$

$$B_5 = 3935.001 - 2434.994 = \text{Rs. } 1500.007$$

## 2. Double declining balance method,

$$\text{FDDB} = \frac{2}{n} = \frac{2}{5} = 0.4$$

$$D_1 = 12000 \times 0.4 = \text{Rs. } 4800$$

$$B_1 = 12000 - 4800 = \text{Rs. } 7200$$

$$D_2 = 7200 \times 0.4 = \text{Rs. } 2880$$

$$B_2 = 7200 - 2880 = \text{Rs. } 4320$$

$$D_3 = 4320 \times 0.4 = \text{Rs. } 1728$$

$$B_3 = 4320 - 1728 = 2592$$

$$D_4 = 2592 \times 0.4 = \text{Rs. } 1036.8$$

$$B_4 = 2592 - 1036.8 = \text{Rs. } 1555.2$$

$$D_5 = 1552.2 \times 0.4 = \text{Rs. } 622.08$$

$$B_5 = 1555.2 - 622.08 = \text{Rs. } 930.12$$

Year (n)	Sinking fund method		Double declining balance method	
	Depreciation (Rs.)	Book value (Rs.)	Depreciation (Rs.)	Book value (Rs.)
1.	1789.793	10210.207	4800	7200
2.	1932.96	8277.231	2880	4320
3.	2087.61	6189.621	1728	2592
4.	2254.62	3935.001	1036.8	1555.2
5.	2434.994	1500.007	622.08	930.12

### Q.7 (b) Solution:

Given:

Diameter of column,  $D = 450 \text{ mm}$

Unsupported length,  $L = 3.5 \text{ m}$

End condition: Column is effectively held in position at both ends but not restrained against rotation

$$\therefore L_{\text{eff}} = 3.5 \text{ m}$$

Factored load,  $P_u = 2250 \text{ kN}$

M25, Fe 415

$$f_{ck} = 25 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

**Slenderness ratio:**  $\lambda = \frac{L_{\text{eff}}}{D} = \frac{3500}{450} = 7.78 < 12$

Hence, the column is designed as short column.

**Minimum eccentricity:**  $e_{\text{min}} = \left( \frac{L}{500} + \frac{D}{30} \right)$  or 20 mm whichever is maximum

$$= \frac{3500}{500} + \frac{450}{30} \text{ or } 20 \text{ mm whichever is maximum}$$

$$= 22 \text{ mm}$$

Also,  $0.05 D = 0.05 \times 450 = 22.5 \text{ mm}$

$\therefore e_{\min} < 0.05 D$

$\Rightarrow$  Column is axially loaded

Hence, the codal formula for axially loaded column can be used

**Longitudinal reinforcement:**  $P_u = 1.05 \left[ 0.4 f_{ck} A_g + (0.67 f_y - 0.4 \times f_{ck}) A_{sc} \right]$

$$\Rightarrow 2250 \times 10^3 = 1.05 \left[ 0.4 \times 25 \times \frac{\pi}{4} \times (450)^2 + (0.67 \times 415 - 0.4 \times 25) A_{sc} \right]$$

$$\Rightarrow A_{sc} = 2060.906 \text{ mm}^2$$

$$A_{sc, \min} = 0.8\% \text{ of } A_g = \frac{0.8}{100} \times \frac{\pi}{4} \times (450)^2 = 1272.35 \text{ mm}^2$$

Also, minimum 6 no. of bars are required to provided for circular column.

Provide 3-20 $\phi$  + 3-25  $\phi$  bars

$$(A_{sc})_{\text{provided}} = 3 \times \frac{\pi}{4} \times 20^2 + 3 \times \frac{\pi}{4} \times 25^2 = 2415.1 \text{ mm}^2 > 2060.9 \text{ mm}^2 \text{ (OK)}$$

$$A_{sc, \max} = 6\% \text{ of } A_g = \frac{6}{100} \times \frac{\pi}{4} (450)^2 = 9542.6 \text{ mm}^2$$

### Spiral reinforcement:

Diameter of spiral ties,

$$\phi_s \geq \text{maximum} \left\{ \begin{array}{l} \frac{\phi_{\text{long, max}}}{4} = \frac{25}{4} = 6.25 \text{ mm} \\ 6 \text{ mm} \end{array} \right.$$

Let us adopt 8 mm diameter spirals.

Assume a clear cover of 40 mm to spirals

Diameter of core,  $D_c = D - (2 \times \text{clear cover})$

$$= 450 - (2 \times 40) = 370 \text{ mm}$$

$$P_s = \frac{\text{Volume of spiral reinforcement}}{\text{Volume of core}}$$

$$P_s = \frac{\frac{\pi}{4}(8)^2 \times \pi(370 - 8)}{\frac{\pi}{4} \times 370^2 \times S} = \frac{0.53166}{S}$$

As per IS 456 : 2000, for helically reinforced column

$$P_s \geq 0.36 \frac{f_{ck}}{f_y} \times \left[ \frac{A_g}{A_c} - 1 \right]$$

$$\Rightarrow \frac{0.53166}{S} \geq 0.36 \frac{25}{415} \left[ \frac{\frac{\pi}{4} \times (450)^2}{\frac{\pi}{4} \times (370)^2} - 1 \right]$$

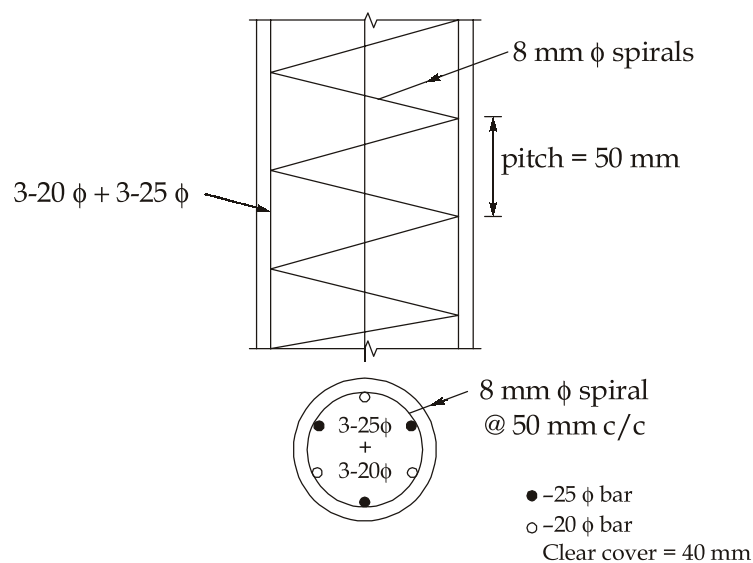
$$S \leq 51.16 \text{ mm}$$

Also, as per IS 456 : 2000

$$S \leq \min. \left\{ \begin{array}{l} 75 \text{ mm} \\ \frac{D_c}{6} = \frac{370}{6} = 61.67 \text{ mm} \end{array} \right. = 61.67 \text{ mm}$$

$$S \geq \max. \left\{ \begin{array}{l} 25 \text{ mm} \\ 3\phi_s = 3 \text{ mm} \end{array} \right. = 25 \text{ mm}$$

∴ Provide 8 mm φ spirals at a pitch of 50 mm c/c.



**Q.7 (c) Solution:**

Given: Cantilever projection,

$$L = 2.4 \text{ m}$$

Materials used: M20, Fe415

$$f_{ck} = 20 \text{ N/mm}^2, \quad f_y = 415 \text{ N/mm}^2.$$

**Depth of slab:**

$$\frac{\text{Span}}{\text{Overall depth}} = 10 \quad (\text{For cantilever slab})$$

$$\Rightarrow \text{Overall depth} = \frac{2.4 \times 1000}{10} = 240 \text{ mm}$$

$$\text{Nominal cover} = 20 \text{ mm}$$

$$\text{Diameter of bar used} = 10 \text{ mm}$$

$$\therefore \text{Effective depth,} \quad d = 240 - 20 - \frac{10}{2} = 215 \text{ mm}$$

Let us provide maximum depth of slab as 240 mm at support and gradually reduce the depth to 120 mm at free end.

**Load calculation:**

$$\text{Self weight of slab} = 0.5 (0.24 + 0.12) \times 25 = 4.5 \text{ kN/m}^2$$

$$\text{Live load} = 2 \text{ kN/m}^2$$

$$\text{Load due to finishes} = 1.5 \text{ kN/m}^2$$

$$\text{Total working load} = 8 \text{ kN/m}^2$$

$$\therefore \text{Factored load } (w_u) = 1.5 \times 8 = 12 \text{ kN/m}^2$$

$$\text{Check for depth:} \quad BM_u = \frac{w_u \times L^2}{2} = \frac{12 \times (2.4)^2}{2} = 34.56 \text{ kN-m/m}$$

$$\text{For Fe 415 steel,} \quad BM_{u, \text{lim}} = 0.138 f_{ck} b d^2$$

$$\therefore \quad BM_u = BM_{u, \text{lim}}$$

$$\Rightarrow \quad 34.56 \times 10^6 = 0.138 \times 20 \times 1000 \times d^2$$

$$\Rightarrow \quad d = 111.9 \text{ mm} < 215 \text{ mm.}$$

Hence, the effective depth provided is sufficient to resist the design moment.

Also, provided depth is more than that required for balanced section and hence section is under-reinforced.

**Reinforcement calculation:**

$$\begin{aligned} A_{st} &= \frac{0.5 f_{ck} b d \left( 1 - \sqrt{1 - \frac{4.6 B M_u}{f_{ck} b d^2}} \right)}{f_y} \\ &= \frac{0.5 \times 20 \times 1000 \times 215 \left( 1 - \sqrt{1 - \frac{4.6 \times 34.56 \times 10^6}{20 \times 1000 \times 215^2}} \right)}{415} \\ &= 466.43 \text{ mm}^2 \end{aligned}$$

$$\therefore \text{Spacing of 10 mm } \phi \text{ bars} = \frac{1000 \times \frac{\pi}{4} (10)^2}{466.43} = 168.385 \text{ mm c/c}$$

$\therefore$  Provide 10 mm  $\phi$  bars @ 160 mm c/c

$$(A_{st})_{\text{provided}} = \frac{1000 \times \frac{\pi}{4} (10)^2}{160} = 490.87 \text{ mm}^2 > (A_{st})_{\text{req.}}$$

**Distribution reinforcement:**  $A_{st} = 0.12\%$  of  $A_g$  (For Fe 415 steel)

$$= \frac{0.12}{100} \times 1000 \times 240 = 288 \text{ mm}^2$$

Let us provide 10 mm  $\phi$  bars as distribution bars

$$\therefore \text{Spacing} = \frac{1000 \times \frac{\pi}{4} (10)^2}{288} = 272.7 \text{ mm c/c}$$

$\therefore$  Provide 10 mm  $\phi$  bars @ 270 mm c/c.

$$(A_{st})_{\text{provided}} = \frac{1000 \times \frac{\pi}{4} (10)^2}{270} = 290.89 \text{ mm}^2$$

**Anchorage length:**  $L_d = \frac{(0.87 \cdot f_y) \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \times 10}{4 \times 1.2 \times 1.6} = 470 \text{ mm}$

Main tension bars are extended into the support to a minimum length of 470 mm including anchorage value of hooks and 90° bends.

**Check for deflection control:**

$$\left( \frac{L}{d} \right)_{\text{max}} = \left( \frac{L}{d} \right)_{\text{Basic}} \times k_t \times k_c \times k_f$$

$$P_t(\%) = \frac{A_{st}}{b d} \times 100 = \frac{490.87}{1000 \times 215} \times 100 = 0.23\%$$

$$\therefore f_s = 0.58 \times 415 \times \frac{466.43}{490.87} = 228.72 \text{ N/mm}^2$$

From graph given modification factor for tension reinforcement)

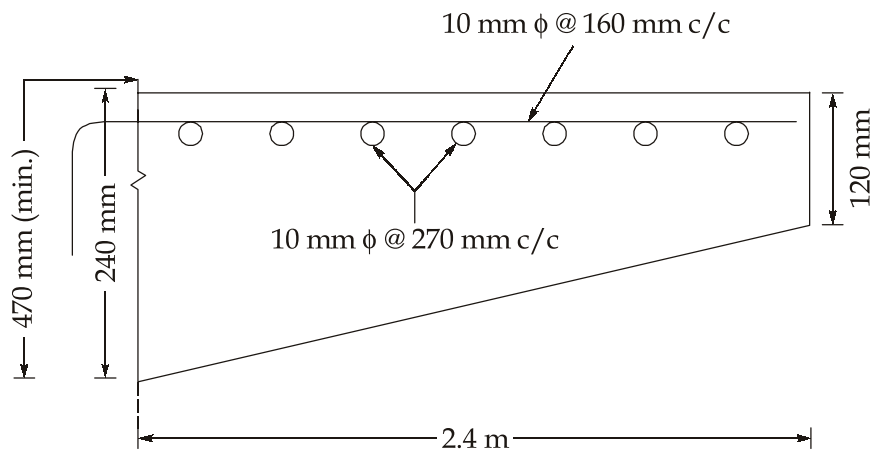
$$k_t = 1.8$$

$$\therefore \left(\frac{L}{d}\right)_{\max} = 7 \times 1.8 \times 1 \times 1 = 12.6$$

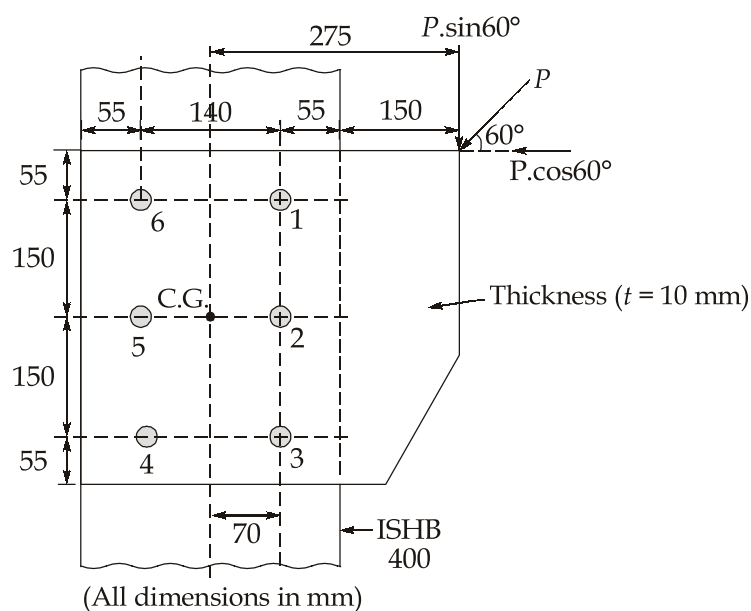
$$\left(\frac{L}{d}\right)_{\text{provided}} = \frac{2400}{215} = 11.16 < 12.6$$

Hence, the cantilever slab satisfies the deflection limit.

**Reinforcement details:**



**Q.8 (a) Solution:**

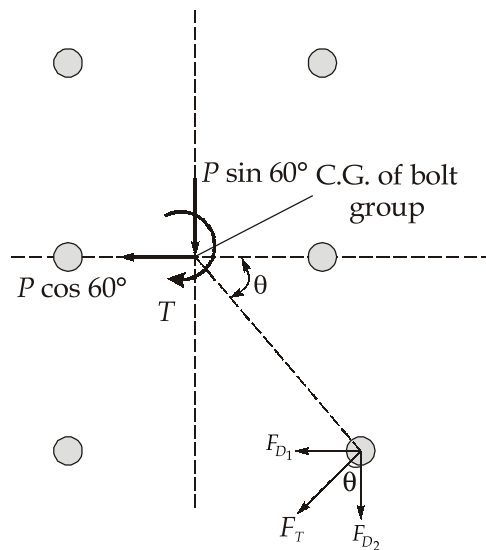


Given: Diameter of bolt,  $d = 20 \text{ mm}$   
 $\therefore$  Diameter of bolt hole,  $d_o = 22 \text{ mm}$   
 Bolt grade is 4.6,  $f_{ub} = 400 \text{ N/mm}^2$   
 $f_{yb} = 240 \text{ N/mm}^2$

Thickness of plate = 10 mm.

Let  $P$  (in kN) be the factored load that can be carried by the bracket.

As we know that critical bolt is that bolt which is farthest from C.G of the bolt group and nearest to the applied force. From figure it is clear that bolt 3 is the most critical.



$$\cos \theta = \frac{70}{\sqrt{70^2 + 150^2}} = \frac{70}{165.53}$$

$$\sin \theta = \frac{150}{\sqrt{70^2 + 150^2}} = \frac{150}{165.53}$$

$$r_i = \sqrt{70^2 + 150^2} = 165.53 \text{ mm}$$

$$\sum r_i^2 = 2 \times (70)^2 + 4 \times (165.53)^2 = 119400.72 \text{ mm}^2$$

Torsional moment

$$T = P \cdot \sin 60^\circ \times 0.275 - P \cdot \cos 60^\circ \times 0.205$$

$$= 0.136 P \text{ kNm}$$

Now, direct shear force in bolts,

$$F_{D1} = \frac{P \cdot \cos 60^\circ}{6} = P / 12$$



$$F_{D_2} = \frac{P \sin 60^\circ}{6} = \frac{P\sqrt{3}}{12}$$

Torsional shear force in bolt,  $F_T = \frac{Tr_i}{\sum r_i^2} = \frac{0.136P \times 165.53 \times 10^{-3}}{119400.72 \times 10^{-6}} = 0.189P$

$$\therefore F_X = F_{D_1} + F_T \cdot \sin \theta = \frac{P}{12} + 0.189P \times \frac{150}{165.53} = 0.255P$$

$$F_Y = F_{D_2} + F_T \cos \theta = \frac{P\sqrt{3}}{12} + 0.188P \times \frac{70}{165.53} = 0.224P$$

Resultant force on bolt,  $F_R = \sqrt{F_X^2 + F_Y^2} = \sqrt{(0.2553P)^2 + (0.224P)^2} = 0.34 P$

### Calculation of design strength of bolt:

$V_{dsb}$  = Design shear strength of bolt

$$= \frac{f_{ub}}{\sqrt{3} \gamma_{mb}} (n_n \cdot A_{nb} + n_s \cdot A_{sb})$$

Here, bolts are in single shear,

$$n_n = 1 ; n_s = 0$$

$$\therefore V_{dsb} = \frac{400}{\sqrt{3} \times 1.25} \left( 1 \times 0.78 \times \frac{\pi}{4} (20)^2 \times 10^{-3} \right) \text{ kN}$$

$$\Rightarrow V_{dsb} = 45.27 \text{ kN}$$

$$V_{dpb} = \text{Design bearing strength of bolt} = 2.5 K_b dt \frac{f_u}{\gamma_{m1}}$$

where,  $K_b = \min. \left\{ \frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{f_{ub}}{f_u}, 1 \right\}$

$$e = 55 \text{ mm}; p = 150 \text{ mm}$$

$$\therefore K_b = \min. \left[ \frac{55}{3 \times 22}, \frac{150}{3 \times 22} - 0.25, \frac{400}{410}, 1 \right] = 0.833$$

$$\therefore V_{dp} = 2.5 \times 0.833 \times 20 \times 10 \frac{410}{1.25} \times 10^{-3} \text{ kN} = 136.612 \text{ kN}$$

$$\therefore V_{db} = \text{minimum} \left\{ \begin{array}{l} V_{dsb} \\ V_{dpb} \end{array} \right. = 45.27 \text{ kN}$$

For safety,  $F_R \leq V_{db}$

$$\Rightarrow 0.34 P \leq 45.27$$

$$\Rightarrow P \leq \frac{45.27}{0.34} = 133.15 \text{ kN}$$

$\therefore$  Maximum factored load that can be carried by the bracket = 133.15 kN

### Q.8 (b) Solution:

Given:

$$\gamma = 16 \text{ kN/m}^3 \quad \phi = 30^\circ$$

$$\gamma_c = 25 \text{ kN/m}^3 \quad \text{M20, Fe 415}$$

$$f_{ck} = 20 \text{ N/mm}^2; f_y = 415 \text{ N/mm}^2$$

$$k_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = \frac{1}{3}$$

Eccentricity of resultant force,  $e = 0.334 \text{ m}$

Summation of vertical forces:

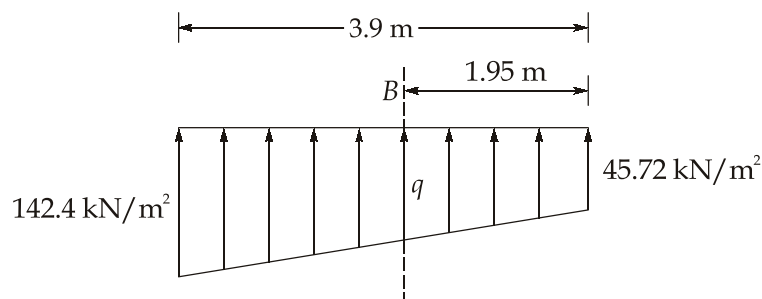
$$\Sigma F_v = 366.8 \text{ kN/m} \quad (\text{Given})$$

Soil pressure at footing base:

$$(\sigma_v)_{\text{Toe}} = \frac{\Sigma F_v}{B} \left(1 + \frac{6e}{B}\right) = \frac{366.8}{3.9} \left(1 + \frac{6 \times 0.334}{3.9}\right) = 142.4 \text{ kN/m}^2$$

$$(\sigma_v)_{\text{Heel}} = \frac{\Sigma F_v}{B} \left(1 - \frac{6e}{B}\right) = \frac{366.8}{3.9} \left(1 - \frac{6 \times 0.334}{3.9}\right) = 45.72 \text{ kN/m}^2$$

**Base pressure distribution:**



$$q = 45.72 + \frac{142.4 - 45.72}{3.9} \times 1.95 = 94.06 \text{ kN/m}^2$$

**Design of heel slab:**

The distributed loading acting downward on the heel slab is given by

(i) Weight of soil (over burden) above heel slab

$$= 16 \times (7.75 - 0.62) \times 1.95 = 222.456 \text{ kN/m}$$

(ii) Weight of heel slab

$$= (1.95 \times 0.62 \times 25) = 30.225 \text{ kN/m.}$$

∴ The total downward load acting on heel slab

$$W = 252.681 \text{ kN/m}$$

Upward pressure on heel slab,  $U$

$$= \frac{1}{2} \times (45.72 + 94.06) \times 1.95 = 136.2855 \text{ kN/m}$$

Location of this upward pressure from  $B$ ,

$$\bar{x} = \frac{2 \times 45.72 + 94.06}{45.72 + 94.06} \times \frac{1.95}{3} = 0.863 \text{ m}$$

∴ Shear force at  $B$ ,

$$V_B = W - U = 252.681 - 136.2855 = 116.4 \text{ kN}$$

∴ Factored shear force,  $V_u = 1.5 \times 116.4 = 174.6 \text{ kN}$

**Bending moment at  $B$ ,**  $BM = 252.681 \times \frac{1.95}{2} - 136.2855 \times 0.863 = 128.75 \text{ kN-m}$

∴ Factored bending moment

$$BM_u = 1.5 \times 128.75 = 193.125 \text{ kNm}$$

As, Clear cover = 75 mm, diameter of bar = 16 mm

∴ Effective depth,  $d = 620 - 75 - 16/2 = 537 \text{ mm}$

Also for Fe415,  $BM_{u, \text{lim}} = 0.138 f_{ck} b d^2$

$$= 0.138 \times 20 \times 1000 \times (537)^2 \text{ Nmm} = 795.898 \text{ kNm}$$

As  $BM_u < BM_{u, \text{lim}}$

Section is under-reinforced and thus,

$$A_{st} = \frac{0.5 \cdot f_{ck} b d}{f_y} \left( 1 - \sqrt{1 - \frac{4.6 BM_u}{f_{ck} b d^2}} \right)$$

$$= \frac{0.5 \times 20 \times 1000 \times 537}{415} \left( 1 - \sqrt{1 - \frac{4.6 \times 193.125 \times 10^6}{20 \times 1000 \times 537^2}} \right)$$

$$= 1037.77 \text{ mm}^2$$

∴ Spacing of 16 mm bars =  $\frac{1000 \times \frac{\pi}{4} (16)^2}{1037.77} = 193.74 \text{ mm c/c}$

Provide 16 mm  $\phi$  bars @ 180 mm c/c

$$\therefore (A_{st})_{\text{provided}} = \frac{1000 \times \frac{\pi}{4} (16)^2}{180} = 1117.01 \text{ mm}^2 > (A_{st})_{\text{required}}$$

**Distribution steel,**  $A_{st} = 0.12\% \text{ of } A_g = \frac{0.12}{100} \times 1000 \times 620 = 744 \text{ mm}^2$

Using 10 mm  $\phi$  bars,

$$\text{Spacing} = \frac{1000 \times \frac{\pi}{4} \times (10)^2}{744} = 105.56 \text{ mm}$$

$\therefore$  Provide 10 mm  $\phi$  bars @ 100 mm c/c.

**Check for shear:**

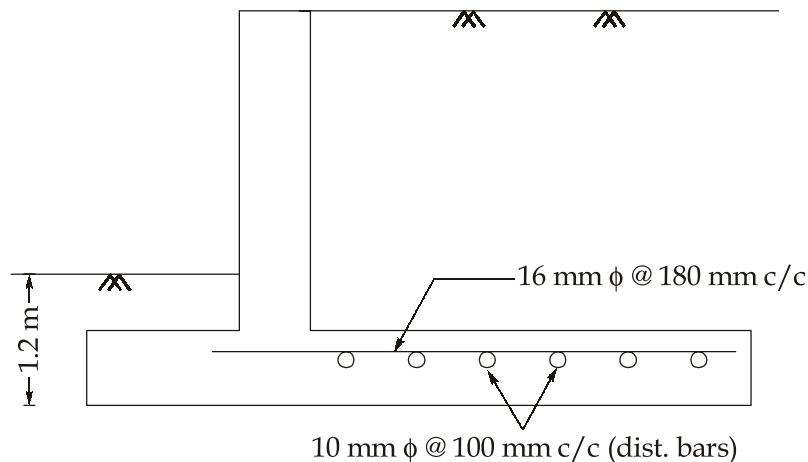
Nominal shear stress,  $\tau_v = \frac{V_u}{bd} = \frac{174.6 \times 10^3}{1000 \times 537} = 0.325 \text{ N/mm}^2$

$$P_t(\%) = \frac{A_{st}}{bd} \times 1000 = \frac{1117.01}{1000 \times 537} \times 100 = 0.21\%$$

From table given

$$\tau_c = 0.28 + \frac{0.36 - 0.28}{0.25 - 0.15} (0.21 - 0.15) = 0.328 \text{ N/mm}^2$$

$$\therefore \tau_v < \tau_c \quad (\text{OK})$$



**Q.8 (c) Solution:**

(i) Floats are of following types:

- 1. Total float:** Total float is the time span by which the starting (or finishing) of an activity can be delayed without delaying the completion of the project. In certain activities, it will be found that there is a difference between maximum time

available and the actual time required to perform the activity. This difference is known as the total float.

2. **Free float:** Free float is that portion of positive total float that can be used by an activity without delaying any succeeding activity (or without affecting the total float of the succeeding activity). The concept of free float is based on the possibility that all the events occur at their earliest times (i.e., all activities start as early as possible).
3. **Independent float:** Independent float gives us an idea about the excess time that exists if the preceding activity ends as late as possible and the succeeding activity starts as early as possible. The independent float is, therefore, defined as the excess of minimum available time over the required activity duration.
4. **Interfering float:** Interfering float is just another name given to the head event slack ( $S_j$ ), specially in CPM networks which are activity oriented. Interfering float is the potential downstream interference of any activity and is equal to the difference between the total float and the free float.

(ii) **Steps in time cost optimisation:**

The time cost optimisation is done in the following steps:

**Establish:** Direct cost-time relationships for various activities of the project, by analyzing past cost records.

**Determine:** Cost slopes for various activities and arrange them in the ascending order of cost slope.

**Compute:** Direct cost for the network with normal duration of activities.

**Crash:** The activities in the critical path as per ranking i.e., starting with the critical activity having the lowest slope.

Parallel non-critical activities which have become critical by the reduction of critical path duration.

**Continue:** Crashing the critical activities in the ascending order of the slope.

**Find:** Total cost of project at every stage by adding indirect costs to the direct cost determined above.

**Plot:** Total cost-duration curve.

**Pick-up:** The optimum duration corresponding to which least total project cost is obtained.

