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Detailed Solutions

ESE-2021 Mains Test Series

Mechanical Engineering Test No : 10

Full Syllabus Test (Paper-I)

Explanations

1. (a) Solution:

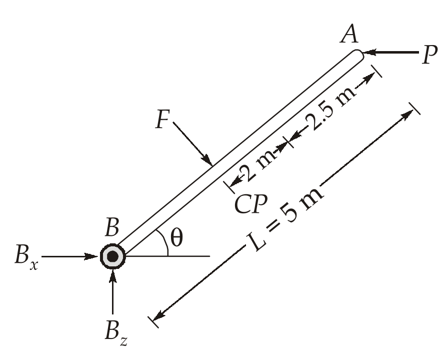
By geometry, length of gate is

$$L = \sqrt{3^2 + 4^2} = 5 \text{ m}$$

Centroid of gate is halfway between or at elevation 1.5 m above point B. The depth h_{CG} = 6 – 1.5 = 4.5 m. The gate area is (1.5 m)(5 m) = 7.5 m^2. Neglecting atmospheric pressure P_a acting on the both sides of gate.

Hydrostatic force on the gate is

$$F = h_{CG}A = 10050 \times 4.5 \times 7.5 = 339187.5 \text{ N}$$



We have to find centre of pressure of F, i.e., CP. As the gate is rectangular, hence

$$h_{CP} = \frac{I_{xx}\cdot \sin^2\theta}{Ah_{CG}} + h_{CG} = \frac{\left(\frac{1.5\times 5^3}{12}\right)\times\left(\frac{3}{5}\right)^2}{4.5\times 7.5} + 4.5$$

$$= 4.67 \text{ m}$$

The distance from point B to force $F = \dfrac{6-4.67}{\sin(36.87)} = 2.222$ m

$\Sigma M_B = 0$

$\Rightarrow PL \sin\theta - F \times 2.222 = P \times (3) - 339187.5 \times 2.222 = 0$

$$P = \frac{339187.5\times 2.222}{3} = 251224.8 \text{ N}$$

$$\Sigma F_x = 0,\ B_x + F\sin\theta - P$$

$$= B_x + 339187.5 \times 0.6 - 251224.875 = 0$$

$$B_x = 47712.375 \text{ N}$$

$$\Sigma F_z = 0,\ B_z - F\cos\theta = B_z - 339187.5 \times 0.8 = 0$$

$$B_z = 271350 \text{ N}$$

Resultant force at hinge B, $B_R = \sqrt{B_x^2 + B_z^2}$

$$B_R = \sqrt{(47712.375)^2 + (271350)^2} = 275.5 \text{ kN}$$

1. (b) Solution:

Given : Pressure head at inlet, $P_1 = 400$ kPa; Pressure drop, $P_2 = 200$ kPa; $l = 1.5$ km $= 1500$ m; $P = 100$ kW and $f = 0.006$

Diameter of the pipe :

Let d = Diameter of the pipe, and

Q = Discharge through the pipe

We know that the pressure head at inlet,

$$H = \frac{p_1}{w} = \frac{400}{9.81} = 40.8 \text{ m}$$

and pressure drop in terms of head (due to friction),

$$h_f = \frac{p_2}{w} = \frac{200}{9.81} = 20.4 \text{ m}$$

$\therefore$ Power to be transmitted (P)

$$100 = wQ(H - h_f) = 9.81 \times Q(40.8 - 20.4) = 200Q$$

or $$Q = \frac{100}{200} = 0.5 \text{ m}^3/\text{s}$$

We also know that the loss of head due to friction (h_f),

$$h_f = \frac{4 f \cdot l\, Q^2}{12.1 \times D^5}$$

$$20.4 = \frac{f\,lQ^2}{3(d)^5} = \frac{0.006 \times 1500 \times (0.5)^2}{3(d)^5} = \frac{0.75}{d^5}$$

$\therefore$ $$D^5 = \frac{0.75}{20.4} = 0.03646 \text{ or } D = 0.0515 \text{ m}$$

We also know that the efficiency of transmission,

$$\eta = \frac{H - h_f}{H} = \frac{40.8 - 20.4}{40.8} = 0.5 = 50\%$$

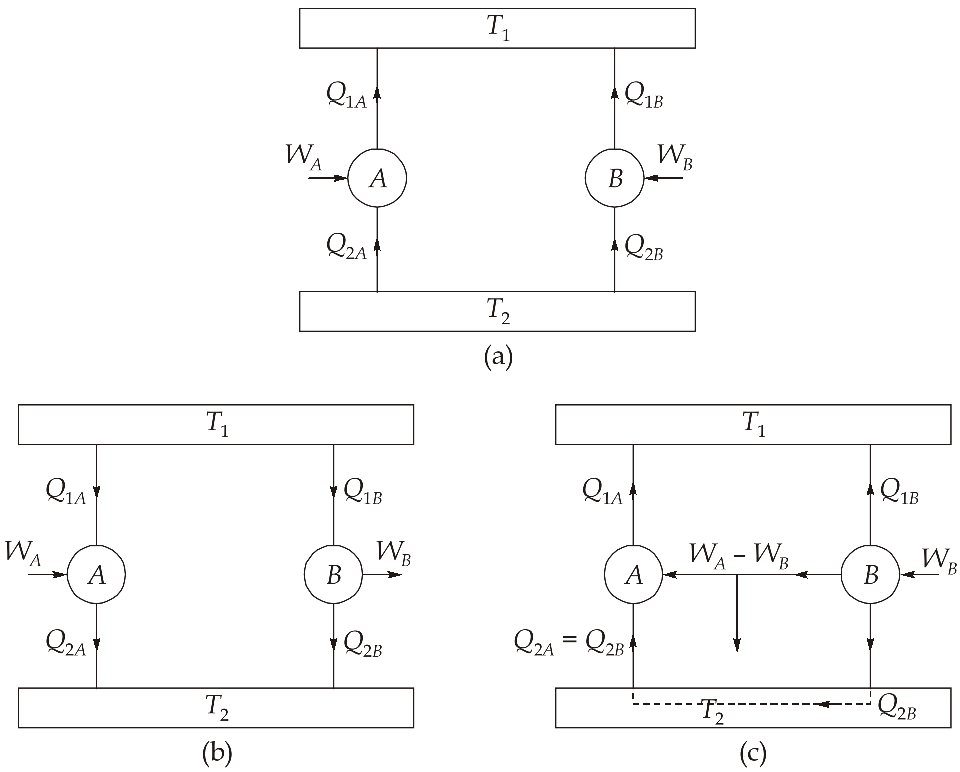
1. (c) Solution:

Let A and B be the two refrigerators operating between temperatures T_1 and T_2 (fig. (a)). Let B be a reversible refrigerator. Let A be any refrigerator reversible or irreversible. We should prove that, $(\text{COP})_B \geq (\text{COP})_A$.

Let us assume that $(\text{COP})_A \geq (\text{COP})_B$. Then,

$$\frac{Q_{2A}}{W_A} \geq \frac{Q_{2B}}{W_B} \qquad \text{...(1)}$$

Let the two refrigerators work in such a way that they draw the same amount of heat from reservoir 2, i.e., $Q_{2A} = Q_{2B}$. Then, from Eq. (1) $W_B \geq W_A$. Since, B is a reversible refrigerator, we can operate it as a heat engine, by reversing the directions of energy interaction as shown in figure (b). As $W_B \geq W_A$, part of W_B can be used to drive the refrigerator A and the reminder $W_B - W_A$ can be delivered as the net work. Since Q_{2A} and Q_{2B} are equal, the sink T_2 can be eliminated (fig. (c)). It is seen that A and B together operating in cycles are producing net work $W_B - W_A$ by exchanging heat with only one reservoir at T_1. This violates Kelvin-Planck statement. Therefore, the assumption $(\text{COP})_A \geq (\text{COP})_B$ is wrong. The COP of refrigerator A cannot be greater than COP of the reversible refrigerator B and $(\text{COP})_A = (\text{COP})_B$ only when the refrigerator A is also reversible.



1. **(d) Solution:**

Advantages :

1. Reflecting surfaces require less material in concentrating collectors than in flat plate collectors.
2. Absorber area is smaller in concentrating collectors. So insulation intensity is greater in concentrating collectors than flat plate collectors.
3. Smaller area of absorber perunit is there in concentrating collectors than in flat plate collectors.
4. Little or no antifreeze is required in concentrating collectors to protect absorber in a concentrator system.
5. Because temperature attainable with concentrator collector is high, amount of heat stored is larger in concentrator collector.
6. Concentrating collector is used for power generation while flat plate collector is not used for power generation.
7. In solar heating and cooling application high temperature of working fluid is attainable.

Disadvantages :

1. Non-uniform flux on absorber is there in concentrating collectors. While in flat plate collectors uniform flux is there.
2. Additional optical losses occur in concentrating collectors but not in flat plate collectors.
3. High initial cost is there for concentrating collectors. Flat plate collectors are cheaper.
4. Additional requirement for maintenance is there in concentrating collectors. While flat plate collectors do not require much maintenance is required.
5. Only beam component is collected in concentrating collectors. While in flat plate collectors, both diffused and beam radiations are collected.
6. It is necessary to have an absorber to track sun image in concentrating collectors.

1. (e) Solution:

The change in kinetic energy :

$$\Delta KE = \frac{V_2^2 - V_1^2}{2000} \text{ kJ/kg}$$

$$= \frac{(50)^2 - (80)^2}{2000} = -1.95 \text{ kJ/kg}$$

There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Taking turbine as control volume, SFEE gives

Rate of net energy transfer by heat, work and mass = Rate of change in interval, kinetic energy, potential energy etc.

$$E_{in} - E_{out} = \Delta E_{system} = 0 \text{ (for steady state)}$$

$$E_{in} = E_{out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{W}_{out} + \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) \qquad [Q = 0]$$

$$\dot{W}_{out} = -\dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}\right)$$

Where, $h_2 = h_f + xh_{fg}$

$$= 19.81 + 0.92 \times (2583.9 - 191.81) = 2392.1 \text{ kJ/kg}$$

Power output of the turbine :

$$\dot{W}_{out} = -12(2392.1 - 3242.4 - 1.95) = 10.2 \text{ MW}$$

Inlet area is determined from the mass flow rate equation :

$$\dot{m} = \frac{A_1 V_1}{v_1} \Rightarrow A_1 = \frac{\dot{m} v_1}{V_1}$$

$$A_1 = \frac{12 \times 0.0279782}{80} = 0.00447 \text{ m}^2$$

2. (a) Solution:

$$\text{Mass of air, } m = \frac{Pv}{RT} = \frac{1500 \times 1}{0.287 \times (273 + 177)} = 11.61 \text{ kg}$$

$$\text{Initial availability, } a_1 = u_1 - u_0 + P_0(v_1 - v_0) - T_0(s_1 - s_0)$$

$$= C_V(T_1 - T_0) + P_0\left[\frac{RT_1}{P_1} - \frac{RT_0}{P_0}\right] - T_0\left[C_P \ln\frac{T_1}{T_0} - R\ln\frac{P_1}{P_0}\right]$$

$$C_V(T_1 - T_0) = 0.718(450 - 300) = 107.7 \text{ kJ/kg}$$

$$P_0\left[\frac{RT_1}{P_1} - \frac{RT_0}{P_0}\right] = 100 \times 0.287\left[\frac{(273 + 177)}{1500} - \frac{(273 + 27)}{100}\right]$$

$$= 0.287\left[\frac{450}{1500} - \frac{300}{100}\right] = -77.49 \text{ kJ/kg}$$

$$T_0\left[C_P \ln\frac{T_1}{T_0} - R\ln\frac{P_1}{P_0}\right] = 300\left[1.005\ln\frac{450}{300} - 0.287\ln\frac{1500}{100}\right]$$

$$= 300[0.4075 - 0.7772]$$

$$= -110.91 \text{ kJ/kg}$$

$$a_1 = 107.7 - 77.49 + 110.91 = 141.1 \text{ kJ/kg}$$

Final availability, $a_2 = u_2 - u_0 + P_0(v_2 - v_0) - T_0(s_2 - s_0)$

$$= C_V(T_2 - T_0) + P_0\left[\frac{RT_2}{P_2} - \frac{RT_0}{P_0}\right] - T_0\left[C_P \ln\frac{T_2}{T_0} - R\ln\frac{P_2}{P_0}\right]$$

Final temperature = Ambient temperature, i.e., $T_2 = T_0$

For process 1 – 2 : $$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2}$$

$$P_2 = \frac{T_2}{T_1} P_1 \text{ (volume remains constant)}$$

$$P_2 = \frac{300}{450} \times 1500 = 1000 \text{ kPa}$$

Final availability, $$a_2 = RT_0\left[\frac{P_0}{P_2} - 1 + \ln\frac{P_2}{P_0}\right]$$

$$= 0.287 \times 300\left[\frac{100}{1000} - 1 + \ln\frac{1000}{100}\right]$$

$$= 120.76 \text{ kJ/kg}$$

Irreversibility, $$I = m(a_1 - a_2)$$

$$= 11.61(141.1 - 120.76)$$

$$= 236.15 \text{ kJ}$$

2. (b) Solution:

Heat conducted from the outside surface of the wire = Heat convected to the air

$$-k\frac{dT}{dr} = h(T_w - T_\infty) \qquad ...(1)$$

where, T_w is the outside surface temperature of the wire. At the centre, T is maximum and $\left(\frac{dT}{dr}\right) = 0.$

From the conduction equation,

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{q_G}{K} = 0$$

$$\therefore \quad \frac{dT}{dr} = -\frac{q_G r}{2K} + \frac{C_1}{r} \qquad ...(2)$$

$$T = -\frac{q_G r^2}{4K} + C_1 \ln r + C_2 \qquad ...(3)$$

At $r = 0$, $\frac{dT}{dr} = \infty$, which is impossible

At $$r = r_o, \left(\frac{dT}{dr}\right)_{r=r_o} = \frac{-qGr_o}{2K} + \frac{C_1}{r_o} \qquad ...(4)$$

Also, $$q_G \pi r_o^2 L = -K 2\pi r_o L\left(\frac{dT}{dr}\right)_{r=r_o}$$

$$\therefore \quad \left(\frac{dT}{dr}\right)_{r=r_o} = -\frac{q_G r_o}{2K} \qquad ...(5)$$

From Eqs. (4) and (5), $C_1 = 0$

Equation (3) reduces to

$$T = -\frac{q_G r^2}{4K} + C_2 \quad \text{...(6)}$$

At $\quad r = r_o,\ T = T_w$

Substituting C_2 in Eq. (6)

$$T = -\frac{q_G r^2}{4K} + \frac{q_G r_o^2}{4K} + T_w$$

$$\therefore \quad T = \frac{q_G r_o^2}{4K}\left(1 - \frac{r^2}{r_o^2}\right) + T_w$$

From Eq. (1), $$\left(\frac{dT}{dr}\right)_{r=r_o} = \frac{-h(T_w - T_\infty)}{K} = -\frac{q_G r_o}{2K}$$

$$\therefore \quad T_w = \frac{q_G r_o}{2h} + T_\infty$$

Substituting T_w in Eq. (7), we get the temperature distribution,

$$T = \frac{q_G r_o^2}{4K}\left(1 - \frac{r^2}{r_o^2}\right) + \frac{q_G r_o}{2h} + T_\infty \quad \text{...(8)}$$

$$\therefore \quad \frac{T - T_\infty}{T_\infty} = \frac{q_G r_o}{2hT_\infty}\left(1 + \frac{hr_o}{2K} + \frac{hr^2}{2r_o K}\right)$$

Putting $r = 0$, $T = T_{max}$

$$\therefore \quad T_{max} = T_\infty + \frac{q_G r_o}{2h}\left(1 + \frac{hr_o}{2K}\right)$$

Now, $$q_G = \frac{I^2 R}{V} = \frac{I^2 R}{\pi r_o^2 L}$$

$$\therefore \quad T_{max} = T_\infty + \frac{I^2}{2\pi r_o h}\frac{R}{L}\left(1 + \frac{hr_o}{2K}\right)$$

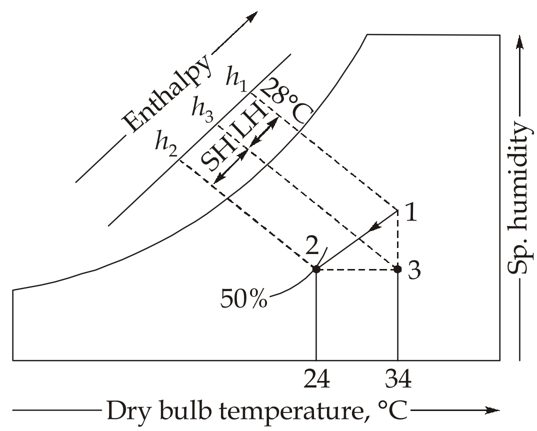
$$200 = 25 + \frac{I^2}{2\pi(0.5 \times 10^{-3}) \times 10} \times 0.037 \times \left(1 + \frac{10 \times 0.5 \times 10^{-3}}{2 \times 204}\right)$$

$$\therefore \quad I_{max} = 12.19 \text{ amperes}$$

2. (c) Solution:

Given : Seating capacity = 25 persons; t_{d_1} = 34°C; tw_1 = 28°C; t_{d_2} = 24°C; ϕ_2 = 50%; v_1 = 0.4 m^3/min/person = 0.4 × 25 = 10 m^3/min; S.H. load = 125600 kJ/h; L.H. load = 42000 kJ/h.

First of all, mark the initial condition of air at 34°C dry bulb temperature and 28°C wet bulb temperature on the psychrometric chart as point 1, as shown in figure. Now mark the final condition of air at 24°C dry bulb temperature and 50% relative humidity on the chart as point 2. Now locate point 3 on the chart by drawing horizontal line through point 2 and vertical line through point 1. From the psychrometric chart, we find that specific volume at point 1,



$$v_{s_1} = 0.9 \text{ m}^3\text{/kg of dry air}$$

Enthalpy of air at point 1,

$$h_1 = 90 \text{ kJ/kg of dry air}$$

Enthalpy of air at point 2,

$$h_2 = 48 \text{ kJ/kg of dry air}$$

and enthalpy of air at point 3,

$$h_3 = 58 \text{ kJ/kg of dry air}$$

We know that mass of air supplied per min.

$$m_a = \frac{v_1}{v_{s1}} = \frac{10}{0.9} = 11.1 \text{ kg/min}$$

and sensible heat removed from the air

$$= m_a(h_3 - h_2) = 11.1(58 - 48) = 111 \text{ kJ/min}$$
$$= 111 \times 60 = 6660 \text{ kJ/h}$$

Total sensible heat of the room SH

$$= 6660 + 125600 = 132260 \text{ kJ/h}$$

We know that latent heat removed from the air

$$= m_a(h_1 - h_3) = 11.1(90 - 58) = 355.2 \text{ kJ/min}$$
$$= 355 \times 60 = 21312 \text{ kJ/h}$$

$\therefore$ Total latent heat of the room,

$$\text{LH} = 21312 + 42000 = 63312 \text{ kJ/h}$$

We know that sensible heat factor,

$$\text{SHF} = \frac{\text{SH}}{\text{SH}+\text{LH}} = \frac{132260}{132260+63312} = 0.676$$

2. (d) Solution:

$$L = \text{Characteristic length} = \frac{r}{3} = \frac{0.02}{3} = 0.0067 \text{ m}$$

$$\text{Bi, Biot number} = \frac{hL}{k} = \frac{100 \times 0.0067}{10} = 0.067$$

As Bi < 0.1, lump theory can be used.

Let T be the temperature to which the egg should be boiled to satisfy the consumer's taste. Therefore,

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hAt}{\rho c V}} = e^{-\frac{100t}{1200\times2\times0.0067\times1000}}$$
$$= e^{-6.219t/1000}$$

$$\frac{T-100}{20-100} = e^{-6.219\times240/1000} = e^{-1.493}$$

$$T = -18 + 100 = 82°\text{C}$$

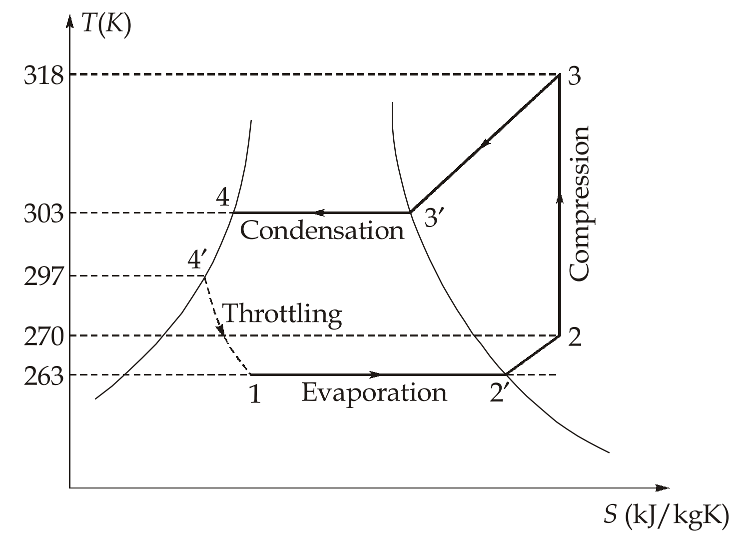
We are to find the time taken for the egg taken from refrigerator at 5°C to be boiled to 82°C.

$$\frac{82-100}{5-100} = e^{-\frac{100\times t}{1200\times2000\times0.0067}} = e^{-6.22\times10^{-3}t}$$

or
$$e^{-6.22\times10^{-3}t} = \frac{-18}{-95} = 0.1895$$

$$e^{6.22\times10^{-3}t} = 5.278$$
$$6.22 \times 10^{-3}t = 1.6635$$
$$t = 267.44\text{s}$$
$$= 4.45 \text{ min.}$$

3. (a) Solution:



Enthalpy at '2′ : $h_2 = h_{2'} + C_P(T_2 - T_{2'})$

From the given table, $h_{2'} = 183.2$ kJ/kg

$(T_2 - T_{2'})$ = Degree of superheat as the vapour enters the compressor = 7°C

$$h_2 = 183.2 + 0.733 \times 7 = 188.33 \text{ kJ/kg}$$

Entropy at 2,
$$S_2 = S_{2'} + C_P \ln\frac{T_2}{T_{2'}}$$

$$= 0.702 + 0.733 \ln\left(\frac{270}{263}\right) = 0.7212 \text{ kJ/kgK}$$

For isentropic process 2–3, we have :

$$S_2 = S_3$$

$$0.7212 = S_{3'} + C_P \ln\frac{T_3}{T_{3'}} = 0.6854 + 0.733 \ln\left(\frac{T_3}{303}\right)$$

$$\ln\left(\frac{T_3}{303}\right) = \frac{0.7212 - 0.6854}{0.733} = 0.0488$$

$$T_3 = 303 \exp(0.0488) = 318 \text{ K}$$

$$h_3 = h_{3'} + C_P(T_3 - T_{3'})$$

$$= 199.6 + 0.733(318 - 303) = 210.6 \text{ kJ/kg}$$

Enthalpy at 4′ : $h_{f4'} = h_{f4} - C_{Pl}(T_4 - T_{4'})$

$$= 64.6 - 1.235 \times 6 = 57.19 \text{ kJ/kg}$$

$$h_{4'} = h_1 = 57.19 \text{ kJ/kg}$$

For specific volume at 2,

$$\frac{v_{2'}}{T_{2'}} = \frac{v_2}{T_2}$$

$$v_2 = \frac{v_{2'}}{T_{2'}} \times T_2 = 0.0767 \times \frac{270}{263} = 0.07874 \text{ m}^3/\text{kg}$$

Refrigerating effect per kg,

$$h_2 - h_1 = 188.33 - 57.19 = 131.14 \text{ kJ/kg}$$

Mass of the refrigerant to be circulated per minute for producing effect of 2400 kJ/min

$$= \frac{2400}{131.14} = 18.3 \text{ kg/min}$$

Theoretical piston displacement per minute

= Mass flow/min. × Specific volume at suction

$$= 18.3 \times 0.07874 = 1.441 \text{ m}^3/\text{min}$$

Theoretical power required to run the compressor

= Mass flow of refrigerant per sec
× Compressor work/kg

$$= \frac{18.3}{60}(h_3 - h_2) = \frac{18.3}{60}(210.6 - 188.33)$$

= 6.79 kJ/s or 6.79 kW

Heat removed through the condenser per min

= Mass flow of refrigerant × Heat removed per kg of refrigerant

$$= 18.3(h_3 - h_{f4'})$$

$$= 18.3(210.6 - 57.19)$$

$$= 2807.4 \text{ kJ/min}$$

Theoretical piston displacement per cylinder

$$= \frac{\text{Total displacement per minute}}{\text{Number of cylinders}}$$

$$= \frac{1.441}{2} = 0.7205 \text{ m}^3/\text{min}$$

Length of stroke = 1.25 × Diameter of piston

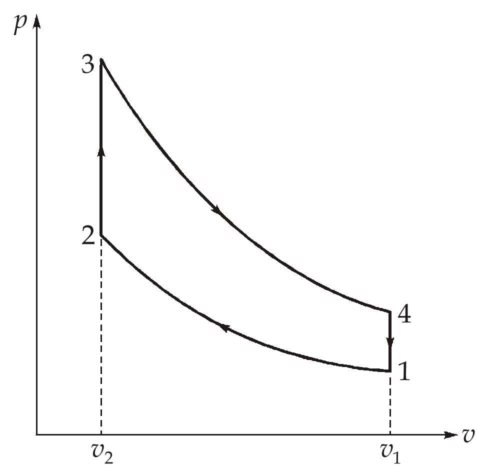
$$0.7205 = \frac{\pi}{4}D^2 \times 1.25D \times 1000$$

$$D = 0.09 \text{ m or } 90 \text{ mm}$$

$$L = 1.25D = 1.25 \times 90 = 112.5 \text{ mm}$$

3. (b) Solution:

(i)



The work done per kg of fluid in the cycle is given by

$$W = Q_s - Q_r = c_v(T_3 - T_2) - c_v(T_4 - T_1)$$

But $$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (r)^{\gamma-1}$$

$\therefore$ $$T_2 = T_1.(r)^{\gamma-1}$$

Similarly, $$T_3 = T_4.(r)^{\gamma-1}$$

$\therefore$ $$W = c_v\left[T_3 - T_1 \cdot (r)^{\gamma-1} - \frac{T_3}{(r)^{\gamma-1}} + T_1\right]$$

This expression is a function of r when T_3 and T_1 are fixed. The value of W will be maximum when

$$\frac{dW}{dr} = 0$$

$\therefore$ $$\frac{dW}{dr} = -T_1.(\gamma - 1)(r)^{\gamma-2} - T_3(1 - \gamma)(r)^{-\gamma} = 0$$

or $$T_3(r)^{-\gamma} = T_1(r)^{\gamma-2}$$

or $$\frac{T_3}{T_1} = (r)^{2(\gamma-1)}$$

$\therefore$ $$r = \left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}$$

$$T_2 = T_1(r)^{\gamma-1} \text{ and } T_4 = T_3/(r)^{\gamma-1}$$

Substituting the value of r in the above equation

$$T_2 = T_1\left[\left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}\right]^{\gamma-1} = T_1\left(\frac{T_3}{T_1}\right)^{1/2} = \sqrt{T_1T_3}$$

Similarly, $$T_4 = \frac{T_3}{\left[\left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}\right]^{\gamma-1}} = \frac{T_3}{\left(\frac{T_3}{T_1}\right)^{1/2}} = \sqrt{T_3T_1}$$

$\therefore$ $$T_2 = T_4 = \sqrt{T_1T_3}\text{ . }\textbf{Proved}$$

(ii)

Power developed, P :

Given : $T_1 = 310$ K; $T_3 = 1450$ K; $m = 0.38$ kg

Work done, $W = c_v[(T_3 - T_2) - (T_4 - T_1)]$

$$T_2 = T_4 = \sqrt{T_1T_3} = \sqrt{310\times1450} = 670.4 \text{ K}$$

$\therefore$ $$W = 0.718[(1450 - 670.4) - (670.4 - 310)]$$
$$= 0.718(779.6 - 360.4) = 300.98 \text{ kJ/kg}$$

$$\text{Work done per second} = 300.98\times\left(\frac{0.38}{60}\right) = 1.906 \text{ kW}$$

Hence, power developed, $P = 1.906$ kW

3. (c) Solution:

Given : $N = 100$; $d_i = 25$ mm $= 0.025$ m; $d_o = 29$ mm $= 0.029$ m; $\dot{m}_w = \dot{m}_c = 500$ kg/min; $t_{c1} = 30°C$; $t_{c2} = 70°C$; h_o (steam side) = 5000 W/m2°C; $t_{h1} = t_{h2} = 100°C$, $R_{fi} = 0.0002$ m^2°C/W.

Overall heat transfer coefficient (U_i); tube length (L) :

Heat gained by water,

$$Q = \dot{m}_c \times c_{pc} \times (t_{c2} - t_{c1})$$

$$= \frac{500}{60} \times 4174 \times (70 - 30) = 1.39 \times 10^6 \text{ W}$$

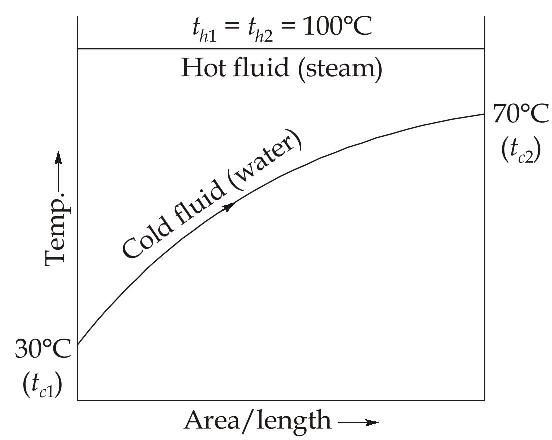
The heat transferred from the steam to the water is given by

$$Q = U_i A_{is} \theta_m \quad \text{...(1)}$$

where, U_i = Overall heat transfer coefficient based on inner surface of the tubes,

A_{is} = Inner surface area of the tubes, and

θ_m = Logarithmic mean temperature difference (LMTD)



$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} = \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\ln[(t_{h1} - t_{c1})/(t_{h2} - t_{c2})]}$$

$$= \frac{(100 - 30) - (100 - 70)}{\ln[(100 - 30)/(100 - 70)]} = \frac{70 - 30}{\ln(70/30)} = 47.2°C$$

U_i is given by

$$\frac{1}{U_i} = \frac{1}{h_i} + R_{fi} + \frac{r_i}{r_o} \times \frac{1}{h_o} \quad \text{...(2)}$$

where, h_i, h_o = Inside and outside heat transfer coefficients respectively, and

R_{fi} = Inside thermal resistance due to fouling.

In order to find out h_i we shall use the following relation :

$$Nu = \frac{h_i d_i}{k} = 0.023(R_e)^{0.8}(Pr)^{0.4}$$

or,

$$h_i = \frac{k}{d_i} \times 0.023(\text{Re})^{0.8}(\text{Pr})^{0.4}$$

where, $$R_e = \frac{Vd_i}{v}$$

V being the average velocity of the water flow which can be calculated by using the following equation.

$$\dot{m}_w (= \dot{m}_c) = \frac{\pi}{4} d_i^2 \times V \times \rho \times N$$

or, $$\frac{500}{60} = \frac{\pi}{4} \times (0.025)^2 \times V \times 988.1 \times 100$$

or, $$V = 0.172 \text{ m/s}$$

$\therefore$ $$\text{Re} = \frac{0.172 \times 0.025}{0.555 \times 10^{-6}} = 7748 \quad \text{[Turbulent flow]}$$

Substituting the values in eqn. (3), we get

$$h_i = \frac{0.6474}{0.025} \times 0.023(7748)^{0.8} \times (3.54)^{0.33} = 1168 \text{ W/m}^2\text{°C}$$

Further, inserting the values in equation (2), we get

$$\frac{1}{U_i} = \frac{1}{1168} + 0.0002 + \frac{0.0125}{0.0145} \times \frac{1}{5000}$$

$$= 0.000856 + 0.002 + 0.000172 = 0.001228$$

$\therefore$ $$U_i = 814.3 \text{ W/m}^2\text{°C}$$

Now, substituting all the values in eqn. (1), we get

$$Q = U_i \times (N \times \pi \times d_i \times L) \times \theta_m$$

$$1.39 \times 10^6 = 814.3 \times (100 \times \pi \times 0.025 \times L) \times 47.2$$

$\therefore$ $$L = \frac{1.39 \times 10^6}{814.3 \times 100 \times \pi \times 0.025 \times 47.2} = 4.6 \text{ m}$$

4. (a) Solution:

(i)

In normal combustion, the flame started by spark travels across the combustion chamber. As the flame front advances, it compresses the unburnt charge in last portion of combustion chamber. If this unburnt charge does not reach its critical temperature for auto-ignition, it will not auto-ignite and flame front will move across this unburnt charges in normal manner. In abnormal combustion called **detonation,** the end charge auto-ignites before the flame front reaches it. In order to auto-ignite, the last unburnt portion of charge must reach above a certain critical temperature and remain at this temperature for a certain length of time. During this period certain chemical reactions take place which prepare the charge for auto-ignition. The time required in this preparation is called **ignition delay**.

In order to limit detonation, we should not allow the unburnt charge to reach its critical temperature. Increase in temperature of mixture reduces delay period of end charge and hence tendency of detonation increases. To avoid the increase temperature, we should limit the compression ratio. Hence, there is a critical compression ratio for a fuel for a given engine setting above which knock occurs. This compression ratio is called highest useful compression ratio.

(ii)

V is a function of : H, D, μ, ρ and g

Mathematically, $$V = f(H, D, \mu, \rho, g) \quad ...(i)$$

or, $$f_1(V, H, D, \mu, \rho, g) = 0 \quad ...(ii)$$

$\therefore$ Total number of variables, $n = 6$

Writing dimensions of each variable, we have :

$$V = LT^{-1}, H = L, D = L, \mu = ML^{-1}T^{-1}, \rho = ML^{-3}, g = LT^{-2}$$

Thus, number of fundamental dimensions, $m = 3$

$\therefore$ Number of π-terms $= n - m = 6 - 3 = 3$

Eqn. (ii) can be written as :

$$f_1(\pi_1, \pi_2, \pi_3) = 0 \quad ...(iii)$$

Each π-term contains $(m + 1)$ variables, where $m = 3$ and is also equal to repeating variables. Choosing H, g, ρ as repeating variables (V being a dependent variable should not be chosen as repeating variable), we get three π-terms as :

$$\pi_1 = H^{a_1}.g^{b_1}\rho^{c_1}.V$$
$$\pi_2 = H^{a_2}.g^{b_2}\rho^{c_2}.D$$
$$\pi_3 = H^{a_3}.g^{b_3}\rho^{c_3}.\mu$$

π_1–term :

$$\pi_1 = H^{a_1}.g^{b_1}\rho^{c_1}.V$$
$$M^0L^0T^0 = L^{a_1}.(LT^{-2})^{b_1}.(ML^{-3})^{c_1}.(LT^{-1})$$

Equating the exponents of M, L and T respectively, we get :

For M : $0 = c_1$

For L : $0 = a_1 + b_1 - 3c_1 + 1$

For T : $0 = -2b_1 - 1$

$\therefore \quad c_1 = 0; b_1 = -\frac{1}{2}$

and, $\quad a_1 = -b_1 + 3c_1 - 1 = \frac{1}{2} + 0 - 1 = -\frac{1}{2}$

Substituting the values of a_1, b_1 and c_1 in π_1, we get :

$$\therefore \quad \pi_1 = H^{-\frac{1}{2}} \cdot g^{-\frac{1}{2}} \cdot \rho^0 \cdot V = \frac{V}{\sqrt{gH}}$$

π_2–term :

$$\pi_2 = H^{a_2}.g^{b_2}\rho^{c_2}.D$$
$$M^0L^0T^0 = L^{a_2}.(LT^{-2})^{b_2}.(ML^{-3})^{c_2}.L$$

Equating the exponents of M, L and T respectively, we get :

For M : $\quad 0 = c_2$

For L : $\quad 0 = a_2 + b_2 - 3c_2 + 1$

For T : $\quad 0 = -2b_2$

$\therefore \quad c_2 = 0; b_2 = 0$

and, $\quad a_2 = -b_2 + 3c_2 - 1 = -1$

Substituting the values of a_2, b_2 and c_2 in π_2, we get :

$$\therefore \quad \pi_2 = H^{-1} \cdot g^0 \cdot \rho^0 \cdot D = \frac{D}{H}$$

π_3–term :

$$\pi_3 = H^{a_3}.g^{b_3}\rho^{c_3}.\mu$$
$$M^0L^0T^0 = L^{a_3}.(LT^{-2})^{b_3}.(ML^{-3})^{c_3}.ML^{-1}T^{-1}$$

Equating the exponents of M, L and T respectively, we get :

For M : $\quad 0 = c_3 + 1$

For L : $\quad 0 = a_3 + b_3 - 3c_3 - 1$

For T : $\quad 0 = -2b_3 - 1$

$\therefore \quad c_3 = -1; b_3 = -\frac{1}{2}$

and, $\quad a_3 = -b_3 + 3c_3 + 1 = \frac{1}{2} - 3 + 1 = -\frac{3}{2}$

Substituting the values of a_3, b_3 and c_3 in π_3, we get :

$$\therefore \qquad \pi_3 = H^{-\frac{3}{2}} \cdot g^{-\frac{1}{2}} \cdot \rho^{-1} \cdot \mu = \frac{\mu}{H^{\frac{3}{2}} \rho \sqrt{g}}$$

$$= \frac{\mu}{H\rho\sqrt{gH}} = \frac{\mu V}{H\rho V\sqrt{gH}} \quad \text{(Multiply and divide by } V\text{)}$$

$$= \frac{\mu}{H\rho V} \cdot \pi_1 \qquad \left(\because \frac{V}{\sqrt{gH}} = \pi_1\right)$$

Substituting the values of π_1, π_2 and π_3 in eqn. (iii), we get :

$$f_1\left(\frac{V}{\sqrt{gH}}, \frac{D}{H}, \frac{\mu}{\rho HV} \cdot \pi_1\right) = 0$$

or
$$\frac{V}{\sqrt{gH}} = \phi\left[\frac{D}{H}, \frac{\mu}{H\rho V} \cdot \pi_1\right]$$

or
$$V = \sqrt{2gH}\phi\left[\frac{D}{H}, \frac{\mu}{\rho VH}\right] \quad \text{(Proved)}$$

(Multiplying or dividing by any constant does not change the character of π-terms).

4. (b) Solution:

Given :

Kinematic viscosity, $\nu = 10^{-6}$ m^2/s

Mean velocity, $V = 1$ m/s

Head loss, $h_f = 5$ m in a length $L = 100$ m

$k_s = 45 \times 10^{-4}$ cm $= 45 \times 10^{-6}$ m

$$f = 0.0055\left[1 + \left(20 \times 10^3 \frac{k_s}{D} + \frac{10^6}{R_e}\right)^{1/3}\right] \quad \text{...(i)}$$

Using Darcy Weisbach equation,

$$h_f = \frac{4 \times f \times L \times V^2}{D \times 2g}$$

or
$$f = \frac{h_f \times D \times 2g}{4 \times L \times V^2} = \frac{5 \times D \times 2 \times 9.81}{4 \times 100 \times 1^2} = 0.2452D$$

Now, the Reynolds number is given by

$$R_e = \frac{\rho VD}{\mu} = \frac{V \times D}{\nu} \qquad \left(\because \nu = \frac{\mu}{\rho}\right)$$

$$= \frac{1 \times D}{10^{-6}} = 10^6 D$$

Substituting the values of f, R_e and k_s in equation (i), we get

$$0.2452D = 0.0055\left[1 + \left(20 \times 10^3 \times \frac{45 \times 10^{-6}}{D} + \frac{10^6}{10^6 D}\right)^{1/3}\right]$$

or
$$\frac{0.2452}{0.0055}D = \left[1 + \left(\frac{0.9}{D} + \frac{1}{D}\right)^{1/3}\right]$$

or
$$44.58D = \left[1 + \left(\frac{1.9}{D}\right)^{1/3}\right] \text{ or } 44.58D - 1 = \left(\frac{1.9}{D}\right)^{1/3}$$

or
$$(44.58D - 1)^3 = \frac{1.9}{D} \text{ or } D(44.58D - 1)^3 = 1.9 \qquad \text{...(ii)}$$

Equation (ii) is solved by hit and trial method.

(i) Let D = 0.1 m, then L.H.S. of equation (ii) becomes

$$\text{L.H.S.} = 0.1(44.58 \times 0.1 - 1)^3 = 0.1 \times 3.458^3 = 4.135$$

This is more than the R.H.S.

(ii) Let D = 0.08 m, then L.H.S. of equation (ii) becomes

$$\text{L.H.S.} = 0.08(44.58 \times 0.08 - 1)^3 = 0.08 \times (2.5664)^3 = 1.352$$

This is less than the R.H.S.

Hence, value of D lies between 0.1 and 0.08.

(iii) Let D = 0.085 m, then L.H.S. of equation (ii) becomes

$$\text{L.H.S.} = 0.085(44.58 \times 0.085 - 1)^3 = 1.844$$

This value is slightly less than R.H.S. Hence, increase the value of D slightly.

(iv) Let D = 0.0854 m, then L.H.S. of equation (ii) becomes

$$\text{L.H.S.} = 0.0854(44.58 \times 0.0854 - 1)^3 = 1.889$$

This value is nearly equal to R.H.S.

$\therefore$ Correct value of D = 0.0854 m.

4. (c) Solution:

$$N = \frac{14000}{60} = \frac{700}{3} \text{ r.p.m.}, W - S = 1470 \text{ N}$$

$$p_{mi} = 7.5 \text{ bar}; V_g = \frac{20000}{3600} = 5.55 \text{ litre/s}$$

$$D = 250 \text{ mm} = 0.25 \text{ m}, L = 400 \text{ mm} = 0.4 \text{ m}$$

$$\pi D_b = 4 \text{ m}, r = 6.5, n = 1$$

(i) **Indicated power, I.P. :**

$$\text{I.P.} = \frac{np_{mi}LANk \times 10}{6}$$

$$Nk = \frac{\left(\frac{14000}{2} - 500\right)}{60} = \frac{6500}{60} \text{ working cycles/min.}$$

$$\therefore \quad \text{I.P.} = \frac{1 \times 7.5 \times 0.4 \times \frac{\pi}{4} \times 0.25^2 \times \left(\frac{6500}{60}\right) \times 10}{6}$$

$$= 26.59 \text{ kW}$$

(ii) **Brake power, B.P. :**

$$\text{B.P.} = \frac{(W - S)\pi D_b N}{60 \times 1000} = \frac{1470 \times 4 \times \left(\frac{700}{3}\right)}{60 \times 1000} = 22.86 \text{ kW}$$

(iii) **Mechanical efficiency, $\eta_{mech.}$:**

$$\eta_{mech.} = \frac{\text{B.P.}}{\text{I.P.}} = \frac{22.86}{26.59} = 0.859 \text{ or } 85.9\%$$

(iv) **Indicated thermal efficiency, $\eta_{th.(I)}$:**

$$\eta_{th(I)} = \frac{\text{I.P.}}{V_g \times C} = \frac{26.59}{5.5 \times 21} = 0.23 \text{ or } 23\%$$

(v) **Relative efficiency, $\eta_{relative}$:**

$$\eta_{th(B)} = \frac{B.P.}{V_g \times C} = \frac{22.86}{5.5 \times 21} = 0.198 \text{ or } 19.8\%$$

$$\eta_{air\text{-}standard} = 1 - \frac{1}{(r)^{\gamma - 1}} = 1 - \frac{1}{(6.5)^{1.4-1}} = 0.527 \text{ or } 52.7\%$$

$$\therefore \qquad \eta_{relative} = \frac{\eta_{th(B)}}{\eta_{air\text{-}standard}} = \frac{19.8}{52.7} = 37.5\%$$

5. (a) Solution:

The declination δ can be approximately determined from the equation,

$$\delta = 23.45 \times \sin\left[\frac{360}{365}(284+n)\right]\text{degree}$$

where n is day of the year counted from 1st January.

Given, ϕ = 28°35′ N = 28.58 degrees

For 20 February 2015:

$$n = 31 + 20 = 51 \text{ (w.e.f 1st January)}$$

$$\delta = 23.45 \sin (330.41) = -11.58°$$

$$E = (9.87 \sin 2B - 7.53 \cos B - 1.5 \sin B) \text{ min}$$

where $B = \frac{360}{364}(n-81) = \frac{360}{364}(51-81) = -29.67$

$$E = 9.87 \sin (2 \times -29.67) - 7.53 \cos (-29.67) - 1.5 \sin (-29.67)$$
$$= -14.29 \text{ min}$$

$$\text{Solar time} = 2:30 \text{ hrs} \pm 4 \times (81°44' - 77°12') \text{ min} - 14.29 \text{ min}$$
$$= 1:57.59 \text{ hrs}$$

$$\omega = [12:00 - 1:57.59] \text{ (in hours)} \times 15 \text{ degrees}$$
$$= -14.398 \text{ degrees}$$

Zenith angle is given as

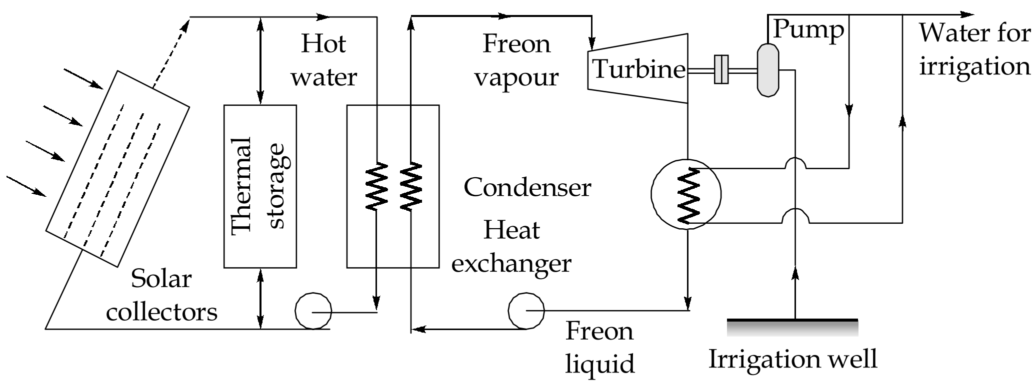
$$\cos\theta_z = \cos\phi\cos\delta\cos\omega + \sin\delta\sin\phi$$
$$= \cos(20°35')\cos(-11.58)\cos(-14.398) + \sin(-11.58)\sin(28°35')$$
$$= 0.7366$$

$$\text{Zenith angle, } \theta_z = \cos^{-1}(0.7366)$$
$$= 42.557°$$

5. (b) Solution:

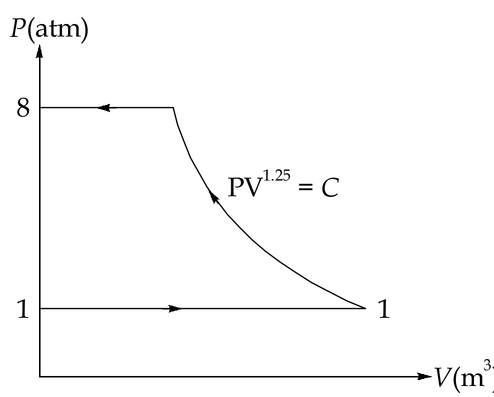
Some features of solar energy make its utilization attractive for irrigation water pumping. These are: (i) more irrigation water is required in summer when solar energy is available most, (ii) intermittent pumping is tolerable and can serve the purpose, (iii) surplus energy can provide pumped storage in the form of a pond. Several solar irrigation pumps have recently been tested in many countries. While in lower range (i.e. 200 W to 5 kW) solar photovoltaic pumps are more successful, in higher ratings (i.e. in range 1 kW to 200 kW) solar thermal pump have become economical and offer superior performance.

A schematic diagram of a type rankine cycle, solar thermal water pump is shown in figure below. Solar collector system may consists of flat plate collectors, non-focusing type (stationary) collectors or sun tracking concentrators. Water is used as heat transport fluid, and yields its heat to a low boiling point organic working fluid (such as Freon R113, R12, Isobutene, etc.) in a heat exchanger. Surplus heat is stored in thermal storage to be used later when sun is not available. The high-pressure vapours of the exchanger (boiler). A part of irrigation pumped water is diverted through condenser for cooling purpose.



Solar thermal water pump

5. (c) Solution:



$$\text{Swept volume, } V_1 = \frac{\pi}{4}d_1^2 \times l = \frac{\pi}{4}\left(\frac{20}{100}\right)^2 \times \frac{24}{100} = 0.00754 \text{ m}^3$$

Work required per cycle,

$$W = \frac{n}{n-1}P_1V_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right\}$$

$$= \frac{1.25}{0.25} \times 1.01325 \times 10^5 \times 0.00764 \left\{ 8^{\frac{0.25}{1.25}} - 1 \right\}$$

$$= 5 \times 1.01325 \times 10^5 \times 0.00754 \times (1.515 - 1)$$

$$= 1970 \text{ J/cycle}$$

Work required per second = Work done per cycle × rps

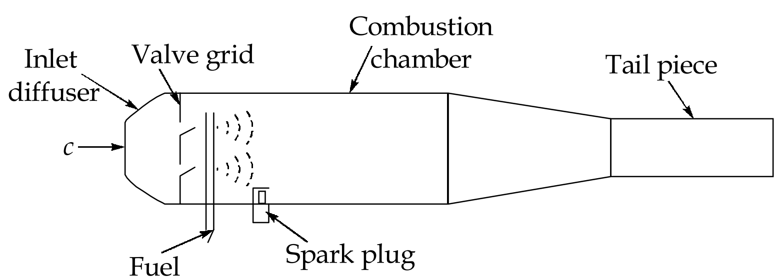
$$= 1970 \times \frac{300}{60} = 9850 \text{ J/sec} = 9850 \text{ W}$$

Indicated power = 9850 W

$$\text{Required power} = \frac{9850}{0.85} = 11588.23 \text{ W} = 11.58 \text{ kW}$$

$$\text{Required power of electric motor} = \frac{11.58}{0.96} = 12.0625 \text{kW}$$

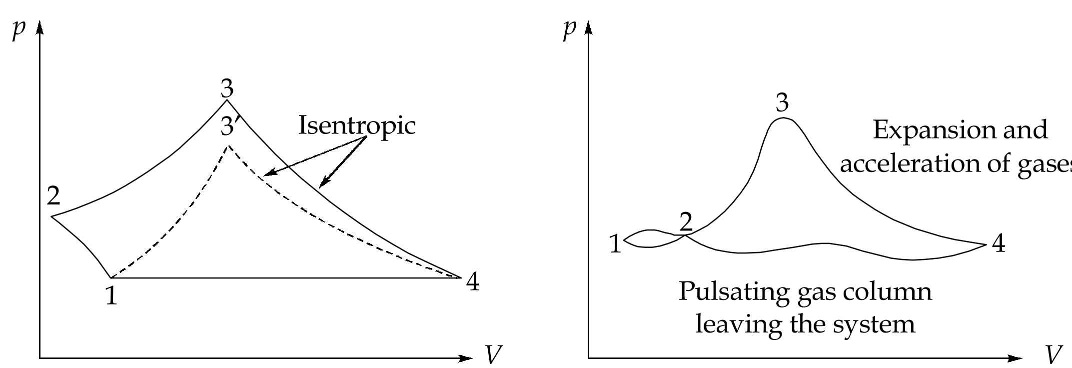
5. (d) Solution:



The pulse jet engine

1. The function of the diffuser is to convert the kinetic energy of the entering air into static pressure rise by slowing down the air velocity. When a certain pressure difference builds up across the valve grid, the valves will open. This makes the fresh air to enter the combustion chamber, when fuel is mixed with the air and combustion starts.
2. A spark plug initiates the combustion process inside the combustion chamber when the inlet valves are closed.
3. Combustion occurs in an enclosed chamber and is approximately a constant volume process.
4. Combustion is nearly an explosion in that enclosed volume and raises the pressure and temperature to high values.
5. The high pressure and temperature forces the gases to flow out of the tail pipe and nozzle.

6. Evacuation of the combustion chamber results in pressure drop, that opens the spring loaded inlet valve and air comes in from the intake.
7. The spring loaded inlet valves are normally closed and open only when the pressure difference across it is attained.
8. The products of combustion surges towards the nozzle. They expand in the nozzle and escape into the atmosphere with a higher velocity so that the exit velocity is much higher than the inlet velocity.
9. Thus, the rate of momentum of the working fluid changed so as to cause a propulsive thrust. Since, the combustion process causes the pressure to increase, the engine can operate even at static conditions once it gets started. When the combustion products accelerate from the chamber, they leave a slight vacuum in the combustion chamber. This, in turn, produces sufficient pressure drop across the valve grid, allowing the valves to open again.
10. A new charge of air enters the combustion chamber which is mixed with fuel that flows continuously. The fresh fuel-air mixture is ignited by the charge leaving and /or by residual charge. New charge need not be ignited with a spark plug again.
11. Proper design allows the duct to fire at a given pulse rate when the fuel flow continuously. The frequency of pulsation is determined by the duct shape and working temperature and may be as high as 500 cycles per second in very small units.
12. The thrust of the pulse jet engine is proportional to the average mass flow rate of gases through the engine multiplied by its increase in velocity.



Actual pulse jet cycle on p-V diagram

5. (e) Solution:

Since the viscosity of the liquid in the model and prototype vary significantly, equality of Reynolds number must apply for dynamic similarity. Let subscripts 1 and 2 refers to prototype and model respectively.

Equating Reynolds number,

$$\frac{N_1 D_1^2}{\nu_1} = \frac{N_2 D_2^2}{\nu_2}$$

or
$$\frac{N_2}{N_1} = \frac{(4)^2}{3} = 5.333$$

Equating the flow coefficients,

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3}$$

or
$$\frac{Q_2}{Q_1} = \left(\frac{N_2}{N_1}\right)\left(\frac{D_2}{D_1}\right)^3 = \frac{5.333}{(4)^3} = 0.0833$$

Equating head coefficients,

$$\frac{H_1}{(N_1 D_1)^2} = \frac{H_2}{(N_2 D_2)^2}$$

or
$$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2$$

$$= \left(\frac{5.33}{4}\right)^2 = 1.776$$

Dimensionless specific speed of the pump can be written as

$$K_{sp} = \frac{N_1 Q_1^{1/2}}{(gH_1)^{3/4}}$$

or
$$N_1 = \frac{K_{sp}(gH_1)^{3/4}}{Q_1^{1/2}} = \frac{0.183(9.81 \times 15)^{3/4}}{2^{1/2}}$$

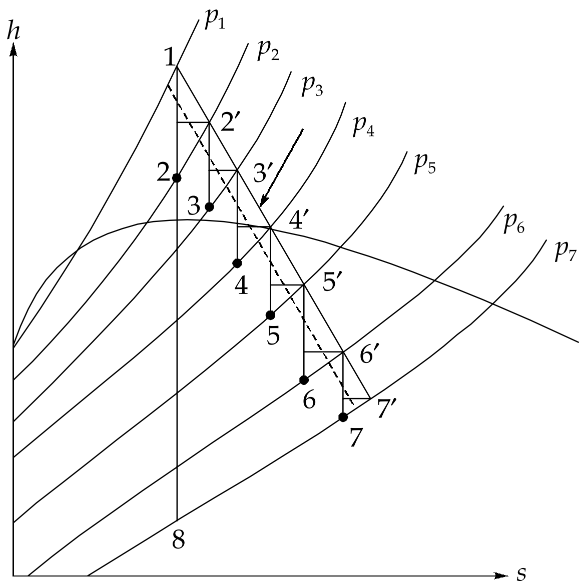
$$= 5.47 \text{ rev/s}$$

Therefore, model speed, $N_2 = 5.47 \times 5.33 = 29.15$ rev/s

and model flow rate $= 0.0833 \times 2 = 0.166$ m^3/s

and model head $= 15 \times 1.776 = 26.64$ m

6. (a) Solution:



Stage point locus in multistage steam turbine

Figure shows the expansion of steam through a multi-stage turbine. Assuming six stages only for simplicity. The initial pressure is p_1 and the back pressure is p_T. If the stage efficiency is known then the locus of the state point may be drawn on the *h-s* diagram.

State 1 is the initial point set down according to the initial condition of steam. 1-2 shows the isentropic expansion of steam in the first stage. But owing to losses the actual state point of the steam after-expansion in the first stage is 2′ and not 2. Thus the actual heat drop is 1-2′ and not 1-2 so, we have

$$h_1 - h_2' = \eta_s(h_1 - h_2)$$

and point 2′ is thus set down with h_2' value on the p_2 line.

When point 2′ is known, isentropic expansion is drawn giving $(h_2' - h_3)$ and h_3' will be calculated from the relation.

$$(h_2' - h_3') = 2'3'$$

$$= \eta_s(h_2' - h_3) = \eta_s \cdot 2'3$$

and the point 3′ is set down. Similarly, other points in the succeeding stages are set down. The isentropic and actual heat drops in the succeeding stages are represented by $3' - 4$ and $3' - 4'$, $4' - 5$ and $4' - 5'$, $5' - 6, 6' - 7$, and $6' - 7'$. The points 1, 2′, 3′, 4′, 5′, 6′, 7′, represent the condition of steam in the wheel chambers of the six stages i.e. at the blade outlets. 1-8 is the rankine heat drop for the initial and final pressure. The curve joining the points 1, 2′, 3′, 4′, 5′, etc. is called the condition line of the expansion of steam because the steam probably expands along some such path. The heat drops $1-2', 2'-3', 3'-4'$, etc. which are actually converted into work may be called useful heat drops in the corresponding stages. The dotted line shows the condition of steam at outlet from the nozzle.

The sum of the isentropic heat drops $1-2', 2'-3, 3'-4'$, etc., called cumulative heat drop is represented by Δh_c. Since the constant pressure line diverge as we move from left to right on the *h-s* diagram, the cumulative heat drop is always greater than direct isentropic or rankine heat drop 1-8.

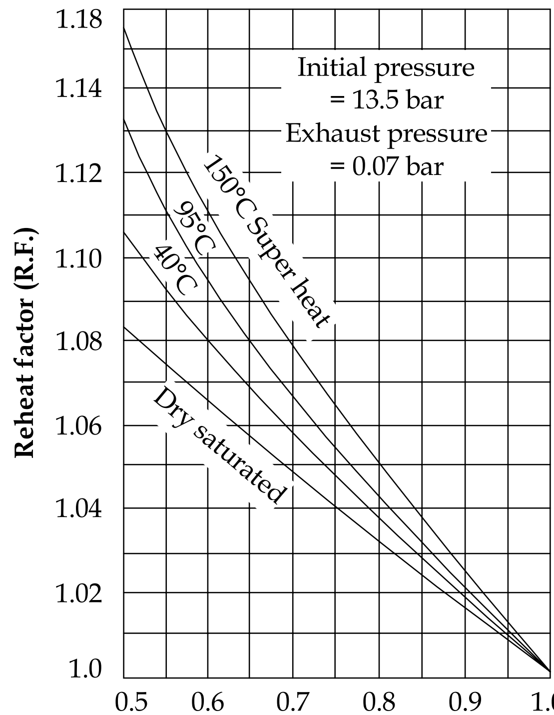
Reheat factor (RF) is defined as the ratio of cumulative heat drop Δh_c to the direct isentropic or rankine heat drop, Δh_{isen} (i.e. 18).

$$RF = \frac{\Delta h_c}{\delta h_{isen}} = \frac{\text{Cumulative heat drop}}{\text{Rankine heat drop}}$$

Referring to figure above:

$$RF = \frac{\Delta h_{12} + \Delta h_{2'3} + \Delta h_{3'4} + \ldots}{\Delta h_{isen}} = \frac{12 + 2'3 + 3'4 + \ldots}{18}$$

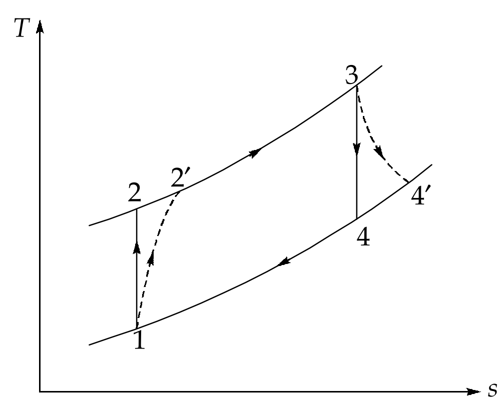
The value of *RF* is always greater than unity because cumulative heat drop is always greater than rankine heat drop. Reheat factor depends upon the turbine stage efficiency, the initial-pressure, and initial superheat, final pressure and number of stages in a given pressure range. The reheat factor is greater if the number of stages are large for a given pressure range and lower the stage efficiency. As is obvious from the figure below that if the initial superheat increases the reheat superheat for a constant value of stage efficiency. For a certain value of initial superheat factor increases as the stage efficiency decreases. Note that lower value of reheat factor is desirable.



Stage efficiency
Effect of stage efficiency and Superheat on Reheat factor

6. (b) Solution:

Optimum pressure ratio for maximum cycle thermal efficiency:



$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}} = x$$

Actual compressor work, $W_{ca} = h_2' - h_1 = \dfrac{h_2 - h_1}{\eta_c} = \dfrac{c_p(T_2 - T_1)}{\eta_c}$ kJ/kg

Actual turbine work, $W_{ta} = h_3 - h_4' = (h_2 - h_4)\eta_t = c_p(T_3 - T_4)\eta_t$ kJ/kg

$$W_{\text{net},\,a} = c_p(T_3 - T_4)\eta_t - \frac{c_p(T_2 - T_1)}{\eta_c}$$

$$= c_p\eta_t T_3\left(1 - \frac{T_4}{T_3}\right) - \frac{c_p T_1}{\eta_c}\left(\frac{T_2}{T_1} - 1\right)$$

$$= c_p\eta_t T_3\left(1 - \frac{1}{x}\right) - \frac{c_p T_1}{\eta_c}(x - 1)$$

Heat supplied = $c_p(T_3 - T_{2'}) \simeq c_p(T_3 - T_2)$. This approximation is only for simplification of the problem without any appreciable error.

Let $\frac{T_3}{T_1} = y$.

So, the heat supplied = $c_p(T_3 - T_1 x) = c_p T_1(y - x)$

$$\eta_{\text{tha}} = \eta_{t'}\frac{T_3}{T_1}\frac{\left(1 - \frac{1}{x}\right)}{(y - x)} - \frac{\frac{(x-1)}{\eta_c}}{(y - x)}$$

$$= \eta_t + \frac{T_3}{T_1}\frac{\left(1 - \frac{1}{x}\right)}{(y - x)} - \frac{\frac{(x-1)}{\eta_c}}{y - x}$$

$$= \frac{\eta_t\left(1 - \frac{1}{x}\right)y - \frac{(x-1)}{\eta_c}}{y - x}$$

For maximum thermal efficiency:

$$\frac{d\eta_{tha}}{dx} = 0$$

i.e. $\left\{\eta_t y\frac{1}{x^2}(y - x) - \frac{1}{\eta_c}(y - x)\right\} - \left\{\eta_t y\left(1 - \frac{1}{x}\right) - \frac{1}{\eta_c}(x - 1)\right\}(-1) = 0$

or, $\left[\frac{\eta_c\eta_t y - x^2}{x^2\eta_c}\right](y - x) + (x - 1)\left[\frac{\eta_c\eta_t y - x}{x\eta_c}\right] = 0$

$$\left[\frac{\eta_c\eta_t y - x^2}{x^2\eta_c}\right](y-x)+(x^2-x)\left[\frac{\eta_c\eta_t y - x}{x^2\eta_c}\right]=0$$

or, $\left(\eta_c\eta_t y - x^2\right)(y-x)+(x^2-x)(\eta_c\eta_t y - x)=0$

or, $\eta_c\eta_t y^2 - \eta_c\eta_t yx - x^2y + x^3 + \eta_c\eta_t yx^2 - x^3 - \eta_c\eta_t yx + x^2 = 0$

or, $\eta_c\eta_t y^2 - 2\eta_c\eta_t yx + \eta_c\eta_t yx^2 - x^2y + x^2 = 0$

Multiplying throughout by $\frac{1}{x^2}$, we have

or, $\frac{1}{x^2}\left(\eta_c\eta_t y^2\right)-\frac{1}{x}(2\eta_c\eta_t y)+\left[y\left(\eta_c\eta_t - 1\right)+1\right]=0$

This is quadratic equation in $\frac{1}{x}$,

$$\frac{1}{x} = \frac{2\eta_c\eta_t y + \sqrt{4(\eta_c\eta_t y^2)\left[y\left(\eta_c\eta_t - 1\right)+1\right]}}{2\eta_c\eta_t y^2}$$

Taking positive root only, we have

$$x = \frac{2\eta_c\eta_t y^2}{2\eta_c\eta_t y + 2\sqrt{4(\eta_c\eta_t y)^2 - \left(\eta_c\eta_t + y^2\right)\left[y\left(\eta_c\eta_t - 1\right)+1\right]}}$$

$$x = \frac{2\eta_c\eta_t y^2}{2\eta_c\eta_t y + \sqrt{\left\{(y-1)\left(\frac{1}{\eta_c\eta_t}\right)-1\right\}}}$$

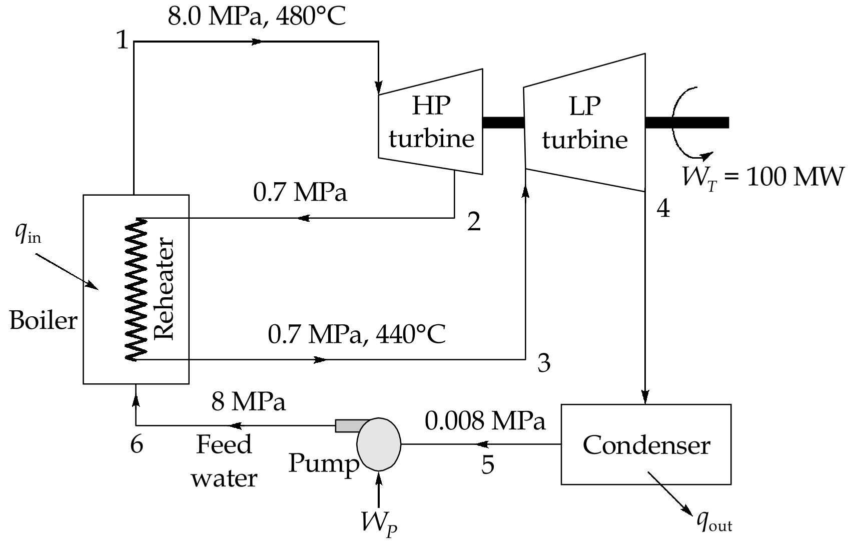
$$x = \frac{y}{1+\sqrt{\left[(y-1)\left(\frac{1}{\eta_c\eta_t}\right)-1\right]}} \qquad \text{Derived}$$

Thus, $$r_{p,\ \text{optimum}} = \left\{\frac{\frac{T_3}{T_1}}{1+\sqrt{\left(\frac{T_3}{T_1}-1\right)\left(\frac{1}{\eta_c\eta_t}-1\right)}}\right\}^{\frac{\gamma}{\gamma-1}}$$

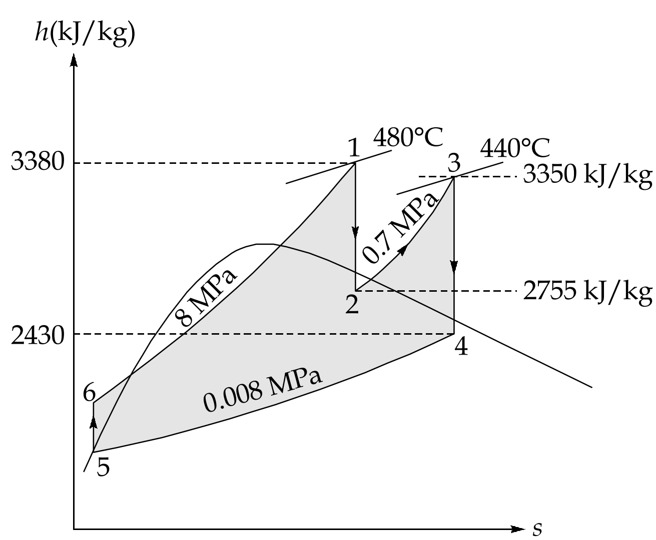
$$= \left\{ \frac{\frac{1152}{288}}{1 + \sqrt{\left(\frac{1152}{288} - 1\right)\left(\frac{1}{0.85 \times 0.8} - 1\right)}} \right\}^{\frac{1.4}{0.4-1}}$$

$$= \left[\frac{4}{1 + \sqrt{3\left(\frac{1}{0.85 \times 0.8} - 1\right)}} \right]^{3.5} = 8.27$$

6. (c) Solution:



Schematic of physical arrangement of steam power cycle



Schematic of cycle of h-s diagram

The pump work: $W_P = v_f(P_1 - P_3)$ [P_1 = 8 MPa, P_2 = 0.7 MPa, P_3 = 0.008 MPa]

$$= 0.001008\,(8 - 0.008) \times 10^3$$
$$= 8.06 \text{ kJ/kg}$$

Enthalpy at state 6: $h_6 = h_5 + W_P = 173.93 + 8.06 = 182$ kJ/kg

Total turbine work per kg of steam:

$$W_T = (h_1 - h_2) + (h_3 - h_4)$$
$$= 3380 - 2755 + 3350 - 2410$$
$$= 1565 \text{ kJ/kg}$$

Net work per kg of steam,

$$W_{\text{net}} = W_T - W_P$$
$$= 1565 - 8.06 = 1556.94 \text{ kJ/kg} \simeq 1557 \text{ kJ/kg}$$

Total heat supplied per kg of steam,

$$Q_{\text{input}} = (h_1 - h_6) + (h_3 - h_2)$$
$$= 3380 - 182 + 3350 - 2755$$
$$= 3793 \text{ kJ/kg}$$

Thermal efficiency:

$$\eta_{\text{th}} = \frac{W_{net}}{Q_{in}} = \frac{1557}{3793} = 0.41 \text{ or } 41\%$$

Mass flow rate of steam, $\dot{m}_s = \dfrac{P}{W_{net}} = \dfrac{100 \times 10^3}{1557} = 64.22$ kg/s

$$= 231.21 \times 10^3 \text{ kg/h}$$

Specific steam consumption,

$$\text{SSC} = \frac{3600}{W_{net}} = \frac{3600}{1557} = 2.31 \text{ kg/kWh}$$

Rate of heat rejection in the condenser,

$$\dot{Q}_{in} = \dot{m}_s(h_4 - h_5) = 64.22(2410 - 173.93)$$
$$= 143.6 \times 10^3 \text{ kW} = 143.6 \text{ MW}$$

7. (a) Solution:

(i)

The power available in wind increases rapidly with wind speed. Therefore main consideration for locating a wind power generation plant is the availability of strong and persistent wind. A suitable site should preferably have some of the following features:

- Not tall obstructions for some distance (about 3 km) in the upwind direction (i.e. the direction of incoming wind) and also as low a roughness as possible in the same direction.
- A wide and open view, i.e. open plain, open shoreline or offshore locations.
- Top of smooth well-rounded hill with gentle slopes (about 1:3 or less) on a flat plain.
- An island in a lake or the sea.
- An island, mountain gap through which wind is channeled.
- The site should be reasonably close to power grid.
- The soil conditions must be such that building of foundations of the turbines and transport of road construction material loaded on heavy trucks must be feasible.
- If there are already wind turbines in the area, their production results are in excellent guide to local wind conditions.

(ii)

Modern wind turbines have two or three blades. Two/three blade rotor HAWT are also known as propeller type wind turbines owing to their similarity with propellers of old aero planes. However, the rotor rpm in case of wind turbine is very low as compared to that for propellers. The relative merits and demerits of two and three blade rotors are as follows:

- Compared to two-blade design, the three-blade machine has smoother power output and balanced gyroscopic force.
- There is no need to teeter the rotor, allowing the use of simple rigid hub. The blades may be cross-linked for greater rigidity.
- Adding third blade increases the power output by about 5 percent only, while the weight and cost of rotor increases by 50 per cent, thus giving a diminished rate of return for additional 50 per cent weight and cost.
- The two-blade rotor is also simpler to erect, since it can be assembled on ground and lifted to the shaft without complicated maneuvers during the lift.

Three blades are more common in Europe and other developing countries including India. The American practice, however, is in favour of two blades.

(iii)

$$\alpha = \frac{c_i}{c_j} = \frac{1450}{2850} = 0.5087$$

$$\text{Thrust, } F = \dot{m}_a(c_j - c_i)$$

$$= 85\ (2850 - 1450) = 85 \times 1400 = 119000 \text{ N}$$

$$\text{Thrust power} = F \times c_i$$

$$= 119000 \times 1450$$

$$= 172.55 \times 10^6 \text{ W}$$

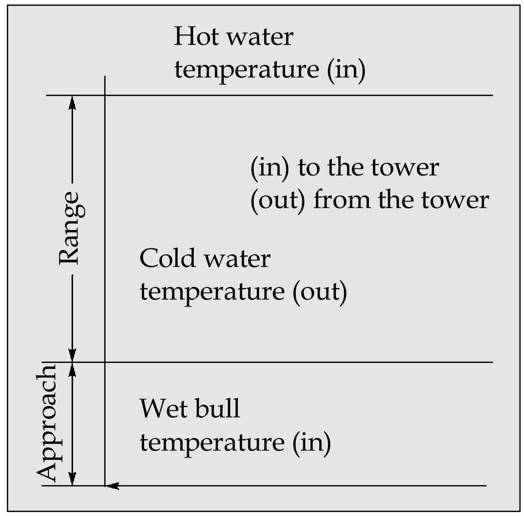
$$\text{Propulsive efficiency, } \eta_p = \frac{2\alpha}{\alpha + 1} = \frac{2 \times 0.5087}{1.5087} = 0.6743$$

$$\eta_p = 67.43\%$$

7. (b) Solution:

(i) A cooling tower is a heat rejection device that rejects waste heat to the atmosphere through the cooling of a water stream to a lower temperature. Cooling tower perform the release of heat from the hot water coming from the condenser. In a power plant condenser acts as the sink and steam from the turbine is being dumped into the condenser. Cooling water is sent into the condenser to condense this steam. As a result, the cooling water temperature rises. To reuse this water again for condenser cooling, the absorbed heat has to be released and cooled. Thermal power plants use cooling towers to cool the circulating water used for condenser cooling. Cooling tower cool the warm water discharged from the condenser and feed the cooled water back to the condenser. They reduce the cooling water demand in the power plant.

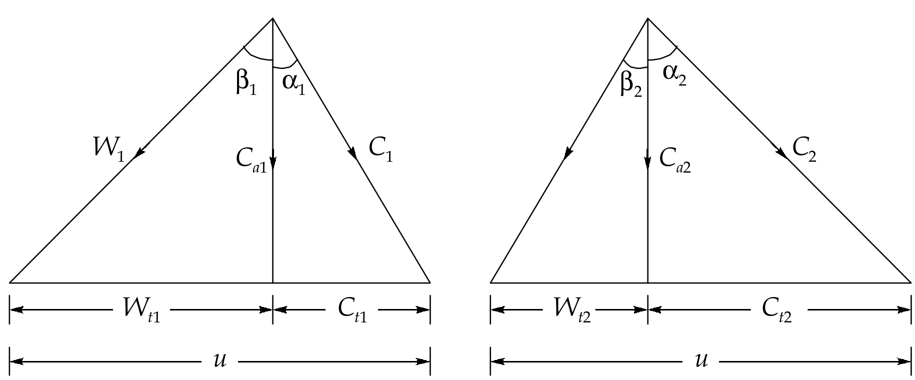
Cooling towers are a very important part of many chemical plants. The primary task of a cooling tower is to reject heat into the atmosphere. They represent a relatively inexpensive and dependable means of removing low-grade heat from cooling water. The make-up water source is used to replenish water lost to evaporation. Hot water from heat exchangers is sent to the cooling tower. The water exits the cooling tower and is sent back to the exchangers or to other units for further cooling.



"Range" is the difference between the cooling tower water inlet and outlet temperature.

"Approach" is the difference between the cooling tower outlet cold water temperature and ambient wet bulb temperature. Although, both range and approach should be monitored, the 'Approach' is a better indicator of cooling tower performance.

(ii)



Inlet velocity triangle ***Outlet velocity triangle***

Since , degree of reaction is 50%, the velocity triangles are symmetrical

$$\alpha_1 = \beta_2 = 15° \text{ and } \alpha_2 = \beta_2 = 45°$$

Degree of reaction, $R = \dfrac{C_a}{2u}(\tan\beta_1 + \tan\beta_2)$

Since, $R = 0.5$

$$C_a = \frac{u}{\tan\beta_1 + \tan\beta_2} = \frac{180}{\tan 45° + \tan 15°} = 141.96 \text{ m/s}$$

Temperature rise per stage is given by

$$\Delta T_s = \frac{\Omega u C_a}{c_p} \times (\tan\theta - \tan\phi)$$

$$= \frac{\Omega u C_a}{c_p} \times (\tan 45° - \tan 15°)$$

$$= \frac{0.84 \times 180 \times 141.96}{1005}(1 - 0.268)$$

$$= 15.634 \text{ K}$$

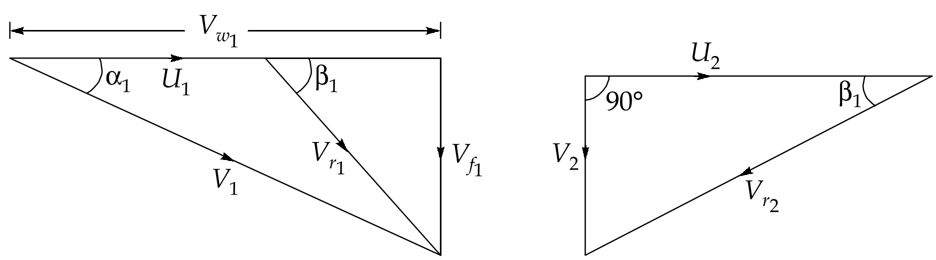
$$\Delta T_{\text{overall}} = \frac{T_1}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right) = \frac{290}{0.82}\left(4^{0.286} - 1\right) \qquad \left[\frac{\gamma - 1}{\gamma} = 0.286\right]$$

$$= 172.084 \text{ K}$$

$$\text{Number of stages, } n = \frac{\Delta T_{overall}}{\Delta T_{stage}} = \frac{172.084}{15.634} = 11$$

7. (c) Solution:

The inlet and outlet velocity triangles are drawn as shown below.



Inlet ***Outlet***

(i) Runner tip speed, $U_1 = \frac{\pi N D}{60} = \frac{\pi \times 430 \times 1.4}{60} = 31.52$ m/s

Since $V_{w2} = 0$,

Power given to the runner by water = $\rho Q V_{w1} U_1$

Hence, $12.25 \times 10^6 = 10^3 \times 12 \times V_{w1} \times 31.52$

which gives $V_{w1} = 32.39$ m/s

Inlet guide vane angle α_1 is given by

$$\tan\alpha_1 = \left[\frac{9.5}{32.39}\right]$$

or $$\alpha_1 = \tan^{-1}\left[\frac{9.5}{32.39}\right] = 16.35°$$

From the inlet velocity diagram, the absolute velocity at runner inlet

$$V_1 = \left[V_{f_1}^2 + V_{w_1}^2\right]^{1/2} = \left[(9.5)^2 + (32.39)^2\right]^{1/2} = 33.75 \text{ m/s}$$

(ii) Runner blade entry angle β_1 is given by

$$\tan\beta_1 = \frac{9.5}{32.39 - 31.52}$$

which gives $$\beta_1 = 84.77°$$

(iii) Total head across the runner = Head transferred to the runner + Head lost in the runner

At inlet, $$H_1 = \left(\frac{p_1}{\rho g}\right) + \left(\frac{V_1^2}{2g}\right) + z_1$$

At outlet, $$H_2 = \left(\frac{p_2}{\rho g}\right) + \left(\frac{V_2^2}{2g}\right) + z_2$$

where, z_1 and z_2 are the elevations of runner inlet and outlet from a reference datum. For zero whirl at outlet, the work done per unit weight of the fluid = $\left[V_{w_1}\dfrac{U_1}{g}\right]$

Hence loss of head in the runner becomes,

$$h_L = H_1 - H_2 - \left[V_{w_1}\frac{U_1}{g}\right]$$

$$= \left[\frac{p_1 - p_2}{\rho g}\right] + \left[\frac{V_1^2 - V_2^2}{2g}\right] + [z_1 - z_2] - \left[V_{w_1}\frac{U_1}{g}\right]$$

It is given that $\left[\dfrac{p_1 - p_2}{\rho g}\right] + [z_1 - z_2] = 62$ m

Therefore,
$$h_L = 62 + \left[\frac{(33.75)^2 - (7)^2}{2 \times 9.81}\right] - \left[\frac{31.52 \times 32.39}{9.81}\right] = 13.49 \text{ m}$$

8. (a) Solution:

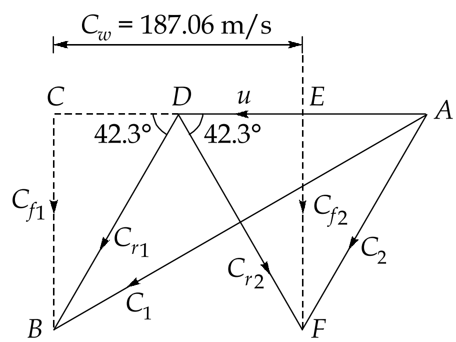
Given: Δh_{isen} = 33.5 kJ/kg, D = 95.5 cm, α_1 = 20°, ϕ = 0.88, η_n = 0.92, K = 0.87, $\beta_1 = \beta_2$

The peripheral speed is $= u = \dfrac{\pi DN}{60} = \dfrac{\pi \times 95.5 \times 3000}{100 \times 60} = 150$ m/s

We know that the nozzle efficiency in a stage of multi-stage turbine is given by

$$\eta_n = \frac{C_1^2 - \phi C_2^2}{2\Delta h_{isen}}$$

From the velocity diagram,



Inlet and outlet velocity triangles

$$C_2^2 = C_{r_2}^2 + u^2 - 2uC_{r_2}\cos\beta_2 = K^2C_{r_1}^2 + u^2 - 2uKC_{r_1}\cos\beta_2$$

$$= K^2\left(C_1^2 - u^2 - 2uC_{r_1}\cos\beta_1\right) + u^2 - 2KuC_{r_1}\cos\beta_2$$

$$= K^2C_1^2 + u^2\left(1 - K^2\right) - 2KuC_{r_1}\cos\beta_1\left(K^2 + \frac{K\cos\beta_2}{\cos\beta_1}\right)$$

Since, $\beta_1 = \beta_2$ and $C_{r_1}\cos\beta_1 = C_1\cos\alpha_1 - u$

$\therefore$
$$C_2^2 = K^2C_1^2 + u^2(1 - K^2) - 2u(C_1\cos\alpha_1 - u)\left(K^2 + K\right)$$

Substituting the values of K, u and α_1, we have

$$C_2^2 = (0.87)^2C_1^2 + (150)^2(1 - 0.87^2) - 2 \times 150(C_1\cos 20° - 150) \times (0.87^2 + 0.87)$$

$$= 0.757C_1^2 - 458C_1 + 78680.25$$

Now putting the values of C_2^2 in the expression of nozzle efficiency

$$0.92 = \frac{C_1^2 - 0.88\left[0.757C_1^2 - 458C_1 + 78680.25\right]}{2 \times 1000 \times 33.5}$$

$$C_1^2 + 1207.28 \times C_1 - 392039.96 = 0$$

or $$C_1 = 266.08 \text{ m/s}$$

$$C_{w_1} = C_1 \cos 20° = 266.08 \times \cos 20° = 250.03 \text{ m/s}$$

$$C_{f_1} = C_1 \sin 20° = 266.08 \times \sin 20° = 91 \text{ m/s}$$

$$\tan\beta_1 = \frac{C_{f_1}}{C_{w_1} - U} = \frac{91}{250.03 - 150}$$

$\Rightarrow$ $$\beta_1 = \beta_2 = 42.29°$$

$$C_{r_1} = \frac{C_{f_1}}{\sin\beta_1} = \frac{91}{\sin 42.29} = 135.23 \text{ m/s}$$

$$C_{r_2} = K \cdot C_{r_1} = 0.87 \times 135.23 = 117.65 \text{ m/s}$$

$$C_{w_2} = U - C_{r_2} \times \cos\beta_2 = 150 - 117.65 \times \cos 42.29°$$

$$= 62.97 \text{ m/s}$$

$$C_w = C_{w_1} - C_{w_2} = 250.03 - 62.97 = 187.06 \text{ m/s}$$

$\therefore$ The blade efficiency, $$\eta_b = \frac{2U \cdot C_w}{C_1^2} = \frac{2 \times 150 \times 187.06}{266.08^2}$$

$$\eta_b = 0.7926$$

So, the gross stage efficiency,

$$\eta_{gs} = \eta_n \times \eta_b$$

$$= 0.7926 \times 0.92 = 72.92\%$$

8. (b) Solution:

(i) Volume of POC at NTP, STP and at 1000°C.

Consider 1 m^3 of gaseous fuel at NTP (1 atm. and 273 K). Let Y m^3 is POC (1 atm 273 K)

Performing Carbon balance:

Carbon in CO + Carbon in CO_2 = Carbon in POC, it follows that

$$0.29 + 0.09 = 0.15Y, \text{ hence}$$

$$Y = 2.53 \text{ m}^3 \text{ at NTP (1 atm and 273 K)}$$

$$Y = 2.53 \times \frac{298}{273} = 2.765 \text{ m}^3 \text{ at STP (1 atm and 298 K)}$$

$$Y = 2.53 \times \frac{1273}{273} \text{m}^3 = 11.797 \text{ m}^3 \text{ at (1 atm and 1273 K)}$$

Note the increase in volume of POC at 1273 K which is around 5 times than at 273 K. This knowledge is important in designing combustion chamber.

(ii) Volume of air at NTP.

Let Z m^3 is volume of air required at NTP. Performing nitrogen balance we get

Nitrogen from air + Nitrogen in gaseous fuel = Nitrogen in POC, we get

$$0.79Z + 0.46 = 0.78 \times 2.53, \text{ hence}$$

$$Z = 1.916 \text{ m}^3 \text{ at NTP}$$

(iii) Percent excess air.

In order to calculate percent excess air, first we have to calculate theoretical air. Theoretical air is the air required for complete combustion of the following reactions:

$$CO + 0.5O_2 = CO_2 \text{ and} \quad \text{...(i)}$$

$$H_2 + 0.5O_2 = H_2O \quad \text{...(ii)}$$

We note that both CO and H_2 require 0.5 mole of oxygen. Hence

Theoretical amount of air 1.071 m^3 at NTP

$$\text{Excess air in \%} = \frac{100(\text{Actual air} - \text{Theoretical air})}{\text{Theoretical air}} = 78.89\%$$

The slight difference in excess air may be due to rounding off.

(iv) %H_2O in POC

From the reaction 2 we get straightway that a mole of hydrogen gives 1 mole of water,

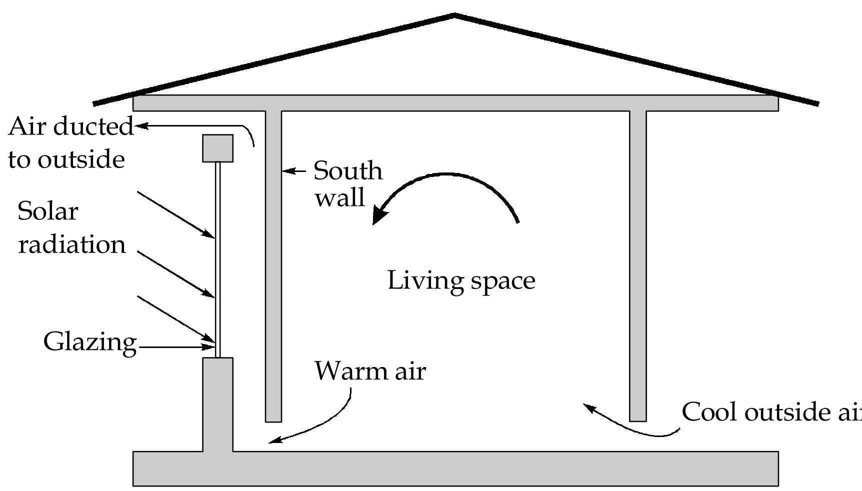
$$\text{hence percent } H_2\text{) in POC} = \frac{(100 \times 0.16)}{(2.53 + 0.16)} = 5.95\%$$

8. **(c) Solution:**

Solar energy is also used for heating or cooling a building to maintain comfortable temperature inside. Passive systems do not require any mechanical device and make use of natural process of convection, radiation and conduction for transport of heat. Use of passive heating/cooling systems put restrictions on the building design to make possible the flow of heat naturaylly. Such a specially designed building is called "solar house”] The state of the art for passive cooling is much less developed than for passive space heating. Natural passive cooling may not always be sufficient to meet the requirement and at peak load, auxiliary means may also be needed, but it greatly reduces the load on the air conditioner plant.

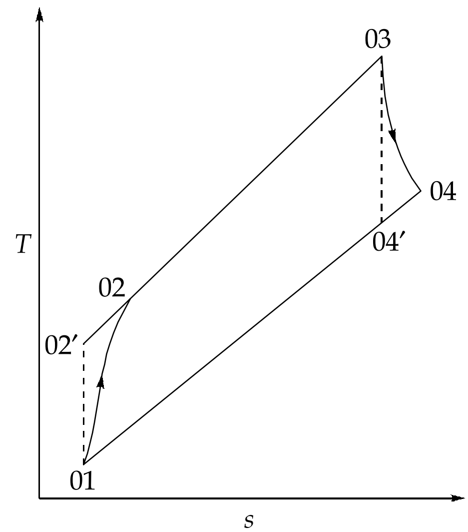
Active heating/cooling systems employ mechanical devices, e.g. pump, blower, etc. to circulate the working fluid for transportation of heat and therefore special building design is not necessary as required in the case of passive heating. Nevertheless, careful building design and insulation is desirable and will be less expensive than additional heating/cooling load due to poor design.

Figure shows the scheme for solar passive cooling through ventilation. This scheme utilizes solar ‘Chimney effect’ and is effective where outside temperatures are moderate. Solar radiation is allowed to heat up the air between the glazing and interior south wall. The heated air rises up and ducted outside and the warm air from the room is drawn into this space due to natural draught thus produced. As a result, cool outside air enters the room from the bottom air vent on the other side of the room.



Solar passive cooling through ventilation

8. (d) Solution:



Work ratio,
$$W_{\text{ratio}} = \frac{(W_{net})_a}{(W_T)_a} = \frac{(W_T)_a - (W_c)_a}{(W_T)_a} = 1 - \frac{(W_c)_a}{(W_T)_a}$$

$$= 1 - \frac{\frac{W_e}{\eta_c}}{\eta_T \cdot W_T} = 1 - \frac{C_P[T_2 - T_1]}{\eta_c \cdot \eta_T \cdot T_3\left[1 - \frac{1}{C}\right]} \qquad \left[\text{where } C = r_P^{\frac{\gamma-1}{\gamma}}\right]$$

$$W_{\text{ratio}} = 1 - \frac{T_1[C-1]}{\eta_c \cdot \eta_T \cdot T_3\left[\frac{C-1}{C}\right]}$$

$$W_{\text{ratio}} = 1 - \frac{c}{t} \cdot \frac{1}{\eta_C \eta_T}$$

$$0.3 = 1 - \frac{12^{0.286}}{4} \cdot \frac{1}{\eta_C \eta_T} = 1 - \frac{0.509}{\eta_C \eta_T}$$

$$\eta_C \eta_T = \frac{0.509}{(1-0.3)} = 0.73$$

For temperature ratio to be minimum work ratio should be zero,

$$W_{\text{ratio}} = 0 = 1 - \frac{c}{t} \cdot \frac{1}{\eta_C \eta_T}$$

$$t_{\min} = \frac{c}{\eta_C \eta_T} = \frac{12^{0.286}}{0.73} = 2.79 \simeq 2.8$$

$$\eta = 1 - \frac{1}{c} = 1 - \frac{1}{12^{0.286}} = 0.508$$

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