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Detailed Solutions

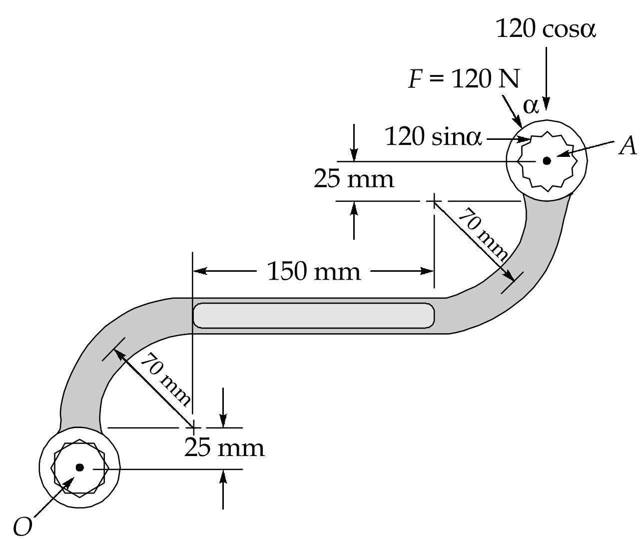
ESE-2021 Mains Test Series

Mechanical Engineering Test No : 11

Full Syllabus Test (Paper-II)

Section-A

1. (a) Solution:



$$F_x = F \sin\alpha = 120 \sin 30° = 60 \text{ N}$$

$$F_y = F \cos\alpha = 120 \cos 30° = 103.923 \text{ N}$$

Vertical distance between point A and point O:

$$d_y = (25 + 70 + 70 + 25) \text{ mm}$$

$$= 190 \text{ mm} = 0.19 \text{ m}$$

Horizontal distance between point A and point O:

$$d_x = (70 + 150 + 70)\text{ mm}$$
$$= 290\text{ mm} = 0.29\text{ m}$$
$$M_0 = F_x \times d_y + F_y \times d_x$$
$$= 60 \times 0.19 + 103.923 \times 0.29 = 41.54\text{ Nm (CW)}$$

Now, moment equation in terms of angle 'α'

$$M_0 = (120\cos\alpha)0.29 + (120\sin\alpha)0.19$$
$$= 22.8\sin\alpha + 34.8\cos\alpha$$

For maximum moment,

$$\frac{dM_0}{d\alpha} = 0 \text{ i.e., } \frac{d}{d\alpha}(22.8\sin\alpha + 34.8\cos\alpha) = 0$$

or, $22.8\cos\alpha - 34.8\sin\alpha = 0$

or, $$\frac{\sin\alpha}{\cos\alpha} = \tan\alpha = \frac{22.8}{34.8} = 0.655$$

i.e. $\alpha = 33.22°$ (for maximum moment)

Magnitude of maximum moment:

$$M_{0,\max} = 22.8\sin 33.22° + 34.8\cos 33.22°$$
$$= 41.60\text{ Nm (CW)}$$ Ans.

1. **(b) Solution:**

(i)

Nanotubes can be classified as armchair nanotubes, zigzag nanotubes or chiral tubes depending on their chirality. Chirality can be described by the chiral vector (n, m), where n and m are integers of the vector equation $R = na_1 + ma_2$; where a_1 and a_2 are the basis vectors of the hexagonal 2D lattice. R is the rolling vector about which the planar graphite sheet (of width equal to the perimeter of the nanotube along the cross section) is rolled to form the CNT. The armchair vector is defined as the vector from a given atom position dividing the hexagons into half. If the armchair vector coincides with the rolling vector (wrapping angle, i.e., angle between the armchair vector and rolling vector, is zero), it results in an armchair nanotube. In other words, the chiral vector R will be given as $R = n(a_1 + a_2)$ (i.e., $n = m$ in the chiral vector). If the wrapping angle is equal to 30° ($R = na_1$, with m being zero), then the tube is of the 'zigzag' type. If the wrapping angle is between zero and 30°, it is called a chiral tube with $R = na_1 + ma_2$; with $n \neq m$. Consider a nanotube that has been unfolded into a planar sheet by splitting along a circumferential vector parallel to the tube axis. The values of n and m determine the chirality, or 'twist' of the nanotube. The chirality in turn affects the conductance of the nanotube, its density, lattice structure and other properties. The value of $n - m$ decides whether SWNT is metallic or semiconducting-metallic if this value is divisible by three, semiconducting otherwise.

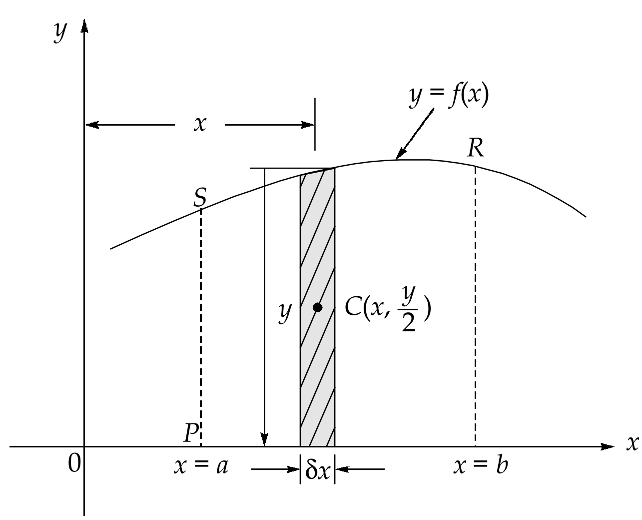
(ii)

Popular techniques are:

- Electric arc discharge
- Chemical vapour deposition
- High pressure carbon monoxide method
- Laser ablation

1. (c) Solution:

The curve $y = x(5 - x)$ cuts the x-axis at 0 and 5 as shown in figure below. Let the coordinates of the centroid be $(\bar{x}, \bar{y})$.



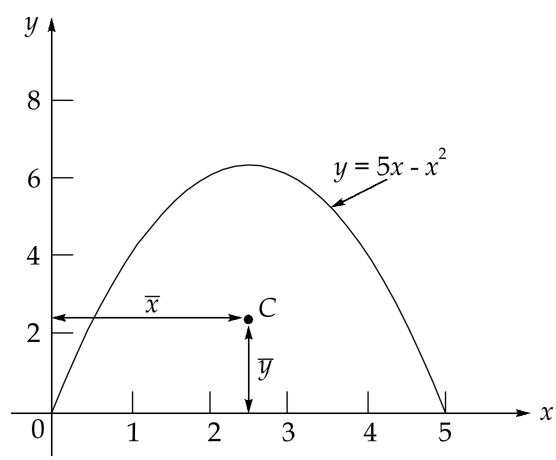
Let $\bar{x}$ and $\bar{y}$ be the distance of the centroid of area A about Oy and Ox respectively then;

$$(\bar{x})(A) = \text{Total first moment of area A about axis}$$

$$Oy = \int_a^b xy\,dx$$

From which, $\bar{x} = \dfrac{\int_a^b xy\,dy}{\int_a^b y\,dy}$ and $(\bar{y})(A)$ = Total moment of area A about axis $Ox = \dfrac{1}{2}\int_a^b y^2 dx$

from which, $\bar{y} = \dfrac{\frac{1}{2}\int_a^b y^2 dx}{\int_a^b y dx}$



$$\bar{x} = \frac{\int_0^5 xydy}{\int_0^5 ydx} = \frac{\int_0^5 x(5x-x^2)dx}{\int_0^5 (5x-x^2)dx}$$

$$= \frac{\int_0^5 (5x^2-x^3)dx}{\int_0^5 (5x-x^2)dx} = \frac{\left|\frac{5x^3}{3} - \frac{x^4}{4}\right|_0^5}{\left|\frac{5x^2}{2} - \frac{x^3}{3}\right|_0^5}$$

$$= \frac{\frac{625}{3} - \frac{625}{4}}{\frac{125}{2} - \frac{125}{3}} = \frac{625}{12} \times \frac{6}{125} = \frac{5}{2} = 2.5$$

$$\bar{y} = \frac{\frac{1}{2}\int_0^5 y^2 dx}{\int_0^5 y dx} = \frac{\frac{1}{2}\int_0^5 (5x - x^2)^2 dx}{\int_0^5 (5x - x^2) dx}$$

$$= \frac{\frac{1}{2}\int_0^5 (25x^2 - 10x^3 + x^4) dx}{\left(\frac{125}{6}\right)} \quad \text{[From the calculation of } \bar{x}\text{]}$$

$$= \frac{\frac{1}{2}\left[\frac{25x^3}{3} - \frac{10x^4}{4} + \frac{x^5}{5}\right]_0^5}{\frac{125}{6}}$$

$$= \frac{\frac{1}{2}\left[\frac{25 \times 125}{3} - \frac{10 \times 625}{4} + 625\right]}{\frac{125}{6}} = \frac{6 \times 625}{2 \times 125}\left[\frac{5}{3} - \frac{10}{4} + 1\right]$$

$$= (3 \times 5)\left[\frac{5 \times 4 - 10 \times 3}{12} + 1\right]$$

$$= 15\left[1 - \frac{10}{12}\right] = 15 \times \frac{2}{12} = \frac{15}{6}$$

$$= \frac{5}{2} = 2.5$$

Hence the centroid of the area lies at (2.5, 2.5) Ans.

1. (d) Solution:

(i)

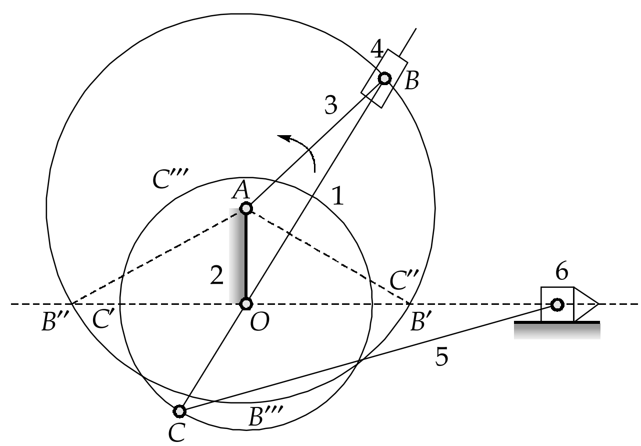
Single slider crank chain

1st inversion applications	:	Reciprocating engine
		Reciprocating compressor
2nd inversion applications	:	Whitworth quick-return mechanism
		Rotary engine
3rd inversion applications	:	Oscillating cylinder engine
		Crank and slotted lever mechanism
4th inversion applications	:	Hand pump

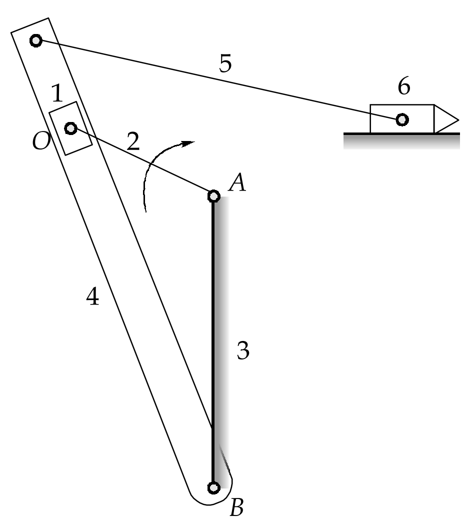
(ii)

Whitworth Quick-Return Mechanism : It is a mechanism used in workshops to cut metals. The forward stroke takes a little longer and cuts the metal whereas the return stroke is idle and takes a shorter period.

Slider 4 rotates in a circle about A and slides on the link 1 as shown in figure. C is a point on the link 1 extended backwards where the link 5 is pivoted. The other end of the link 5 is pivoted to the tool, the forward stroke of which cuts the metal. The axis of motion of the slider 6 (tool) passes through O and is perpendicular to OA, the fixed link. The crank 3 rotates in the counter-clockwise direction.



Crank and Slotted-Lever Mechanism : If the cylinder of an oscillating cylinder engine is made in the form of a guide and the piston in the form of a slider, the arrangements as shown in figure below, is obtained. As the crank rotates about A, the guide 4 oscillates about B. At a point C on the guide, the link 5 is pivoted, the other end of which is connected to the cutting tool through a pivoted joint.



Comparing a crank and slotted-lever quick-return mechanism with a Whitworth quick-return mechanism, the following observations are made:

- Crank 3 of the Whitworth mechanism is longer than its fixed link 2 whereas the crank 2 of the slotted-lever mechanism is shorter than its fixed link 3.
- Coupler link 1 of the Whitworth mechanism makes complete rotations about its pivoted joint O with the fixed link. However, the coupler link 4 of the slotted-lever mechanism oscillates about its pivot B.
- The coupler link holding the tool can be pivoted to the main coupler link at any convenient point C in both cases. However, for the same displacement of the tool, it is more convenient if the point C is taken on the extension of the main coupler link (towards the pivot with the fixed link) in case of the Whitworth mechanism and beyond the extreme position of the slider in the slotted-lever mechanism.

1. (e) Solution:

$$\text{Initial volume, } V = \frac{\pi}{4}d^2 l = \frac{\pi}{4}(40 - 2\times 0.6)^2 \times 90 = 106427 \text{ cc}$$

E = 100 GPa, ; μ = 0.33; K_{oil} = 2.6 GPa

Change in volume:
$$\frac{\delta V_1}{V} = \frac{Pd}{2tE}\left[\frac{5}{2} - 2\mu\right]$$
$$= \frac{5\times 10^6 (40 - 2\times 0.6)\times 10^{-2}}{2\times 6\times 10^{-3}\times 100\times 10^9}[2.5 - 2\times 0.33]$$
$$= 0.002964$$
$$\delta V_1 = 0.002964 \times 106427 = 315.45 \text{ cc}$$

The liquid shrinkage volume can be calculated as:

$$\frac{5\times 10^6}{\frac{\delta V_2}{V}} = 2.6 \times 10^9 \qquad \left[\frac{\sigma_n}{e_v} = \frac{P}{e_v} = K\right]$$

$$\delta V_2 = \frac{5\times 10^6 \times 106427}{2.6\times 10^9} = 204.67 \text{ cc}$$

Net addition of oil which must be pumped into the shell,

$$\delta V = \delta V_1 + \delta V_2$$
$$= 315.45 + 204.67$$
$$= 520.12 \text{ cc (or cm}^3\text{)}$$

Ans.

2. (a) Solution:

The inside diameter of the brass sleeve after change in temperature can be expressed as

$$d_{\text{final }B} = d_{initial\,B} + \Delta d_B$$

$$= d_{initial\,B} + \alpha_B \Delta T d_{initial\,B} \quad \text{...(i)}$$

Similarly, the outside diameter of the steel shaft after the same change in temperature is given by

$$D_{\text{Final }S} = D_{initial\,S} + \Delta D_S$$

$$= D_{initial\,S} + \alpha_S \Delta T D_{initial\,S} \quad \text{...(ii)}$$

From the problem statement, we are trying to determine the temperature change that will cause the inside the diameter of the brass sleeve to be 0.05 mm greater than the outside diameter of the steel shaft. This requirement can be expressed as

$$d_{\text{final }B} = D_{\text{final }S} + 0.05 \text{ mm} \quad \text{...(iii)}$$

Substituting equation (i) and (ii) into equation (iii) to obtain the following relationship

$$d_{initial\,B} + \alpha_B \Delta T\, d_{initial\,B} = D_{initial\,S} + \alpha_S \Delta T\, D_{initial\,S} + 0.05 \text{ mm}$$

Collect the terms with ΔT on the left-hand side of the equation

$$\alpha_B \Delta T\, d_{initial\,B} - \alpha_S \Delta T\, D_{initial\,S} = D_{initial\,S} - d_{initial\,B} + 0.05 \text{ mm}$$

Factor out ΔT,

$$\Delta T\left[\alpha_B d_{initial\,B} - \alpha_S D_{initial\,S}\right] = D_{initial\,S} - d_{initial\,B} + 0.05 \text{ mm}$$

and thus, ΔT can be expressed as

$$\Delta T = \frac{D_{initial\,S} - d_{initial\,B} + 0.05 \text{ mm}}{\alpha_B d_{initial\,B} - \alpha_S D_{initial\,S}}$$

Solve for the temperature change,

$$\Delta T = \frac{300 \text{ mm} - 299.75 \text{ mm} + 0.05 \text{ mm}}{(17.6 \times 10^{-6}/°\text{C})(299.75 \text{ mm}) - (11.9 \times 10^{-6}/°\text{C})(300 \text{ mm})}$$

$$= \frac{0.30 \text{ mm}}{0.005276 \text{ mm}/°\text{C} - 0.003480 \text{ mm}/°\text{C}}$$

$$= \frac{0.30 \text{ mm}}{0.001796 \text{ mm}/°\text{C}} = 167.04°\text{C}$$

Initially, the shaft and sleeve were at a temperature of 25°C. With the temperature change determine above, the temperature at which the sleeve fits over the shaft with a gap of 0.05 mm is

$$T_{\text{final}} = T_{initial} + \Delta T = 25°C + 167.04°C = 194.04°C \quad \text{Ans.}$$

2. (b) Solution:

Length, L = 2 m = 200 mm

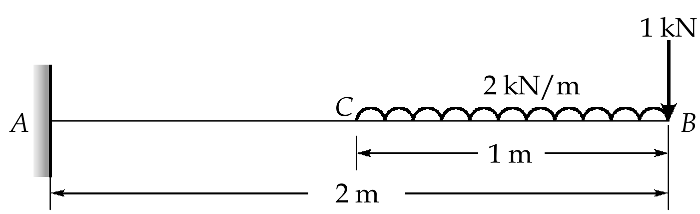
UDL, $w = 2\text{ kN/m} = \dfrac{2\times1000}{1000}\text{N/mm} = 2\text{ N/mm}$

Length BC, a = 1 m = 1000 mm

Point load, W = 1 kN = 1000 N

Value of $E = 2.1 \times 10^5\ \text{N/mm}^2$

Value of $I = 6.667 \times 10^7\ \text{mm}^4$



Deflection at the free end,

Let y_1 = Deflection at the free end due to point load of 1000 N; y_2 = Deflection at the free end due to UDL on length BC

The deflection at the free end due to point load 1000 N is given as

$$y_1 = \frac{WL^3}{3EI} \qquad [\because \text{Here } y_1 = y_B]$$

$$= \frac{1000\times2000^3}{3\times2.1\times10^5\times6.667\times10^7} = 0.1904 \text{ mm}$$

The deflection at the free end due to UDL of 2 N/mm over a length of 1 m from the free end is given as

$$y_2 = \frac{wL^4}{8EI} - \left[\frac{w(L-a)^4}{8EI} + \frac{w(L-a)^3}{6EI}\times a\right]$$

$$= \frac{2\times2000^4}{8\times2.1\times10^5\times6.667\times10^7} - \left[\frac{2(2000-1000)^4}{8\times2.1\times10^5\times6.667\times10^7} + \frac{2(2000-1000)^3\times1000}{6\times2.1\times10^5\times6.667\times10^7}\right]$$

$$= 0.2857 - [0.01785 + 0.0238] = 0.244 \text{ mm}$$

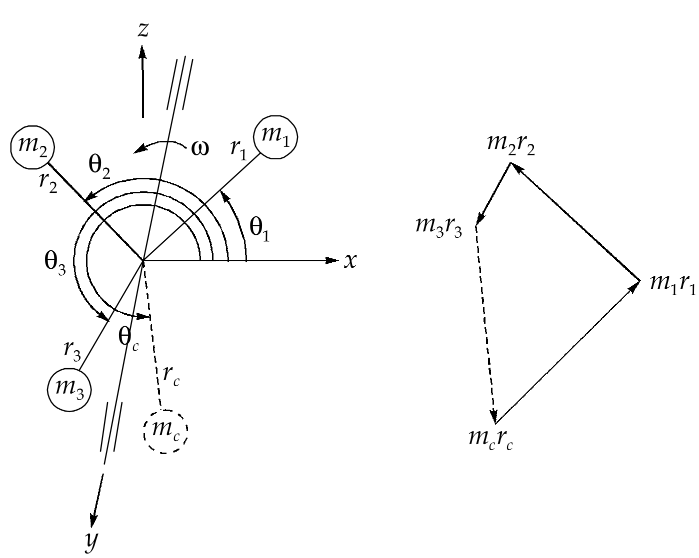
$\therefore$ Total deflection at the free end = $y_1 + y_2$ = 0.1904 + 0.244 = 0.4344 mm Ans.

2. (c) Solution:

Microconstituent	Phases present	Arrangement of phases	Mechanical properties (Relative)
Spheroidite	α Ferrite + Fe_3 C	Relatively small Fe_3C sphere-like particles in an α-ferrite matrix	Soft and ductile
Coarse pearlite	α Ferrite + Fe_3 C	Alternating layers of α-ferrite and Fe_3C that are relatively thick	Harder and stronger than spheroidite, but not as ductile as spheroidite
Fine pearlite	α Ferrite + Fe_3 C	Alternating layers of α-ferrite and Fe_3C that are relatively thin	Harder and stronger than coarse pearlite, but not as ductile as coarse pearlite
Bainite	α Ferrite + Fe_3 C	Very fine and elongated particles of Fe_3C in an α-ferrite matrix	Harder and strength greater than fine pearlite; hardness less than martensite; ductility greater than martensite
Tempered martensite	α Ferrite + Fe_3 C	Very small Fe_3C sphere-like particles in an α-ferrite matrix	Strong; not as hard as martensite, but much more ductile than martensite
Martensite	Body-centered tetragonal, single phase	Needle-shaped grains	Very hard and very brittle

2. (d) Solution:

Following figure shown the various masses:



$$m_1r_1 = 5 \times 75 = 375$$

$$m_1r_1 = 4 \times 85 = 340$$

$$m_3 r_3 = 3 \times 50 = 150$$

$$\sum mr + m_c r_c = 0$$

$$375 \cos 45° + 340 \cos 135° + 150 \cos 240° + m_c r_c \cos\theta_c = 0$$

$$265.165 - 240.416 - 75 + m_c r_c \cos\theta_c = 0$$

$$m_c r_c \cos\theta_c = 240.416 + 75 - 265.165$$

$$\Rightarrow \quad m_c r_c \cos\theta_c = 50.25 \quad \text{...(i)}$$

Also, $375 \sin 45° + 340 \sin 135° + 150 \sin 240° + m_c r_c \sin\theta_c = 0$

$$265.165 + 240.416 - 129.9 + m_c r_c \sin\theta_c = 0$$

$$m_c r_c \sin\theta_c = 129.9 - 265.165 - 240.416$$

$$m_c r_c \sin\theta_c = -375.68 \quad \text{...(ii)}$$

Squaring and adding equation (i) and equation (ii)

$$m_c r_c = \sqrt{(50.25)^2 + (375.68)^2} = 379$$

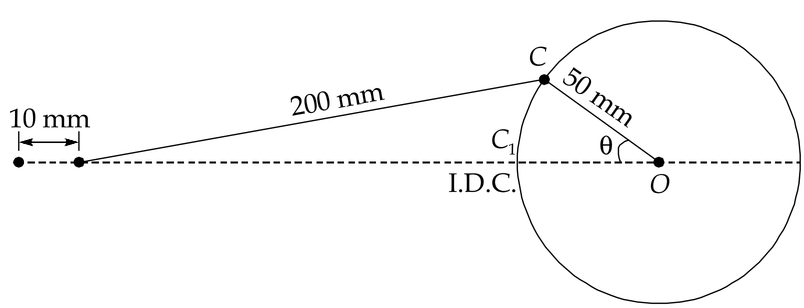
$$m_c = \frac{379}{75} = 5.0533 \text{ kg} \quad \text{Ans.}$$

$$\tan\theta_c = \frac{-375.68}{50.25} = -7.4762$$

$$\theta_c = 277.62°$$

θ_c lies in the fourth quadrant as numerator is negative and denominator is positive.

3. (a) Solution:



Given data: $N = 1800$ rpm, $\omega = 2\pi N/60 = 188.52$ rad/s, $r = 50$ mm $= 0.05$ m,
$l = 200$ mm $= 0.2$ m, $D = 80$ mm, $m_R = 1$ kg, $P = 0.7$ N/mm^2, $x = 10$ mm

$$\text{Piston load, } F_L = \frac{\pi}{4} D^2 P = \frac{\pi}{4} \times (80)^2 \times 0.7 = 3520 \text{ N}$$

We know that, $$x = r\left[(1-\cos\theta)+\frac{\sin^2\theta}{2n}\right] \qquad \left[n = \frac{l}{r} = 4\right]$$

$$= r\left[(1-\cos\theta)+\frac{1-\cos^2\theta}{2n}\right]$$

$$10 = 50\left[(1-\cos\theta)+\frac{1-\cos^2\theta}{2\times 4}\right]$$

$$10 = 40\left[1-\cos\theta+\frac{1-\cos^2\theta}{8}\right]$$

$$10 = 50 - 50\cos\theta + 6.25 - 6.25\cos^2\theta$$

$$6.25\cos^2\theta + 50\cos\theta - 46.25 = 0$$

Solving this quadratic equation, we get

$$\theta = 33.14° \simeq 33°$$

Inertia force on reciprocating parts,

$$F_I = m_R Q_R = m_R\omega^2 r\left(\cos\theta+\frac{\cos 2\theta}{n}\right)$$

$$= 1\times(188.52)^2\times 0.05\left(\cos 33°+\frac{\cos 66°}{4}\right)$$

$$= 1671 \text{ N}$$

Net load on the gudgeon pin,

$$F_P = F_L - F_I = 3520 - 1671 = 1849 \text{ N}$$

Let ϕ = Angle of inclination of the connecting rod to the line of stroke

We know that, $$\sin\phi = \frac{\sin\theta}{n} = \frac{\sin 33°}{4} = \frac{0.5446}{4} = 0.1361$$

or, $$\phi = \sin^{-1}(0.1361) = 7.82°$$

Thrust in the connecting rod,

$$F_Q = \frac{F_P}{\cos\phi} = \frac{1849}{\cos 7.82°} = 1866.3 \text{ N}$$

Reaction between the piston and cylinder:

$$F_N = F_P\tan\phi = 1849\tan 7.82° = 254 \text{ N}$$

Net load, $F_P = F_L - F_I = 0$

i.e. $F_L = F_I$

$$m_R(\omega_1)^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right] = \frac{\pi}{4}D^2P$$

$$(\omega_1)^2 \times 0.05\left[\cos 33° + \frac{\cos 66°}{4}\right] = \frac{\pi}{4}(80)^2 \times 0.7$$

$$0.047\omega_1^2 = 3520$$

$$\omega_1^2 = \frac{3520}{0.047} = 74894 \text{ or } \omega_1 = 273.6 \text{ rad/s}$$

$$N_1 = \frac{\omega_1 \times 60}{2\pi} = \frac{273.6 \times 60}{2\pi} = 2612 \text{ rpm}$$

3. (b) Solution:

Given: $G = \frac{T_G}{T_P} = \frac{D_G}{D_P} = 10$; $L = 660$ mm; P = 500 kW = 500 × 10^3 W; $N_P = 1800$ rpm;

$\phi = 22.5°$; $W_N = 175$ N/mm width

(i)

Let m = Required module, T_P = Number of teeth on the pinion

T_G = Number of teeth on the gear

T_P = Number of teeth on the pinion,

D_P = Pitch circle diameter of the pinion, and

D_G = Pitch circle diameter of the gear

We know that minimum number of teeth on the pinion in order to avoid interference,

$$T_P = \frac{2A_W}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]}$$

$$= \frac{2 \times 1}{10\left[\sqrt{1 + \frac{1}{10}\left(\frac{1}{10} + 2\right)\sin^2 22.5°} - 1\right]}$$

$$= \frac{2}{0.15} = 13.3 \simeq 14 \qquad ...(\because A_W = 1 \text{ module})$$

$\therefore$ $T_G = G \times T_P = 10 \times 14 = 140$ $\quad ...\left(\because \frac{T_G}{T_P} = 10\right)$

We know that $$L = \frac{D_G}{2} + \frac{D_P}{2} = \frac{10D_P}{2} + \frac{D_P}{2} = 5.5D_P \qquad \ldots\left(\because \frac{D_G}{D_P} = 10\right)$$

$\Rightarrow$ $$660 = 5.5\,D_P$$

$\Rightarrow$ $$D_P = 120 \text{ mm}$$

$\therefore$ $$m = \frac{D_P}{T_P} = \frac{120}{14} = 8.6 \text{ mm}$$

Since, the nearest standard value of the module is 8 mm, therefore we shall take

$$m = 8 \text{ mm}$$

(ii) Number of teeth on each wheel

We know that number of teeth on the pinion,

$$T_P = \frac{D_P}{m} = \frac{120}{8} = 15$$

and number of teeth on the gear,

$$T_G = G \times T_P = 10 \times 15 = 150$$

(iii) Necessary width of the pinion

We know that the torque acting on the pinion

$$T = \frac{P \times 60}{2\pi N_P} = \frac{500 \times 10^3 \times 60}{2\pi \times 1800} = 2652 \text{ Nm}$$

$\therefore$ Tangential load, $$W = \frac{T}{D_P/2} = \frac{2652}{0.12/2} = 44200 \text{ N} \qquad \ldots[\because D_P \text{ is taken in metres}]$$

and normal load on the tooth,

$$W_N = \frac{W_T}{\cos\phi} = \frac{44200}{\cos 22.5°} = 47840 \text{ N}$$

Since the normal pressure between teeth is 175 N per mm of width, therefore necessary width of the pinion.

$$b = \frac{47840}{175} = 273.4 \text{ mm}$$

(iv) Load on the bearing of the wheels

We know that the radial load on the bearings due to the power transmitted.

$$W_R = W_N \cdot \sin\phi = 47840 \times \sin 22.5°$$
$$= 18308 \text{ N} = 18.308 \text{ kN}$$

3. (c) Solution:

(i)

Given : D = 240 mm, d = 120 mm, μ = 0.2, P_a = 1 N/mm^2, n = 750 rpm

$$\text{Operating force, } P = \frac{\pi P_a d}{2}(D - d)$$

$$= \frac{\pi}{2} \times 1 \times 120(240 - 120) = 22619.467 \text{ N}$$

$$\text{Torque, } M_t = \frac{\mu P}{4}(D + d) = \frac{0.2 \times 22619.467}{4}(240 + 120)$$

$$= 407150.4 \text{ Nmm}$$

$$\text{Power} = \frac{2\pi n M_t}{60 \times 10^6} = \frac{2\pi \times 750 \times 407150.4}{60 \times 10^6} = 31.97 \text{ kW}$$

(ii)

Given: hollow cylindrical shaft d_0 = 60 mm,

d_i = 40 mm, l = 1.5 m, τ_{max} = 150 N/mm^2

(a)

Let T be the largest torque that can be applied to the shaft.

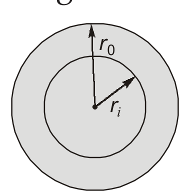
$$\tau_{max} = \frac{16T}{\pi d_o^3 (1 - K^4)}$$

where, $$K = \frac{d_i}{d_o} = \frac{40}{60} = 0.66$$

$$\tau_{max} = \frac{16T}{\pi (60)^3 [1 - 0.66^4]} = 150$$

$$T = 5.1025 \text{ kN-m}$$

(b) The minimum value of the shearing stress in the shaft is at its inner radius (r_i)



$$\frac{\tau_{max}}{r_{max}} = \frac{\tau_{min}}{r_{min}}$$

$$\Rightarrow \quad \frac{150}{30} = \frac{\tau_{min}}{20}$$

$$\Rightarrow \quad \tau_{min} = 100 \text{ MPa}$$

4. (a) Solution:

The differential equation of motion is given by

$$m\ddot{x} + kx = 0$$

Since, m = 0.5 kg and k = 300 N/m, the above differential equation can be written as

$$0.5\ddot{x} + 300x = 0$$

The natural frequency ω is given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{300}{0.5}} = 24.495 \text{ rad/s}$$

The frequency f is given by

$$f = \frac{\omega}{2\pi} = \frac{24.495}{2\pi} = 3.898 \text{ Hz}$$

The response of the system is given by

$$x = X\sin(\omega t + \phi)$$

where,
$$X = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega}\right)^2} = \sqrt{(0.01)^2 + 0} = 0.01$$

$$\phi = \tan^{-1}\left(\frac{x_0}{\dot{x}_0/\omega}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

that is,
$$x = 0.01 \cos 24.495t$$

For this simple conservative system, the system total energy is equal to the maximum kinetic energy or the maximum strain energy. Therefore, has

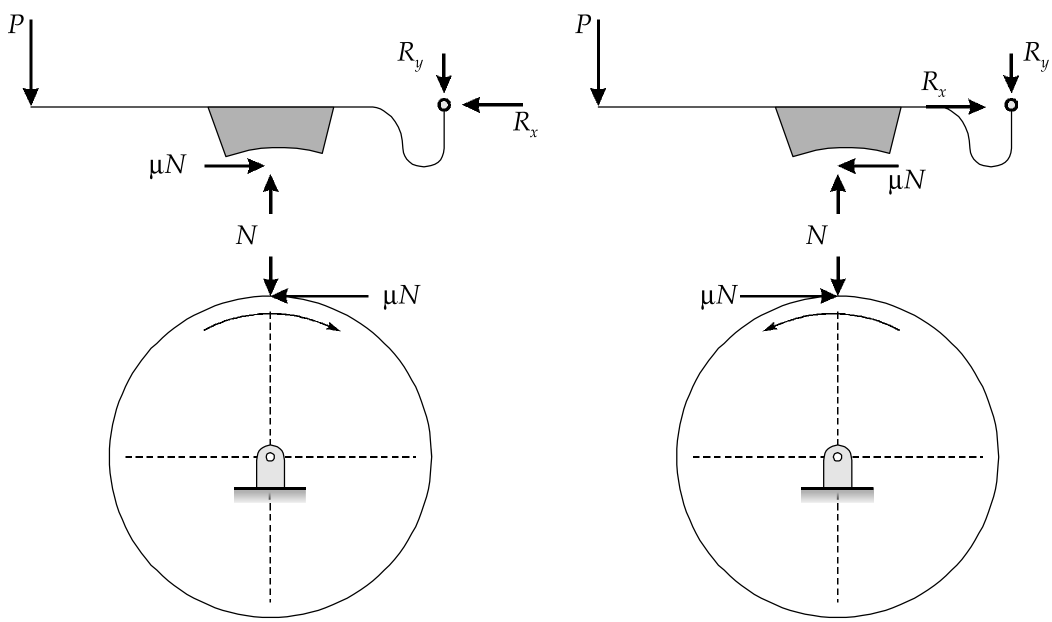
$$E = \frac{1}{2}kX^2 = \frac{1}{2}(300)(0.01)^2 = 0.015 \text{ Nm}$$

This is the same as the maximum kinetic energy given by

$$E = \frac{1}{2}m(\omega X)^2 = \frac{1}{2}(0.5)(24.495 \times 0.01)^2 = 0.015 \text{ Nm}$$

4. (b) Solution:

Refer following figure:



(i) FBD for CW Rotation **(ii) FBD for CCW Rotation**

M_t = 250 Nm, n = 120 rpm, μ = 0.3, P = 1 N/mm^2, l = 2w

For clockwise rotation (figure (i))

$$N = \frac{M_t}{\mu R} = \frac{250 \times 1000}{0.3 \times 200} = 4166.67 \text{ N}$$

Taking moments about the hinge-pin,

$$\mu N(50) + P(500) - \mathbf{N}(200) = 0$$

$$0.3 \times 4166.67 \times 50 + P(500) - 4166.67 \times 200 = 0$$

$$P = 1541.67 \text{ N}$$

$$R_x = \mu N = 0.3 \times 4166.67 = 1250 \text{ N}$$

$$R_y = N - P = 4166.67 - 1541.67 = 2625 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(1250)^2 + (2625)^2}$$

$$= 2907.426 \text{ N}$$

For Anti-clockwise rotation [Figure (ii)]

Taking moments about the hinge-pin;

$$-\mu N(50) + P(500) - N(200) = 0$$

$$-0.3 \times 4166.67 \times 50 + P(500) - 4166.67 \times 200 = 0$$

$$P = 1791.67 \text{ N}$$

$$R_x = \mu N = 0.3 \times 4166.67 = 1250 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(1250)^2 + (2916.67)^2} = 3173.24 \text{ N}$$

Rate of Heat Generation

$$\text{Initial velocity of drum, } \omega r = \frac{2\pi \times 120}{60} \times 0.2 = 2.513 \text{ m/s}$$

The rate of heat generated during the braking period is equal to the rate of work done by the frictional force.

$$\begin{aligned} \text{Rate of heat generated} &= \text{Frictional force} \times \text{Average velocity} \\ &= \mu N \times 1.2565 \\ &= 0.3 \times 4166.67 \times 1.2565 \\ &= 1570.63 \text{ Nm/s or W} \end{aligned}$$

Dimensions of the block

$$N = plw$$

$$4166.67 = 1 \times 2w \times w$$

or $$w = 45.64 \simeq 46 \text{ mm}$$

$$l = 2w = 2 \times 46 = 92 \text{ mm}$$

Self locking property

$$a = 200 \text{ mm}, c = 50 \text{ mm}, \mu = 0.3$$

$$\frac{a}{c} = 4$$

$$\frac{a}{c} > \mu$$

The brake is not self-locking.

4. (c) Solution:

The circular frequency of the system is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4000}{10}} = 20 \text{ rad/s}$$

The frequency ratio r is given by

$$r = \frac{\omega_r}{\omega} = \frac{40}{20} = 2$$

The critical damping coefficient C_c is defined as

$$C_c = 2\,m\omega = 2 \times 10 \times 20 = 400 \text{ N.s/m}$$

The damping factor ξ is then given by

$$\xi = \frac{c}{C_c} = \frac{40}{400} = 0.1$$

which is the case of an underdamped system, and the complete solution can be written in the following form

$$x(t) = x_h + x_p$$

$$= Xe^{-\xi\omega t}\sin(\omega_d t + \phi) + X_0\beta\sin\left(\omega_f t - \psi\right)$$

where ω_d is the damped circular frequency

$$\omega_d = \omega\sqrt{1-\xi^2} = 20\sqrt{1-(0.1)^2} = 19.8997 \text{ rad/s}$$

The constants X_0, β and ψ are

$$X_0 = \frac{F_0}{k} = \frac{60}{4000} = 0.015 \text{ m}$$

$$\beta = \frac{1}{\sqrt{\left(1-r^2\right)^2 + (2r\xi)^2}} = \frac{1}{\sqrt{\left(1-(2)^2\right)^2 + (2\times 2\times 0.1)^2}}$$

$$= 0.3304$$

$$\psi = \tan^{-1}\left(\frac{2r\xi}{1-r^2}\right) = \tan^{-1}\frac{2\times 2\times 0.1}{1-(2)^2}$$

$$= \tan^{-1}(-0.13333) = -0.13255 \text{ rad}$$

The displacement can then be written as a function of time as

$$x(t) = Xe^{-2t}\sin(19.8997t + \phi) + 0.004956\sin(40t + 0.13255)$$

The constants X and ϕ can be determined using the initial conditions. The transmissibility β_t is defined as

$$\beta_t = \frac{\sqrt{1+(2r\xi)^2}}{\sqrt{\left(1-r^2\right)^2 + (2r\xi)^2}}$$

$$= \frac{\sqrt{1+(2\times2\times0.1)^2}}{\sqrt{\left(1-(2)^2\right)^2+(2\times2\times0.1)^2}} = 0.35585$$

The amplitude of the force transmitted is given by

$$|F_t| = F_0\beta_t = 60 \times 0.35858 = 21.351 \text{ N}$$

5. **(a) Solution:**

(i)

Material properties depend upon the crystal structure because properties are directly linked with the slip systems. The slip systems are the number of directional crystal planes along which the dislocations move with an ease. In FCC crystal system, there are 12 such slip system, so there are 12 different directions on crystal plane along which the dislocations can move very easily. So whatever may be type adn direction of applied load, there will be always at least one direction along which the dislocations may be moving. Since failure due to dislocation motion is ductile failure, so FCC crystal will always fail in ductile mode.

Although BCC has 24 slip systems but it not called a ductile material because atomic packing factor of BCC is lower, so the average distance between atoms is more than FCC crystal structure. So to jump the edge dislocation from one side to another side, larger stressess will be required. When larger stresses are required to move dislocation, it is said that strength of the material has increased. This is the reason BCC crystals are always stronger in comparision to FCC and less ductile.

(ii)

As we know that there are five phases of iron, if we want a stronger material at room temperature, since α-phase has BCC structure. To obtain ferrite structure, we add chromium as an alloying element. Since the lines on phase diagram and TTT diagram change by adding alloying elements, chromium addition results in shifting the solves line to the right. Thus more amount of ferrite is found at room temperature.

5. **(b) Solution:**

Oil flow rate from pump, $Q = 0.002 \text{ m}^3/\text{s}$

Diameter of the cylinder, $D = 50 \text{ mm} = 0.05 \text{ m}$

Diameter of the rod, $d = 20 \text{ mm} = 0.02 \text{ m}$

Load during the extension and retraction, $F = 6000 \text{ N}$

Piston velocity during extension stroke, $V_E = \dfrac{Q}{A_P}$

$$V_E = \frac{0.002}{\frac{\pi}{4}\left(0.05^2 - 0.02^2\right)} = 1.2 \text{ m/s}$$

Cylinder pressure during extension stroke,

$$P_E = \frac{F}{A_P} = \frac{6000}{\frac{\pi}{4} \times 0.05^2} = 30.6 \text{ bar}$$

Cylinder pressure during retraction stroke,

$$P_R = \frac{F}{A_P - A_R}$$

$$P_R = \frac{6000}{\frac{\pi}{4}\left(0.05^2 - 0.02^2\right)} = 36.4 \text{ bar}$$

Cylinder power during extension stroke $= \frac{P_E Q}{1000} = \frac{30.6 \times 10^5 \times 0.002}{1000} = 6.12 \text{ kW}$

Cylinder power during retraction stroke $= \frac{P_E Q}{1000} = \frac{36.4 \times 10^5 \times 0.002}{1000} = 7.28 \text{ kW}$

5. (c) Solution:

First of all, we have to convert these engineering stress and strain into true stress and true strain,

$$\sigma_T = \sigma(1 + e)$$

$$\sigma_{T_1} = 235(1 + 0.194) = 280 \text{ MPa}$$

$$\sigma_{T_2} = 250(1 + 0.296) = 324 \text{ MPa}$$

Similarly, true strain is given as

$$\in_T = \ln(1 + e)$$

$$\in_{T_1} = \ln(1 + 0.194) = 0.177$$

$$\in_{T_2} = \ln(1 + 0.296) = 0.259$$

True stress and true strain are related as

$$\sigma_T = K \in_T^n$$

Taking logarithm of this equation:

$$\log \sigma_T = \log K + n \log \in_T$$

Substituting the values:

$$\log(280) = \log K + n\log(0.177) \quad \text{...(i)}$$

$$\log(324) = \log K + n\log(0.259) \quad \text{...(ii)}$$

Solving equations (i) and (ii), we get

$$K = 543 \text{ MPa and } n = 0.383$$

$$\in_T(\text{for } e = 0.25) = \ln(1 + 0.25) = 0.223$$

Corresponding true stress can be calculated as

$$\sigma_T = K \in_T^n$$

$$= 0.543(0.223)^{0.383} = 306 \text{ MPa}$$

Converting this σ_T into engineering stress, we get

$$\sigma = \frac{\sigma_T}{1+e} = \frac{306}{1.25} = 245 \text{ MPa}$$

5. (d) Solution:

(i)

Low Carbon Steels

Properties: nonresponsive to hear treatments; relatively soft and weak; machinable and weldable. Typical applications: automobile bodies, structural shapes, pipelines, buildings, bridges and tin cans.

Medium Carbon Steels

Properties: hear treatable; relatively large combinations of mechanical characteristics. Typical applications: railway wheels and tracks, gears, crankshafts, and machine parts.

High Carbon Steels

Properties: hard, strong, and relatively brittle.

Typical applications: chisels, hammers, knives, and hacksaw blades.

High Alloy Steels (Stainless and Tool)

Properties: hard and wear resistant; resistant to corrosion in a large variety of environments. Typical applications: cutting tools, drills, cutlery, food processing, and surgical tools.

(ii)

***(a)* With respect to composition and heat treatment:**

Gray iron - 2.5 to 4.0 wt% C and less than 1.0 wt% Si. White iron is heated in a nonoxidizing atmosphere and at a temperature between 800 and 900°C for an extended time period.

***(b)* With respect to microstructure:**

Gray iron - Graphite flakes are embedded in a ferrite or pearlite matrix.

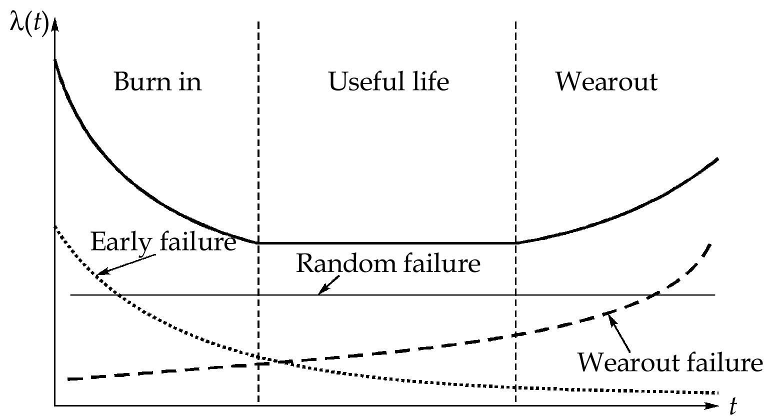
Malleable iron - Graphite clusters are embedded in a ferrite or pearlite matrix.

***(c)* With respect to mechanical characteristics:**

Gray iron - Relatively weak and brittle in tension; good capacity for damping vibrations.

Malleable iron - Moderate strength and ductility.

5. (e) Solution:



The bath tub curve

	Characterized by	Caused by	Reduced by
Burn-in	DFR	Manufacturing defects: welding flaws, cracks, defective parts, poor quality control, contamination, poor workmanship	Burn-in testing Screening Quality control acceptance testing
Useful life	CFR	Environment Random loads Human error "Acts of God" Chance events	Radundancy Excess strength
Wear-out	IFR	Fatigue Corrosion Aging Friction Cyclical loading	Derating Preventive maintenance Parts replacement Technology

6. (a) Solution:

Sound Fields

The sound waves that emanate from a machinery source travel in all directions. However, depending on the interaction at the boundaries, they are absorbed, or reflected, or even transmitted further, or a combination of these phenomena occurs depending on the nature of the boundary. This is due to the impedance of the boundary. The sound waves are thus amplified or attenuated depending upon the boundary, and these conditions lead to different sound fields.

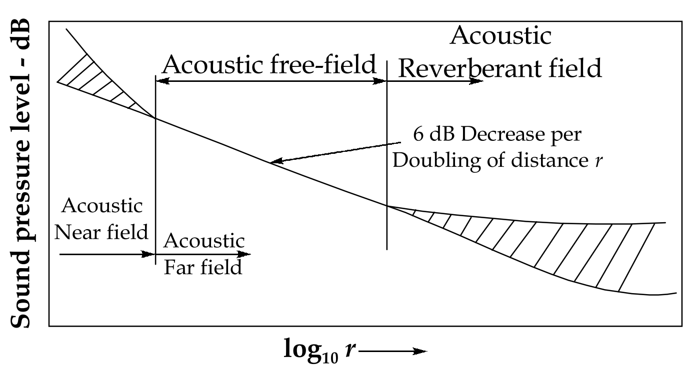
Near-Field Condition

The region very close to the source of sound has a sound level that is almost constant and equal to the maximum level of the generated sound. Thus, while monitoring the noise of a machine, measurements in this region should be avoided. The approximate distance from the machine where such a near-field condition exists is equal to one length of the largest dimension of the machine.

Far-Field Condition

Any region beyond the near field of the machinery is known as its far field. The far field can be further divided into two distinct regions-the free field region and the reverberant field region. In the free-field region it is assumed that there are no strong reflected waves, and for a point spherical source, the sound pressure level reduces by 6 dB for every doubling of the distance from the source. While performing noise monitoring of machines, it is preferred that the noise is measured in the free-field conditions where there are no reflections from the boundary.

The second region in the far field close to the boundary is known as the reverberant field where, because of strong reflections from hard walls, the sound level increases. On a shop floor where machinery noise is to be monitored, measurements away from hard reflecting walls should be done. The free condition of decrease of 6 dB for every doubling of the distance for a three-dimensional point source is violated. Figure below shows the variation of the noise field from a distance of r from the source.



Variation of sound pressure level at different field conditions.

6. (b) Solution:

(i)

Many industrial robots today move to goal points that have been taught. A taught point is one that the manipulator is moved to physically, and then the joint position sensors are read and the joint angles stored. When the robot is commanded to return to that point in space. each joint is moved to the stored value. In simple "teach and playback" manipulators such as these, the inverse kinematic problem never arises, because goal points are never specified in Cartesian coordinates. When a manufacturer specifies how precisely a manipulator can return to a taught point, he is specifying the repeatability of the manipulator.

Any time a goal position and orientation are specified in Cartesian terms, the inverse kinematics of the device must be computed in order to solve for the required joint angles. Systems that allow goals to be described in Cartesian terms are capable of moving the manipulator to points that were never taught-points in its workspace to which it has perhaps never gone before. We will call such points computed points. Such a capability is necessary for many manipulation tasks. For example, if a computer vision system is used to locate a part that the robot must grasp, the robot must be able to move to the Cartesian coordinates supplied by the vision sensor. The precision with which a computed point can be attained is called the accuracy of the manipulator.

(ii)

Electric actuators are generally those where an electric motor drives robot links through some mechanical transmission, e.g., gears, etc. In the early years of industrial robotics, hydraulic robots were the most common, but recent improvements in electric-motor design have meant that most new robots are of all-electric construction. The first commercial electrically driven industrial robot was introduced in 1974 by ABB. The advantages and disadvantages of an electric motor are the following:

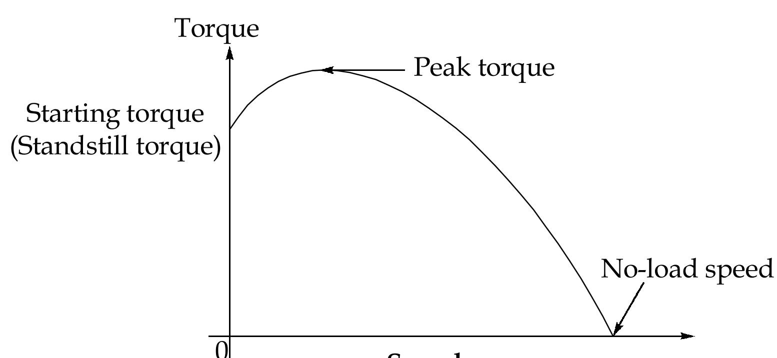
Advantages

- Widespread availability of power supply.
- Basic drive element in an electric motor is usually lighter than that for fluid power, i.e., pressurized fluid or compressed air.
- High power-conversion efficiency.
- No pollution of working environment.
- Accuracy and repeatability of electric drive robots are normally better than fluid power robots in relation to cost.
- Being relatively quiet and clean, they are very acceptable environmentally.

- They are easily maintained and repaired.
- Structural components can be lightweight.
- The drive system is well suited to electronic control.

Disadvantages

- Electrically driven robots often require the incorporation of some sort of mechanical transmission system.
- Additional power is required to move the additional masses of the transmission system.
- Unwanted movements due to backlash and plays in the transmission elements.
- Due to the increased complexity with the transmission system, we need complex control requirement and additional cost for their procurement and maintenance.
- Electric motors are not intrinsically safe, mainly, in explosive environments.



Torque-speed characteristics of a stepper motor

6. (c) Solution:

(i)

$$F_{\text{object}} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} ? & 0 & ? & 3 \\ 0.5 & ? & ? & 9 \\ 0 & ? & ? & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$n_x o_x + n_y o_y + n_z o_z = 0 \text{ or } n_x(0) + 0.5(o_y) + 0(o_z) = 0$$

$$n_x a_x + n_y a_y + n_z a_z = 0 \text{ or } n_x a_x + 0.5 a_y + o = 0$$

$$a_x o_x + a_y o_y + a_z o_z = 0 \text{ or } a_x(0) + a_y o_y + a_z o_z = 0$$

$$n_x^2 + n_y^2 + n_z^2 = 1 \text{ or } n_x^2 + 0.25 + 0 = 1$$

$$o_x^2 + o_y^2 + o_z^2 = 1 \text{ or } o + o_y^2 + o_z^2 = 1$$

$$a_x^2 + a_y^2 + a_z^2 = 1$$

$$n_z = \sqrt{0.75} = 0.866,\ o_y = 0,\ o_z = 1$$

$$\vec{n} \times \vec{o} = \vec{a}$$

$$\left(n_x\vec{i} + n_y\vec{j} + n_z\vec{k}\right) \times \left(o_x\vec{i} + o_y\vec{j} + o_z\vec{k}\right) = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$\left(0.866\vec{i} + 0.5\vec{j} + o\vec{k}\right) \times \left(0.1\vec{i} + 0\vec{j} + \vec{k}\right) = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$0.5\vec{i} - 0.866\vec{j} + 0\vec{k} = a_x\vec{i} + a_y\vec{i} + a_z\vec{k}$$

$$a_x = 0.5,\ a_y = -0.866,\ a_z = 0$$

Complete matrix representation of the frame:

$$F = \begin{bmatrix} 0.866 & 0 & 0.5 & 3 \\ 0.5 & 0 & -0.866 & 9 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(ii)

$$F = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & \cos 45^\circ & -\sin 45^\circ & 5 \\ 0 & \sin 45^\circ & \cos 45^\circ & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0.707 & -0.707 & 5 \\ 0 & 0.707 & 0.707 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6. (d) Solution:

(i)

1. The tool is perfectly sharp and there is no contact along the clearance face.
2. The shear surface is a plane extending upward from the cutting edge.
3. The cutting edge is a straight line extending perpendicular to the direction of motion and generates a plane surface as the work moves past it.
4. The chip does not flow to either side (plane strain).
5. The depth of cut is constant.
6. The width of the tool is greater than that of the workpiece.
7. The work moves relative to the tool with uniform velocity.
8. A continuous chip is produced with no built-up edge.
9. The shear and normal stresses along shear plane and tool are uniform (strength of materials approach). Such an ideal two-dimensional cutting operation is referred to as orthogonal cutting.

(ii)

Assuming Taylor's tool life equation, $VT^n = C$

$$V_1T_1 = V_2T_2 = V_3T_3 = \ldots\ldots\ldots\ldots\ldots = C$$

Here, $V_1 = 60$ m/min; $T_1 = 80$ min

$V_2 = 120$ m/min; $T_2 = 20$ min

$V_3 = ?$ (to be determined); $T_3 = 40$ min

Taking, $$V_1T_1^n = V_2T_2^n$$

i.e. $$\left(\frac{T_1}{T_2}\right)^n = \left(\frac{V_1}{V_2}\right)$$

or $$\left(\frac{80\,\text{min}}{20\text{ min}}\right)^n = \left(\frac{120\,\text{m/min}}{60\text{ m/min}}\right)$$

From which, $n = 0.5$

Again $$V_3T_3^n = V_1T_1^n$$

i.e. $$\left(\frac{V_3}{V_1}\right) = \left(\frac{T_1}{T_3}\right)^n$$

or $$V_3 = \left(\frac{80}{40}\right)^{0.5} \times 60 = 84.84\text{ m/min}$$

7. (a) Solution:

(i)

Vibration signals provide useful information that lead to insights on the operating condition of the equipment under test [1, 2]. By inspecting the physical characteristics of the vibration signals, one is able to detect the presence of a fault in an operating machine, to localise the position of a crack in gear, to diagnose the health state of a ball bearing, etc. For decades, researchers are looking at means to diagnose automatically the health state of rotating machines, from the smaller bearings and gears to the larger combustion engines and turbines. With the advent of wireless technologies and miniature transducers, we are now able to monitor machine operating condition in real time and, with the aid of computational intelligence and pattern recognition technique, in an automated fashion. This paper draws from a collection of past and recent works in the area of automatic machine condition monitoring using vibration signals.

Typically, vibration signals are acquired through vibration sensors. The three main classes of vibration sensors are displacement sensors, velocity sensors, and accelerometers. Displacement sensor can be non-contact as in the case of optical sensors and they are more sensitive in the lower frequency range, typically less than 1 kHz. Velocity sensors, on the other hand, operate more effectively with flat amplitude response in the 10 Hz to 2 kHz range. Among these sensors, accelerometers have the best amplitude response in the high frequency range up to tens of kHz. Usually, accelerometers are built using capacitive sensing, or more commonly, a piezoelectric mechanism. Accelerometers are usually light weight ranging from 0.4 gram to 50 gram.

Advantages of vibration signal monitoring

Vibration signal processing has some obvious advantages. First, vibration sensors are non-intrusive, and at times non-contact. As such, we can perform diagnostic in a non-destructive manner. Second, vibration signals can be obtained online and in-situ. This is a desired feature for production lines. The trending capability al o provide mean to predictive maintenance of the machineries. As such, unnecessary downtime for preventive maintenance can be minimized, Third, the vibration sensors are inexpensive and widely available. Modern mobile smart devices are equipped with one tri-axial accelerometer typically. Moreover, the technologies to acquire and convert the analogue outputs from the sensors are affordable nowadays.

Limitations of vibration signals

Although techniques based on vibration signals are generally versatile, their usefulness is limited by several factors. As mentioned above, different vibration sensors have different operating characteristic, and attention should be paid when selecting a suitable sensor. This would require the engineer to understand the physical characteristics of the vibration source. Also, useful characteristics of the vibration signal could be easily masked by inappropriate mounting of the sensors. The adhesion mechanism may also damp the high frequency component if it is not done correctly. Vibration sensors also come with a wide variety of dynamic ranges and sensitivities. Careful perusal of the datasheet is always recommended when choosing a suitable sensor.

(ii)

1. Break-even volume

Fixed cost (FC) = ₹6000 month

Variable cost (VC) = ₹2 per unit

Selling price (SP) = ₹7 per unit

For Q to be break-even volume,

$$\text{Break even volume} = \frac{\text{Fixed cost}}{\text{Contribution per unit}}$$

Contribution per unit = Selling price per unit – Variable price per unit

= ₹(7 – 2) = 5

$$= \frac{6000}{5} = 1200 \text{ units/month}$$

Selling price (SP) = ₹7 per unit

2. For Q = 1000,

Profit = Sales Revenue – Total cost

= SR – [FC + VC × Q]

= (7 × 1000) – [6000 + 2 × 1000]

= (7000) – (6000 + 2000)

= (₹7000 – 8000) = – ₹1000 (i.e. loss of ₹1000)

3. For profit of ₹4000 and *Q* =

Contribution per unit = Selling price per unit – Contribution per unit

= 7 – 2 = 5

Total contribution = FC + Profit

= 6000 + Profit

= (6000 + 4000) = 10000

$$\frac{10000}{5} = 2000 \text{ units}$$

7. (b)

Let x and y be the number of items M and N respectively.

Total profit on the production = Rs. $(600x + 400y)$

Mathematical formulation of the given problem is as follows:

Maximize, $Z = 600x + 400y$

Subject tot he constraints:

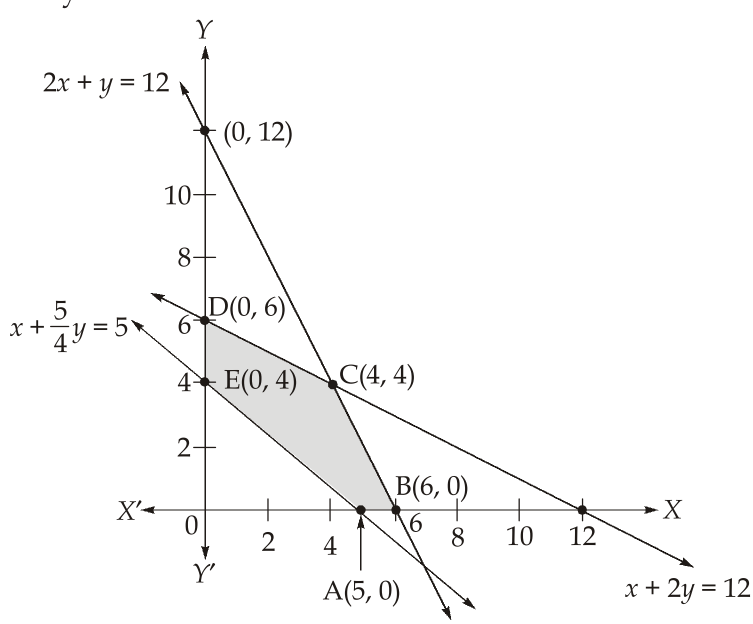
$$x + 2y \leq 12 \text{ (Constraint on machine I)} \quad \text{... (i)}$$

$$2x + y \leq 12 \text{ (Constraint on machine II)} \quad \text{... (ii)}$$

$$x + \frac{5}{4}y \geq 5 \text{ (Constraint on machine III)} \quad \text{... (iii)}$$

$$x \geq 0, y \geq 0 \quad \text{... (iv)}$$

Let us draw the graph of constraints (i) to (iv). ABCDE is the feasible region (shaded) as shown in figure, determined by the constraints (i) to (iv). Observe that the feasible region is bounded, coordinates of the corner points A, B, C, D and E are (5, 0) (6, 0) (4, 4) (0, 6) and (0, 4) respectively.



Let us evaluate, $Z = 600x + 400y$ at these corner points.

Corner point	$Z = 600x + 400y$	
(5, 0)	3000	
(6, 0)	3600	
(4, 4)	4000	Maximum
(0, 6)	2400	
(0, 4)	1600	

We see that the point (4, 4) is giving the maximum value of Z. Hence, the manufacturer has to produce 4 units of each item to get the maximum profit of Rs. 4000.

7. (c) Solution:

(i)

$\frac{V}{A}$ should be maximized for maximizing solidification time.

$$\text{Cylinder volume, } V = \frac{\pi D^2}{4},\ L = \frac{4V}{\pi D^2}$$

$$\text{Cylinder area, } A = 2\times\frac{\pi D^2}{4} + \pi DL$$

$$A = \frac{\pi D^2}{2} + \pi DL$$

Substituting L into this equation:

$$A = \frac{\pi D^2}{2} + \pi D\left(\frac{4V}{\pi D^2}\right) = \frac{\pi D^2}{2} + \frac{4V}{D}$$

Differentiating A w.r.t. D :

$$\frac{dA}{dD} = \pi D - \frac{4V}{D^2} = 0$$

$$\pi D = \frac{4V}{D^2}$$

$$D^3 = \frac{4V}{\pi} \text{ i.e. } D = \left(\frac{4V}{\pi}\right)^{0.333}$$

$$L = \left(\frac{4V}{\pi}\right)^{0.333}$$

Therefore optimal values are

$$D = L = \left(\frac{4V}{\pi}\right)^{0.333}$$

and therefore optimal $\frac{D}{L}$ ratio = 1.0

(ii)

$$\text{Draft} = 20 - 18 = 2 \text{ mm}$$

$$\text{Contact length, } L = [\text{Roll radius} \times \text{Draft}]^{1/2}$$

$$= (250 \times 2)^{0.5} = 22.36 \text{ mm} = 0.02236 \text{ m}$$

$$\text{True strain, } \in = \ln\left(\frac{20}{18}\right) = 0.1054$$

$$\text{Flow strain, } \overline{Y}_f = \frac{K \in^n}{1+n}$$

$$= \frac{600(0.1054)^{0.22}}{1.22} = 300 \text{ MPa}$$

$$\text{Rolling force, } F = 0.2\ \overline{Y}_f\ L$$

$$= 0.2 \times 300 \times 0.02236 = 1341600 \text{ N}$$

$$\text{Torque per roll, } T = 0.5 \times F \times L$$

$$= 0.5 \times 1341600 \times 0.02236 = 15000 \text{ Nm}$$

$$\text{Power per roll, } P = \frac{2\pi N}{60} \times 2T = \frac{2\pi 12}{60} \times 2 \times 15000$$

$$= 37700 \text{ W}$$

8. (a)

(i)

Inventory Carrying Costs (Holding Costs): These are the costs associated with holding a given level of inventory on hand and this cost vary in direct proportion to the amount of holding and period of holding the stock in stores. The holding costs include.

1. Storage costs (rent, heating, lighting, etc).
2. **Handling costs:** Costs associated with moving the items such as cost of labour, equipment for handling.
3. Depreciation, taxes and insurance.
4. Costs on record keeping.
5. Product deterioration and obsolescence.
6. Spoilage, breakage, pilferage and loss due to the perishable nature.

Shortage Costs: When there is a demand for the product and the item needed is not in stock, then we incur shortage cost or cost associated with stock out. The shortage costs include:

1. Backorder costs.
2. Loss of future sales.
3. Loss of customer goodwill.
4. Extra cost associated with urgent, small quantity ordering costs.
5. Loss of profit contribution by lost sales revenue.

The unsatisfied demand can be satisfied at a later stage (by means of back orders) or unfulfilled demand is lost completely (no back ordering, the shortage costs become proportional to only the shortage quantity).

(ii)

$$\text{Annual requirement, } D = 12500 \text{ units}$$

$$\text{Ordering costs, } C_o = \text{Rs. } 2000/\text{year}$$

$$\text{Inventory carrying costs, } C_h = \text{Rs. } 48/\text{unit}/\text{annum}$$

$$\text{Economic order quantity, } Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2 \times 12500 \times 2000}{48}} = 1020.62$$

$$= 1021 \text{ numbers}$$

$$\text{No. of orders to be placed in a year} = \frac{D}{Q^*} = \frac{12500}{1021} = 12.24 = 12$$

The re-order point (ROP) = Lead time consumption

Lead time is given as 1/2 month.

$$\text{Re-order point (ROP)} = \frac{12500}{24} = 520.83$$

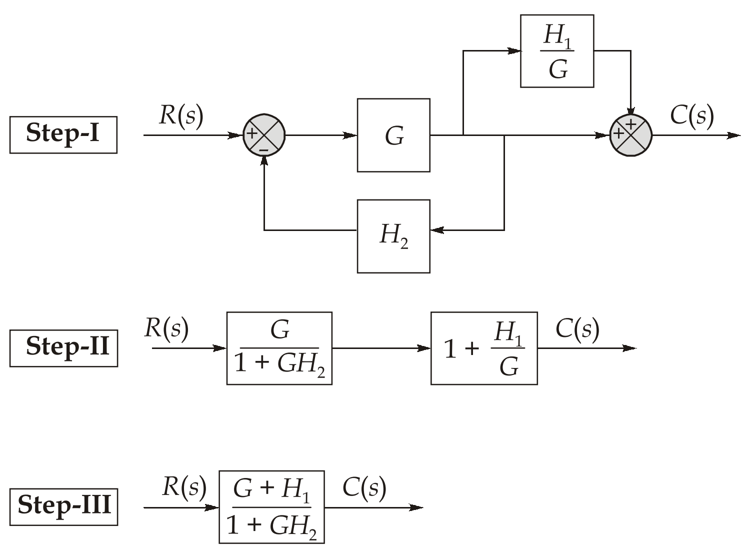
$$= 521 \text{ units (without considering safety stock)}$$

8. (b)

(i)

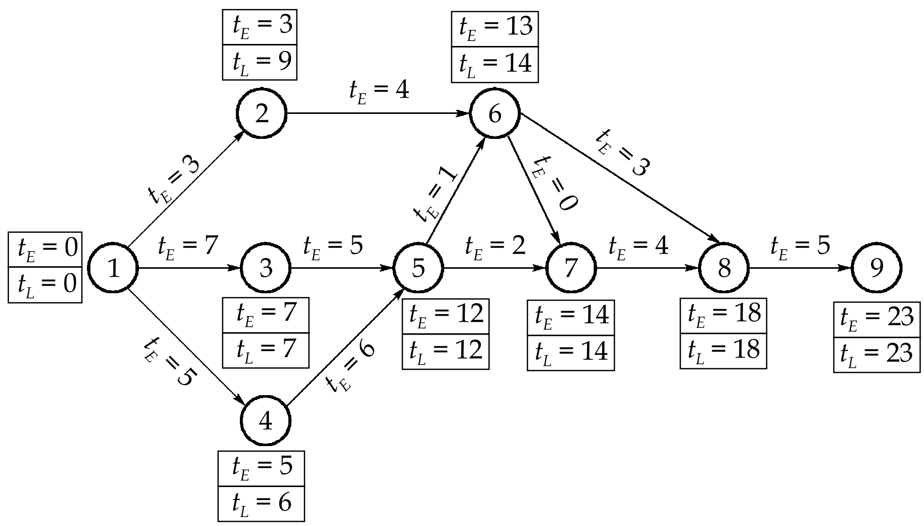
Electrical systems	Thermal systems	Liquid-level systems	Pneumatic systems
Charge, q	Heat flow, J	Liquid flow, m^3	Air flow, m^3
Current, A	Heat flow rate, J/min	Liquid flow rate, m^3/min	Air flow rate, m^3/min
Voltage, V	Temperature, °C	Head, m	Pressure, N/m^2
Resistance, Ω	Resistance, °C/(J/min)	Resistance, (N/m^2)/(m^3/min)	Resistance, (N/m^2)/(m^3/min)
Capacitance, C	Capacitance, J/°C	Capacitance, (m^2)	Capacitance, (m^3)/(N/m^2)

(ii)



8. (c)

(i)



Event No.	Earliest expected time			Latest occurence time			
	Predecessor event (i)	Activity time (t_{Eij})	T_E	Successor event (j)	Activity time (t_{Eij})	(T_L)	Slack (s) ($s = t_L - t_E$)
1.	–	–	0	2, 3, 4	3, 7, 5	0	$\boxed{0}$
2.	1	3	3	6	4	10	7
3.	1	7	7	5	5	7	$\boxed{0}$
4.	1	5	5	5	6	6	1
5.	3 4	5 6	12 11	6 7	1 2	12	$\boxed{0}$
6.	2 5	4 1	7 13	7 8	0 3	14	1
7.	5 6	2 0	14 13	8	4	14	$\boxed{0}$
8.	6 7	3 4	16 18	9	5	18	$\boxed{0}$
9.	8	5	23	–	–	23	$\boxed{0}$

Since, slack = 0 i.e. $t_L - t_E = 0$ for 1 - 3 - 5 - 6 - 7 - 8 - 9 path, also the longest path $\Rightarrow$ critical path is 1 - 3 - 5 - 6 - 7 - 8 - 9.

(ii)

Variance in time estimates is calculated as

$$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$$

In case of expert A, $$\sigma^2 = \left(\frac{8-4}{6}\right)^2 = \frac{4}{9}$$

In case of expert B, $$\sigma^2 = \left(\frac{10-4}{6}\right)^2 = 1$$

So, the variance is less in the case of expert A. Hence, it is concluded that the expert A is more certain about his estimates of time.

❍❍❍❍