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Detailed Solutions

ESE-2021 Mains Test Series

Electrical Engineering Test No : 10

Section-A

Q.1 (a) Solution:

We have,

$$\nabla f = (2xy^3 + y)\hat{i} + (3x^2y^2 + x)\hat{j} \text{ and at } (2, 1)$$

$$\nabla f = 5\hat{i} + 14\hat{j}$$

The unit vector is given by,

$$\hat{b} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j} + \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}, \qquad \text{since } \theta = \frac{\pi}{3}$$

$$\text{Therefore, directional derivative} = (5\hat{i} + 14\hat{j})\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$$

$$= \frac{5 + 14\sqrt{3}}{2} = 14.62$$

Q.1 (b) Solution:

By KCL,

$$\frac{v(t)}{R} + i\left(0^-\right) + \frac{1}{L}\int_0^t v\,dt + C\frac{dv}{dt} = 1$$

Taking Laplace transform,

$$V(s\left[\frac{1}{R}+\frac{1}{sL}+sC\right] = \frac{I}{s}$$

Putting the values,

$$V(s)\left[2+\frac{2}{s}+\frac{s}{2}\right]=\frac{2}{s}$$

$$V(s) = \frac{4}{s^2+4s+4}=\frac{4}{(s+2)^2}$$

Taking inverse Laplace transform,

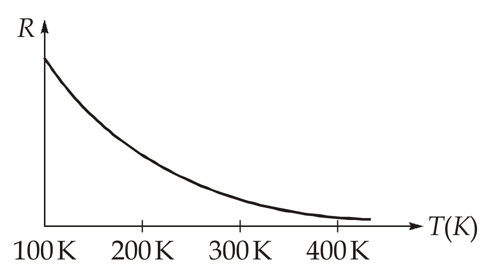
$$v(t) = 4te^{-2t} \qquad V;\ t > 0$$

The response is critically damped as $\xi = 1$

Q.1 (c) Solution:

The resistance of intrinsic semiconductor is dependent on temperature. Its value decreases with increase in temperature, as conductivity increases with temperature,

$$R \propto T^{-3/2}e^{E_g/KT}$$



For pure semiconductor,

$$R \propto \rho$$

So, $$R \propto T^{-3/2}e^{E_g/KT}$$

Volume resistivity (or electrical resistivity) or specific electric resistance,

$$\rho = \frac{R_1 A}{l} = 7 \times 10^{16} \text{ ohm-m}$$

$$R_1 = \frac{\rho l}{A}$$

$$A = 3 \times 4 = 12 \times 10^{-4}\ \text{m}^2$$

$$l = 5\ \text{cm} = 5 \times 10^{-2}\ \text{m}$$

$$R_1 = \frac{5\times10^{-2}\times7\times10^{16}}{12\times10^{-4}} = 2.92 \times 10^{18}\ \Omega$$

Surface resistivity = Surface resistance = $2.4 \times 10^{18}\ \Omega$

$\therefore$ Net resistance = $R_1 + R_2 = 2.92 \times 10^{18} + 2.4 \times 10^{18}\ \Omega$

$= 5.32 \times 10^{18}\ \Omega$

Q.1 (d) Solution:

At balance,

$$(R_1 + j\omega L_1)\left(\frac{R_4}{1+j\omega C_4 R_4}\right) = R_2R_3$$

Separating the real and imaginary terms, we have

$$R_1 = \frac{R_2R_3}{R_4} = \frac{400\times 600}{1000} = 240\ \Omega$$

and

$$L_1 = R_2R_3C_4$$
$$= 400 \times 600 \times 0.5 \times 10^{-6} = 0.12\text{ H}$$

Storage factor,
$$Q = \frac{\omega L_1}{R_1} = \frac{2\pi\times 1000\times 0.12}{240} = 3.14$$

Q.1 (e) Solution:

Given,
$$T = 200^\circ\text{ K}$$
$$N_C = 4.7 \times 10^{17} \text{ and } N_V = 7 \times 10^{18}/\text{cm}^3$$

AT 200 K,
$$KT = 0.0259\times\left(\frac{200}{300}\right) = 0.01727\text{ eV}$$

$$n_i^2 = N_C N_V e^{-\frac{E_g}{KT}}$$

But N_C and N_V are given at 300° K and the temperature of interest is 200° K

N_C and N_V both are dependent of T as $T^{3/2}$

$$n_i^2 = N_{C(300)}N_{V(300)}\times\left(\frac{200}{300}\right)^3 e^{-\frac{E_g}{KT}}$$

Also, $E_g = 1.42$ eV

$$n_i^2 = (4.7\times 10^{17})(7\times 10^{18})\times\left(\frac{200}{300}\right)^3\times e^{-\frac{1.42}{0.01727}} = 1.9041$$

$$n_i = \sqrt{1.9041} = 1.38/\text{cm}^2$$

Now,
$$n_0 = 5p_0$$
$$n_i^2 = n_0p_0 = 5p_0^2 \quad \text{(at equilibrium)}$$

$$P_0 = \sqrt{\frac{n_i^2}{5}} = \frac{ni}{\sqrt{5}} = \frac{1.38}{\sqrt{5}} = 0.6171/\text{cm}^3$$

and $$n_0 = 5p_0 = 3.08/\text{cm}^3$$

Q.2 (a) Solution:

We have, $$M = 3x^2y^3 e^y + y^3 + y^2$$

and $$N = x^3y^3e^y - xy$$

$$\frac{\partial M}{\partial y} = 9x^2y^2e^y + 3x^2y^3e^y + 3y^2 + 2y$$

$$\frac{\partial N}{\partial x} = 3x^2y^3e^y - y$$

The equation is not exact, we have

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 9x^2y^2e^y + 3y^2 + 3y$$

$$= 3(3x^2y^2e^y + y^2 + y)$$

and $$\frac{1}{M}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] = \frac{3(3x^2y^2e^y + y^2 + y)}{y(3x^2y^2e^y + y^2 + y)} = \frac{3}{y}$$

Which is a function of y alone. The integrating factor is

$$\text{I.F.} = e^{-\int \frac{3}{y}dy} = e^{-3\ln y} = \frac{1}{y^3}$$

Multiplying the given differential equation throughout by the integrating factor $\frac{1}{y^3}$,

We get,

$$\left(3x^2e^y + 1 + \frac{1}{y}\right)dx + \left(x^3e^y - \frac{x}{y^2}\right)dy = 0$$

or $$\left(3x^2e^y dx + x^3e^y dy\right) + dx + \left(\frac{1}{y}dx - \frac{x}{y^2}dy\right) = 0$$

or $$d(x^3e^y) + dx + d\left(\frac{x}{y}\right) = 0$$

Integrating, we get the solution as,

$$x^3e^y + x + \frac{x}{y} = c$$

$$y(x^3e^y + x) + x = cy$$

Q.2 (b) (i) Solution:

Given, Bandwidth = 10 Mbps, frame size = 512 bits. In order to detect collision

$$\text{Transmission time} \geq \text{round trip time}$$

$$\frac{L}{B} \geq 2 \times \text{Propagation time}$$

$$L \geq 2 \times \frac{d}{V} \times B$$

$$512 \text{ bits} \geq 2 \times \left(\frac{d}{V}\right) \times 10 \text{ Mbps}$$

$$\left(\frac{d}{V}\right) \leq \frac{512 \text{ bits}}{10 \text{ Mbps} \times 2} \leq \frac{2^9 \text{ bits}}{10 \times 2^{21} \text{bit/sec}}$$

$$\left(\frac{d}{V}\right) \leq \frac{1}{10 \times 2^{12}} \text{sec}$$

Thus, propagation time, $\leq \dfrac{1}{10 \times 2^{12}} \text{sec}$

When band width becomes 10 Mbps and all other parameters remains as it is.

$$T_t \geq 2 \times T_P$$

$$L \geq 2 \times T_P \times B$$

$$L \geq 2 \times \frac{1 \times 100 \times 2^{20}}{10 \times 2^{12}}$$

$$L \geq 20 \times 2^8 \text{ bits} = 5120 \text{ bits}$$

So minimum frame size should be 5120 bits.

Q.2 (b) (ii) Solution:

Process	Arrival Time	Service Time
P_1	0	5
P_2	1	7
P_3	3	3
P_4	4	6

At t = 0, ready queue P_1

P_1	

0 5

At t = 5, ready queue P_2, P_3, P_4

P_1	P_2	

0 5 10

At t = 10, ready queue P_3, P_4, P_2

P_1	P_2	P_3	

0 5 10 13

At t = 13, ready queue P_4, P_2

P_1	P_2	P_3	P_4	

0 5 10 13 18

At t = 18, ready queue, P_2, P_4

P_1	P_2	P_3	P_4	P_2	

0 5 10 13 18 20

At t = 20, ready queue P_4

P_1	P_2	P_3	P_4	P_2	P_4

0 5 10 13 18 20 21

$$\text{Waiting time} = CT - AT - BT$$

$$WT_1 = 5 - 0 - 5 = 0$$

$$WT_2 = 20 - 1 - 7 = 12$$

$$WT_3 = 13 - 3 - 3 = 7$$

$$WT_4 = 21 - 4 - 6 = 11$$

$$\text{Average waiting time} = \frac{WT_1 + WT_2 + WT_3 + WT_4}{\text{No. of processes}}$$

$$= \frac{0 + 12 + 7 + 11}{4} = \frac{30}{4} = 7.5 \text{ unit}$$

Q.2 (c) (i) Solution:

Inductance, $L = 200 + 40\theta - 4\theta^2 - \theta^3\ \mu H$

$\therefore$ rate of change of inductance, with deflection

$$\frac{dL}{d\theta} = 40 - 8\theta - 3\theta^2\ \mu H/rad$$

$$\frac{dL}{d\theta} \text{ for } \theta = \frac{\pi}{2}$$

$$40 - 8 \times \frac{\pi}{2} - 3\left(\frac{\pi}{2}\right)^2 = 20\ \mu H/rad$$

Deflection, $\theta = \frac{1}{2}\frac{I^2}{K}\frac{dL}{d\theta}$

with $I = 1.5$ A,

We have $\theta = \frac{\pi}{2}$ and $\frac{dL}{d\theta} = 20 \times 10^{-6}$ H/rad

$\therefore$ $$\frac{\pi}{2} = \frac{1}{2}\frac{(1.5)^2}{K} \times 20 \times 10^{-6}$$

$\therefore$ Spring constant, $K = 14.32 \times 10^{-6}$ Nm/rad

For $I = 1$ A,

$$\text{Deflection, } \theta = \frac{1}{2} \times \frac{(1)^2}{14.32 \times 10^{-6}}(40 - 8\theta - 3\theta^2) \times 10^{-6}$$

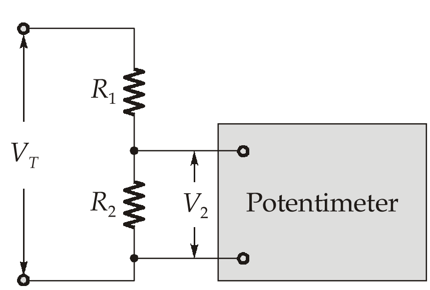
$$28.64\,\theta = 40 - 8\theta - 3\theta^2$$

$$3\theta^2 + 36.64\,\theta - 40 = 0$$

or $\theta = 1.008$ rad $= 57.8°$

Q.2 (c) (ii) Solution:

The basic volt ratio box explained in the question is shown in figure,



Given, $V_T = 100$ V,

$$V_2 = 2 \text{ V}$$

$$R_1 + R_2 = 10 \text{ M}\Omega$$

$$\text{Volt box ratio} = \frac{V_T}{V_2} = \frac{100}{2} = 50$$

Also,
$$V_2 = \left(\frac{R_2}{R_1 + R_2}\right) \times V_T$$

or,
$$\left(\frac{R_1 + R_2}{R_2}\right) = 50$$

or,
$$R_1 + R_2 = 50R_2$$

$$R_1 = 49R_2$$

Also,
$$R_1 + R_2 = 10 \text{ M}\Omega$$

Solving for R_1 and R_2 we have

$$R_1 = 9.8 \text{ M}\Omega \text{ and } R_2 = 0.2 \text{ M}\Omega$$

Q.3 (a) Solution:

The position vector,
$$\vec{r} = 3\hat{a}_x + 4\hat{a}_y + 12\hat{a}_z$$

$$r = |\vec{r}| = 13 \text{ m}$$

The dipole moment,
$$\vec{p} = 1.6 \times 10^{-19} \times 10^{-11}\hat{a}_z = 1.6 \times 10^{-30}\hat{a}_z$$

Thus, the potential at point (3, 4, 12) is given as

$$V = \frac{\vec{p} \cdot \vec{r}}{4\pi\varepsilon_0 r^3} = \frac{(1.6 \times 10^{-30}\hat{a}_z) \cdot (3\hat{a}_x + 4\hat{a}_y + 12\hat{a}_z)}{4\pi \times \dfrac{10^{-9}}{36\pi} \times 13^3}$$

$$= \frac{1.6 \times 10^{-30} \times 12 \times 9}{10^{-9} \times 13^3} = 7.865 \times 10^{-23} \text{ V}$$

The electric field intensity at point (3, 4, 12) is given as

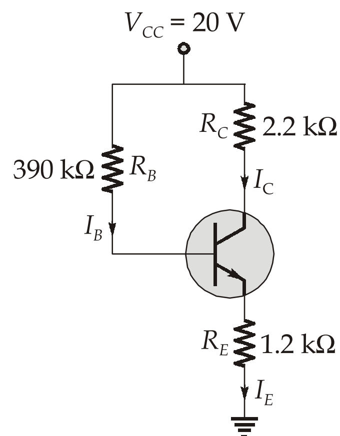
$$\vec{E} = \frac{1}{4\pi\varepsilon_0 r^3}\left[\frac{3(\vec{p} \cdot \vec{r})\vec{r}}{r^2} - \vec{p}\right]$$

$$= \frac{1}{4\pi \times \dfrac{10^{-9}}{36\pi} \times 13^3}\left[\frac{(3 \times 1.6 \times 10^{-30} \times 12) \times (3\hat{a}_x + 4\hat{a}_y + 12\hat{a}_z)}{13^2} - 1.6 \times 10^{-30}\hat{a}_z\right]$$

$$= (4.189\hat{a}_x + 5.585\hat{a}_y + 10.2\hat{a}_z) \times 10^{-24} \text{ V/m}$$

Q.3 (b) Solution:

(i) DC equivalent circuit :



By KVL in input loop,

$$V_{CC} - I_B R_B - V_{BE} - I_E R_E = 0$$

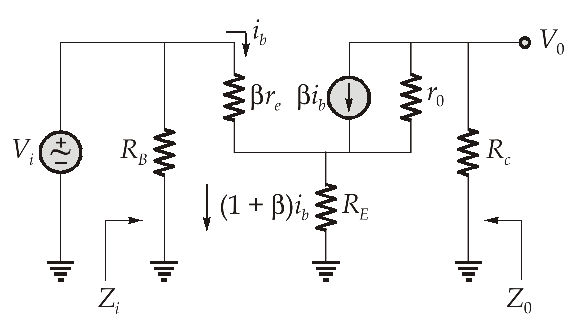
$$I_B[R_B + (1+\beta)R_E] = V_{CC} - V_{BE}$$

$$I_B = \frac{20 - 0.7}{390 + 141 \times 1.2} = 34.51\ \mu\text{A}$$

$$I_C = \beta I_B = 140 \times 34.51 = 4.83\text{ mA}$$

$$r_e = \frac{V_T}{I_{EQ}} = \frac{25}{141 \times 34.51 \times 10^{-3}} = 5.137\ \Omega$$

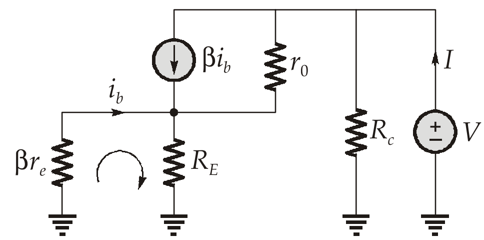
(ii) AC equivalent circuit :



Input impedance, $(Z_i) = \left[(1+\beta)R_E + \beta r_e\right] \| R_B$

$$= [141 \times 1.2 + 140 \times 0.00517]\ ||\ 390$$

$$= 118.35\text{ k}\Omega$$



Applying KVL in loop,

$$\beta r_e i_b + (1+\beta)i_b R_E = 0$$

$$i_b = 0,$$

So, $$\beta i_b = 0$$

$$Z_0 = \frac{V}{I} = R_c = 2.2\text{ k}\Omega$$

So, output impedance $(Z_0) = 2.2\text{ k}\Omega$

(iii) Voltage gain:

$$V_0 = -\beta\, i_b(R_c \,||\, r_0) \quad \text{...(i)}$$

and $$V_i = [(1+\beta)R_E + \beta r_e]i_b$$

$$i_b = \frac{V_i}{(1+\beta)R_E + \beta r_e}$$

On putting value of 'i_b' in equation (i),

$$V_0 = \frac{-V_i\beta(R_c \,\|\, r_0)}{(1+\beta)R_E + \beta r_e}$$

$$A_V = \frac{V_0}{V_i} = \frac{-\beta(R_c \,\|\, r_0)}{(1+\beta)R_E + \beta r_e} = \frac{-140\times[2.2 \,\|\, 100]}{141\times 1.2 + 140\times 0.00517}$$

Voltage gain, $$A_V = -1.77$$

Q.3 (c) (i) Solution:

Polarization, $$P = N_p = N\,\alpha\,E$$

Where, N = Concentration of atoms

α = Polarizability

E, electrical field $= 4\times 10^5$ V/m

$$P = 2.4\times 10^{25}\times 0.21\times 10^{-40}\times 4\times 10^5$$

$$= 2.016\times 10^{-10}\ \text{Coulumb/m}^2$$

$$P = Np = 2.016\times 10^{-10}\ \text{C/m}^2$$

$$p = \frac{2.016\times10^{-10}\,\text{C/m}^2}{2.4\times10^{25}} = 0.84 \times 10^{-35}\ \text{C-m}$$

$$\text{Charge of He} = 2\ (\text{electrical charge})$$

$$p = 2e \times d$$

$$d = \frac{p}{2e} = \frac{0.84\times10^{-35}}{2\times1.6\times10^{-19}} = 2.625 \times 10^{-17}\ \text{m}$$

Q.3 (c) (ii) Solution:

Eddy current loss,
$$P_e = \frac{\pi^2 f^2 B_m^2 t^2}{\beta\rho}\ \text{Watt/m}^3$$

$$= \frac{\pi^2 \times f^2 \times B_m^2 \times t^2}{6\rho\times7800}$$

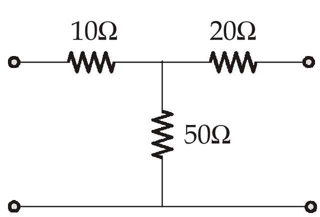
$$= \frac{\pi^2 \times 50\times50\times(1.1)^2\times(0.0005)^2}{6\times30\times10^{-8}\times7800} = 0.5316\ \text{W/kg}$$

$$\text{Hysteresis loss} = \frac{380\times50}{7800} = 2.436\ \text{watt/kg}$$

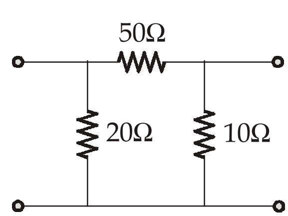
$$\text{Total iron loss} = \text{Eddy current loss} + \text{hysteresis loss}$$
$$= 2.436 + 0.5316$$
$$= 2.967\ \text{watt/kg}$$

Q.4 (a) Solution:

This two-port network can be considered as the cascade connection of two two-port networks as shown below:



Network (*a*)



Network (*b*)

For the network (*a*), as this is a *T*-network, the *z*-perameters are given as

$$Z_{11} = 60\ \Omega\ ;\ Z_{12} = 50\ \Omega\ ;\ Z_{22} = 70\ \Omega$$

$\therefore$
$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$$
$$= 60 \times 70 - 50^2 = 1700$$

$$\therefore \qquad A_a = \frac{Z_{11}}{Z_{21}} = \frac{60}{50} = \frac{6}{5}$$

$$B_a = \frac{\Delta Z}{Z_{21}} = \frac{1700}{50} = 34\Omega$$

$$C_a = \frac{1}{Z_{21}} = \frac{1}{50}\mho$$

$$D_a = \frac{Z_{22}}{Z_{21}} = \frac{70}{50} = \frac{7}{5}$$

For the network (b), as this is a π-network the y-parameters are given as

$$y_{11} = \left(\frac{1}{50} + \frac{1}{20}\right) = \frac{7}{100}\mho$$

$$y_{12} = y_{21} = -\frac{1}{50}\mho$$

$$y_{22} = \frac{1}{50} + \frac{1}{10} = \frac{3}{25}\mho$$

$$\therefore \qquad \Delta y = (y_{11}\,y_{22} - y_{12}\,y_{21}) = \frac{7}{100} \times \frac{3}{25} - \left(-\frac{1}{50}\right)^2 = \frac{1}{125}$$

$$\therefore \qquad A_b = \frac{-y_{22}}{y_{21}} = \frac{-\frac{3}{25}}{-\frac{1}{50}} = 6$$

$$B_b = -\frac{1}{y_{21}} = \frac{-1}{-\frac{1}{50}} = 50\Omega$$

$$C_b = \frac{-\Delta y}{y_{21}} = \frac{-\frac{1}{125}}{-\frac{1}{50}} = \frac{2}{5}\mho$$

$$D_b = \frac{-y_{11}}{y_{21}} = \frac{-\frac{7}{100}}{-\frac{1}{50}} = \frac{7}{2}$$

For the entire network, the ABCD parameters are given as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \times \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

$$= \begin{bmatrix} \frac{6}{5} & 34 \\ \frac{1}{50} & \frac{7}{5} \end{bmatrix} \times \begin{bmatrix} 6 & 50 \\ \frac{2}{5} & \frac{7}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 20.8 & 179 \\ 0.68 & 5.9 \end{bmatrix}$$

Q.4 (b) (i) Solution:

Given, $R_P = 25\ \Omega$; $L_P = 15$ mH ;

$V_L = 415$ V ; $f = 50$ Hz

Phase voltage, $V_P = V_L = 415$ V

Per phase reactance, $X_P = 2\pi f L_P = 2\pi \times 50 \times 0.015$

$= 4.71\ \Omega$

$\therefore$ Per phase impedance,

$$Z_P = \sqrt{R_p^2 + X_p^2} = \sqrt{(25)^2 + (4.71)^2} = 25.44\ \Omega$$

Power factor, $\cos\phi = \dfrac{R_p}{Z_p} = \dfrac{25}{25.44} = 0.98$

$\phi = 10.67°$

Phase current, $I_P = \dfrac{V_p}{Z_p} = \dfrac{415}{25.44} = 16.31$ A

and line current, $I_L = \sqrt{3}I_P = \sqrt{3} \times 16.31 = 28.25$ A

$\therefore$ Reading of the wattmeters are

$$W_1 = V_L I_L \cos(30° - \phi)$$

$$= 415 \times 28.25 \cos(30° - 10.67°)$$

$$= 11.065 \text{ kW}$$

$$W_2 = V_L I_L \cos(30° + \phi)$$

$$= 415 \times 28.25 \cos(30° + 10.67°)$$

$$= 8.893 \text{ kW}$$

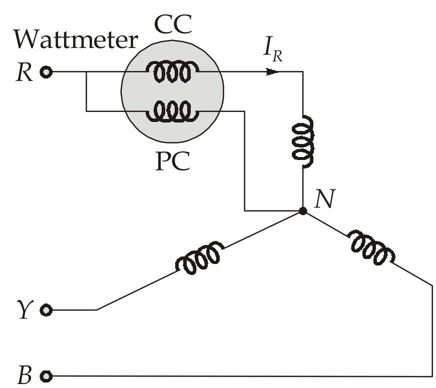
Q.4 (b) (ii) Solution:

Taking V_R as the reference and assuming phase to neutral voltage = V

$$V_R = V\angle 0°$$

$$V_Y = V\angle -120°$$

and $$V_B = V\angle -240°$$



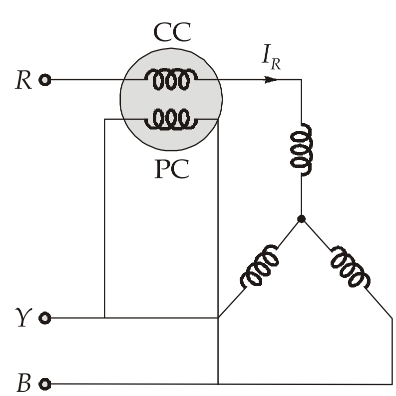
$$V_R I_R \cos\phi = 400 \text{ W}$$

$$V I_R \times 0.8 = 400 \text{ W}$$

$$V I_R = 500 \text{ W} \quad ...(i)$$

$$I_R = I_R\angle -36.87°$$

(Power factor = 0.8 lagging inductive load)



$$V_{YB} = V_Y - V_B$$

$$= V\angle -120° - V\angle -240°$$

$$= \sqrt{3}V\angle -90° V$$

$$I_R = I_R\angle -36.87° \text{ A}$$

Angle between V_{YB} and I_R

$$\theta = -90° - (-36.87°) = 53.13°$$

As pressure coil connected between Y and B phases

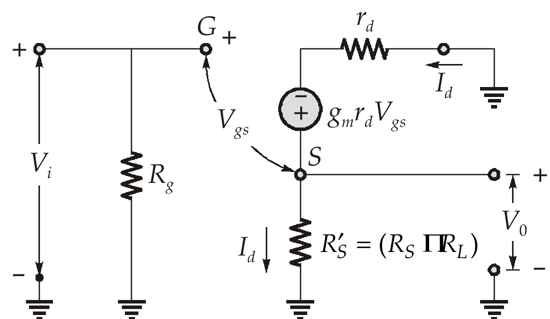
$$\text{Reading of wattmeter} = V_{YB} I_R \cos\theta$$

$$= \sqrt{3}\text{V} \times I_R \cos(-53.13°)$$

$$= \sqrt{3} \times 500 \times 0.6 = 519.6 \text{ W}$$

Q.4 (c) Solution:

(i) Assuming that all the externally connected capacitors are short-circuited under ac condition, the ac-equivalent small signal low frequency can be given as



$$R_s' = R_S \,||\, R_L = \frac{4.7 \times 10}{4.7 + 10} = 3.20 \text{ k}\Omega$$

Applying KVL in the circuit,

$$g_m r_d V_{gs} = I_d (r_d + R_s')$$

Since $V_{gs} = V_i - I_d R_s'$, we get from the above equation

$$g_m r_d (V_i - I_d R_s') = I_d (r_d + R_s')$$

or
$$g_m r_d V_i = I_d (r_d + R_s' + g_m r_d R_s')$$

Which gives,
$$I_d = \frac{g_m r_d V_i}{(r_d + R_s' + g_m r_d R_s')} \quad \text{...(i)}$$

Output voltage of amplifier is

$$V_0 = I_d R_s'$$

on putting value of I_d in above expression

$$V_0 = \frac{g_m r_d V_i R_s'}{r_d + R_s' + g_m r_d R_s'}$$

$$A_V = \frac{V_0}{V_i} = \frac{g_m r_d R_s'}{r_d + R_s' + g_m r_d R_s'}$$

(ii) Given, $V_i = 2\text{ V}$, $g_m = 2\text{ mA/V}$

$r_d = 100\text{ k}\Omega$

and $R_s = 3.2\text{ k}\Omega$

On putting respective parameters values in equation (i),

$$I_d = \frac{g_m r_d V_i}{r_d + R_s' + g_m r_d R_s'} = \frac{2\times 2\times 100}{100 + 3.2 + 2\times 100\times 3.2}$$

$$= 0.5382\text{ mA}$$

(iii) The output voltage of circuit is obtained as,

$$V_0 = I_d R_s' = 0.5382 \times 3.2 = 1.72\text{ Volt}$$

$$A_v = \frac{V_0}{V_i} = \frac{1.72}{2} = 0.86\text{ V/V}$$

Note that, the gain of a common-drain amplifier is always less than unity.

Section-B

Q.5 (a) Solution:

```
        Main ( )
{       int avg,  sum = 0;
        int i ;
        int marks [30] ;              /* array declaration*/
        for (i = 0 ; i < = 29; i ++)
{       print f ("\n Enter Marks");
        scanf ("%d,&marks [i]);  /*store data in array*/
}
        for (i = 0 ; i < = 29 ; i++)
        sum = sum + marks [i];   /* read data from an array*/
        avg = sum/30
        printf ("\n averge marks = %d", avg);
}
```

Q.5 (b) Solution:

Denote the given integral by I. The integrand of I is not analytic at the points $z = 0$ and $z = \frac{1}{2}$ both of which lie inside c.

We write
$$\frac{z^2+1}{z(2z-1)} = (z^2+1)\left[\frac{1}{z-\frac{1}{2}} - \frac{1}{z}\right]$$

Therefore,
$$\oint_c \frac{z^2+1}{z(2z-1)}dz = \oint_c \frac{z^2+1}{z-\frac{1}{2}}dz - \oint_c \frac{z^2+1}{z}dz = I_1 - I_2$$

Now, the integrands of I_1 and I_2 are not analytic at a point which lies inside c, using the Cauchy integral formula to I_1 and I_2, we get

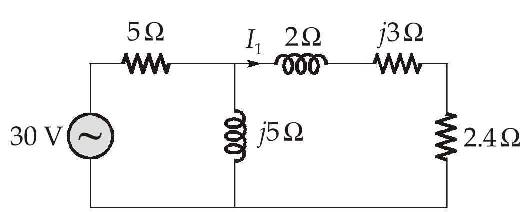
$$I_1 = \oint_c \frac{z^2+1}{z-\frac{1}{2}}dz = \left\{2\pi i\left[z^2+1\right]_{z=\frac{1}{2}}\right\} = \frac{5\pi i}{2}$$

$$I_2 = \oint_c \frac{z^2+1}{z}dz = \left\{2\pi i\left[z^2+1\right]_{z=0}\right\} = 2\pi i$$

Hence,
$$I = I_1 - I_2 = \frac{5\pi i}{2} - 2\pi i = \frac{\pi i}{2}$$

Q.5 (c) Solution:

When the 30 V source is acting alone, let the current the branch $(2 + j3)\Omega$ be I_1.



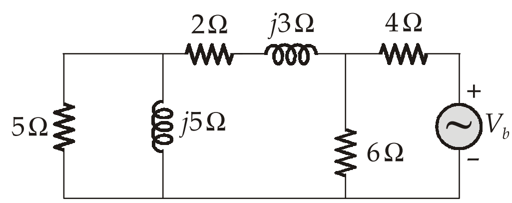
Impedance,
$$Z = 5 + \frac{j5\times(4.4+j3)}{4.4+j8} = \left(\frac{7+j62}{4.4+j8}\right)\Omega$$

$\therefore$
$$I = \frac{30}{Z} = \frac{30(4.4+j8)}{7+j62}$$

$$\therefore \qquad I_1 = I \times \frac{j5}{4.4 \times j8} = \frac{30(4.4+j8)}{7+j62} \times \frac{j5}{4.4+j8}$$

$$= \frac{j150}{7+j62} A$$

When the V_b source is acting alone, let the current through the branch (2 + j3)Ω be I_2.



Impedance,

$$Z = 4 + \frac{6 \times (4.5 + j5.5)}{10.5 + j5.5} = \left(\frac{69 + j55}{10.5 + j5.5}\right)\Omega$$

$$\therefore \qquad I' = \frac{V_b}{Z} = \frac{V_b(10.5 + j5.5)}{69 + j55}$$

$$\therefore \qquad I_2 = I' \times \frac{6}{10.5 + j5.5} = \frac{V_b(10.5 + j5.5)}{69 + j55} \times \left(\frac{6}{10.5 + j5.5}\right)$$

$$= \frac{6V_b}{69 + j55} A$$

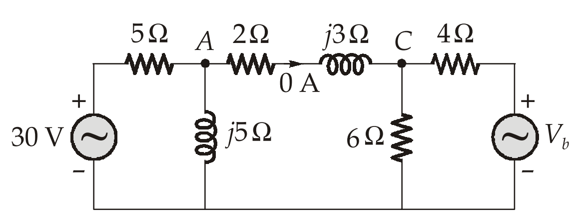
Current through the branch (2 + j3)Ω will be zero, if

$$I_1 = I_2$$

$$\frac{j150}{7+j62} = \frac{6V_b}{69+j55}$$

$$V_b = 25 + j25 = 35.35\angle 45°\text{V}$$

Alternate Solution:



The current through (2 + j3) is '0 A' means potential difference between A and C is zero

$$V_A - V_C = 0$$

$$V_A = V_C$$

$$V_A = \frac{30 \times j5}{5 + j5} = \frac{j30}{1 + j1}$$

$$V_C = V_b \times \frac{6}{10} = V_b \times \frac{3}{5}$$

$$V_A = V_C$$

$$\frac{j30}{1 + j1} = V_b \times \frac{3}{5}$$

$$V_A = V_C$$

$$\frac{j30}{1 + j1} = V_b \times \frac{3}{5}$$

$$V_b = (25 + j25)$$

$$V_b = 25\sqrt{2}\angle 45^\circ = 35.35\angle 45^\circ \text{V}$$

Q.5 (d) Solution:

Total energy of system = Net work done is bringing charges from infinity to their respective position.

$$W = W_1 + W_2 + W_3$$

$$= 0 + Q_2 V_{21} + Q_3(V_{31} + V_{32})$$

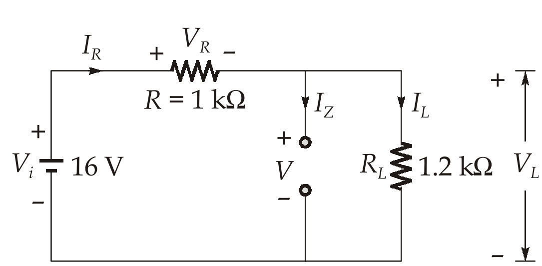
$$= Q_2 \cdot \frac{Q_1}{4\pi\varepsilon_0 |(0,0,1) - (0,0,0)|} + \frac{Q_3}{4\pi\varepsilon_0}\left[\frac{Q_1}{|(1,0,0) - (0,0,0)|} + \frac{Q_2}{|(1,0,0) - (0,0,1)|}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left(Q_1 Q_2 + Q_1 Q_3 + \frac{Q_2 Q_3}{\sqrt{2}}\right)$$

$$= \frac{1}{4\pi \times \frac{10^{-9}}{36\pi}}\left(-4 - 3 + \frac{12}{\sqrt{2}}\right) \times 10^{-18} = 9\left(\frac{12}{\sqrt{2}} - 7\right)\text{nJ} = 13.37 \text{ nJ}$$

Q.5 (e) Solution:

(i) Assume Zener diode to be OFF :



$$V = \frac{R_L}{R + R_L} \times V_i = \frac{1.2}{1 + 1.2} \times 16 = 8.73 \text{ V}$$

Since $V < V_Z$ (8.73 < 10 V), the Zener diode is in 'OFF' state

So, $$V_L = V = 8.73 \text{ V}$$

$$V_R = V_i - V_L = 16 - 8.73 = 7.27 \text{ V}$$

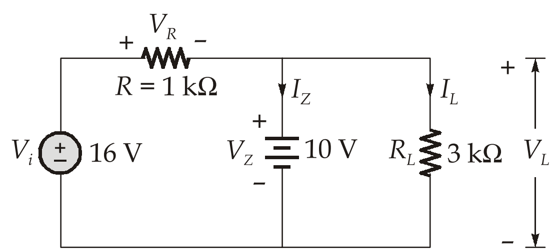
$$I_Z = 0 \text{ A}$$

$$P_Z = V_Z I_Z = V_Z (0 \text{ A}) = 0 \text{ W}$$

(ii) Now with, $$R_L = 3 \text{ k}\Omega$$

$$V = \frac{3}{1+3} \times 16 = 12 \text{ V}$$

Since V = 12 V is greater than V_Z = 10 V, the diode is in the 'ON' state and the equivalent network can be given as



$$V_L = V_L = 10 \text{ V}$$

$$V_R = V_i - V_L = 16 - 10 = 6 \text{ V}$$

$$I_L = \frac{V_L}{R_L} = \frac{10}{3\text{k}\Omega} = 3.33 \text{ mA}$$

$$I_R = \frac{V_R}{R} = \frac{6}{1\text{k}\Omega} = 6 \text{ mA}$$

$$I_Z = I_R - I_L$$
$$= 6 - 3.33 = 2.67 \text{ mA}$$

The power dissipated is

$$P_Z = V_Z I_Z = 10 \times 2.67 = 26.7 \text{ mW}$$

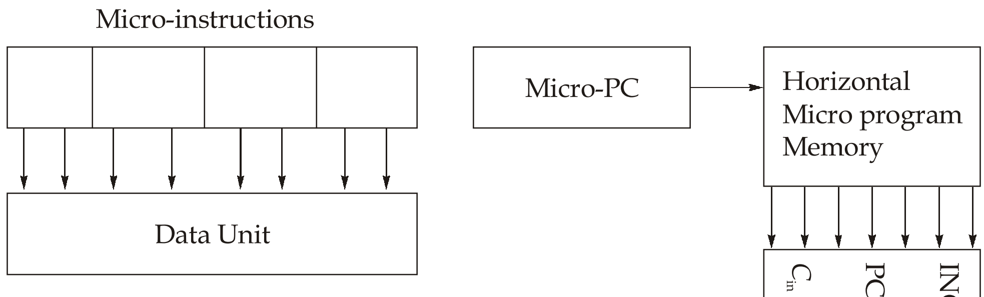
Which is less than the specified

$$P_{ZM} = 30 \text{ mW}$$

Q.6 (a) (i) Solution:

Horizontal Microprogramming:

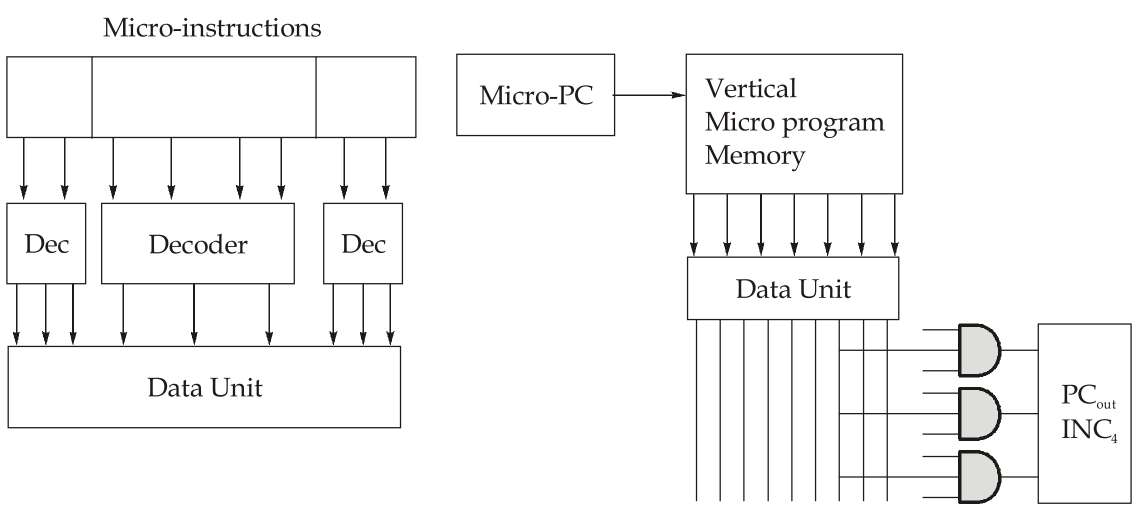
- There is no intermediate decoders and the control word bits are directly connected to their destination.
- Each bit in the control word is directly connected to some control signal.
- The total number of bits in the control word is equal to the total number of control signal in the CPU.
- Each micro-instruction specifies many different micro-operations to be performed in parallel.
- All control signals are present directly in micro code.
- Due to a lot of signals, there are many bits in the microinstruction.



Horizontal Micro Code

Horizontal Micro Programming

Vertical Micro programming:



- Vertical microcode schemes employ an extra level of decoding to control the work width.

- An 'n' bit control word may have 2^n bit signal values.
- It takes less space but may be slower.
- Actions needed to decode into signals at execution time.
- Each micro instruction specifies single micro operation to be performed.

Q.6 (a) (ii) Solution:

$$\text{Virtual address} = 47 \text{ bits}$$

$$\text{Page size} = 16 \text{ kB}$$

$$\text{Page table entry size (PTES)} = 8 \text{ B}$$

We have to perform paging until page table size ≤ page size

1st time paging:

$$\text{Number of pages} = \frac{\text{Virtual address}}{\text{Page size}} = \frac{2^{47}}{2^{14}} = 2^{33}$$

$$\text{Page table size} = \text{No. of pages} \times \text{PTES}$$

$$= 2^{33} \times 8 = 64 \text{ GB}$$

Since page table size > page size, so

IInd time paging:

$$\text{Page table size} = \frac{64 \text{ GB}}{2^{14}} \times 8 \text{B}$$

$$= \frac{2^{36}}{2^{14}} \times 2^3 \text{B} = 32 \text{ MB}$$

Since page table > page size, so

IIIrd time paging:

$$\text{Page table size} = \frac{32 \text{ MB}}{2^{14}} \times 8 \text{ B} = \frac{2^{25}}{2^{14}} \times 2^3 \text{ B} = 2^{14} \text{ B} = 16 \text{ kB}$$

$$\text{Now, page table size} = \text{Page size}$$

So, 3 levels are required to map logical address space if every page table is required to fit in single page.

Q.6 (b) Solution:

The characteristic equation of A is given by

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{vmatrix}$$

or $$(1 - \lambda)(2 - \lambda)(3 - \lambda) = 0$$

$$\lambda = 1, 2, 3$$

corresponding to the eigen value $\lambda = 1$, we have

$$(A - I)X = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 + x_3 = 0$$

$$x_1 + x_3 = 0$$

We obtain two equations in three unknown one of the variables x_1, x_2, x_3 can be chosen arbitrarily.

Taking $x_3 = 1$, we obtain the eigen vector as $[-1, -1, 1]^T$

Corresponding to the eigen value,

$$\lambda = 2$$

We have, $$(A - 2I)X = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

or $x_1 = 0, x_3 = 0$ and x_2 arbitrary

Taking $x_2 = 1$, we obtain the eigen vector as $[0, 1, 0]^T$

Corresponding to the eigen value, $\lambda = 3$, have

$$(A - 3I)X = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 1 \\ 2 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

or $$x_1 = 0$$

$$-x_2 + x_3 = 0$$

Choosing $x_3 = 1$, we obtain the eigen vector as $[0, 1, 1]^T$.

Q.6 (c) (i) Solution:

Conductors have low value of resistivity and are broadly employed in transmission and distribution of power as they exhibit low power loss. Copper and aluminium are two materials which are used and commercially accepted. Conductor material's properties desirable for low resistivity materials are as follows:

- The materials should exhibit a low temperature coefficient which means that the resistivity of such materials does not appreciably change with temperature. This is essential so as to avoid variation in voltage drop and power loss with the change of temperature. Such property of material is helpful and desirable in construction of winding of electrical machines.
- Material should have sufficient mechanical strength. This is necessary because the overhead lines of the conductors used for transmission of power should be capable of bearing stresses due to wind and their own weight.
- Ductility is another desirable property as it allows the material to be drawn in desired shape as some applications need circular cross-section and other need rectangular cross section.
- Material should offer minimum contact resistance which is also reduced by process of soldering of material. Hence material should be selected which allows proper soldering.
- The conducting material should be resistant to corrosion when used in outdoor atmosphere.
- The conducting material should be cheap.

Q.6 (c) (ii) Solution:

Insulation resistance, $R' = 1820\ \text{M}\Omega$

Core radius, $r_1 = \dfrac{1.8\ \text{cm}}{2} = 0.009\ \text{m}$

Sheath radius, $r_2 = \dfrac{5\ \text{cm}}{2} = 2.5\ \text{cm} = 0.025\ \text{m}$

Length of cable, $l = 3000$ metre

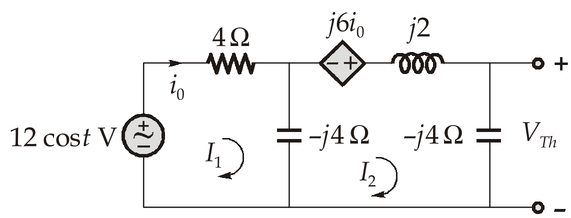
Resistivity of dielectric,
$$\rho = \frac{2\pi l \times R'}{\log_e \frac{r_2}{r_1}} = \frac{2\pi \times 3000 \times 1820 \times 10^6}{\log_e \frac{0.025}{0.009}}$$

$$= 33.57 \times 10^{12}\ \Omega\text{-m}$$

Q.7 (a) Solution:

To find V^{th} :

Removing the 2 Ω resistor and open circuiting the terminals and then converting the dependent current source into dependent voltage source, we redraw the circuit as follows:



By KVL for the two loops (here $i_0 = I_1$)

$$(4 - j4)\, I_1 + j4\, I_2 = 12$$

$$-j2\, I_1 + (-j6) I_2 = 0$$

Solving for I_2,

$$I_2 = \frac{\begin{vmatrix} (4-j4) & 12 \\ j2 & 0 \end{vmatrix}}{\begin{vmatrix} (4-j4) & j4 \\ -j2 & -j6 \end{vmatrix}} = \frac{j24}{-j24 - 24 - 8} = 0.6\ \angle{-126.87}\ \text{A}$$

Therefore, Thevenin voltage is

$$V_{Th} = I_2 \times (-j4) = \frac{12}{4 + j3} = 2.4\angle - 143.13\text{V}$$

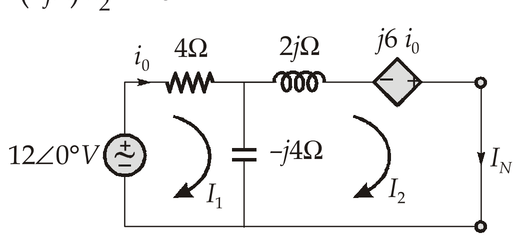
To find I_{sc}

Removing the 2 Ω resistor and short-circuiting the terminals and then converting the dependent current source into dependent voltage source, we redraw the circuit as shown below:

By KVL for the two loops,

$$(4 - j4)\, I_1 + j4\, I_2 = 12$$

$$-j2\, I_1 + (-j2)\, I_2 = 0$$



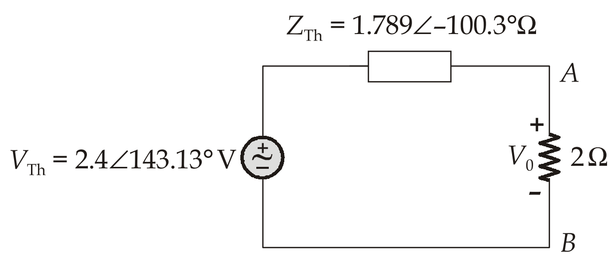
Solving for I_2,

$$I_2 = \frac{\begin{vmatrix}(4-j4) & 12 \\ -j2 & 0\end{vmatrix}}{\begin{vmatrix}(4-j4) & j4 \\ -j2 & -j2\end{vmatrix}} = \frac{j24}{-8-j8-8} = 1.341\angle -116.56\text{A}$$

Therefore, Thevenin impedance, is

$$Z_{Th} = \frac{V_{\text{Th}}}{I_N} = \frac{2.4\angle -36.87°}{1.341\angle 63.435°} = 1.789\angle -100.3°\Omega$$

Thus, Thevenin's equivalent circuit becomes as shown below:



Thus, the required voltage,

$$V_0 = \left(\frac{V_{\text{Th}}}{Z_{\text{Th}}+2}\right)\times 2 = \left(\frac{2.4\angle -36.87°}{1.789\angle -100.3° + 2}\right)\times 2$$

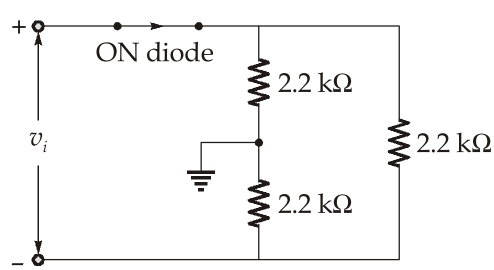
$$= 1.97\angle{-170.53°}\text{ V}$$

Q.7 (b) (i) Solution:

For positive half cycle of v_i :

$$D_1 \rightarrow \text{ON}, \qquad D_2 \rightarrow \text{OFF}$$

Equivalent network can be redrawn as



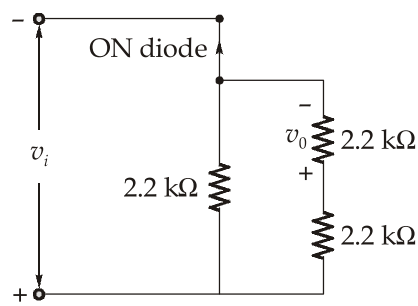
Voltage divider rule :

$$V_{0\,(\text{peak})} = \frac{2.2 \times V_{i(\text{peak})}}{2.2+2.2} = \frac{2.2\times 100}{2.2+2.2} = 50\text{ V (peak)}$$

For negative half cycle of v_i :

$$D_1 \rightarrow \text{OFF}, \qquad D_2 \rightarrow \text{ON}$$

Equivalent circuit can be redrawn as



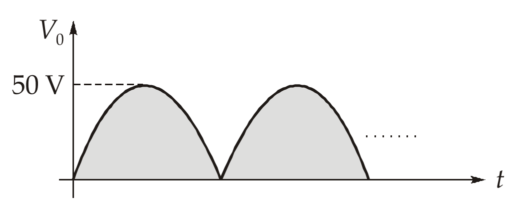
The polarity of v_0 across 2.2 kΩ resistor acting as a load is same as previous (+ve)

Voltage divider rule :

$$V_{0\,(\text{peak})} = \frac{2.2 \times V_{i(\text{peak})}}{2.2+2.2} = \frac{1}{2}(V_{i\,\text{peak}})$$

$$= \frac{1}{2}\times(-100) = -50\text{ V}$$

Negative sign shows that polarity of v_0 is opposite to that of v_i



$$V_{0\,(\text{avg})} = \frac{2\times V_{0(\max)}}{\pi} = \frac{2}{\pi}\times 50 = 31.83\text{ volts}$$

$$V_{0\,(\text{dc})} = 31.83\text{ Volts}$$

Q.7 (b) (ii) Solution:

$$V_E = I_E R_E \approx I_C R_E$$

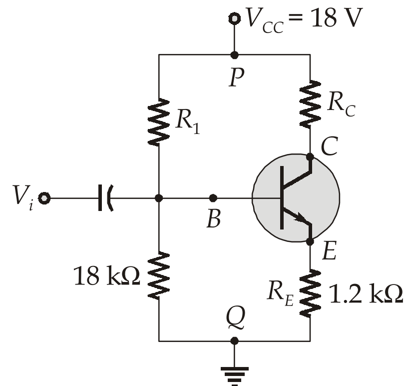
$$= 2 \times 1.2 = 2.4$$

$$V_B = V_{BE} + V_E = 0.7 + 2.4 = 3.1\text{ V}$$

$$V_B = \frac{R_2 V_{CC}}{R_1+R_2} = \frac{18}{18+R_1}\times 18 = 3.1\text{ V}$$

$$R_1 = \frac{18 \times 18}{3.1} - 18 = 86.51\ \text{k}\Omega$$

Now, applying KVL in outer loop ($PCEQ$),



$$V_{CC} - I_C R_C - V_{CEQ} - I_E R_E = 0$$

$$R_C = \frac{V_{CC} - V_{CEQ} - I_E R_E}{I_C} = \frac{18 - 10 - 2 \times 1.2}{2} = 2.8\ \text{k}\Omega$$

So, $R_1 = 86.51\ \text{k}\Omega$

$R_C = 2.8\ \text{k}\Omega$

Q.7 (c) Solution:

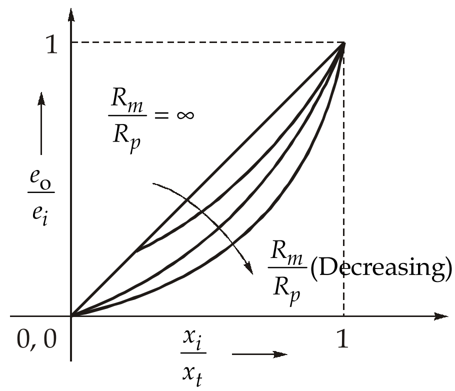
Merits of Potentiometer:

- They are inexpensive.
- They are simple to operate and very useful for applications where the requirements are not particularly severe.
- They are very useful for measurement of large amplitudes of displacement (in cm scale).
- Their electrical efficiency is very high and they provide sufficient output to permit control operation without further amplification.
- While the frequency response of wire wound potentiometers is limited, the other types of potentiometers are free from this problem.
- In wire wound potentiometers the resolution is limited, while in cermet and metal film potentiometers, the resolution in infinite.

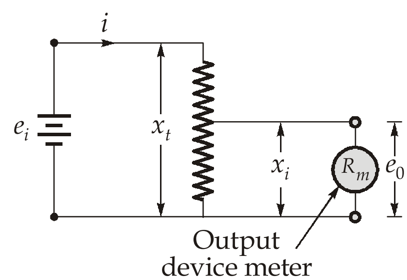
Demerits:

- The main disadvantage of using a linear potentiometer is that they require a large force to move their sliding contacts (wipers).
- The other problems with sliding contacts are that they can be contaminated, can wear out, become misaligned and generate noise. So the life of the transducer is

limited. However, recent developments have produced a roller contact wiper which (it is claimed that it) increases the life of the transducer upto 40 times.



$$\text{Sensitivity, } S = \frac{\text{output}}{\text{input}} = \frac{e_0}{x_i} = \frac{e_i}{x_t}$$



$$e_0 = \frac{x_i}{x_t} e_i$$

$$\frac{e_0}{e_i} = \frac{e_i}{x_t}$$

Let, $$K = \frac{x_i}{x_t}$$

The total resistance seen by the source is:

$$R = R_p(1-K) + \frac{KR_pR_m}{KR_p + R_m}$$

$$= \frac{KR_p^2(1-K) + R_pR_m}{KR_p + R_m}$$

$\therefore$ $$\text{Current, } i = \frac{e_i}{R} = \frac{e_i(KR_p + R_m)}{KR_p^2(1-K) + R_pR_m}$$

The output voltage under load condition is:

$$e_0 = i\frac{KR_pR_m}{KR_p + R_m}$$

$$= \frac{e_i(KR_p + R_m)}{KR_p^2(1-K) + R_pR_m} \times \frac{KR_pR_m}{(KR_p + R_m)}$$

$$= \frac{e_iK}{K(1-K)(R_p / R_m) + 1}$$

The ratio of output voltage to input voltage under load condition is:

$$\frac{e_0}{e_i} = \frac{K}{K(1-K)(R_p / R_m) + 1}$$

Q.8 (a) Solution:

Consider projection of S on the x-y plane. The projection is the circular region $x^2 + y^2 \leq 16$, $z = 0$ and the boundary curve C is the circle $z = 0$, $x^2 + y^2 = 16$, we have

$$\oint_c \vec{V} \cdot \vec{dr} = \oint_c (3x - y)dx - 2yz^2dy - 2y^2z\,dz$$

$$= \oint_c (3x - y)dx$$

Since $z = 0$,

Putting, $x = 4\cos\theta$,

$y = 4\sin\theta$

We obtain, $$\oint_c (3x - y)dx = \int_0^{2\pi} 4(3\cos\theta - \sin\theta)(-4\sin\theta)d\theta$$

$$= -16\int_0^{2\pi}\left(\frac{3}{2}\sin 2\theta - \frac{1}{2}(1 - \cos 2\theta)\right)d\theta$$

$$= 16\left(\frac{1}{2}\right)2\pi = 16\pi$$

Now, $$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x - y & -2yz^2 & -2y^2z \end{vmatrix}$$

$$= \hat{i}(-4yz + 4yz) - \hat{j}(0) + \hat{k}(1) = \hat{k}$$

$$\hat{n} = \frac{2(x\hat{i} + y\hat{j} + z\hat{k})}{2\sqrt{x^2 + y^2 + z^2}} = \frac{1}{4}(x\hat{i} + y\hat{j} + z\hat{k})$$

$$(\nabla \times \vec{V}) \cdot \hat{n} = \frac{z}{4}$$

Therefore, $$\iint_s (\nabla \times \vec{V}) \cdot \hat{n} dA = \iint_s \frac{z}{4} dA = \iint_R \frac{z}{4} \frac{dx\,dy}{n \cdot k} = \iint \frac{z}{4} \frac{dx\,dy}{\left(\frac{z}{4}\right)} = \iint dx\,dy = 16\pi$$

It is the area of the circular region in the x-y plane.

Hence, Stoke's theorem is proved.

Q.8 (b) (i) Solution:

Sampling oscilloscope : A sampling oscilloscope is used to examine very fast signals. It is similar in principle to the used of stroboscopic light to look at fast mechanical motion. Samples are taken at different portions of the waveform, over successive cycles and then the total picture is stretched, amplified by relatively low bandwidth amplifiers and displayed as continuous wave on the screen. The display may be made up from as many 1000 dots of luminescence. The vertical deflection for each dot is obtained from progressively later points in each successive cycles of the input waveform as shown in figure below. The horizontal deflection of the electron beam is obtained by application of a staircase waveform to X-deflection plates.

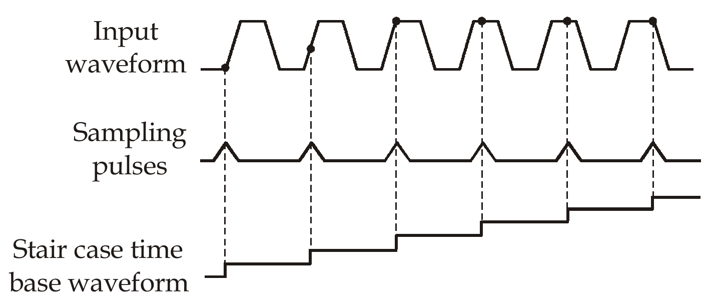
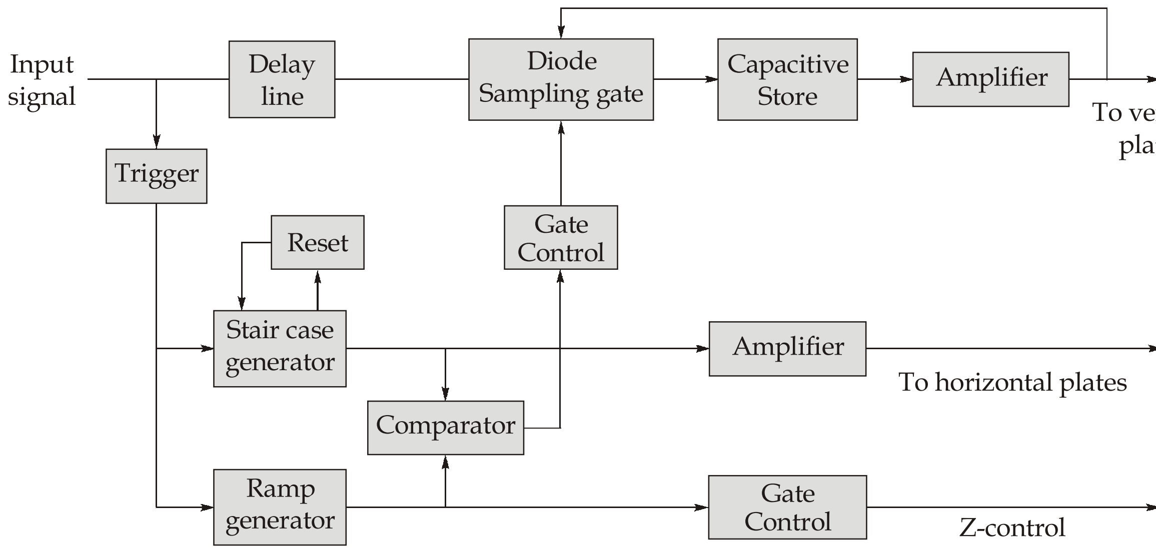


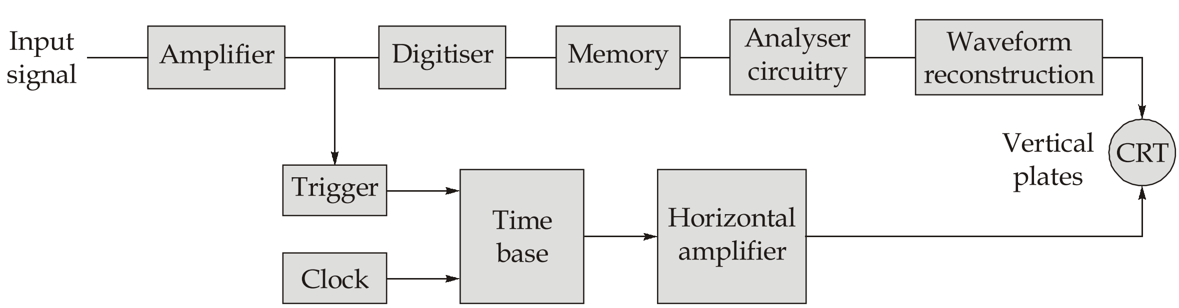
Figure shows a block schematic for a typical oscilloscope the input signal is delayed and then sampled by a diode gate. The sampled signal is saved on a capacitor store, then amplified and fed to the vertical plates. Unity feedback is used from the amplifier to the sampling diode gate. This ensures that the voltage on the capacitor store is only increased by the incremental value of the input voltage change, between each sample.



The staircase is reset after a certain number of steps, typically 100 to 1000 points are used to create the waveform on the screen.

Storage oscilloscopes : Storage oscilloscopes, capable of retaining the image on the screen for longer than that possible with conventional high persistence phosphors have many applications. Two techniques are used to store signals in an oscilloscope and these are called analog and digital storage. Analog storage is capable of higher speeds, but is less versatile than digital storage.

Digital storage oscilloscopes : A digital oscilloscope digitizes the input signal, so that all subsequent signals are digital. A conventional CRT is used and storage occurs in electronic digital memory. Figure shows a block diagram of a basic digital storage oscilloscope. The input signal is digitized and stored in memory in digital form. In this state it is capable of being analyzed to produce a variety of different information. To view the display on the CRT the data from memory is reconstructed in analog form.



Q.8 (b) (ii) Solution:

Given $P_1 = 2000$ W

and $P_2 = -500$ W

$$\phi = \tan^{-1}\left[\sqrt{3}\frac{(2000-(-500))}{(2000+(-500))}\right] = 70.89°$$

$$\text{Power factor} = \cos\phi = \cos 70.89° = 0.327$$

Q.8 (c) (i) Solution:

There are four kinds of polarizations:

1. Electronic polarization, P_e :
2. Ionic polarization, P_i :
3. Orientational polarization, P_0 :
4. Space charge polarization, P_s :

1. Electronic polarization, P_e :

It occurs in materials containing nonpolar molecules and monatomic gages. When material is subjected to external electric field, a displacement in negatively charged electron cloud is observed. Which leads to induced dipole moment. On removal of electric field, the induced polarization is lost. This type of polarization is independent of temperature.

2. Ionic polarization, P_i :

When ionic materials are placed under electric field cations and anious are displaced in opposite direction. This polarization is also independent of temperature.

3. Orientational polarization, P_0 :

Such polarization is seen in materials having permanent electric dipole moment or polar molecules. On applying external electric field each molecule of the material tends to align itself completely in the field direction acquiring some net electric dipole moment. This alignment is partial at ordinary temperature and complete at low temperature. Eg. water, polymer, Ceramics.

4. Space charge polarization, P_s :

When electric field is applied to a dielectric medium at high temperature the electric charge gets accumulated at the interface. Due to sudden change in conductivity there is a tendency of redistribution of charges in the dielectric medium in presence electric field. The value is very small and hence not significant in most of the materials.

Total polarization or polarizability (α)

$$\alpha = \alpha_e + \alpha_i + \alpha_0$$

or $$\alpha = \alpha_d + \alpha_0$$

Where, α_d is deformation polarizability

$$= \alpha_e + \alpha_i$$

Non polar dielectrics show only α_d

Polar dielectrics show α_d and α_0

By Langevin, $$\alpha_0 = \frac{P_0^2}{3KT}$$

So, $$\alpha = \alpha_e + \alpha_i + \frac{P_0^2}{3KT}$$

Total polarization, $$P = P_e + P_i + P_0$$

$$P = N\left(\alpha_e + \alpha_i + \frac{P_0^2}{3KT}\right)E = P_e + P_i + P_0$$

As for single phase dielectric,

$$P_s = 0$$

Q.8 (c) (ii) Solution:

CsCl structure has one cation (Cs^+) and one anion (Cl^-) in a unit cell,

Given, lattice parameter, $a = 0.412 \times 10^{-9}$ m

Number of ion pair per unit volume,

$$N_i = \frac{1}{a^3} = \frac{1}{(0.412 \times 10^{-9})^3} = 1.43 \times 10^{28} \text{ m}^{-3}$$

Ni is also concentration of cations and anions individually,

From Clausius-Mossotti equation,

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{1}{3\epsilon_0}\left[N_i\alpha_e(Cs^+) + N_i\alpha_e(Cl^-) + N_i\alpha_i\right]$$

So, $$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{1.43 \times 10^{28}(3.35 \times 10^{-40} + 3.40 \times 10^{-40} + 6 \times 10^{-40})}{3 \times 8.85 \times 10^{-12}\text{Fm}^{-1}}$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = 0.686$$

Simplifying we get, $$\epsilon_r = 7.56$$

For optical frequencies,

$$\frac{\epsilon_{r(\text{op})}-1}{\epsilon_{r(\text{op})}+2} = \frac{1}{3\epsilon_0}\left[N_i\alpha_e(Cs^+) + N_i(\alpha_e)Cl^-\right]$$

$$= \frac{(1.43\times10^{28}\,\text{m}^{-3})(3.35\times10^{-40}+3.40\times10^{-40})}{3\times8.85\times10^{-12}\,\text{Fm}^{-1}}$$

$$\frac{\epsilon_{r(\text{op})}-1}{\epsilon_{r(\text{op})}+2} = 0.363$$

On solving $\epsilon_{r\,(\text{op})} = 2.71$

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