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Detailed Solutions

ESE-2021 Mains Test Series

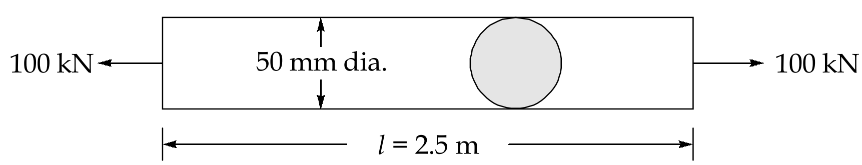
Mechanical Engineering Test No : 13

Full Syllabus Test (Paper-II)

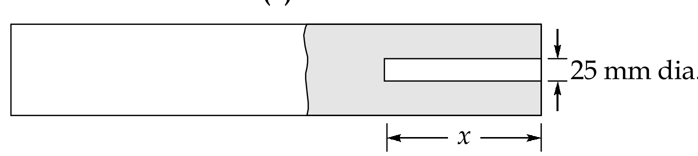
Explanations

1. (a) Solution:

Refer to figure (a) and (b),



(a)



(b)

Diameter of the steel tie rod = 50 mm = 0.05 m

Length of the steel rod, l = 2.5 m

Magnitude of the pull, P = 100 kN

Diameter of the bore = 25 mm = 0.025 m

Modulus of elasticity, $E = 200 \times 10^9$ N/m^2

Length of the bore = x

Stress in the solid rod, $\sigma = \frac{P}{A} = \frac{100 \times 1000}{(\pi/4) \times (0.05)^2} = 50.92 \times 10^6 \text{ N/m}^2$

Elongation of the solid, rod, $\delta l = \frac{\sigma l}{E} = \frac{50.92 \times 10^6 \times 2.5}{200 \times 10^9}$

$= 0.000636$ m or 0.636 mm

Elongation after the rod is bored = 1.15 × 0.636 = 0.731 mm

Area at the reduced section = $\frac{\pi}{4}(0.05^2 - 0.025^2) = 0.001472 \text{ m}^2$

Stress in the reduced section, $\sigma' = \frac{100 \times 1000}{0.001472} = 67.9 \times 10^6 \text{ N/m}^2$

$\therefore$ Elongation of the rod = $\frac{\sigma(2.5 - x)}{E} + \frac{\sigma' \cdot x}{E} = 0.731 \times 10^{-3}$

$$= \frac{50.92 \times 10^6 (2.5 - x)}{200 \times 10^9} + \frac{67.9 \times 10^6 x}{200 \times 10^9} = 0.731 \times 10^{-3}$$

$= 50.92 \times 10^9 (2.5 - x) \times 67.9 \times 10^6 x$

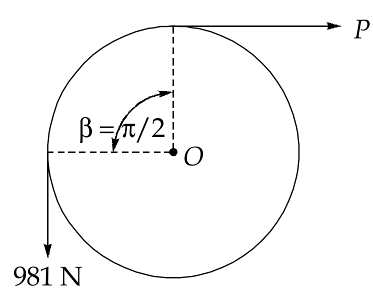
$= 200 \times 10^9 \times 0.731 \times 10^{-3}$

$= (2.5 - x) + 1.33x = 2.87$

$x = 1.12$ m

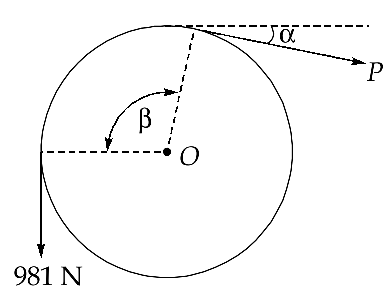
Hence, length of the bore = 1.12 m

1. (b) Solution:



(a) $\alpha = 0$

FBD for case (a)



(b) $P = 500$ N

FBD for case (b)

Impending slippage of the cable over the fixed drum is given by

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

When $\alpha = 0°$, the angle of contact becomes $\beta = \frac{\pi}{2}$. For impending upward motion of the load, $T_2 = P_{max}$, $T_1 = 981$ N, we have $\frac{P_{max}}{981} = \exp\left[0.30\left(\frac{\pi}{2}\right)\right] = 1.602$

$$P_{max} = 981 \times 1.602 = 1572 \text{ N}$$

For impending downward motion of the load,

$$T_2 = 981 \text{ N and } T_1 = P_{min}$$

$$\frac{981}{P_{min}} = \exp\left[0.30\left(\frac{\pi}{2}\right)\right] = 1.602$$

$$P_{min} = \frac{981}{1.602} = 612 \text{ N}$$

For case (b):

$$T_2 = 981 \text{ N and } T_1 = P = 500 \text{ N}$$

$$\frac{981}{500} = \exp\,[0.3\beta]$$

$$0.30\beta = \ln\left(\frac{981}{500}\right) = 0.674$$

$$\beta = \frac{0.674}{0.3} = 2.25 \text{ rad}$$

$$= 2.25 \times \frac{360}{2\pi} = 128.7°$$

From the geometry, $\alpha = 128.7° - 90° = 38.7°$

1. (c) Solution:

Length of the cantilever, $l = 2$ m

External diameter of the steel tube,

$$D = 150 \text{ mm} = 0.15$$

Thickness of the tube = 10 mm

Internal diameter of the tube,

$$d = D - 2r = 150 - 2 \times 10 = 130 \text{ mm} = 0.13 \text{ m}$$

Maximum bending stress, $\sigma_b = 150$ MN/m^2

Maximum bending moment,

$$M = Wl + 2W(l - a) = W(3l - 2a)$$
$$= W(3 \times 2 - 2 \times 0.5) = 5\text{ W Nm}$$

Moment of inertia, $I = \frac{\pi}{64}\left(D^2 - d^4\right)$

$$= \frac{\pi}{64}\left[(0.15)^4 - (0.13)^4\right] = 10.8 \times 10^{-6}\text{m}^4$$

Using the bending equation, we have

$$\frac{M}{I} = \frac{\sigma_b}{y} \text{ or } \sigma_b = \frac{M \times y}{I}$$

$$150 \times 10^6 = \frac{5W \times \left(\frac{D}{2}\right)}{10.8 \times 10^{-6}} = \frac{5W \times 0.075}{10.8 \times 10^{-6}}$$

$\therefore$ $$W = \frac{150 \times 10^6 \times 10.8 \times 10^{-6}}{5 \times 0.075} = 4320\text{ N}$$

Hence, $W = 4320$ N

1. (d) Solution:

(i)

A governor is said to be sensitive when it readily responds to a small change of speed. The movement of the sleeve for a fractional change of speed is the measure of sensitivity.

As a governor is used to limit the change of speed of the engine between minimum to full-load conditions, the sensitiveness of a governor is also defined as the ratio of the difference between the maximum and the minimum speeds (range of speed) to the mean equilibrium speed. Thus,

$$\text{Sensitiveness} = \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

When N = mean speed; N_1 = minimum speed corresponding to full load conditions;

N_2 = maximum speed corresponding to no-load conditions

Sensitiveness of a governor is a desirable quality. However, if a governor is too sensitive, it may fluctuate continuously, because when the load on the engine falls, the sleeve rises rapidly to a maximum position. This shuts off the fuel supply to the extent to affect a sudden fall in the speed. As the speed falls to below the mean value, the sleeve again moves rapidly and falls to a minimum position to increase the fuel supply. The speed subsequently rises and becomes more than the average with the result that the sleeve again rises to reduce the fuel supply. This process continues and is known as hunting.

A governor is said to be stable if it brings the speed of the engine to the required value and there is not much hunting. The ball masses occupy a definite position for each speed of the engine within the working range.

Obviously, the stability and the sensitivity are two opposite characteristics.

(ii)

$$N_1 = 64 \text{ rpm (Initial speed)}$$

$$N_2 = 65 \text{ rpm (Changed speed)}$$

$$\text{Initial height, } h_1 = \frac{895}{N_1^2} = \frac{895}{64^2} = 0.2185 \text{ m}$$

$$\text{Final height, } h_2 = \frac{895}{N_2^2} = \frac{895}{65^2} = 0.2118 \text{ m}$$

$$\text{Change in vertical height} = h_1 - h_2$$

$$= 0.2185 - 0.2118$$

$$= 6.7 \times 10^{-3} \text{ m}$$

$$= 6.7 \text{ mm}$$

1. (e) Solution:

Case A:

Length of the longest link = 7

Length of the shortest link = 3

Length of other links = 5 and 6

Since 7 + 3 < 5 + 6, it belongs to the class-I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

Case B:

Length of the longest link = 8

Length of the shortest link = 4

Length of other links = 6 and 7

Length of (longest link + shortest link) < Length of other links

8 + 4 < 6 + 7, so the mechanism belongs to class-I mechanism.

As the shortest link is fixed, it is double crank mechanism.

Case C:

Length of the longest link = 8

Length of the shortest link = 4

Length of other links = 5 and 6

Since 8 + 4 > 5 + 6, it belongs to class-II mechanism.

Therefore, it is double-rocker mechanism.

2. (a) Solution:

Given : BP = 200 mm = 0.2 m; BD = 250 mm = 0.25 m, M = 15 kg; m = 2 kg; F = 24 N; α_1 = 30°; α_2 = 40°

First of all, let us find the minimum and maximum speed of the governor.

The minimum and maximum position of the governor is shown figure (a) and (b) respectively.

Let, N_1 = Minimum speed, and N_2 = Maximum speed

From figure (a), we find the minimum radius of rotation,

$$r_1 = BG = BP \sin 30° = 0.2 \times 0.5 = 0.1 \text{ m}$$

Height of the governor,

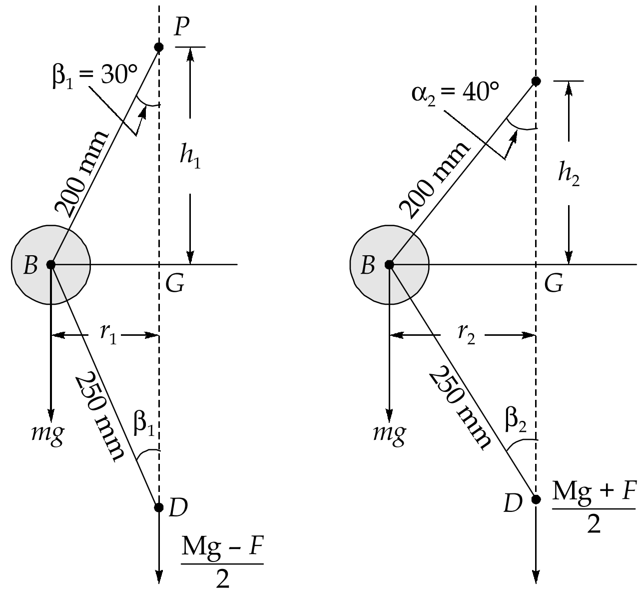
$$h_1 = PG = BP \cos 30° = 0.2 \times 0.866 = 0.1732 \text{ m}$$

and $$DG = \sqrt{(BD)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.1)^2} = 0.23 \text{ m}$$

$\therefore$ $$\tan\beta_1 = \frac{BG}{DG} = \frac{0.1}{0.23} = 0.4348$$

and $$\tan\alpha_1 = \tan 30° = 0.5774$$

$\therefore$ $$q_1 = \frac{\tan\beta_1}{\tan\alpha_1} = \frac{0.4348}{0.5774} = 0.753$$



(a) Minimum position **(b)** Maximum position

We know that when the sleeve moves downwards, the frictional force (F) acts upwards and the minimum speed is given by

$$(N_1)^2 = \frac{m.g. + \left(\frac{Mg - F}{2}\right)(1 + q_1)}{mg} \times \frac{895}{h_1}$$

$$= \frac{2 \times 9.81 + \left(\frac{15 \times 9.81 - 24}{2}\right)(1 + 0.753)}{mg} \times \frac{895}{0.1732} = 33596$$

$\therefore$ $N_1 = 183.3$ rpm

Now from figure (b), we find that maximum radius of rotation,

$$r_2 = BG = BP \sin 40° = 0.2 \times 0.643 = 0.1268 \text{ m}$$

Height of the governor, $h_2 = \text{PG} = \text{BP} \cos 40° = 0.2 \times 0.766 = 0.1532$ m

and $DG = \sqrt{(BD)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.1268)^2} = 0.2154$ m

$\therefore$ $$\tan\beta_2 = \frac{BG}{DG} = \frac{0.1268}{0.2154} = 0.59$$

and $\tan\alpha_2 = \tan 40° = 0.839$

$\therefore$ $$q_2 = \frac{\tan\beta_2}{\tan\alpha_2} = \frac{0.59}{0.839} = 0.703$$

We know that when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum speed is given by

$$(N_2)^2 = \frac{m.g. + \left(\frac{mg + F}{2}\right)(1 + q_2)}{mg} \times \frac{895}{h_2}$$

$$= \frac{2 \times 9.81 + \left(\frac{15 \times 9.81 + 24}{2}\right)(1 + 0.703)}{mg} \times \frac{895}{0.1532} = 49236$$

$\therefore \quad N_2 = 222$ rpm

We know that range of speed = $N_2 - N_1 = 222 - 183.3 = 38.7$ rpm

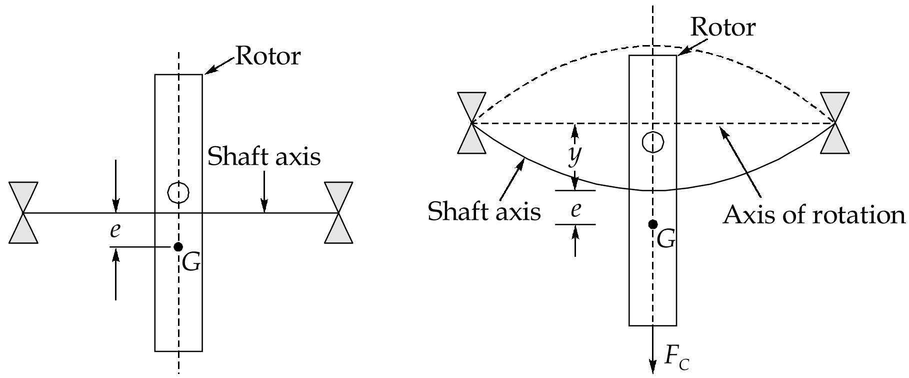
2. **(b) Solution:**

(i)

Critical or Whirling speed of a shaft

In actual practice, a rotating shaft carries different mountings and accessories in the form of gears, pulleys, etc. When the gears or pulleys are put on the shaft, the centre of gravity of the pulley or gear does not coincide with the centre line of the bearings or with the axis of the shaft, when the shaft is stationary. This means that the centre of gravity of the pulley or gear is at a certain distance from the axis of rotation and due to this, the shaft is subjected to centrifugal force. This force will bent the shaft which will further increase the distance of centre of gravity of the pulley or gear from the axis of rotation. This correspondingly increases the value of centrifugal force, which further increases the distance of centre of gravity from the axis of rotation. This effect is cumulative and ultimately the shaft fails. The bending of shaft not only depends upon the value of eccentricity (distance between centre of gravity of the pulley and the axis of rotation) but also depends upon the speed at which the shaft rotates.

The speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as critical or whirling speed.



(a) When shaft is stationary (b) When shaft is rotating

Critical or whirling speed of a shaft

(ii)

Given: d = 100 mm = 0.1 m, L = 1 m, m = 1000 kg, E = 200 × 10^9 N/m^2

$$\text{Cross-section area of shaft, } A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(0.1)^2 = 7.854\times10^{-3}\,\text{m}^2$$

$$\text{Moment of inertia of shaft, } I = \frac{\pi}{64}d^4 = \frac{\pi}{64}\times(0.1)^4 = 4.9\times10^{-6}\,\text{m}^4$$

Frequency of longitudinal vibration,

$$\text{Static deflection, } \delta = \frac{WL}{AE} = \frac{1000\times9.81\times1}{7.854\times10^{-3}\times200\times10^9}$$

$$= 6.245 \times 10^{-6}\text{ m}$$

$$f_n = \frac{0.4958}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{6.245\times10^{-6}}} = 199.47 \simeq 200\text{ Hz}$$

Frequency of transverse vibration,

$$\text{Static deflection, } \delta = \frac{WL^3}{3EI} = \frac{1000\times9.81\times1}{3\times200\times10^9\times4.9\times10^{-6}}$$

$$= 3.3367 \times 10^{-3}\text{ m}$$

$$f_n = \frac{0.4958}{\sqrt{\delta}} = \frac{0.4958}{\sqrt{3.3367\times10^{-3}}} = 8.63\text{ Hz}$$

(iii)

$$\frac{1}{k_1} = \frac{1}{(2k+2k)} + \frac{1}{k} + \frac{1}{(1.5k+1.5k)}$$

$$\frac{1}{k_1} = \frac{1}{4k} + \frac{1}{k} + \frac{1}{3k}$$

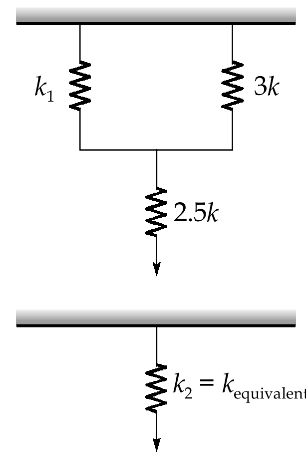
$$\frac{1}{k_1} = \frac{(3+12+4)}{12k} = \frac{19}{12k}$$

$$k_1 = \frac{12}{19}k$$

$$\frac{1}{k_2} = \frac{1}{(k_1+3k)} + \frac{1}{2.5k}$$

$$= \frac{19}{69k} + \frac{1}{2.5k} = \frac{0.6753}{k}$$

$$k_2 = k_{equivalent} = 1.48 \text{ k}$$



2. (c) Solution:

Given: m = 2500 kg; x = 1.5 m; R = 30 m; v = 24 km/h = 6.67 m/s; d_w = 0.75 m or r_w = 0.375 m; $G = \frac{\omega_E}{\omega_W} = 5$; I_W = 18 kgm^2; I_E = 12 kgm^2; h = 0.9 m

The weight of the trolley ($W = mg$) will be equally distributed over the four wheels, which will act downwards. The reaction between the wheels and the road surface of the same magnitude will act upwards.

$\therefore$ Road reaction over each wheel $= \frac{W}{4} = \frac{mg}{4} = 2500 \times \frac{9.81}{4} = 6131.25$ N

We know that angular velocity of the wheels,

$$\omega_W = \frac{v}{r_W} = \frac{6.67}{0.375} = 17.8 \text{ rad/s}$$

and angular velocity of precession,

$$\omega_P = \frac{v}{R} = \frac{6.67}{30} = 0.22 \text{ rad/s}$$

$\therefore$ Gyroscopic couple due to the rotating parts of the motor and gears,

$$C_E = 2I_W \cdot \omega_P = 2 \times 18 \times 17.8 \times 0.22 = 141 \text{ Nm}$$

and gyroscopic couple due to the rotating parts of the motor and gears,

$$C_E = 2I_E \cdot \omega_P \cdot \omega_P = 2I_E\, G\, \omega_W\, \omega_P \qquad (\because \omega_E = G\,\omega_W)$$

$$= 2 \times 12 \times 5 \times 17.8 \times 0.22 = 470 \text{ Nm}$$

$\therefore$ Net gyroscopic couple, $C = C_W - C_E = 141 - 470 = -329$ Nm

[–ve sign is used due to opposite direction of motor]

Due to this net gyroscopic couple, the vertical reaction on the rails will be produced. Since C_E is greater than C_W, therefore the reaction will be vertically downwards on the outer wheels and vertically upwards on the inner wheels. Let the magnitude of this reaction at each of the outer or inner wheels be $\frac{P}{2}$ newton.

$$\frac{P}{2} = \frac{C}{2x} = \frac{329}{2 \times 1.5} = 109.7 \text{ N}$$

We know that the centrifugal force,

$$F = \frac{mV^2}{R} = \frac{2500 \times (6.67)^2}{30} = 3707 \text{ N}$$

$$C_0 = F_c \times h = 3707 \times 0.9 = 3336.3 \text{ Nm}$$

This overturning couple is balanced by the vertical reactions which are vertically upwards on the outer wheel and vertically downwards on the inner wheels.

Let the magnitude of this reaction at each of the outer or inner wheels be $\frac{Q}{2}$ newton.

$$\therefore \qquad \frac{Q}{2} = \frac{C_0}{2x} = \frac{3336.3}{2 \times 1.5} = 1112.1 \text{ N}$$

We know that vertical force exerted on each outer wheel,

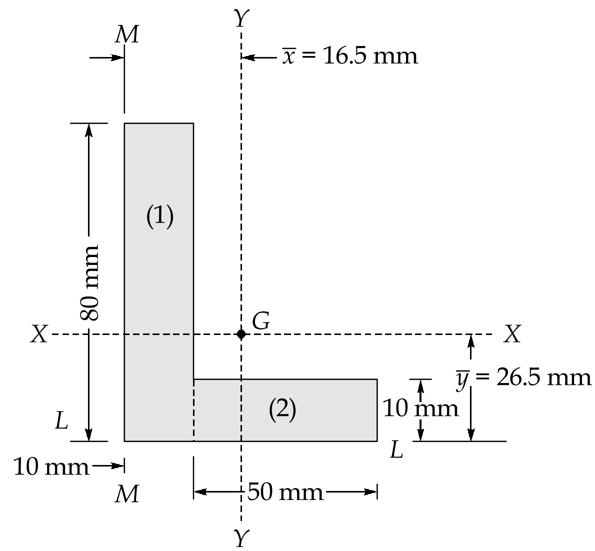
$$P_0 = \frac{W}{4} - \frac{P}{2} + \frac{Q}{2} = 6131.25 - 109.7 + 1112.1 = 7142.65 \text{ N}$$

and vertical force exerted on each inner wheel,

$$P_1 = \frac{W}{4} + \frac{P}{2} - \frac{Q}{2} = 6131.25 + 109.7 - 1112.1 = 5128.85 \text{ N}$$

3. (a) Solution:

(i)



To determine the location of centroid of the section we have the following table:

Components	Area, a(mm^2)	Centroidal distance 'x' from MM (mm)	Centroidal distance 'y' from L(mm)	ax (mm^3)	ay (mm^3)
Rectangle (1)	80 × 10 = 800	5	40	4000	32000
Ractangle (2)	50 × 10 = 500	35	5	17500	2500
Total	Σa = 1300	-	-	Σax = 21500	Σay = 34500

$$\therefore \quad \bar{x} = \frac{\sum ax}{\sum a} = \frac{21500}{1300} = 16.5 \text{ mm}$$

and,
$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{34500}{1300} = 26.5 \text{ mm}$$

To find I_{XX} and I_{YY}, use theorem of parallel axes, as follows:

$$I_{XX} = ?$$

$$I_{XX} = I_{XX_1} + I_{XX_2}$$

$$= \left[\frac{10 \times 80^3}{12} + 80 \times 10 \times (40 - 26.5)^2\right] + \left[\frac{50 \times 10^3}{12} + 50 \times 10 \times (26.5 - 5)^2\right]$$

$$= 572466 + 235291$$

$$= 807757 \text{ mm}^4$$

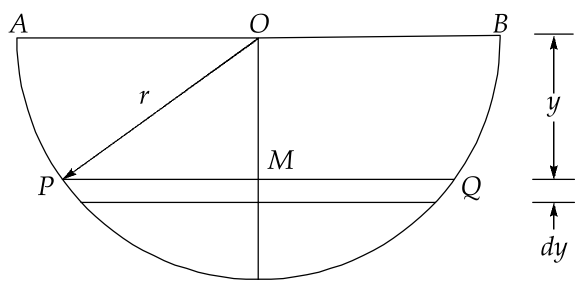
$$I_{YY} = ?$$

$$I_{YY} = I_{YY_1} + I_{YY_2}$$

$$= \left[\frac{80\times 10^3}{12} + 80\times 10\times(16.5-5)^2\right] + \left[\frac{10\times 50^3}{12} + 50\times 10\times(35-16.5)^2\right]$$

$$= 112466 + 275291 = 387757 \text{ mm}^4$$

(ii)



Let figure represents a simple circular plate of radius r,

The length of the elementary strip $PQ = 2\times PM = 2\sqrt{(r^2-y^2)}$

$$\text{Area of } PQ = 2\sqrt{(r^2-y^2)}\,dy$$

If w be the weight per unit area of the material, weight of $PQ = 2\sqrt{(r^2-y^2)}\,dy\,w$

The c.g. of PQ lies on OC at distance y from O.

$$\bar{y} = \frac{\int_0^r 2\sqrt{r^2-y^2}\,dy\,wy}{\int_0^r 2\sqrt{r^2-y^2}\,dy\,w} = \frac{\int_0^r y\sqrt{r^2-y^2}\,dy}{\int_0^r \sqrt{r^2-y^2}\,dy}$$

Now,
$$\int_0^r y\sqrt{r^2-y^2}\,dy = -\frac{1}{2}\int_0^r \left(r^2-y^2\right)^{1/2}(-2y)dy$$

$$= -\frac{1}{2}\left|\frac{2}{3}\left(r^2-y^2\right)^{3/2}\right|_0^r = \frac{r^3}{3} \text{ and } \int_0^r \sqrt{r^2-y^2}\,dy$$

$$= \left|y\frac{\left(r^2-y^2\right)}{2} + \frac{r^2}{2}\sin^{-1}\frac{y}{2}\right|_0^r = \frac{r^2}{2}\times\frac{\pi}{2} = \frac{\pi r^2}{4}$$

$$\therefore \qquad \bar{y} = \frac{\frac{r^3}{3}}{\frac{\pi r^2}{4}} = \frac{4r}{3\pi}$$

Hence the c.g. of a semi-circular lamina lies on the central radius at distance $\frac{4r}{3\pi}$ from the bounding diameter, where r is the radius of the plate.

3. (b) Solution:

(i)

- Rolling bearing
  - Ball bearings
    - Deep-Grrove ball bearings
    - Self-Aligning ball bearings
    - Angular contact ball bearings
    - Thrust ball bearings
  - Roller bearing
    - Tapered roller bearings
    - Spherical roller bearings
    - Cylindrical roller bearings
    - Needle roller bearings

(ii)

- **Abrasive wear :** Abrasive wear occurs when the bearing is made to operate in an environment contaminated with dust, foreign particles, rust or spatter. Remedies against this type of wear are provision of oil seals, increasing surface hardness and use of high viscosity oils. The thick lubricating film developed by these oils allows fine particles to pass without scratching.
- **Corrosive wear :** The corrosion of the surfaces of bearing parts is caused by the entry of water or moisture in the bearing. It is also caused due to corrosive elements present in the extreme. Pressure (EP) additives that are added in the lubricating oils. These elements attack the surfaces of the bearing, resulting in fine wear uniformly distributed over the entire surface. Remedies against this type of wear are, providing complete enclosure for the bearing free from external contamination, selecting proper additives and replacing the lubricating oil at regular intervals.

- **Pitting :** Pitting is the main cause of the failure of antifriction bearings. Pitting is a surface fatigue failure which occurs when the load on the bearing part exceeds the surface endurance strength of the material. This type of failure is characterised by pits which continue to grow resulting in complete destruction of the bearing surfaces. Pitting depends upon the magnitude of Hertz' contact stress and the number of stress cycles. The surface endurance hardness.

(iii)

$$F_r = 5 \text{ kN};\ n = 1450 \text{ rpm};\ L_{99h} = 8000h$$

Step I : Bearing life with 99% reliability

$$L_{99} = \frac{60nL_{99h}}{10^6} = \frac{10(1450)(8000)}{10^6}$$

$$= 696 \text{ million rev.}$$

Step II : Bearing life with 90 reliability

$$\left(\frac{L_{99}}{L_{10}}\right) = \left[\frac{\log_e\left(\frac{1}{R_{99}}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/1.17} = \left[\frac{\log_e\left(\frac{1}{0.99}\right)}{\log_e\left(\frac{1}{0.90}\right)}\right]^{1/1.17} = 0.1342$$

Therefore,

$$L_{10} = \frac{L_{99}}{0.1342} = \frac{696}{0.1342} = 5186.29 \text{ million rev.}$$

Step III : Dynamic load carrying capacity of bearing

$$C = P(L_{10})^{1/3} = 5000(5186.29)^{1/3} = 86547.7 \text{ N}$$

3. **(c) Solution:**

(i)

$$P = 2500 \text{ N},\ \tau = 50 \text{ N/mm}^2$$

Total area of two vertical welds is given by,

$$A = 2 \times 50t = 100t \text{ mm}^2$$

Primary shear stress in the weld,

$$\tau_1 = \frac{P}{A} = \frac{2500}{100t} = \frac{25}{t} \text{ N/mm}^2 \quad \text{...(i)}$$

The moment of inertia of two parallel welds about the x-axis is given by,

$$I = 2\left[\frac{t(50)^3}{12}\right] = 20833.33t\text{ mm}^4$$

$$\sigma_b = \frac{M_b y}{I} = \frac{2500 \times 150 \times 25}{20833.33t} = \frac{450}{t}\text{ N/mm}^2 \qquad \text{...(ii)}$$

From (i) and (ii), we can calculate maximum principal shear stress in the weld given as:

$$\tau = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_1^2}$$

$$= \sqrt{\left(\frac{225}{t}\right)^2 + \left(\frac{25}{t}\right)^2} = \frac{226.385}{t}\text{ N/mm}^2$$

The permissible shear stress in the weld is 50 N/mm^2. Therefore

$$\frac{226.385}{t} = 50 \text{ or } t = 4.53\text{ mm}$$

(ii)

Given data: P = 12 kN, t = 3 mm, σ_t = 80 N/mm^2, τ = 60 N/mm^2, σ_c = 120 N/mm^2

There are four rivets in the lap joint, which are subjected to single shear from shear considerations, we can find diameters as below:

$$4\left[\frac{\pi d^2}{4}\tau\right] = P \text{ or } \pi d^2 \tau = P$$

$$d = \left[\frac{P}{\pi\tau}\right]^{1/2} = \left[\frac{12000}{\pi \times 60}\right]^{0.5} = 7.9788\text{ mm}$$

or $\qquad d = 8$ mm

From crushing consideration:

$$4dt\sigma_c = P$$

$$4d \times 3 \times 120 = 12 \times 10^3$$

$$d = 8.33\text{ mm or } d = 10\text{ mm} \quad \text{[next available diameter]}$$

From above two design considerations, we can observe that crushing becomes the criterion of design. Therefore,

$$d = 10\text{ mm}$$

To calculate width of band, let us consider tensile strength of plate along the section XX,

$$(w - 2d)t\sigma_t = P$$

$$(w - 2 \times 10)(3) = 12 \times 10^3$$

$$w = 50 + 20 = 70 \text{ mm}; \; m = 1.5d = 1.5 \times 10 = 15 \text{ mm}$$

$$P + 2m = w$$

$$P + 2 \times 15 = 70$$

$$P = 40 \text{ mm}$$

4. (a) Solution:

(i)

Given: $kW = 50$, $n = 300$ rpm, $m = 0.15$

For bolts, $S_{yt} = 380$ N/mm^2; $fs = 3$; $N = 6$

For flanges, $D_0 = 200$ mm; $D_i = 150$ mm

Step I : Permissible tensile stress

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{380}{3} = 126.67 \text{ N/mm}^2$$

Step II : Preload in bolts

The torque transmitted by the shaft is given by,

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} = \frac{60 \times 10^6 (50)}{2\pi(300)}$$

$$= 1591549.4 \text{ N-mm}$$

$$R_f = \frac{2\left(R_o^3 - R_i^3\right)}{3\left(R_o^3 - R_i^3\right)} = \frac{2\left(100^3 - 75^3\right)}{3\left(100^2 - 75^2\right)} = 88.1 \text{ mm}$$

Initial tension in each bolt:

$$P_i = \frac{M_t}{\mu N R_f} = \frac{1591549.4}{0.15(6)(88.1)} = 20072.51 \text{ N}$$

Step III : Diameter of bolts

Due to pre-load of 20072.51 N, the bolts are subjected to tensile stresses,

$$P_i = \left(\frac{\pi}{4}\right) d_1^2 \sigma_t$$

or

$$d_1^2 = \frac{4P_i}{\pi\sigma_t} = \frac{4(20072.51)}{\pi(126.67)}$$

$\therefore$ $d = 14.2$ mm

(ii)

From figure for 3ϕ hole, $\dfrac{d}{w} = \dfrac{3}{25} = 0.12, K_t = 2.68$

For 5ϕ hole, $\dfrac{d}{w} = \dfrac{5}{25} = 0.20,\ K_t = 2.52$

For 10ϕ hole, $\dfrac{d}{w} = \dfrac{10}{25} = 0.40,\ K_t = 2.25$

$$\text{Stress for } 3\phi \text{ hole} = \frac{P}{(w-d)t}K_t$$

$$= \frac{20\times10^3\times2.68}{(25-3)\times15} = 162.42 \text{ MPa}$$

$$\text{Stress for } 5\phi \text{ hole} = \frac{20\times10^3\times2.52}{(25-3)\times15} = 168 \text{ MPa}$$

$$\text{Stress for } 10\phi \text{ hole} = \frac{20\times10^3\times2.25}{(25-3)\times15} = 200 \text{ MPa}$$

4. (b) Solution:

(i)

A gear tooth has involute profile only outside the base circle. In fact, the involute profile begins at the base circle. In some cases, the dedendum is so large that it extends below this base circle. In such situations, the portion of the tooth below the base circle is not involute. The tip of the tooth on the mating gear, which is involute, interferes with this non-involute portion of the dedendum. This phenomenon of tooth profiles overlapping and cutting into each other is called 'interference'. In this case, the tip of the tooth overlaps and digs into the root section of its mating gear. Interference is non-conjugate action and results in excessive wear, vibrations and jamming.

When the gears are generated by involute rack cutters, this interference is automatically eliminated because the cutting tool removes the interfering portion of the flank. This is called 'undercutting'. Undercutting solves the problem of interference. However, an undercut tooth is considerably weaker. Undercutting not only weakens the tooth, but also removes a small involute portion adjacent to the base circle. This loss of involute profile may cause a serious reduction in the length of the contact.

(ii)

Given: $\phi = 16°$; $m = 6$ mm; $t = 16$; $N_1 = 240$ rpm or $\omega_1 = 2\pi \times \frac{240}{60} = 25.136$ rad/s;

$$G = \frac{T}{t} = 1.75 \text{ or } T = G.t = 1.75 \times 16 = 28$$

1. Addenda on pinion and gear wheel

We know that addendum on pinion

$$= \frac{mt}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6\times 16}{2}\left[\sqrt{1+\frac{28}{16}\left(\frac{28}{16}+2\right)\sin^2 16°} - 1\right]$$

$$= 48(1.224 - 1) = 10.76 \text{ mm}$$

$$\text{and addendum on wheel} = \frac{mt}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6\times 28}{2}\left[\sqrt{1+\frac{16}{28}\left(\frac{16}{28}+2\right)\sin^2 16°} - 1\right]$$

$$= 84\,(1.054 - 1) = 4.56 \text{ mm}$$

2. Length of path of contact

We know that the pitch circle radius of wheel,

$$R = \frac{mT}{2} = \frac{6\times 28}{2} = 84 \text{ mm}$$

and pitch circle radius of pinion

$$r = \frac{mt}{2} = \frac{6\times 16}{2} = 48 \text{ mm}$$

$\therefore$ Addendum circle radius of wheel,

$$R_A = R + \text{Addendum of wheel} = 84 + 10.76 = 94.76 \text{ mm}$$

and addendum circle radius of pinion,

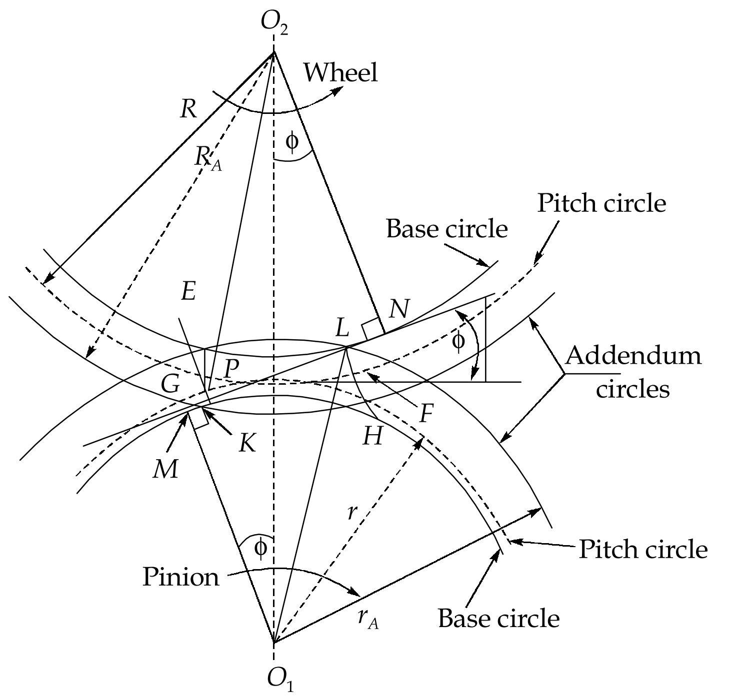
$$r_A = r + \text{Addendum of pinion} = 48 + 4.56 = 52.56 \text{ mm}$$

We know that the length of path of approach,

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{(94.76)^2 - (84)^2\cos^2 16} - 84\sin 16°$$

$$= 49.6 - 23.15 = 26.45 \text{ mm}$$



Length of path of contact

and the length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(52.56)^2 - (48)^2\cos^2 16°} - 48\sin 16°$$

$$= 25.17 - 13.23 = 11.94 \text{ mm}$$

∴ Length of the path of contact,

$$KL = KP + PL = 26.45 + 11.94 = 38.39 \text{ mm}$$

3. Maximum velocity of sliding of teeth on either side of pitch point

Let ω_2 = Angular speed of gear wheel

We know that $\dfrac{\omega_1}{\omega_2} = \dfrac{T}{t} = 1.75$

or $$\omega_2 = \frac{\omega_1}{1.75} = \frac{25.136}{1.75} = 14.28 \text{ rad/s}$$

$\therefore$ Maximum velocity of sliding of teeth on the left side of pitch point i.e. at point K

$$= (\omega_1 + \omega_2)KP = (25.136 + 14.28)26.45 = 1043 \text{ mm/s}$$

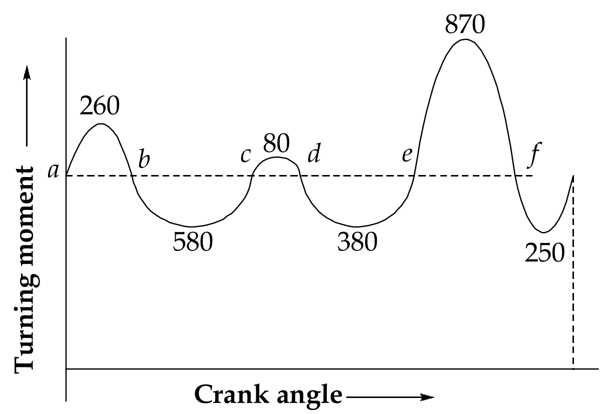
and maximum velocity of sliding of teeth on the right side of pitch point i.e. at point L

$$= (\omega_1 + \omega_2)PL = (25.136 + 14.28)11.94 = 471 \text{ mm/s}$$

4. (c) Solution:

(i)

Let flywheel KE at $a = E$



At $b = E + 260$

At $c = E + 260 - 580 = E - 320$

At $d = E - 320 + 80 = E - 240$

At $e = E - 240 - 380 = E - 620$

At $f = E - 620 + 870 = E + 250$

At $g = E + 250 - 250 = E$

Maximum energy $= E + 260$ (at b)

Minimum energy $= E - 620$ (at e)

Maximum fluctuation of energy,

$$e_{max} = (E + 260) - (E - 620) \times \text{Horizontal scale} \times \text{Vertical scale}$$

$$= 880 \times \left(3 \times \frac{\pi}{180}\right) \times 500 = 23038 \text{ Nm}$$

$$K = \frac{e_{max}}{I\omega^2} = \frac{e_{max}}{mk^2\omega^2} = \frac{23038}{55 \times 2.1 \times \left(\frac{2\pi \times 1600}{60}\right)^2}$$

$$= 0.0034 \text{ or } 0.34\%$$

(ii)

m = 2.5 kg, F_0 = 35 N, A = 15 mm, T = 0.2s

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\pi = 31.4 \text{ rad/s}$$

At resonance,
$$\omega = \omega_n = \sqrt{\frac{k}{m}}$$

$$k = m\omega_n^2 = 2.5 \times 100\pi^2 = 2464.9 \text{ N/m} = 2.464 \text{ N/mm}$$

$$A = \frac{F_0 / k}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\xi\omega_0}{\omega_n}\right)^2}}$$

At resonance, $\frac{\omega}{\omega_n} = 1$

$$A = \frac{[F_0 / k]}{[2\xi]}$$

$$\xi = \frac{F_0}{2kA} = \frac{35}{2 \times 2.465 \times 15} = 0.4733$$

$$C = 2\, m\omega_n\xi = 2 \times 2.5 \times 31.4 \times 0.4733$$
$$= 74.308 \text{ N/m/s}$$

5. **(a) Solution:**

(i)

Single-walled carbon nanotubes (SWNT) have been shown to exhibit piezo-resistive effect, i.e. when they are bent or stretched, their electrical resistance changes. Based on this principle, carbon nanotube-based pressure and strain sensors have been developed. The pressure sensor consists of an ultra thin aluminum oxide membrane to which carbon nanotubes are attached. To calibrate the device, the deformation of the membrane in response to applied pressure was measured using white-light interferometry. They then monitored changes in nanotube resistance as a function of strain. They could detect a change even for strains as small as a hundredth of a percent, which in this case were induced by pressures of a few tens of kilopascals.

The sensing nanotube was in this case metallic so that the gauge factor was positive. It had a value close to that of the best silicon devices. Flow sensors have also been realised using SWNT. The SWNT bundles were packed between two metal electrodes and it was observed that they produced electrical signals in response to fluid flow. This is due to the direct scattering of the free carriers from the fluctuating coulombic fields of the ions or polar molecules in the flowing liquid. It was found through experiments that the ionic strength of the flowing liquid significantly affected the induced voltage.

(ii)

There are different ways of classifying the synthesis routes for nanostructured materials.

One of them is based on the starting state of material, namely, gas, liquid and solid. Techniques such as vapour condensation [physical vapour deposition (PVD) and chemical vapour deposition (CVD) and variants of these techniques) use the gaseous state of matter as the starting material for synthesizing nanoparticles. Techniques such as sol-gel, chemical and electrochemical (electrolytic) deposition and rapid solidification processing use liquids as the starting material. Severe plastic deformation processes such as high-energy ball milling, equichannel angular extrusion, etc., and nano-lithography, start with solids for synthesizing nanocrystalline materials.

However, the most popular way of classifying the synthesis routes is based on how the nanostructures are built, and such an approach leads to two routes, namely, the 'bottom-up' and the 'top-down' approaches. In the bottom-up approach, individual atoms and molecules are brought together or self-assembled to form nanostructured materials in at least one dimension. All the techniques that start with liquid and gas as the starting material fall into this category. In the second approach (top-down approach), a microcrystalline material is fragmented to yield a nanocrystalline material, All the solid state routes fall into this category.

Usually, the bottom-up techniques can give very fine nanostructures of individual nanoparticles, nanoshells, etc., with narrow size distributions, if the process parameters are effectively controlled. The top-down techniques do not usually lead to individual nanoparticles; however, they can produce bulk nanostructured materials. Many of the bottom-up approaches have difficulties in scale up, while the top-down approaches can be easily scaled up. Thus, one can see that both these approaches are complementary to each other, depending on the requirement of a particular application. The most prominent techniques to synthesize nanostructured materials are described.

5. (b) Solution:

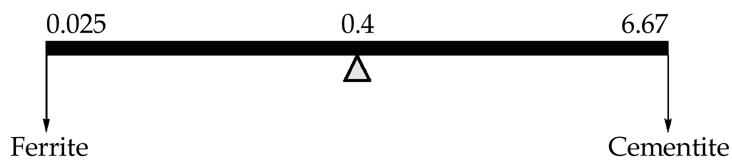
(i)

The limitations of equilibrium phase diagrams are:

1. Stability of the phases under equilibrium condition only.
2. It does not give any information about other metastable phases. i.e. bainite, martensite.
3. It does not indicate the possibilities of suppression of proeutectoid phase separation.
4. No information about kinetics.
5. No information about size.
6. No information on properties.

(ii)

(a) the amount of Fe_3C, ferrite (α) and pearlite.



$$\text{Percentage of } Fe_3C = \frac{0.4 - 0.025}{6.67 - 0.025} \times 100$$

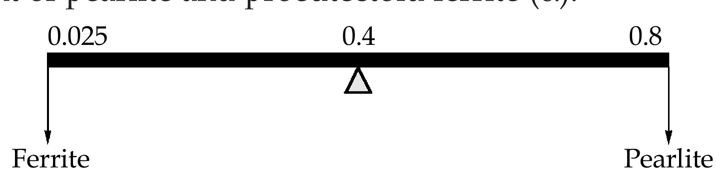
Percentage of Fe_3C in 0.4% C steel is 5.46%

Percentage of ferrite (α) in 0.4 %C steel = 94.36%

or

$$\text{Percentage of ferrite} = \frac{6.67 - 0.4}{6.67 - 0.025} \times 100 = 94.36\%$$

(b) the amount of pearlite and proeutectoid ferrite (α).



$$\text{Percentage of pearlite} = \frac{0.4 - 0.025}{0.8 - 0.025} \times 100$$

Percentage of pearlite = 48%

Percentage of proeutectoid ferrite (α) in 0.4 %C steel = (100 – 48)%

Percentage of proeutectoid ferrite (α) = 52%

or

$$\text{Percentage of proeutectoid ferrite} = \frac{0.8-0.4}{0.8-0.025}\times 100 = 52\%$$

5. (c) Solution:

(i)

- If the break-even point is low and angle of incidence is large. The margin of safety is large and the business enjoys financial stability. A low break-even point indicates that the business could be run profitably even if there is a fall in sales, unless the sales are very low.
- If the break-even point is low and angle of incidence is small, the conclusions are the same as in above except that the rate of profit earning capacity is not so high as in above condition.
- If the break-even point is high and angle of incidence is small. The margin of safety is low. The business is very vulnerable, even a small reduction in activity may result in a loss.
- If the break-even point is high and angle of incidence is large. This shows that the margin of safety is low. The business is likely to incur losses through a small reduction in activity. However, after the break-even point, the business makes the profit at a high rate.

(ii)

$$\text{Fixed cost, } F = ₹40000$$

$$\text{Variable cost, } V = ₹10 \text{ per unit}$$

$$\text{Sales cost, } S = ₹20 \text{ per unit}$$

$$\text{Total cost} = F + QV$$

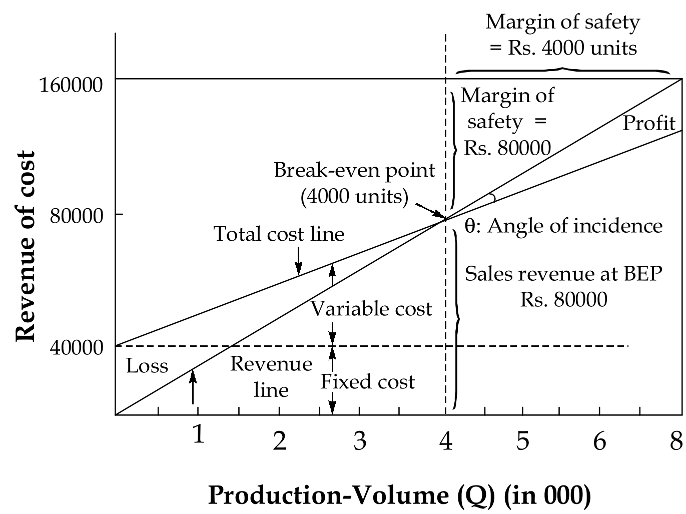
$$\text{Revenue} = QS$$

At break-even point (BEP), total cost is equal to total revenue, thus

$$F + QV = QS$$

$$Q = \frac{F}{S-V} = \frac{40000}{20-10} = 4000 \text{ units}$$

Break-even chart is shown below:



5. (d) Solution:

Commonly used NDT techniques

Technique	Capabilities	Limitations
Visual inspection	Macroscopic surface flaws	Small flaws are difficult to detect, no subsurface flaws
Microscopy	Small surface flaws	Not applicable to larger structures; no subsurface flaws.
Radiography	Subsurface flaws	Smallest defect detectable is 2% of the thickness; radiation protection. No subsurface flaws not for porous materials.
Dye penetrate	Surface flaws	No subsurface flaws not for porous materials
Ultrasonic	Subsurface flaws	Material must be good conductor of sound.
Magnetic particle	Surface/near surface and layer flaws	Limited subsurface capability, only for ferromagnetic materials.
Eddy current	Surface and near surface flaws	Difficult to interpret in some applications; only for metals.
Acoustic emission	Can analyze entire structure	Difficult to interpret, expensive equipments.

5. (e) Solution:

$$\text{Reliability} = \text{P(no failure before T)} = \exp\left[\frac{-T}{MTTF}\right]$$

The probability that failure will occur before time T is 1 minus reliability

$$P(\text{failure before } T) = 1 - \exp\left[\frac{-T}{MTTF}\right]$$

$$\text{MTTF} = 4 \text{ years}$$

Part (i) :

$$P(\text{no failure before } T) = \exp\left[\frac{-4}{4}\right] = e^{-1} = 0.3679$$

Part (ii):

Probability of failure before T = 4 years is

$$= 1 - e^{-1} = 1 - 0.3679 = 0.6321$$

Part (iii):

$$P(\text{no failure before 6 years}) = \exp\left[\frac{-6}{4}\right] = e^{-1.5} = 0.2231$$

6. (a) Solution:

(i)

$$\text{Transducer sensitivity, } K_1 = 0.3 \text{ ohm/°C}$$

$$\text{Wheatstone bridge sensitivity, } K_2 = 0.01 \text{ V/ohm}$$

$$\text{Amplifier gain, } K_3 = 80 \text{ V/V}$$

$$\text{Pen recorder, } K_4 = 1.2 \text{ mm/V}$$

$$\text{Overall sensitivity, } K = K_1 \times K_2 \times K_3 \times K_4$$

$$= 0.3 \times 0.01 \times 80 \times 1.2$$

$$= 0.288 \text{ mm/°C because } \left[\frac{\text{ohm}}{\text{C}} \times \frac{V}{\text{ohm}} \times \frac{V}{V} \times \frac{\text{mm}}{\text{V}}\right]$$

$$\text{Static sensitivity, } K = \frac{\text{Change of output signal}}{\text{Change of input signal}}$$

$$0.288 = \frac{30}{\text{Change of input signal}}$$

$$\text{Temperature change} = \frac{30}{0.288} = 104.167°\text{C}$$

(ii)

$$\text{Range of pressure} = \text{Maximum pressure} - \text{Minimum pressure}$$
$$= 20 - 0 = 20 \text{ bar}$$
$$\text{Percentage of error} = \frac{\text{Calibration pressure}}{\text{Range of pressure}}$$
$$= \frac{0.125}{20} \times 100 = \pm 0.625\%$$
$$\text{Possible error at 2.5 bar} = \frac{0.125}{2.5} \times 100 = 5\%$$

(iii)

Comparision between production planning and production control

Production Planning	Production Control
Production planning is a pre-production activity.	Production conrol will be in action when production activity begins.
Planning involves the collection, maintenance and analysis of data with respect to time standards, materials and their specification, machines and their process capabilities.	Control is concerned with communication of their information and producing reports like output reports, productivity rejection rate, etc.
Planning is useful to anticipate the problems and devising remedial measure in case the problem arises.	Control involves in taking corrective steps in case of error to match actual performance against the planned performance.
Planning is a centralised activity and includes functions like materials control, tool control, process planning and control.	Control is a widespread activity. Includes functions such as dispatching programming and inspection, etc.
Planning sees that all the necessary resources are available to make the production at right quality and time.	Control keeps track of the activities and sees whether everything is going as per schedule or not.

6. (b) Solution:

(i)

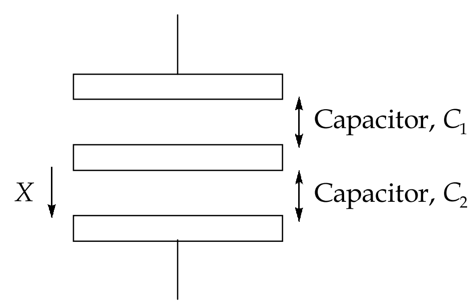
The capacitance is directly proportional to both the plate area and the relative permittivity of the dielectric material and inversely proportional to the distance separating the plates. Therefore, the capacitance can be altered by changing either of the influencing parameters. Variable distance capacitive transducers are generally more sensitive than variable area transducers. A push-pull displacement sensor is a variable distance capacitive transducer. This can be made using three plates in which a central plate is free to move relative to two fixed outer plates. Thus, the upper pair forms one capacitor and the lower pair forms another capacitor. There is a non-linear relationship between the change in capacitance ΔC and the displacement x. As a result of this, the central plate moves downward to increase the plate separation of the upper capacitor and decrease the separation of the lower capacitor.

Therefore, the capacitance of a parallel plate capacitor is given by

$$C_1 = \frac{\varepsilon_0 \varepsilon_r A}{d+x}$$

and
$$C_2 = \frac{\varepsilon_0 \varepsilon_r A}{d-x}$$

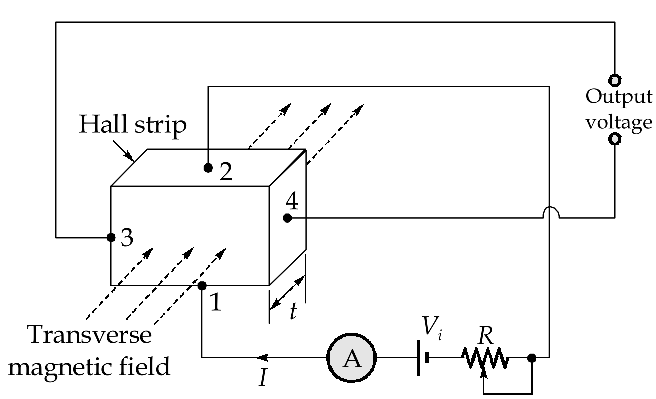
where ε_r is the relative permittivity of the dielectric between the plates; ε_0 is the permittivity of free space constant; x is the displacement of central plate; A is the area of overlap between the two plates and d is the distance of the place separation.



Push pull displacement sensor

(ii)

When a beam of charged particles passes through a magnetic field, the beam is deflected from its straight line path due to the forces acting on the particles. A current flowing in a conductor, such as a beam, is deflected by a magnetic field. This effect is called Hall effect.



Hall effect sensor

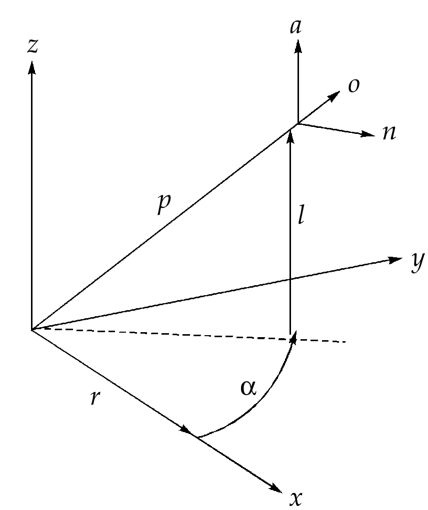
The working principle of a Hall effect sensor is that if a strip of conducting material carries a current in the presence of a transverse magnetic field as shown in figure, the difference of potential is produced between the opposite edges of the conductor. The magnitude of the voltage depends upon the current and the magnetic field. The current is passed through leads 1 and 2 of the strip and the output leads 3 and 4 are connected with a hall strip. When a transverse magnetic field passes through the strip, the voltage difference occurs in the output leads. The hall effect sensor has the advantage of being able to operate as a switch and it can operate upto 100 kHz.

Applications of Hall Effect Sensor

- It is used as a magnetic switch for electric transducer.
- It is used for the measurement of the position, displacement and proximity.
- It is used for measurement of current.
- It is used for measurement of power.

6. (c) Solution:

A cylindrical coordinate system includes two linear translations and one rotation. The sequence is a translation of 1 along the z-axis, as shown in figure below. Since these transformations are all relative to the universe frame. The total transformation caused by these three transformations is found by pre-multiplying by each matrix as follows:



Cylindrical coordinates

$$^{R}T_{P} = T_{cyl}(r,\alpha,l) = Trans(0,0,l)\,Rot(z,\alpha)\,Trans(r,0,0)$$

$$^{R}T_{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c\alpha & -s\alpha & 0 & 0 \\ s\alpha & c\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & r \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{R}T_{P} = T_{cyl}(r,\alpha,l) = \begin{bmatrix} c\alpha & -s\alpha & 0 & rc\alpha \\ s\alpha & c\alpha & 0 & rs\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

First three columns represent the orientation of the frame after this series of transformations. In this case, we are only interested in the position of the origin of the frame, or the last column.

The original orientation of the frame can be restored by rotating the n, o, a frame about the a-axis an angle of $-\alpha$, which is equivalent to post multiplying the cylindrical coordinate matrix by a rotation matrix of Rot $(a, -\alpha)$.

$$T_{\text{cyl}} \times \text{Rot}\,(a, -\alpha) = \begin{bmatrix} c\alpha & -s\alpha & 0 & rc\alpha \\ s\alpha & c\alpha & 0 & rs\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c(-\alpha) & -s(-\alpha) & 0 & 0 \\ s(-\alpha) & c(-\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & rc\alpha \\ 0 & 1 & 0 & rs\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore,

$$T = \begin{bmatrix} 1 & 0 & 0 & -2.394 \\ 0 & 1 & 0 & 6.578 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & rc\alpha \\ 0 & 1 & 0 & rs\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By comparing the matrix:

$$l = 9$$

$$rc\alpha = -2.39 = r\cos\alpha$$

$$rs\alpha = 6.578 = r\sin\alpha$$

$$\tan\alpha = -\frac{6.578}{2.394} = -2.7477$$

$$\alpha = -70° \text{ or } 110°$$

As $s\alpha$ and $c\alpha$ are positive and negative, respectively. Therefore α is in the second quadrant.

$$\alpha = 110°$$

$$r\sin\alpha = 6.578$$

$\Rightarrow$ $$r = 7$$

As derived above, the original orientation of the robot will be given as

$$^{R}T_{P} = \begin{bmatrix} c\alpha & -s\alpha & 0 & rc\alpha \\ s\alpha & c\alpha & 0 & rs\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Substituting the value of r, α and l, we get

$$^{R}T_{P} = \begin{bmatrix} -0.342 & -0.9397 & 0 & -2.394 \\ 0.9397 & -0.342 & 0 & 6.578 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. (a) Solution:

(i)

Volume of steel plate, $V = 7.5 \times 12.5 \times 2 = 187.5 \text{ cm}^3$

Surface area of plate, $A = 2(7.5 \times 12.5 + 7.5 \times 2.0 + 12.5 \times 2.0)$

$$= 267.5 \text{ cm}^2$$

Solidification time, $T_{Ts} = 1.6$ min

$$T_{Ts} = C_m\left(\frac{V}{A}\right)^2 \qquad [C_m : \text{mold constant}]$$

$$C_m = \frac{T_{Ts}}{\left(\frac{V}{A}\right)^2} = \frac{1.6}{\left(\frac{187.5}{267.5}\right)^2} = 3.26 \text{ min/cm}^2$$

Now, we have to design the riser dimension.

For riser, $T_{Ts} = 2$ min

Volume of riser, $V = \dfrac{\pi D^2 H}{4} = \dfrac{\pi D^3}{4}$ [Given $H/D = 1$]

$$\text{Surface area of riser, } A = \pi DH + 2\left(\frac{\pi D^2}{4}\right)$$

$$= \pi D^2 + \frac{\pi D^2}{2} = 1.5\pi D^2$$

$$\frac{V}{A} \text{ ratio for riser } = \frac{\pi D^3/4}{1.5\pi D^2} = \frac{D}{6}$$

$$2 = 0.09056D^2$$

$$D^2 = \frac{2}{0.09056} = 22.086 \text{ cm}^2$$

$$D = 4.7 \text{ cm}$$

(ii)

Simple exponential smoothing can be seen as a form of weighted moving average. Expanding the general equation $F_{t+1} = \alpha D_t + (1 - \alpha)F_t$, we get

$$F_{t+1} = \alpha D_1 + \alpha(1-\alpha)D_{t-1} + \alpha(1-\alpha)^2 D_{t-2} + \ldots + \alpha(1-\alpha)^{t-1} D_1 + (1-\alpha)^t F_1$$

Ast is large and tends to infinity, the term $(1 - \alpha)^t$ tends to zero. The rest of the terms are all terms involving D_j. It can be seen that the F_{t+1} value is a weighted average of the terms D_t to D_1 with weights $\alpha, \alpha(1-\alpha), \alpha(1-\alpha)^2, \ldots + \alpha(1-\alpha)^{t-1}$. If $0 \le \alpha \le 1$, each weight is smaller than 1 and is decreasing. The highest weight is given to the most recent point and the weights progressively decrease by a factor $(1 - \alpha)$ as the data gets older. As t tends to infinity, the weights are α, $\alpha(1 - \alpha)$, $\alpha(1 - \alpha)^2$,... This is an infinite geometric series whose first term is α and the common term is $(1 - \alpha)$.

The sum of all the terms of the progression is $\frac{\alpha}{(1-(1-\alpha)} = 1$.

A small value of α implies that initial weight given to the recent data is small and the subsequent weights are smaller. This means that more terms contribute to the forecast. This also means that more weight is given to the forecast than to the demand.

7. **(b) Solution:**

A microprocessor or a microcontroller is told what to do by means of some instructions which we call as software. The instruction set contains a list of instructions the microprocessor recognizes and carries out a specific operation such as addition, multiplication, etc. intended by the programmer. A series of instructions written, which can be understood by the microprocessor, to carry out a specific task is called a program.

Microprocessors understand instructions written in binary code - called the machine code. It is very time consuming to write programs in machine code, therefore, a set of codes called mnemonic code is given by the manufacturer for each microprocessor which is easy to learn and use. The programs written using the mnemonic code are called assembly level programs and the language is referred to as the assembly language. Each instruction in assembly language is an abbreviated statement of the operation that is intended to be carried out and as such becomes easily comprehendible during the program writing process as well as when checking the program for errors. As we have already mentioned, the microprocessor would only recognize the machine code. To perform a series of operations we will have to convert these instructions in assembly language into the machine code. For this we can use manufacturer's instructions/data sheets which give the binary code for each mnemonic code used in the assembly language. This work can be very time consuming and also extremely error-prone and as such is not feasible when the programs are long. To overcome this we use computer programs that convert assembly level programs into machine code programs.

The set of commands that is understood by the microprocessors is called instruction set of that particular microprocessor or the microcontroller. This is given by the manufacturer. In general, the instructions are classified as the sets referring to data transfer, arithmetic operations, logical decisions and program control.

As already mentioned, the instruction sets are particular to each microprocessor but certain instructions are common to most microprocessors available in the market.

Some of the common instructions are as follows:

1. **Load:** This instruction reads the contents of a particular memory location and copies to a specific register in the processor. Example: Say a data is stored in memory location 1110. This data would be copied to the accumulator {register).
2. **Store:** This is the reverse of load instruction. This copies data in a register to a specified memory location.
3. **Add:** This instruction adds the data or the contents of a specified memory location to the data in some particular register.
4. **Decrement:** This instruction subtracts 1 from the contents of a specified location. For example, we may have the accumulator as the specified location.
5. **Compare:** This instruction compares the contents of register and a specified memory location and indicates whether it is greater than, less than or equal to the contents of the memory location. This result appears as flag in the status register.

6. **AND, Exclusive-OR:** These instructions AND and Exclusive-OR do the logical operations of ANDing and EX-ORing, respectively, the respective bits of the data element of a specified memory location and the data in some other register.
7. **Logical Shift (Left or Right):** These instructions move the pattern of bits one place to the left or to the right by moving a 0 to the end of the number. Logical shift right shifts in 0 into the MSB. For example, for logical shift right a 0 is shifted to the MSB and the LSB is shifted to the carry flag in the status register.
8. **Arithmetic Shift {Left or Right):** Arithmetic shift instruction involves moving the pattern of bits in the register one place to the right or left. Arithmetic shift right retains the MSB and shifts all others to the right. Arithmetic shift left shifts every bit one position left and the MSB is shifted to carry and whatever is there in carry is dropped.
9. **Rotate (Left or Right):** Rotate moves the pattern of bits in the register one place to the left or to the right and the bit that spills over is written back into the ocher end.
10. **Jump:** Jump instruction changes the sequence in which the program is being carried out. For example: Jump to the instruction ...if the accumulator is not zero.
11. **Branch:** This instruction makes the program to take a different branch of instructions when a particular condition is satisfied.
12. **Halt:** This instruction stops all further activity.

7. (c) Solution:

Let x_1 and x_2 be the amount invested in bonds A and B, respectively. Using the given data, we may state the problem as follows:

Maximise $$Z = 0.07x_1 + 0.10x_2$$

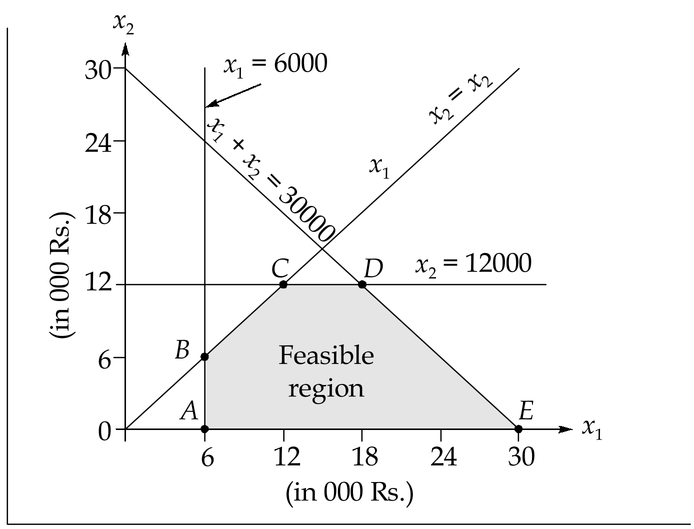
Subject to
$$x_1 + x_2 \leq 30000$$
$$x_1 \geq 12000$$
$$x_1 - x_2 \geq 0$$
$$x_1, x_2 \geq 0$$

The constraints are plotted graphically in figure. The feasible region is shown shaded and is bound by points, A, B, C, D and E.

The extreme points are evaluated here,

Point	x_1	x_2	$Z = 0.07x_1 + 0.10x_2$
A	6000	0	420
B	6000	6000	1020
C	12000	12000	2040
D	18000	12000	2460
E	30000	0	2100

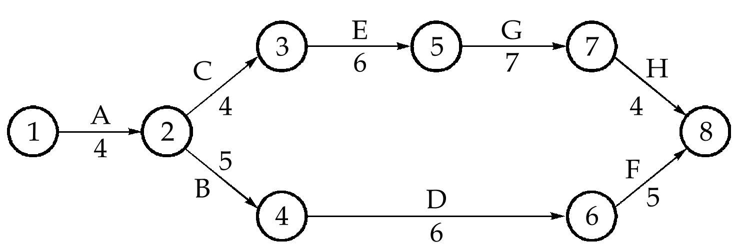
The Z-value is maximum at point D. Accordingly, the optimal solution is invest Rs.18000 in Bond A and Rs. 12000 in Bond B. It would yield a return of Rs. 2460.



Graphic determination of investment mix

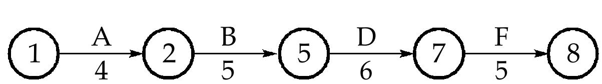
8. **(a) Solution:**

We have the following network diagram for the given project with normal costs:



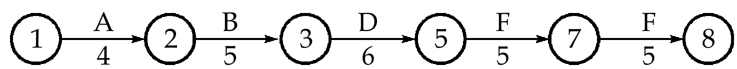
Beginning from the Start Node and terminating with the End Node, there are two paths for the network as detailed below:

Path I



The time for the path = 4 + 5 + 6 + 5 = 20 weeks

Path II



The time for the path = 4 + 4 + 6 + 7 + 4 = 25 weeks

Maximum of {20, 25} = 25

Therefore Path II is the critical path and the critical activities are A, C, E, G and H. The non-critical activities are B, D and F.

Given that the normal time of activity A is 4 weeks while its crash time is 3 weeks. Hence the time of this activity can be reduced by one week if the management is prepared to spend an additional amount. However, the time cannot be reduced by more than one week even if the management may be prepared to spend more money. The normal cost of this activity is Rs. 8,000 whereas the crash cost is Rs. 9,000. From this, we see that crashing of activity A by one week will cost the management an extra amount of Rs. 1,000. In a similar fashion, we can work out the crash cost per unit time for the other activities also. The results are provided in the following table.

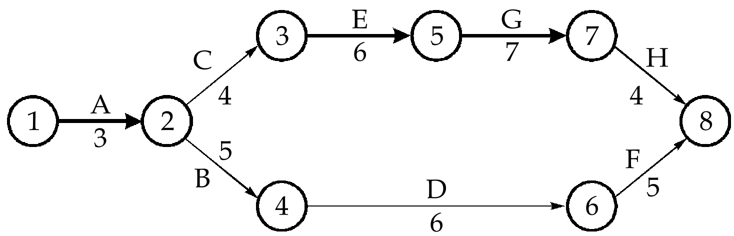
Activity	Normal time	Crash time	Normal cost	Crash cost	Normal time- Crash cost	Crash cost per unit time
A	4	3	8000	9000	1	1000
B	5	3	16000	20000	2	2000
C	4	3	12000	13000	1	1000
D	6	5	34000	35000	1	1000
E	6	4	42000	44000	2	1000
F	5	4	16000	16500	1	500
G	7	6	66000	72000	1	6000
H	4	3	2000	5000	1	3000

A non-critical activity can be delayed without delaying the execution of the whole project. But, if a critical activity is delayed, it will delay the whole project. Because of this reason, we have to select a critical activity for crashing. Here we have to choose one of the activities A, C, E, G and H. The crash cost per unit time works out as follows:

Rs. 1,000 for A; Rs. 1,000 for C; Rs. 1,000 for E; Rs. 6,000 for G; Rs. 3,000 for H.

The maximum among them is Rs. 1,000. So we have to choose an activity with Rs.1,000 as the crash cost per unit time. However, there is a tie among A, C and E. The tie can be resolved arbitrarily. Let us select A for crashing. We reduce the time of A by one week by spending an extra amount of Rs. 1,000.

After this step, we have the following network with the revised times for the activities:



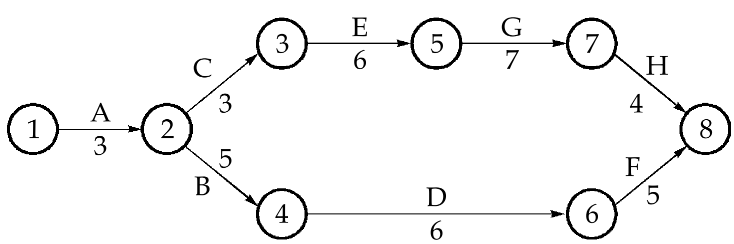
The revised time for Path I = 3 + 5 + 6 + 5 = 19 weeks.

The time for Path II = 3 + 4 + 6 + 7 + 4 = 24 weeks.

Maximum of {19, 24} = 24.

Therefore Path II is the critical path and the critical activities are A, C, E, G and H. However, the time for A cannot be reduced further. Therefore, we have to consider C, E, G and H for crashing. Among them, C and E have the least crash cost per unit time. The tie between C and E can be resolved arbitrarily. Suppose we reduce the time of C by one week with an extra cost of Rs. 1,000.

After this step, we have the following network with the revised times for the activities:



The time for path I = 3 + 5 + 6 + 5 = 19 weeks

The time for path II = 3 + 3 + 6 + 7 + 4 = 23 weeks

Maximum of {19, 23} = 23

Therefore path II is the critical path and the critical activities are A, C, E, G and H. Now the time for A or C cannot be reduced further. Therefore, we have to consider E, G and H for crashing. Among them, E has the least crash cost per unit time. Hence we reduce the time of E by one week an extra cost of Rs. 1000.

By the given condition, we have to reduce the project time by 3 weeks. Since this has been accomplished, we stop with the step.

Result : We have arrived at the following crashing scheme for the given project:

Reduce the time of A, C and E by one week each.

Project time after crashing is 22 weeks.

Extra amount required = 1000 + 1000 + 1000 = Rs. 3000

8. (b) Solution:

(i)

$$\text{Tooling cost } C_t = \text{Tool change cost} + \text{Tool regrind cost} + \text{Tool depreciation}$$

$$= \frac{5}{60}\times 8 + \frac{5}{60}\times 5 + 0.30 = \text{Rs.}1.38$$

$$\text{Machining cost, } C_m = \text{Rs.}\frac{5}{60}$$

$$\therefore \quad V_T = C\left[\frac{C_m}{C_t}\cdot\frac{n}{1-n}\right]^n = 150\left[\frac{5}{60\times 1.38}\cdot\frac{0.25}{0.75}\right]^{0.25}$$

$$= 56.5 \text{ m/min}$$

(ii)

(a) $$\text{Shear angle, } \phi = \tan^{-1}\left(\frac{r\cos\alpha}{1-r\sin\alpha}\right)$$

$$r = \frac{t}{t_c} = \frac{0.127}{0.228} = 0.557$$

$$\alpha = 10°$$

$$\therefore \quad \phi = \tan^{-1}\left(\frac{0.557\times 0.985}{1-0.557\times 0.1736}\right)$$

$$= \tan^{-1}(0.607) = 31.25°$$

(b) $$\mu = \tan\beta = \frac{F_c\sin\alpha + F_t\cos\alpha}{F_c\cos\alpha - F_t\sin\alpha}$$

$$= \frac{567\times 0.1736 + 227\times 0.985}{567\times 0.985 - 227\times 0.1736}$$

$$= \frac{98.4\times 223.6}{558.5+39.4} = \frac{332}{519.1} = 0.64$$

or $$\beta = \tan^{1}(0.64) = 32.62°$$

(c) $$\text{Shear force, } F_s = F_c\cos\phi - F_t\sin\phi$$

$$= 567 \times 0.855 - 227 \times 0.519$$

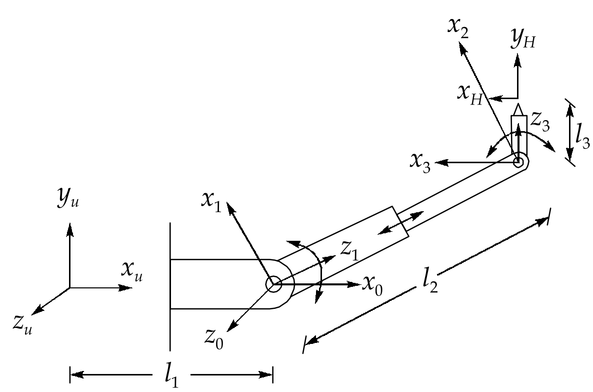
$$= 484.8 - 117.8 = 367 \text{ N}$$

$$\therefore \qquad \tau_s = \frac{F_s}{A_s} = \frac{367 \times \sin\phi}{bt}$$

$$= \frac{367 \times 0.519}{6.35 \times 0.127} = \frac{190.5}{0.806} = 236.5 \text{ N/mm}^2$$

(d) $$\text{Cutting power} = \frac{F_c V}{1000} = \frac{567 \times 2}{1000} = 1.134 \text{ kW}$$

8. (c) Solution:



We can note that there is no particular reset position.

Coordinate frames are assigned as under:

D-H parameter table:

#	θ	d	α
0 - 1	$90° + \theta_1$	0	90°
1 - 2	0	l_2	–90°
2 - 3	θ_3	0	90°
3 - H	0	l_3	0°

Link	a_i	θ_i	d_i	α
0 - 1	l_1	$90° + \theta_1$	0	90°
1 - 2	0	0	l_2	–90°
2 - 3	0	θ_2	0	90°
3 - H	0	0	l_3	0°

$$^{U}T_0 = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -s_1 & 0 & c_1 & 0 \\ c_1 & 0 & s_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad [c_1 : \cos\theta_1 ; s_1 = \sin\theta_1]$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{U}T_H = {}^{U}T_0 {}^{0}T_H = {}^{U}T_0 A_1 A_2 A_3 A_4$$

The total transformation can be summarized by multiplying these matrices.

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