Detailed Solutions

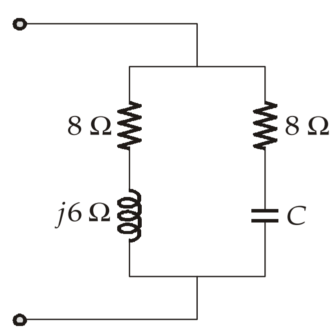
ESE-2021 Mains Test Series

Electrical Engineering Test No : 12

Section-A

Q.1 (a) Solution:

The input admittance of the circuit is given by



$$Y_{in} = \frac{1}{8+j6} + \frac{1}{8-jX_c}$$

Where,

$$X_c = \frac{1}{\omega C}$$

$$Y_{in} = \frac{1}{8+j6} \times \frac{8-j6}{8-j6} + \frac{1}{8-jX_c} \times \frac{8+jX_c}{8+jX_c}$$

$$= \frac{8-j6}{100} + \frac{8+jX_c}{64+X_c^2}$$

$$= \left(\frac{8}{100} + \frac{8}{64+X_c^2}\right) + j\left(\frac{X_c}{64+X_c^2} - \frac{6}{100}\right)$$

At resonance, the imaginary part of admittance must be zero.

$$\therefore \quad \frac{X_c}{64+X_c^2} - \frac{6}{100} = 0$$

$$\frac{X_c}{64+X_c^2} = \frac{6}{100}$$

Where, $X_c = \frac{1}{\omega C}$ and $\omega = 5000$ rad/sec

$$6X_c^2 - 100X_c + 384 = 0$$

$$X_c = 10.666 \text{ and } 6$$

$$\frac{1}{\omega C} = 10.666 \text{ and } 6$$

$$C = \frac{1}{5000 \times 10.66} \text{ and } \frac{1}{5000 \times 6} = 18.76\ \mu\text{F and } 33.33\ \mu\text{F}$$

Q.1 (b) Solution:

(i) We know, electric displacement factor

$$D = \varepsilon_o E + P$$

$$\varepsilon_o \varepsilon_r E = \varepsilon_o E + P$$

or $$P = \varepsilon_o E(\varepsilon_r - 1)$$

Since $$P = \chi_e \varepsilon_o E$$

$\therefore$ $$\chi_e \varepsilon_o E = \varepsilon_o E(\varepsilon_r - 1)$$

or, $$\chi_e = \varepsilon_r - 1$$

$\therefore$ We can say electric susceptibility is a measure of the extent to which the material can be polarized by applying the electric field.

(ii) Capacitance of formed capacitor,

$$C = \frac{\varepsilon A}{d}$$

$$= \frac{\varepsilon_o \varepsilon_r A}{d}$$

$$= \frac{8.85 \times 10^{-12} \times 6 \times 6.45 \times 10^{-4}}{1 \times 10^{-3}}$$

$$= 3.42 \times 10^{-11} \text{ F}$$

Charge, $q = CV = 3.42 \times 10^{-11} \times 10$

$= 3.42 \times 10^{-10}$ C

Displacement vector, $D = \varepsilon E = \varepsilon_o \varepsilon_r E$

$$= \frac{8.85 \times 10^{-12} \times 6 \times 10}{1 \times 10^{-3}}$$

$= 5.31 \times 10^{-7}$ C/m^2

$P = D - \varepsilon_o E$

$$= 5.31 \times 10^{-7} - \frac{8.85 \times 10^{-12} \times 10}{1 \times 10^{-3}}$$

$= 5.31 \times 10^{-7} - 8.85 \times 10^{-8}$

$= 4.425 \times 10^{-7}$ C/m^2

Q.1 (c) Solution:

Apply KVL around Q_1

$$-V_{GS1} + V_{GS2} + V_{DS1} = 0$$

$$V_{GS1} = V_{GS2} + V_{DS1} \quad \text{...(i)}$$

But, $I_{D1} = I_{D2} = I_D$

So, $$I_{DSS}\left[1 - \frac{V_{GS1}}{V_P}\right]^2 = I_{DSS}\left[1 - \frac{V_{GS2}}{V_P}\right]^2$$

So, $$V_{GS1} = V_{GS2} \quad \text{...(ii)}$$

On putting equation (ii) in equation (i),

$$V_{DS1} = 0 \text{ V}$$

Apply KVL in input loop,

$$V_{GS1} + 2I_D = 0$$

$$V_{GS1} = -2I_D \quad \text{...(iii)}$$

and $$I_D = 8\left[1 + \frac{V_{GS}}{4}\right]^2 \quad \text{...(iv)}$$

From equation (iii) and (iv),

$$I_D = 8\left[1 - \frac{I_D}{2}\right]^2 = 2[2 - I_D]^2$$

$$I_D = 2I_D^2 - 8I_D + 8$$

$$2I_D^2 - 9I_D + 8 = 0$$

$$I_D = 1.22 \text{ mA}, 3.28 \text{ mA}$$

$$I_D = 1.22 \text{ mA (valid)}$$

$$V_{GS1} = V_{GS2} = -2I_D = -2.44 \text{ V}$$

KVL in outer loop,

$$15 = 5I_D + V_{DS2} + V_{DS1} + 2I_D$$

$$V_{DS2} = 15 - 7I_D = 15 - 7 \times 1.22 = 6.46 \text{ V}$$

Q.1 (d) Solution:

Divide the interval (0, 6) into six parts each of width 1.

x	0	1	2	3	4	5	6
$y = f(x)$	$y_0 = 1$	$y_1 = 0.5$	$y_2 = 0.2$	$y_3 = 0.1$	$y_4 = 0.05882$	$y_5 = 0.0385$	$y = 0.027$

By Trapezoidal rule,

$$\int_0^6 \frac{1}{1+x^2} = \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$= \frac{1}{2}\left[(1 + 0.027) + 2(0.5 + 0.2 + 0.1 + 0.058 + 0.0385)\right]$$

$$= 1.4108$$

Q.1 (e) Solution:

The deflection of a thermoelectric ammeter is

$$\theta = kI^2$$

Suppose θ_F is the full scale deflection.

$$\therefore \quad \theta_F = k \times (10)^2$$

or

$$\frac{\theta_F}{k} = 100$$

The deflection at half scale is $\frac{\theta_F}{2}$.

$$\therefore \quad \frac{\theta_F}{2} = kI^2$$

or current required to give half scale deflection

$$I = \left(\frac{\theta_F}{2k}\right)^{1/2} = \left(\frac{100}{2}\right)^{1/2} = 7.07 \text{ A}$$

Q.2 (a) Solution:

The composition of an alloy is expressed in terms of its constituent elements as follows: (1) Composition by wt. %; (2) Composition by atom percent %

1. **Composition by weight percent :** It is relative weight of a particular element relative to the total alloy weight. For example, an alloy that contains two hypothetical atoms denoted by 1 and 2, the concentration of 1 in weight %.

$$C_1 = \frac{m_1}{m_1 + m_2} \times 100$$

where m_1 and m_2 represent the weight (or mass) of element 1 and 2 respectively.

2. **Composition by atom percent (atom %) :** The number of moles of an element in relation to the total moles of the elements in an alloy. For example, in two element alloy number of moles of element 1 can be computed,

$$n_{m1} = \frac{m_1'}{A_1}$$

where m_1' and A_1 denotes the mass (in grams) and atomic weight respectively.

So, concentration in terms of atom percent

$$C_1' = \frac{n_{m1}}{n_{m1} + n_{m2}} \times 100$$

Derivation of composition : Conversion equation from (wt. %) to (atom %).

Assuming the masses are expressed in units of grams and denoted with prime.

$$M' = m_1' + m_2'$$

Concentration,
$$C_1' = \frac{n_{m1}}{n_{m1} + n_{m2}} \times 100 = \frac{\frac{m_1'}{A_1}}{\frac{m_1'}{A_1} + \frac{m_2'}{A_2}} \times 100$$

By using relation of rearrangement of mass in gram.

$$m_1' = \frac{C_1 M'}{100}$$

Substitution of this expression and its m_2' equivalent in equation.

$$C_1' = \frac{\frac{C_1 M'}{100 A_1}}{\frac{C_1 M'}{100 A_1} + \frac{C_2 M'}{100 A_2}} \times 100$$

Upon simplification, $$C_1' = \frac{C_1A_2}{C_1A_2 + C_2A_1}$$

(ii) If we denote the respective weight percent compositions as $C_{Al} = 97$ and $C_{Cu} = 3$.

$$C_{Al}' = \frac{C_{Al}A_{Cu}}{C_{Al}A_{Cu} + C_{Cu}A_{Al}} \times 100$$

$$= \frac{97 \times 63.55}{(97) \times (63.55 \text{ g/mol}) + (3)(26.98 \text{ g/mol})} \times 100$$

$$= 98.7 \text{ atom\%}$$

and $$C_{Cu}' = \frac{C_{Cu} \cdot A_{Al}}{C_{Cu}A_{Al} + C_{Al}A_{Cu}} \times 100$$

$$= \frac{(3)(26.98 \text{ g/mol})}{(3)(26.98 \text{ g/mol}) + (97)(63.55 \text{ g/mol})} \times 100$$

$$= 1.30 \text{ atom \%}$$

Q.2 (b) Solution:

Voltage across instrument for full scale deflection = 100 mV

Current in instrument for full scale deflection

$$I = \frac{V}{R} = \frac{100 \times 10^{-3}}{20} = 5 \times 10^{-3} \text{ A}$$

Deflecting torque, $$T_d = NBldI = 100 \times B \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 5 \times 10^{-3}$$

$$= 375B \times 10^{-8} \text{ N-m}$$

Controlling torque for a deflection θ = 120°

$$T_c = k\theta = 0.375 \times 10^{-6} \times 120 = 45 \times 10^{-6} \text{ N-m}$$

At final steady position, $$T_d = T_c$$

or $$375 \times 10^{-6}B = 45 \times 10^{-6}$$

∴ Flux density in the air gap

$$B = \frac{45 \times 10^{-6}}{375 \times 10^{-6}} = 0.12 \text{ Wb/m}^2$$

Resistance of coil winding

$$R_c = 0.3 \times 20 = 6\ \Omega$$

Length of mean turn

$$L_{mt} = 2(l + d) = 2(30 + 25) = 110 \text{ mm}$$

Let a be the area of cross-section of wire and ρ be the resistivity.

Resistance of coil, $R_c = N\rho\frac{Lmt}{a}$

∴ Area of cross-section of wire, $a = \frac{100 \times 1.7 \times 10^{-8} \times 110 \times 10^{-3} \times 10^{6}}{6}$

$= 31.37 \times 10^{-3}\ \text{mm}^2$

Diameter of wire, $d = \left[\frac{4/\pi}{(31.37 \times 10^{-3})}\right]^{1/2} = 0.2\ \text{mm}$

Q.2 (c) (i) Solution:

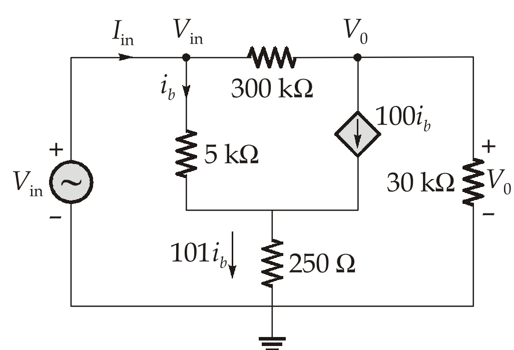
Since, $\beta = 100,$

$$I_C = \frac{\beta}{1+\beta} \times I_E = \frac{100 \times 0.5}{101} = 0.495\ \text{mA}$$

$$I_E = 0.5\ \text{mA}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = \frac{100 \times 25}{0.495} = 5.05\ \text{k}\Omega$$

r_π - model of given amplifier circuit,



Applying KVL at input side,

$$V_{in} = 5.05\ i_b + 0.25 \times 101\ i_b$$

$$V_{in} = 30.3\ i_b$$

$$i_b = \frac{V_{in}}{30.3\ \text{k}\Omega}$$

On applying KCL at output node 'V_0'

$$\frac{V_0 - V_{in}}{300} + \frac{V_0}{30} + 100 i_b = 0$$

$$V_0\left[\frac{1}{300}+\frac{1}{30}\right]-\frac{V_{in}}{300}+100i_b = 0$$

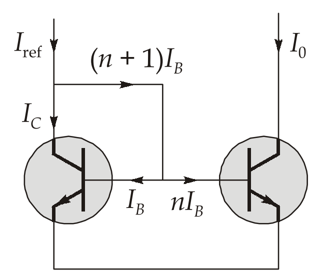
$$V_0\left[\frac{1}{300}+\frac{1}{30}\right] = V_{in}\left[\frac{1}{300}-\frac{100}{30.3}\right]$$

Voltage gain,
$$\frac{V_0}{V_{in}} = \frac{\left(\frac{1}{300}-\frac{100}{30.3}\right)}{\left(\frac{11}{300}\right)} = -89.91$$

Q.2 (c) (ii) Solution:

Area of Q_2 is n-times, so base current of Q_2 is n-times than that of Q_1

Now,



$$I_{ref} = I_C + (n+1)I_B$$

$\because$
$$I_C = \beta I_B$$

$$I_{ref} = \beta I_B + (n+1)I_B = (n+1+\beta)I_B$$

Also,
$$I_0 = n\beta I_B$$

$$\frac{I_0}{I_{ref}} = \frac{n\beta I_B}{(n+1+\beta)I_B}$$

$$= \frac{n\beta}{(n+1+\beta)}$$

$$I_0 = \frac{nI_{ref}}{1+\left(\frac{n+1}{\beta}\right)}$$

Q.3 (a) (i) Solution:

Input Output Interface : The method that is used to transfer information between internal storage and external I/O devices is known as I/O interface. The CPU is interfaced using special communication links by the peripherals connected to any computer system. These communication links are used to resolve the speed differences between CPU and peripheral. There exist special hardware components between CPU and peripherals to supervise and synchronize all the input and output transfers that are called interface units.

Data transfer to and from the peripherals may be done in any of the three possible ways

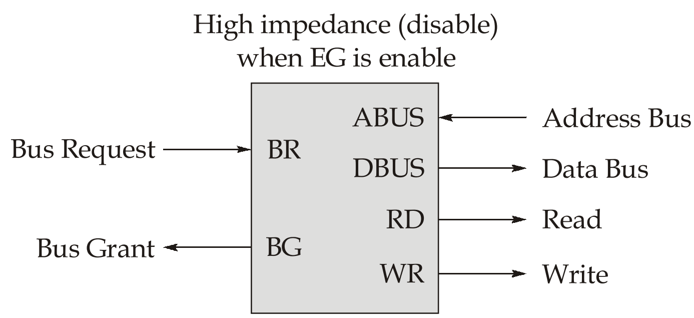
1. Programmed I/O.
2. Interrupt- initiated I/O.
3. Direct memory access(DMA).

Programmed I/O: It is due to the result of the I/O instructions that are written in the computer program. Each data item transfer is initiated by an instruction in the program. Usually the transfer is from a CPU register and memory. In this case it requires constant monitoring by the CPU of the peripheral devices.

Example of Programmed I/O: In this case, the I/O device does not have direct access to the memory unit. A transfer from I/O device to memory requires the execution of several instructions by the CPU, including an input instruction to transfer the data from device to the CPU and store instruction to transfer the data from CPU to memory. In programmed I/O, the CPU stays in the program loop until the I/O unit indicates that it is ready for data transfer. This is a time consuming process since it needlessly keeps the CPU busy.

Interrupt- initiated I/O: Since in the above case we saw the CPU is kept busy unnecessarily. This situation can very well be avoided by using an interrupt driven method for data transfer. By using interrupt facility and special commands to inform the interface to issue an interrupt request signal whenever data is available from any device. In the meantime the CPU can proceed for any other program execution. The interface meanwhile keeps monitoring the device. Whenever it is determined that the device is ready for data transfer it initiates an interrupt request signal to the computer. Upon detection of an external interrupt signal the CPU stops momentarily the task that it was already performing, branches to the service program to process the I/O transfer, and then return to the task it was originally performing.

Direct Memory Access: The data transfer between a fast storage media such as magnetic disk and memory unit is limited by the speed of the CPU. Thus we can allow the peripherals directly communicate with each other using the memory buses, removing the intervention of the CPU. This type of data transfer technique is known as DMA or direct memory access. During DMA the CPU is idle and it has no control over the memory buses. The DMA controller takes over the buses to manage the transfer directly between the I/O devices and the memory unit.

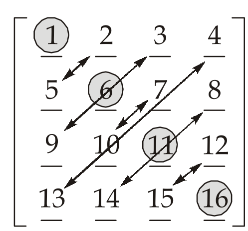


Bus Request: It is used by the DMA controller to request the CPU to relinquish the control of the buses.

Bus Grant: It is activated by the CPU to Inform the external DMA controller that the buses are in high impedance state and the requesting DMA can take control of the buses. Once the DMA has taken the control of the buses it transfers the data. This transfer can take place in many ways, like burst mode, short burst mode, interleaved DMA.

Q.3 (a) (ii) Solution:

Let the 4 × 4 matrix is




```
#include <stdio.h>
#define ROW 4
#define COL 4
int M[ROW][COL] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16};
main( ){
    int i, j, t;
    for (i = 0; i < 4; ++i){
        for(j = i; j < 4; ++j){
        t = M[i][j];
        M[i][j] = M[j][i];
```

```
            M[j][i] = t;
            }
        }
        for (i = 0; i < 4; ++i){
            for (j = 0; j < 4; ++j){
                printf ("%d", M[i][j]);
        }
    }
    }
```

$$\begin{bmatrix} \textcircled{1} & 5 & 9 & 13 \\ 2 & \textcircled{6} & 10 & 14 \\ 3 & 7 & \textcircled{11} & 15 \\ 4 & 8 & 12 & \textcircled{16} \end{bmatrix}$$

Q.3 (b) Solution:

Let, A = Area of plates

Let $C_1 = C_3$ be the capacitance formed with dielectric having dielectric constant ε_1.

$C_{equivalent}$ be the equivalent capacitance. C_2 be the capacitance formed with dielectric having dielectric constant ε_2.

Then,
$$C_1 = C_3 = \frac{\varepsilon_0\,\varepsilon_1\,A}{d/4} = \frac{4\,\varepsilon_0\,\varepsilon_1\,A}{d} \text{ and}$$

$$C_2 = \frac{\varepsilon_0\,\varepsilon_2\,A}{d/2} = \frac{2\,\varepsilon_0\,\varepsilon_2\,A}{d}$$

Also, equivalent capacitance = $C_{equivalent}$

$$\therefore \quad \frac{1}{C_{equivalent}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{2}{C_1} + \frac{1}{C_2}$$

($\because C_1 = C_3$) (C_1, C_2, C_3 are in series)

or
$$\frac{1}{C_{equivalent}} = \frac{2d}{4\varepsilon_0\varepsilon_1 d} + \frac{d}{2\varepsilon_0\varepsilon_2 A}$$

or
$$C_{equivalent} = \frac{2\varepsilon_1\,\varepsilon_2\,\varepsilon_0\,A}{d(\varepsilon_1 + \varepsilon_2)}$$

Given,

$$V_{\text{equivalent}} = \text{Total voltage} = 10\text{V},$$

$$V_1 = V_3 = 2\text{V}$$

We know that

$$C \propto \frac{1}{V}$$

$$\therefore \quad \frac{C_{\text{equivalent}}}{C_1} = \frac{V_1}{V_{\text{equivalent}}}$$

or $$\frac{C_{\text{equivalent}}}{C_1} = \frac{1}{5}$$

or $$\frac{2\,\varepsilon_0\,\varepsilon_1\,\varepsilon_2 A}{d(\varepsilon_1+\varepsilon_2)} \times \frac{d}{4\,\varepsilon_0\,\varepsilon_1\,A} = \frac{1}{5}$$

or $$\frac{\varepsilon_2}{2(\varepsilon_1+\varepsilon_2)} = \frac{1}{5}$$

or $$5\varepsilon_2 = 2\varepsilon_1 + 2\varepsilon_2$$

$$2\varepsilon_1 = 3\varepsilon_2$$

or $$\varepsilon_1 : \varepsilon_2 = 3:2$$

Q.3 (c) Solution:

Putting $y^3 = z$ and $3y^2 \frac{dy}{dx} = \frac{dz}{dx}$, the given equation becomes.

$$x(1-x^2)\frac{dz}{dx} + (2x^2-1)z = ax^3,$$

or $$\frac{dz}{dx} + \frac{2x^2-1}{x-x^3}z = \frac{\alpha x^3}{x-x^3} \quad \text{...(i)}$$

Which is Leibitz's equation in z

$$\therefore \quad \text{I.F.} = e^{\left(\int \frac{2x^2-1}{x-x^3}dx\right)}$$

Now,

$$\int \frac{2x^2-1}{x-x^3}dx = \int\left(-\frac{1}{x} - \frac{1}{2}\frac{1}{1+x} + \frac{1}{2}\frac{1}{1-x}\right)dx$$

$$= -\log x - \frac{1}{2}\log(1+x) - \frac{1}{2}\log(1-x)$$

$$= -\log\left[x\sqrt{(1-x^2)}\right]$$

$$\text{I.F.} = e^{-\log\left[x\sqrt{(x-1-x^2)}\right]} = \left[x\sqrt{(1-x^2)}\right]^{-1}$$

Thus the solution of (i) is

$$Z(\text{I.F.}) = \int \frac{ax^3}{x-x^3}(I.F.)\,dx + C$$

$$\frac{Z}{\left[x\sqrt{(1-x^2)}\right]} = a\int \frac{x^3}{x(1-x^2)} \times \frac{1}{x\sqrt{(1-x^2)}}dx + C$$

$$= a\int x(1-x^2)^{-3/2}dx$$

$$= \frac{-a}{2}\int(-2x)(1-x^2)^{-3/2}\,dx + C$$

$$= a(1-x^2)^{-\frac{1}{2}} + C$$

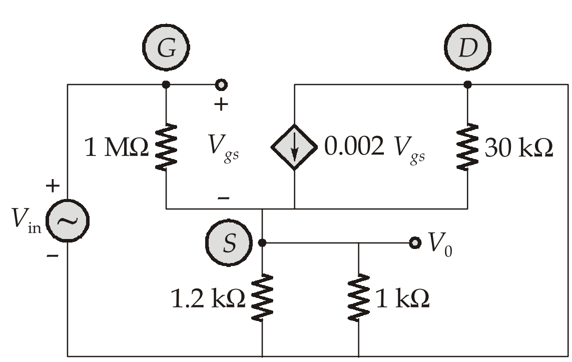
Hence the solution of the given equation is

$$y^3 = ax + cx\sqrt{(1-x^2)}$$

Q.4 (a) Solution:

(i) AC equivalent circuit :

$$g_m V_{gs} = 2 \times 10^{-3}\ V_{gs} = 0.002\ V_{gs}$$



on applying KCL at 'V_0'

$$\frac{V_0}{1.2\text{k}} + \frac{V_0}{1\text{k}} + \frac{V_0 - V_{in}}{1000\text{k}} - 0.002V_{gs} + \frac{V_0}{30\text{k}} = 0$$

$$\frac{V_0}{1.2\text{k}} + V_0 + \frac{V_0 - V_{in}}{1000\text{k}} - 0.002[V_{in} - V_0] + \frac{V_0}{30\text{k}} = 0 \qquad [\because V_{gs} = V_{in} - V_0]$$

$$V_0\left[\frac{1}{1\text{k}} + \frac{1}{1.2\text{k}} + \frac{1}{1000\text{k}} + 0.002 + \frac{1}{30\text{k}}\right] = \left[\frac{1}{1000\text{k}} + 0.002\right]V_{in}$$

$$\frac{V_0}{V_{in}} = \frac{\left(\frac{1}{1000\text{k}} + 0.002\right)}{\left(\frac{1}{1\text{k}} + \frac{1}{1.2\text{k}} + \frac{1}{1000\text{k}} + 0.002 + \frac{1}{30\text{k}}\right)}$$

$$= 0.517 \text{ V/V}$$

(ii) Current gain,

$$\frac{I_L}{I_{in}} = \frac{\frac{V_0}{1\text{k}}}{\frac{V_{in} - V_0}{1\text{M}}} = 1000\left(\frac{V_0}{V_{in} - V_0}\right)$$

$$= \frac{1000}{\frac{V_{in}}{V_0} - 1} = \frac{1000}{\frac{1}{A_V} - 1}$$

$$A_I = 1071.96$$

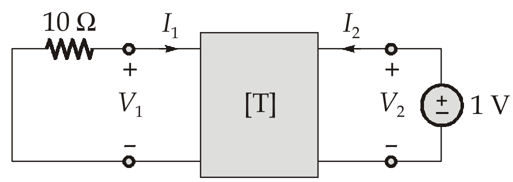
(iii) Input resistance,

$$R_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{in}}{\frac{V_{in} - V_0}{1\text{M}}} = 10^6\left(\frac{V_{in}}{V_{in} - V_0}\right)$$

$$= 10^6\left(\frac{1}{1 - A_V}\right) = \frac{10^6}{1 - 0.517} = 2.07 \text{ M}\Omega$$

Q.4 (b) Solution:

We find Z_{Th} using the circuit below:



Substituting the given ABCD parameters

$$V_1 = A\,V_2 - B\,I_2$$
$$I_1 = C\,V_2 - D\,I_2$$

We obtain,

$$V_1 = 4V_2 - 20\,I_2 \quad \text{...(i)}$$
$$I_1 = 0.1\,V_2 - 2\,I_2 \quad \text{...(ii)}$$

At the input port $V_1 = -10I_1$ substituting this in equation (i) gives

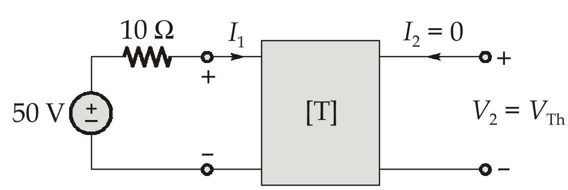
$$-10\,I_1 = 4V_2 - 20\,I_2$$

Or $$I_1 = -0.4\,V_2 + 2\,I_2 \quad \text{...(iii)}$$

Hence,

$$Z_{Th} = \frac{V_2}{I_2} = \frac{4}{0.5} = 8\Omega$$

To find V_{Th}, We use the circuit shown below:

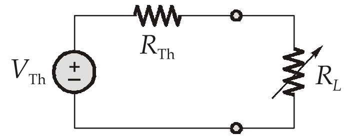


At the output port $I_2 = 0$ and at the input port

$$V_1 = 50 - 10I_1$$
$$50 - 10\,I_1 = 4V_2 \quad \text{...(iv)}$$
$$I_1 = 0.1\,V_2 \quad \text{...(v)}$$
$$50 - V_2 = 4V_2$$
$$V_2 = 10$$

Thus, $$V_{Th} = V_2 = 10\,V$$

The equivalent circuit is shown in figure.



$$R_L = R_{Th} = 8\Omega$$

The maximum power is

$$P = I^2R_L = \left(\frac{V_{Th}}{2R_L}\right)^2 R_L$$

$$= \frac{V_{Th}^2}{4R_L} = \frac{100}{4\times 8} = 3.125\,\text{W}$$

Q.4 (c) Solution:

Total secondary circuit resistance = 1.2 + 0.2 = 1.4 Ω

Total secondary circuit reactance = 0.5 + 0.3 = 0.8 Ω

Secondary circuit phase angle, $\delta = \tan^{-1}\frac{0.8}{1.4} = 29°42'$

$\cos\delta = 0.8686$ and $\sin\delta = 0.4955$

Primary winding turns, $N_P = 1$

Secondary winding turns, $N_S = 200$

Magnetizing current, $I_m = \frac{\text{Magnetizing mmf}}{\text{Primary turns}} = \frac{100}{1} = 100\text{ A}$

Loss component, $I_e = \frac{\text{mmf equivalent to iron loss}}{\text{Primary turns}}$

$$= \frac{50}{1} = 50\text{ A}$$

∴ Actual ratio, $R = n + \frac{I_e\cos\delta + I_m\sin\delta}{I_s}$

$$= 200 + \frac{50\times 0.8686 + 100\times 0.4955}{5}$$

$$= 218.6$$

Primary current, I_P = Actual transformation ratio × Secondary current

$= 218.6 \times 5 = 1093\text{ A}$

In order to eliminate the ratio error, we must reduce the secondary winding turns or in other words we must reduce the turns ratio.

The nominal ratio is 200 and therefore for zero ratio error the actual transformation ratio should be equal to the nominal ratio.

Nominal ratio, $K_n = 200$

Actual ratio, $R = n + \dfrac{I_e \cos\delta + I_m \sin\delta}{I_s}$

∴ For zero ratio error, $K_n = R$

or $200 = n + \dfrac{50 \times 0.8686 + 100 \times 0.4955}{5}$

$= n + 18.6$

Turns ratio, $n = 181.4$

Hence, secondary winding turns

$N_s = nN_p = 181.4 \times 1 = 181.4$

Reduction in secondary winding turns

$= 200 - 181.4 = 18.6 \approx 19$

Section-B

Q.5 (a) Solution:

$$H = -\nabla V_m$$

$$\therefore \quad H = -\left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_2\right)\left(x^2y + y^2x + z\right)$$

$$= -\left(2xy + y^2\right)\hat{a}_x - \left(x^2 + 2xy\right)\hat{a}_y - 1\left(\hat{a}_z\right)$$

$$H(1, 1, 1) = -3\hat{a}_x - 3\hat{a}_y - \hat{a}_z$$

$$B = \mu_0 \mu_r\ H = 4\pi \times 10^{-7} \times 1 \times \sqrt{\left(3^2 + 3^2 + 1^2\right)}$$

$$= 5.5\ \mu T$$

Q.5 (b) Solution:

Given $\left(\dfrac{W}{L}\right)_1 = 1.5\left(\dfrac{W}{L}\right)_2 = 20$

$V_{GS1} = V_{GS2} = -V_3$

Assume that both Q_1 and Q_2 are operating in saturation region

$$I_{D1} = K_n'\left(\frac{W}{L}\right)_1 \cdot \frac{[V_{GS1} - V_t]^2}{2}$$

Similarly,
$$I_{D2} = K_n'\left(\frac{W}{L}\right)_2 \cdot \frac{[V_{GS2} - V_t]^2}{2}$$

$$\frac{I_{D1}}{I_{D2}} = \frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2} \times \frac{(V_{GS1} - V_t)^2}{(V_{GS2} - V_t)^2}$$

Since, $V_{GS1} = V_{GS2}$

So,
$$\frac{I_{D1}}{I_{D2}} = 1.5 \quad \text{...(i)}$$

Also
$$I_{D1} + I_{D2} = 200\ \mu\text{A} \quad \text{...(ii)}$$

From equation (i) and (ii),

$$2.5\, I_{D2} = 200$$
$$I_{D2} = 80\ \mu\text{A}$$

and
$$I_{D1} = 120\ \mu\text{A}$$
$$V_1 = 5 - 40\, I_{D1} = 5 - 40 \times 0.120 = 0.2\text{ V}$$
$$V_2 = 5 - 40\, I_{D2} = 5 - 40 \times 0.08 = 1.8\text{ V}$$

Again,
$$I_{D1} = K_n'\left(\frac{W}{L}\right)_1 \cdot \frac{[V_{GS} - V_t]^2}{2}$$

$$80 = 100 \times 20 \cdot \frac{[V_{GS1} - 1]^2}{2}$$

$$\sqrt{\frac{2}{25}} = (V_{GS1} - 1)$$
$$V_G - V_S = 1.28$$
$$0 - V_S = 1.28$$
$$V_3 = -1.28\text{ V}$$

In both transistors $V_{DS} > V_{GS} - V_t$ so both transistors are in saturation region

$$V_1 = 0.20\text{ V}$$
$$V_2 = 1.80\text{ V}$$
$$V_3 = -1.28\text{ V}$$

Q.5 (c) Solution:

Main memory is divided in page size = 16 bytes

We know that, page size = frame size

So in virtual address least 4 bit represent page size and remaining bits represents page number.

So, page number request are:

0 = 00 0000 i.e. Page number = 0

4 = 00 0100 i.e. Page number = 0

8 = 00 1000 i.e. Page number = 0

20 = 01 0100 i.e. Page number = 1

24 = 01 1000 i.e. Page number = 1

36 = 10 0100 i.e. Page number = 2

44 = 10 1100 i.e. Page number = 2

12 = 00 1100 i.e. Page number = 0

68 = 100 0100 i.e. Page number = 4

72 = 100 1000 i.e. Page number = 4

80 = 101 0000 i.e. Page number = 5

84 = 101 0100 i.e. Page number = 5

28 = 1 1100 i.e. Page number = 1

32 = 010 0000 i.e. Page number = 2

88 = 101 1000 i.e. Page number = 5

92 = 101 1100 i.e. Page number = 5

Request are:

0	0	0	1	1	4	2	0	4	4	5	5	1	2	5	5
						2	2	2	2	2	2	1	1	1	1
					4	4	4	4	4	4	4	4	4	4	4
			1	1	1	1	1	1	1	5	5	5	5	5	5
0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2
F	H	H	F	H	F	F	H	H	H	F	H	F	F	H	H

So, at the end pages 1, 2, 4, 5 are in memory and 7 page fault are present.

Q.5 (d) Solution:

Let, $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & -2 \\ -5 & 3 \end{bmatrix}$

and $D = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$

Then, $BAC = D$

$AC = B^{-1} D$

$\therefore \quad A = B^{-1} DC^{-1}$

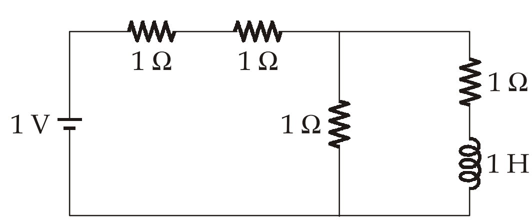
Now, $B^{-1} = \dfrac{\text{adj} B}{|B|} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

$C^{-1} = \dfrac{\text{adj} C}{|C|} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

Hence, $A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 14 & 8 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 24 & 13 \\ -34 & -18 \end{bmatrix}$

Q.5 (e) Solution:



$$i_L(0^-) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}\text{A}$$

$$i_L(\infty) = \frac{1}{2} \times \frac{1}{2+0.5} = \frac{1}{5}\text{ A}$$

$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))e^{-t/\tau}$$

$$i_L(t) = \frac{1}{5} + \left(\frac{1}{3} - \frac{1}{5}\right)e^{-5t/3}$$

$$V_L = \frac{L\, di_L(t)}{dt} = \frac{-2}{15} \times \frac{5}{3} e^{-5t/3} = -\frac{2}{9} e^{-5t/3}$$

Q.6 (a) (i) Solution:

Important characteristic of antiferromagnetic materials are as follows:

- Magnetic moment are aligned antiparallel to each other resulting in a net zero magnetic dipole.
- Antiferromagnetic property is observed only below a certain temperature known as Neel temperature. This happens due to ordered antiparallel alignment of spin magnetic moments.
- Above Neel temperature, antiferromagnetic materials become paramagnetic.
- This behavior is exhibited only in very few elements like Mn, Cr and ferro and nickel oxides.

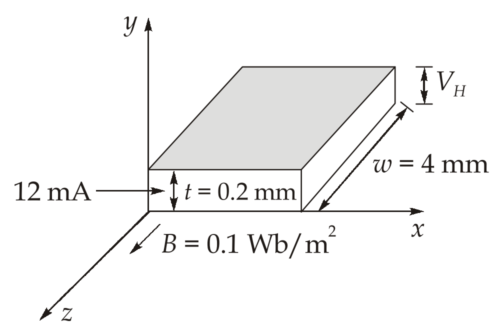
Q.6 (a) (ii) Solution:

Given, width, $w = 4$ mm ; thickness, $t = 0.2$ mm

current, $I = 12$ mA ; magnetic field, $B = 0.1\ \text{Wb/m}^2$

Hall voltage, $V_H = -1.0$ mV ;

or $|V_H| = 1 \times 10^{-3}$ V



We know,

$$\text{Voltage due to hall effect, } V_H = \frac{BI}{\rho w}$$

Where, B = magnetic field, I = current flowing in specimen, w = width of specimen, ρ = charge density of specimen c/m^3

So, $$\frac{1}{\rho} = R_H$$

where, $R_H \rightarrow$ Hall coefficient

$$V_H = \frac{BI}{w} R_H$$

$$R_H = \frac{V_H \cdot w}{BI} = \frac{10^{-3} \times 4 \times 10^{-3}}{0.1 \times 12 \times 10^{-3}}$$

$$= \frac{4 \times 10^{-6}}{1.2 \times 10^{-3}} = 3.33 \times 10^{-3} \text{ m}^3/\text{C}$$

$$\rho = \frac{1}{R_H} = 300 \text{ C/m}^3$$

Let concentration of electron be N_e/m^3

$$N_e \times q = \rho$$

$$N_e = \frac{300}{1.6 \times 10^{-19}} = 1.875 \times 10^{21}/\text{m}^3$$

Q.6 (b) Solution:

(i) Inductor and resistor circuit,

$$X_L = 2\pi fL = 2\pi \times 60 \times 530 \text{ mH}$$

$$= 200\ \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(200)^2 + (200)^2}$$

$$= 283\ \Omega$$

$$\cos\phi = \frac{R}{Z} = \frac{200}{283} = 0.707$$

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$I_L = I_P = \frac{V_P}{Z} = \frac{231\text{ V}}{283} = 816 \text{ mA}$$

$$P_L = \sqrt{3} V_L I_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 816 \text{ mA} \times 0.707 = 400 \text{ W}$$

$$\sin\phi = \frac{X_L}{Z} = \frac{200}{238} = 0.707$$

$$Q_L = \sqrt{3} V_L I_L \sin\phi$$

$$= \sqrt{3} \times 400 \times 816 \times 0.707$$

$$= 400 \text{ VAR}$$

(ii) Capacitor circuit,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 60 \times 1\,\mu F} = 2.65\ k\Omega$$

$$I_P = \frac{V_L}{X_C} = \frac{400}{2.65\ k\Omega} = 151\ mA$$

$$I_L = \sqrt{3}I_P = \sqrt{3}I_P = \sqrt{3} \times 151\ mA = 261\ mA$$

$$\cos\phi = \frac{R}{Z} = \frac{0}{2.65 k\Omega} = 0$$

$$P_C = \sqrt{3}V_L I_L \cos\phi = \sqrt{3}V_L I_L \times 0 = 0$$

$$\sin\phi = \frac{X_L}{Z} = \frac{2.65\ k\Omega}{2.65\ k\Omega} = 1$$

$$Q = \sqrt{3}V_L I_L \sin\phi$$

$$= \sqrt{3} \times 400 \times 261 \times 1 = 181\ VAR$$

(iii) All components in the circuit,

$$P = P_L + P_C = 400\ W + 0 = 400\ W$$

$$Q = Q_L - Q_C = 400 - 181 = 219\ VAR$$

$$\phi = \tan^{-1}\frac{Q}{P} = \tan^{-1}\frac{219}{400} = 28.70$$

$$\cos\phi = \cos 28.7° = 0.877$$

$$I_L = \frac{P}{\sqrt{3}V_L \cos\phi} = \frac{400}{\sqrt{3} \times 400 \times 0.877} = 658\ mA$$

Q.6 (c) (i) Solution:

$f(z)$ has simple poles at

$$z = 0, \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$$

$\therefore$

$$\text{Res } f(0) = \lim_{z\to 0}\left[z \times f(z)\right] = \lim_{z\to 0}\left(\frac{\sin z}{\cos z}\right) = 0$$

$$\text{Res } f\left(\frac{\pi}{2}\right) = \lim_{Z\to\frac{\pi}{2}}\left\{\frac{\left(z-\frac{\pi}{2}\right)\sin z}{z\cos z}\right\}$$

$$= \lim_{Z \to \frac{\pi}{2}} \frac{\left(z - \frac{\pi}{2}\right)\cos z + \sin z}{\cos z - z \sin z}$$

$$= \frac{1}{-\frac{\pi}{2}} = -\frac{2}{\pi}$$

And $$\text{Res } f\left(-\frac{\pi}{2}\right) = \lim_{z \to -\frac{\pi}{2}} \left\{ \frac{\left(z + \frac{\pi}{2}\right)\sin z}{z \cos z} \right\}$$

$$= \lim_{z \to -\frac{\pi}{2}} \frac{\left(z + \frac{\pi}{2}\right)\cos z + \sin z}{\cos z - z \sin z}$$

$$= \frac{-1}{-\frac{\pi}{2}} = \frac{2}{\pi}$$

Hence, $$\text{sum of residues} = 0 - \frac{2}{\pi} + \frac{2}{\pi} = 0$$

Q.6 (c) (ii) Solution:

If X is a random variable, then

$$\sum_{i=0}^{7} P(x_i) = 1,$$

$$0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + k = 1$$

i.e., $$10K^2 + 9K - 1 = 0$$

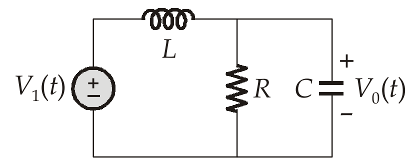
$$K = \frac{1}{10}$$

$$P(0 < X < 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$$

$$= K + 2K + 2K + 3K$$

$$= 8K = \frac{8}{10} = \frac{4}{5}$$

Q.7 (a) Solution:



The transfer function is,

$$H(s) = \frac{V_0}{V_i} = \frac{R \parallel \frac{1}{sC}}{sL + R \parallel \frac{1}{sC}}, \; s = j\omega \qquad \text{... (i)}$$

But
$$R \parallel \frac{1}{sC} = \frac{R / sC}{R + \frac{1}{sC}} = \frac{R}{1 + sRC} \qquad \text{...(ii)}$$

Substituting this into equation (i) gives

$$H(s) = \frac{R/(1+sRC)}{sL + R/(1+sRC)} = \frac{R}{s^2 RLC + sL + R}; \; s = j\omega$$

or
$$H(\omega) = \frac{R}{-\omega^2 RLC + j\omega L + R} \qquad \text{...(iii)}$$

Since, $H(0) = 1$ and $H(\infty) = 0$, we conclude that circuit is second-order low-pass filter. The magnitude of H is

$$H = \frac{R}{\sqrt{(R - \omega^2 RLC)^2 + \omega^2 L^2}} \qquad \text{...(iv)}$$

The corner frequency is the same as the half-power frequency i.e.,

Where H is reduced by a factor of $\frac{1}{\sqrt{2}}$. Since the dc value of $H(\omega)$ is 1, at the corner frequency equation 3 becomes after squaring

$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2}$$

or
$$2 = (1 - \omega_c^2 LC)^2 + \left(\frac{\omega_c L}{R}\right)^2$$

Substituting the value of R, L and C we obtain.

$$2 = \left(1-\omega_c^2 \times 4 \times 10^{-6}\right)^2 + \left(\omega_c \times 10^{-3}\right)^2$$

Assuming that ω_c is in K rad/sec,

$$2 = \left(1-4\omega_c^2\right)^2 + \omega_c^2$$

$$16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

Solving the quadratic equation we get $\omega_c^2 = 0.5509$ and –0.1134

Since ω_c is real, ω_c = 0.742 K rad/sec

= 742 rad/sec.

Q.7 (b) Solution:

We have,

$$f(x, y) = x^3 + y^3 - 3axy$$

$$p = \frac{\partial f}{\partial x} = 3x^2 - 3ay$$

$$q = \frac{\partial f}{\partial y} = 3y^2 - 3ax$$

$$r = \frac{\partial^2 f}{\partial x^2} = 6x,$$

$$s = \frac{\partial^2 f}{\partial x \partial y} = -3a$$

$$t = \frac{\partial^2 f}{\partial y^2} = 6y$$

For maxima and minima,

$$\frac{\partial f}{\partial x} = 0$$

and

$$\frac{\partial f}{\partial y} = 0$$

$$3x^2 - 3ay = 0$$

$$x^2 = ay$$

$$y = \frac{x^2}{a} \quad \text{...(i)}$$

$$3y^2 - 3ax = 0$$

$$y^2 = ax \quad \text{...(ii)}$$

Putting the value of y from (i) in (ii), we get

$$x^4 = a^3x$$

$$x(x^3 - a^3) = 0$$

$$x(x - a)(x^2 + ax + a^2) = 0$$

$$x = 0, a$$

Putting $x = 0$ in equation (i),

We get $y = 0$

Putting $x = a$,

We get $y = a$

Stationary points	(0, 0)	(a, a)
r	0	$6a$
s	$-3a$	$-3a$
t	0	$6a$
$rt - s^2$	$-9a^2 < 0$	$27a^2 > 0$

At (0, 0) there is no extremum value, since $rt - s^2 < 0$

When $a < 0$, f is maximum

When $a > 0$, f is minimum

Q.7 (c) Solution:

$$\text{Load Power} = 100 \times 9 \times 0.1 = 90 \text{ W}$$

$$\text{Load power factor } \cos\phi = 0.1$$

$$\therefore \quad \phi = 84.26° \text{ and } \sin\phi = 0.995 \text{ and } \tan\phi = 9.95$$

Resistance of pressure coil circuit,

$$R_p = 3000\ \Omega$$

Reactance of pressure coil circuit

$$= 2\pi \times 50 \times 30 \times 10^{-3}$$

$$= 9.42\ \Omega$$

As the phase angle of pressure coil circuit is small

$$\beta = \tan\beta = \frac{9.42}{3000} = 0.00314 \text{ rad}$$

(i) When pressure coil is connected on the load side, the wattmeter measures power loss in pressure coil circuit in addition to load power.

$$\text{True power} = VI\cos\phi$$
$$= 100 \times 9 \times 0.1 = 90\text{ W}$$

Let us consider only the effect of inductance.

Reading of wattmeter

$$= \text{True power }(1 + \tan\phi\tan\beta)$$
$$= 90(1 + 9.95 \times 0.00314)$$
$$= 92.81\text{ W}$$

Power loss in pressure coil circuit

$$= \frac{V^2}{R_P} = \frac{(100)^2}{3000} = 3.33\text{ W}$$

∴ Reading of wattmeter considering the power loss in pressure coil circuit

$$= 92.81 + 3.33 = 96.14\text{ W}$$

$$\text{Percentage error} = \frac{96.14 - 90}{90} \times 100 = 6.82\%$$

(ii) When the current coil is on the load side, the wattmeter measures the power in the load plus the power loss in the current coil. In fact, the current coil acts as a load.

∴ $$\text{Total power} = \text{Power consumed in load} + I^2R_C$$
$$= 90 + 9^2 \times 0.1 = 98.1\text{ W}$$

$$\text{Impedance of load} = \frac{100}{9} = 11.1\ \Omega$$

$$\text{Resistance of load} = 11.1 \times 0.1 = 1.11\ \Omega$$

$$\text{Reactance of load} = 11.1 \times 0.995 = 11.05\ \Omega$$

Resistance of load plus resistance of current coil

$$= 1.11 + 0.1 = 1.21\ \Omega$$

Reactance of load plus reactance of current coil

$$= 11.05\ \Omega$$

Impedance of load including current coil

$$= \sqrt{(1.21)^2 + (11.05)^2} = 11.1\ \Omega$$

Power factor of load including current coil

$$= \frac{1.21}{11.1} = 0.109$$

$$\therefore \qquad \phi = 83.74° \text{ and } \tan\phi = 9.12$$

$$\text{Reading of wattmeter} = 98.1 \times (1 + 9.2 \times 0.00314) = 100.9 \text{ W}$$

$$\text{Percentage error} = \frac{100.9 - 90}{90} \times 100 = 12.1\%$$

Q.8 (a) (i) Solution:

Standard conductivity formula is

$$\sigma = q\,\mu_n n_0 + p\,\mu_p p_0$$

Now, from mass action law

$$n_0\,p_0 = n_i^2$$

$$n_0 = \frac{n_i^2}{p_0}$$

then

$$\sigma = \frac{q\mu_n n_i^2}{p_0} + q\mu_p p_0 \qquad \text{...(i)}$$

To find minimum conductivity

$$\frac{d\sigma}{dp_0} = 0$$

$$\frac{-q\mu_n n_i^2}{p_0^2} + q\mu_p = 0$$

$${p_0}^2 = n_i^2\left(\frac{\mu_n}{\mu_0}\right)$$

$$p_0 = n_i\sqrt{\frac{\mu_n}{\mu_p}} = n_i\left(\frac{\mu_n}{\mu_p}\right)^{1/2} \qquad \text{...(ii)}$$

Now put equation (ii) in equation (i),

$$\sigma = \sigma_{min} = \frac{q\mu_n n_i^2}{\left[n_i\sqrt{\frac{\mu_n}{\mu_p}}\right]} + q\mu_p\left[n_i\sqrt{\frac{\mu_n}{\mu_p}}\right]$$

On simplifies,
$$\sigma_{\min} = \frac{q\mu_n n_i^2 + q n_i^2 \mu_p \left(\frac{\mu_n}{\mu_p}\right)}{n_i\sqrt{\frac{\mu_n}{\mu_p}}}$$

$$= \frac{2q\mu_n n_i^2}{n_i\sqrt{\frac{\mu_n}{\mu_p}}} = 2q n_i \sqrt{\mu_n \mu_p} \quad \text{...(iii)}$$

The intrinsic conductivity σ_i is defined is
$$\sigma_i = q\, n_i(\mu_n + \mu_p)$$
$$q\, n_i = \frac{\sigma_i}{\mu_n + \mu_p} \quad \text{...(iv)}$$

Now, put equation (iv) in equation (iii),
The minimum conductivity can be written as
$$\sigma_{\min} = \frac{2\sigma_i\sqrt{\mu_n\mu_p}}{\mu_n + \mu_p}$$

Q.8 (a) (ii) Solution:

Given,
$$n(x) = 10^{16}\left(1 - \frac{x}{L}\right),$$
$$L = 10\ \mu\text{m} = 10 \times 10^{-4}\ \text{cm}$$
$$J_n = -80\ \text{A/cm}^2 \text{ for } 0 \le x \le L$$
$$J_n = q\mu_n n E + q D_n \frac{dn}{dx}$$
$$-80 = (1.6\times10^{-19})(1000)(10^{16})\left(1-\frac{x}{L}\right)E + (1.6\times10^{-19})(25.9)\times\left(\frac{-10^{16}}{L}\right)$$

Where,
$$L = 10 \times 10^{-4}\ \text{cm} = 10^{-3}\ \text{cm}$$
$$-80 = 1.6E[1 - 10^3 x] - 41.47$$

On solving for electric field, we get
$$E = \frac{-38.56}{[1-10^3 x]}\ \text{V/cm}$$

Q.8 (b) (i) Solution:

Let
$$f_1 = ax^2 - byz - (a+2)x = 0$$
$$f_2 = 4x^2y + z^3 - 4 = 0$$

Then

$$\vec{\nabla}_{f1} = (2ax - a - 2)\hat{i} - bz\,\hat{j} + by\,\hat{k} \quad \text{...(i)}$$
$$= (a-2)\hat{i} - 2b\hat{j} + b\hat{k} \text{ at } (1, -1, 2)$$
$$\vec{\nabla}_{f2} = 8xy\hat{i} + 4x^2\hat{j} + 3z^2\hat{k} \quad \text{...(ii)}$$
$$= -8\hat{i} + 4\hat{j} + 12\hat{k} \text{ at } (1, -1, 2)$$

The surfaces (i) and (ii) will cut orthogonally if

$$\vec{\nabla}_{f1} \cdot \vec{\nabla}_{f2} = 0$$

i.e.,
$$-8(a-2) - 8b + 12b = 0$$
$$-2a + b + 4 = 0 \quad \text{...(iii)}$$

Also since the point (1, –1, 2) lies on (i) and (ii),
$$a + 2b - (a+2) = 0$$
or
$$b = 1$$
From (iii),
$$-2a + 5 = 0$$
$$a = 5/2$$
Hence
$$a = 5/2 \text{ and } b = 1.$$

Q.8 (b) (ii) Solution:

As there are 7 students in the class, the first examined must be a boy

∴ Probability that first is a boy $= \frac{4}{7}$

Then the probability that the second is a girl $= \frac{3}{6}$

∴ Probability of the next boy $= \frac{3}{5}$

Similarly the probability that the fourth is a girl $= \frac{2}{4}$

The probability that the fifth is a boy $= \frac{2}{3}$

The probability that the sixth is a girl = $\frac{1}{2}$ and the last is a boy $\frac{1}{1}$

Thus, $$P = \frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times\frac{1}{2}\times\frac{1}{1} = \frac{1}{35}$$

Q.8 (c) (i) Solution:

Applying the first condition of balance for magnitude

$$Z_1Z_4 = Z_2Z_3$$

Now, $$Z_1Z_4 = 400 \times 400 = 1,60,000$$

and $$Z_2Z_3 = 200 \times 800 = 1,60,000$$

$\therefore$ $$Z_1Z_4 = Z_2Z_3$$

Applying the second condition for balance required for phase,

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$$

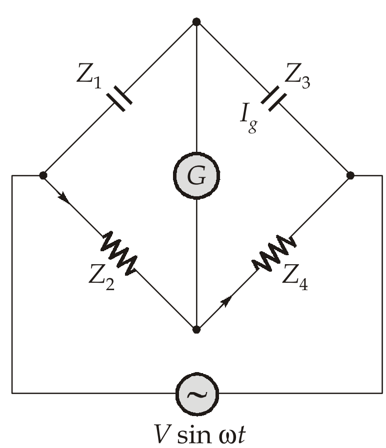
Now, $$\angle\theta_1 + \angle\theta_4 = 50° + 20° = 70°$$

$$\angle\theta_2 + \angle\theta_3 = 40° - 50° = -10°$$

This indicates that the condition for phase relationship is not satisfied and therefore, the bridge is unbalanced even though the condition for equality of magnitude is satisfied.

Q.8 (c) (ii) Solution:

Redrawing the given bridge circuit, we have :



$$Z_1 = \frac{1}{j\omega C_1}\Omega \text{ and } Z_2 = 70 \text{ k}\Omega$$

$$Z_3 = \frac{10^6}{j0.3\omega}\Omega \text{ and } Z_4 = 210 \text{ k}\Omega$$

At balance, current through galvanometer,

$$I_g = 0$$

and $$|Z_1||Z_4| = |Z_2||Z_3|$$

$\therefore$ $$\left(\frac{1}{\omega C_1}\right) \times (210\ \text{k}\Omega) = 70\ \text{k}\Omega \left(\frac{10^6}{0.3\omega}\right)$$

or $$C_1 = \frac{210 \times 0.3}{70} = 0.9\ \mu\text{F}$$

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