India's Best Institute for IES, GATE & PSUs

Detailed Solutions

ESE-2021 Mains Test Series

Mechanical Engineering Test No : 15

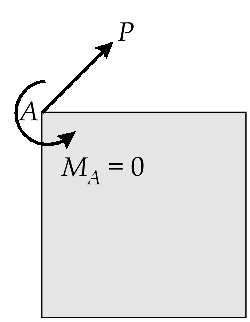
Full Syllabus Test (Paper-II)

Explanations

1. (a) Solution:

(a) We must have, $M_A = 0$

$$(P\sin\alpha)a - Q\cdot a = 0$$

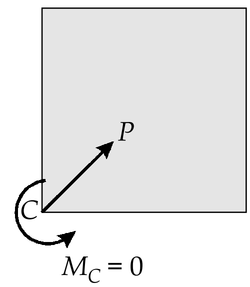


$$\sin\alpha = \frac{Q}{P} = \frac{Q}{2Q} = \frac{1}{2}$$

$$\alpha = 30°$$

(b) We must have $M_C = 0$

$$(P\sin\alpha)a - (P\cos\alpha)a - Q(a) = 0$$



$$\sin\alpha - \cos\alpha = \frac{Q}{P} = \frac{Q}{2Q} = \frac{1}{2}$$

$$\sin\alpha = \cos\alpha + \frac{1}{2} \qquad \text{...(i)}$$

$$\sin^2\alpha = \cos^2\alpha + \cos\alpha + \frac{1}{4}$$

$$1 - \cos^2\alpha = \cos^2\alpha + \cos\alpha + \frac{1}{4}$$

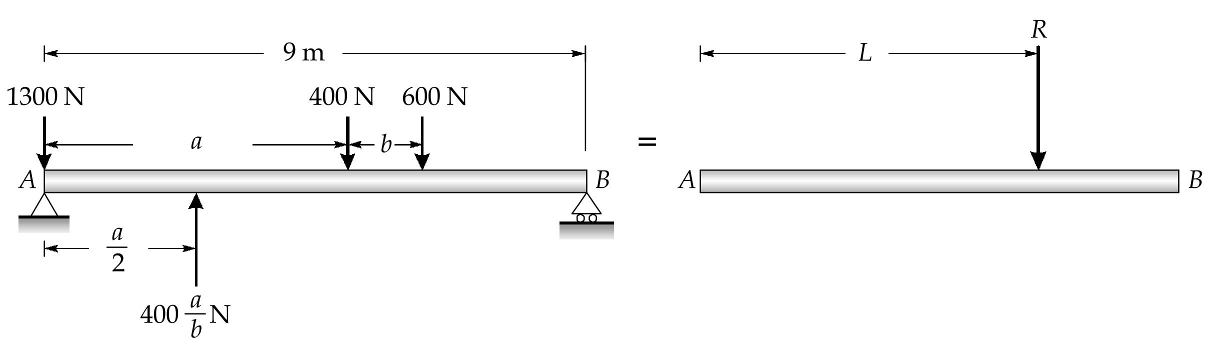
$$2\cos^2\alpha + \cos\alpha - 0.75 = 0$$

Solving the quadratic in $\cos\alpha$:

$$\cos\alpha = \frac{-1 \pm \sqrt{7}}{4}; \qquad \alpha = 65.7° \text{ or } 155.7°$$

Only the first value of α satisfies equation (i), therefore $\alpha = 65.7°$

1. (b) Solution:



For equivalence,

$$\Sigma F_y: \qquad -1300 + 400\frac{a}{b} - 400 - 600 = -R$$

or $$R = \left(2300 - 400\frac{a}{b}\right)N \qquad \text{...(1)}$$

ΣM_A : $\quad \frac{a}{2}\left(400\frac{a}{b}\right) - a(400) - (a+b)(600) = -LR$

or $$L = \frac{1000a + 600b - 200\frac{a^2}{b}}{2300 - 400\frac{a}{b}}$$

Then with $b = 1.5$ m, $$L = \frac{10a + 9 - \frac{4}{3}a^2}{23 - \frac{8}{3}a} \quad ...(2)$$

where a, L are in m.

(i) Find value of a to maximize L.

$$\frac{dL}{da} = \frac{\left(10 - \frac{8}{3}a\right)\left(23 - \frac{8}{3}a\right) - \left(10a + 9 - \frac{4}{3}a^2\right)\left(-\frac{8}{3}\right)}{\left(23 - \frac{8}{3}a\right)^2}$$

or $230 - \frac{184}{3}a - \frac{80}{3}a + \frac{64}{9}a^2 + \frac{80}{3}a + 24 - \frac{32}{9}a^2 = 0$

or $\quad 16a^2 - 276a + 1143 = 0$

Then, $$a = \frac{276 \pm \sqrt{(-276)^2 - 4(16)(1143)}}{2(16)}$$

or $\quad a = 10.3435$ m and $a = 6.9065$ m

Since $\quad AB = 9$ m, a must be less than 9 m; ($a = 6.91$ m)

(ii) Using Eq. (1), $\quad R = 2300 - 400\frac{6.9065}{1.5}$ or R = 458 N

and using Eq. (2), $$L = \frac{10(6.9065) + 9 - \frac{4}{3}(6.9065)^2}{23 - \frac{8}{3}(6.9065)} = 3.16 \text{ m}$$

R is applied 3.16 m to the right of A.

1. (c) Solution:

$$r_D = r_A + D_B \qquad \text{(From geometrical configuration)}$$

$$\frac{mT_D}{2} = \frac{mT_A}{2} + mT_B$$

$$\frac{T_D}{2} = \frac{15}{2} + 20$$

$$T_D = 15 + 40 = 55 \text{ teeth}$$

$$r_E = r_A + r_B + r_C \qquad \text{(From geometrical configuration)}$$

$$T_E = T_A + T_B + T_C = 15 + 20 + 15 = 50 \text{ teeth}$$

Sl.No.	Condition	Arm	A	B/C	D	E
1.	Arm fix 1 rev. to A	0	1	$-\frac{T_A}{T_B}$	$-\frac{T_B}{T_D}\times\frac{T_A}{T_B}$	$-\frac{T_C}{T_E}\times\frac{T_A}{T_B}$
2.	x	0	x	$-\frac{xT_A}{T_B}$	$-\frac{xT_A}{T_D}$	$-\frac{T_C}{T_E}\times\frac{xT_A}{T_B}$
3.	y	y	y	y	y	y
4.	Total	y	$x+y$	$y-\frac{xT_A}{T_B}$	$y-\frac{xT_A}{T_D}$	$y-\frac{T_C}{T_E}\times\frac{xT_A}{T_B}$

$$N_C = N_B = y - x\cdot\frac{T_A}{T_B} \qquad \text{(Integral Gear } B\text{–}C)$$

$$N_E = y - x\cdot\frac{T_C}{T_E}\times\frac{T_A}{T_B}$$

$$N_D = y - x\cdot\frac{T_A}{T_D} = 0$$

$$y - x\times\frac{15}{55} = 0$$

$$\therefore \quad y = \frac{3x}{11}$$

$$1000 = x + y = N_A \qquad \text{[Given]}$$

$$x = 785.7;\ y = 214.3$$

$$y - x\times\frac{T_A}{T_D} = 0$$

$$N_E = y - x \times \frac{T_C}{T_E} \times \frac{T_A}{T_B}$$

$$N_E = 37.5 \text{ rpm}$$

1. (d) Solution:

Let position A be at the beginning of contact and position B be at maximum deflection.

$$T_A = \frac{1}{2} m V_o^2 \text{ (Kinetic energy)}$$

$$V_A = 0 \text{ (Zero force in spring at its free length)}$$

$$T_B = 0 \text{ (Zero velocity at maximum deflection)}$$

$$V_B = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2$$

where x_1 = deflection of spring k_1

x_2 = deflection of spring k_2

Conservation of energy :

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} m V_o^2 + 0 = 0 + \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2$$

$$k_1 x_1^2 + k_2 x_2^2 = m V_o^2 \quad \text{...(i)}$$

Case (i) : When springs are in series

Let F be the force carried by the two springs.

Then, $x_1 = \dfrac{F}{k_1}$ and $x_2 = \dfrac{F}{k_2}$

From equation (i) :

$$F^2 \left[\frac{1}{k_1} + \frac{1}{k_2} \right] = m V_o^2$$

So, $$F = V_o \sqrt{\frac{m}{\left\{ \frac{1}{k_1} + \frac{1}{k_2} \right\}}}$$

The maximum deflection is

$$\delta = x_1 + x_2 = \left[\frac{1}{k_1} + \frac{1}{k_2} \right] F$$

$$= \left[\frac{1}{k_1} + \frac{1}{k_2}\right] V_o \sqrt{\frac{m}{\left\{\frac{1}{k_1} + \frac{1}{k_2}\right\}}}$$

$$= V_o \sqrt{m\left[\frac{1}{k_1} + \frac{1}{k_2}\right]}$$

$$= V_o \sqrt{m\left(\frac{k_1 + k_2}{k_1 k_2}\right)}$$

Case (ii) : When springs are in parallel

$$x_1 = x_2 = \delta$$

Equation (i) becomes,

$$k_1\delta^2 + k_2\delta^2 = mV_o^2$$

$$\delta^2 = \frac{mV_o^2}{k_1 + k_2}$$

$$\delta = V_o\sqrt{\frac{m}{k_1 + k_2}}$$

1. (e) Solution:

Bar AB : (Rotation about A)

$$\vec{\omega}_{AB} = 4 \text{ rad/s} \circlearrowright = -(4 \text{ rad/s})\hat{k}$$

$$\vec{r}_{B/A} = -(175 \text{ mm})\hat{i}\,,\ \vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A} = (-4\hat{k}) \times (-175\hat{i})$$

$$\vec{v}_B = (700 \text{ mm/s})\hat{j}$$

Bar BD : (Plane motion = Translation with B + Rotation about B)

$$\vec{\omega}_{BD} = \vec{\omega}_{BD}\hat{k}\,;\ \vec{r}_{D/B} = -(200 \text{ mm})\hat{j}$$

$$\vec{v}_D = \vec{v}_B + \vec{\omega}_{BD} \times \vec{r}_{D/B} = 700\hat{j} + (\vec{\omega}_{BD}\hat{k}) \times (-200\hat{j})$$

$$\vec{v}_D = 700\hat{j} + 200\vec{\omega}_{BD}\hat{i}$$

Bar DE : (Rotation about E)

$$\vec{\omega}_{DE} = \vec{\omega}_{DE}\hat{k}$$

$$\vec{r}_{D/E} = -(275\text{ mm})\hat{i} + (75\text{ mm})\hat{j}$$

$$\vec{v}_D = \vec{\omega}_{DE} \times \vec{r}_{D/E} = (\vec{\omega}_{DE}\hat{k}) \times (-275\hat{i} + 75\hat{j})$$

$$\vec{v}_D = -275\vec{\omega}_{DE}\hat{j} - 75\vec{\omega}_{DE}\hat{i}$$

Equating components of the two expressions for $\vec{v}_D$,

$\hat{j}$:

$$700 = -275\vec{\omega}_{DE}$$

$$\vec{\omega}_{DE} = -2.5455\text{ rad/s} \qquad (\vec{\omega}_{DE} = 2.55\text{ rad/s})$$

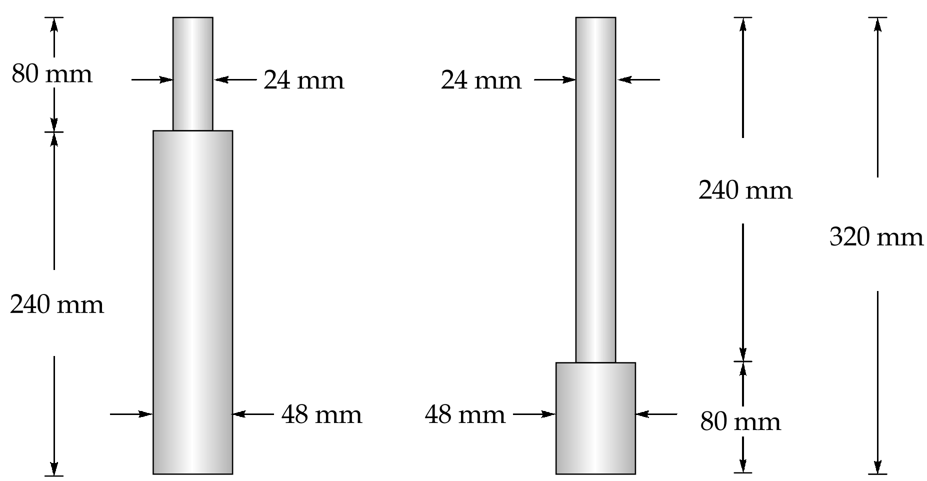
$\hat{i}$:

$$200\vec{\omega}_{BD} = -75\vec{\omega}_{DE}$$

$$\vec{\omega}_{BD} = -\frac{3}{8}\vec{\omega}_{BD}$$

$$\vec{\omega}_{BD} = -\left(\frac{3}{8}\right)(-2.5455) = 0.95455\text{ rad/s} \; (\vec{\omega}_{BD} = 0.955\text{ rad/s})$$

2. (a) Solution:



The larger diameter of the bars = 2 × diameter of the bars

∴ Larger areas of cross-sections = 4 × smaller area of cross-sections

For the same blow to bar *B*, the strain energy produced by the blow should equal to that produced by the blow to the first bar.

In Bar *A* :

Maximum instantaneous stress in the smaller cross-section = 160 MPa

Maximum instantaneous stress in the larger cross-section = $\frac{160}{4}$ = 40 MPa

In Bar B :

Let maximum instantaneous stress in the smaller cross-section = σ

Then maximum instantaneous stress in the larger cross-section = $\frac{\sigma}{4}$

Equating Strain Energies of Bars :

Strain energy of Bar A = Strain energy of Bar B

$$\frac{40^2}{2E}\times\frac{\pi}{4}(48)^2\times240+\frac{160^2}{2E}\times\frac{\pi}{4}(24)^2\times80 = \frac{(\sigma/4)^2}{2E}\times\frac{\pi}{4}(48)^2\times80+\frac{\sigma^2}{2E}\times\frac{\pi}{4}(24)^2\times240$$

Dividing throughout by $\frac{\pi}{4}\times24^2\times\frac{80}{2E}$,

$$40^2(12+16) = \sigma^2\left(\frac{1}{4}+3\right) \text{ or } \sigma = 117.4 \text{ MPa}$$

Ratio of Strain Energies :

Let maximum stress in the smaller cross-section = σ

Then stress in the larger cross-section = $\frac{\sigma}{4}$

$$\frac{U_B}{U_A} = \frac{\frac{(\sigma/4)^2}{2E}\times\frac{\pi}{4}(48)^2\times80+\frac{\sigma^2}{2E}\times\frac{\pi}{4}(24)^2\times240}{\frac{(\sigma/4)^2}{2E}\times\frac{\pi}{4}(48)^2\times240+\frac{\sigma^2}{2E}\times\frac{\pi}{4}(24)^2\times80}$$

Dividing throughout by $\frac{\sigma^2}{2E}\times\frac{\pi}{4}(24)^2\times80$

$$\frac{U_B}{U_A} = \frac{(1/16)\times2^2+3}{(1/16)2^2\times3+1} = 1.857$$

Ratio of strain energies per unit volume :

The ratio of strain energies per unit volume of two bars for the same maximum stress in the smaller cross-sections,

$$\text{Volume of Bar } A = \frac{\pi}{4}(48)^2\times240+\frac{\pi}{4}(24)^2\times80 \text{ mm}^3;$$

$$\text{Volume of Bar } B = \frac{\pi}{4}(48)^2\times80+\frac{\pi}{4}(24)^2\times240 \text{ mm}^3;$$

$$\frac{U_B}{U_A} = \frac{\left[\frac{(\sigma/4)^2}{2E} \times \frac{\pi}{4}(48)^2 \times 80 + \frac{\sigma^2}{2E} \times \frac{\pi}{4}(24)^2 \times 240\right] \frac{4/\pi}{48^2 \times 80 + 24^2 \times 240}}{\left[\frac{(\sigma/4)^2}{2E} \times \frac{\pi}{4}(48)^2 \times 240 + \frac{\sigma^2}{2E} \times \frac{\pi}{4}(24)^2 \times 80\right] \frac{4/\pi}{48^2 \times 240 + 24^2 \times 80}}$$

$$= 1.857 \times \frac{24^2 \times 80(2^2 \times 3 + 1)}{24^2 \times 80(2^2 + 3)} = 1.857 \times 1.857 = 3.448$$

2. (b) Solution:

(i) Filler materials : These are used in polymers to improve tensile and or compressive strengths, so provide abrasion resistance, toughness, dimensional stability and thermal stability, for example, wood flour, silica flour, sand particles, glass powder, clay, talc, limestone and even some synthetic polymers. Carbon black consists of very small and spherical particles of carbon, when added to vulcanized rubber, this extremely inexpensive material enhances, tensile strength, toughness and tear and abrasion resistance of rubber. Automobile tires contain 15-30 vol% of carbon black. A strong bond exists between rubber matrix and carbon particles.

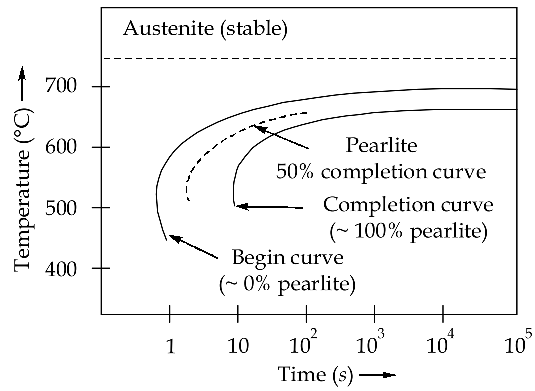
Plasticizers : The flexibility, ductility and toughness of polymers can be improved by the addition of plasticizers. The addition of plasticizers reduces hardness and stiffness of the composite. Moreover, they reduce the glass transition temperature. They are commonly used in polymers that are intrinsically brittle at room temperature such as polyvinyl chloride (PVC). Plasticizers are used in the manufacture of sheets or films, tubes, raincoats and curtains.

Some polymers under normal environmental conditions are subjected to rapid deterioration as a result of exposure to light, UV rays and also to oxidation. Addition of stabilizers counteracts these deteriorations processes.

Colourants : These impart a specific colour to a polymer and may be added as dyes or pigments. Dyes dissolve and become part of the molecular structures, but pigments are filer materials that do not dissolve but remains as a separate phase. Pigments are transparent or opaque.

Flame retardant : In the manufacture of textiles and children's toys, flammability of a polymer is of serious concern. Flame retardants enhance the flammability resistance of combustible polymers. These retardants may function by interfering with the combustion process, when the gas phase and combustion region may be cooled.

(ii) TTT diagram are Time-Temperature Transformation diagrams. This diagram shows the percentage of austenite transformed into pearlite as a function of temperature and time. The higher the temperature or the longer the time, the greater is the percentage of austenite transformed to pearlite. For each temperature, there is a minimum time for the transformation to begin. This period defines the critical cooling rate; with longer time Austenite begins to transform into pearlite.

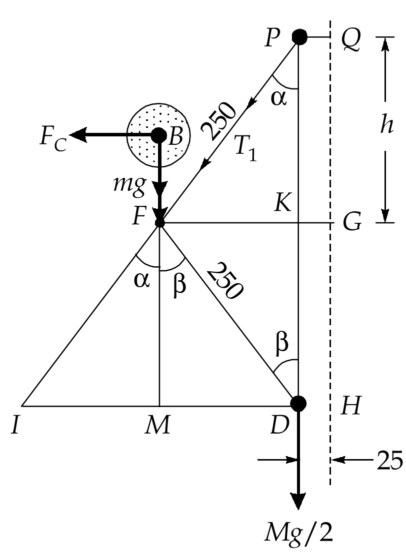


TTT diagram is used in designing heat treatment cycle for steels.

Depending upon the rate of cooling in quenching medium, Austenite

transformation takes place to different structures. When austenite is cooled slowly as in furnace, coarse pearlite is produced. If the rate of cooling is relatively high, as it is in air, austenite is transformed into fine pearlite. When it is cooled rapidly at a high rate, such as by quenching it in water, its FCC structure is transformed to a body-centered tetragonal (BCT) structure. This microstructure is called martensite. Thus, with the help of TTT diagram, heat treatment cycle can be determined.

2. (c) Solution:



Given : $PF = DF = 250$ mm $= 0.25$ m; $PQ = DH = KG = 25$ mm $= 0.025$ m; $M = 25$ kg; $m = 3.2$ kg; $r = FG = 175$ mm $= 0.175$ m; $h = QG = PK = 200$ mm $= 0.2$ m; $N = 160$ rpm

1. Length of the extension link

Let BF = Length of the extension link

The Proell governor in its mid-position is shown in figure.

From the figure, we find that

$$FM = GH = QG = 200 \text{ mm} = 0.2 \text{ m}$$

We know that

$$N^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h} \quad ...(\because \alpha = \beta \text{ or } q = 1)$$

$$(160)^2 = \frac{0.2}{BM}\left(\frac{3.2+25}{3.2}\right)\frac{895}{0.2} = \frac{7887}{BM}$$

$$\therefore \quad BM = \frac{7887}{(160)^2} = 0.308 \text{ m}$$

From figure, $BF = BM - FM = 0.308 - 0.2 = 0.108$ m $= 108$ mm

2. Tension in the upper arm

Let T_1 = Tension in the upper arm

$$PK = \sqrt{(PF)^2 - (FK)^2} = \sqrt{(PF)^2 - (FG - KG)^2}$$

$$= \sqrt{(250)^2 - (175-25)^2} = 200 \text{ mm}$$

$$\cos\alpha = \frac{PK}{PF} = \frac{200}{250} = 0.8$$

and

$$T_1 \cos\alpha = mg + \frac{Mg}{2} = 3.2 \times 9.81 + \frac{25 \times 9.81}{2}$$

$$= 154 \text{ N}$$

$$\therefore \quad T_1 = \frac{154}{\cos\alpha} = \frac{154}{0.8} = 192.5 \text{ N}$$

3. (a) Solution:

Thin cylinders are designed on the criterion of circumferential or hoop stress which is assumed constant over the thickness. However, in case of thick cylinders, stresses in all the three directions exist. Considering the case of a thick cylinder subjected to internal pressure only, maximum values of three principal stresses at the inner radius are

- Radial stress or pressure, p_i (compressive)
- Hoop stress, $\sigma_c = \dfrac{d_o^2 + d_i^2}{d_o^2 - d_i^2} \cdot p_i = \dfrac{k^2+1}{k^2-1} \cdot p_i$ where $k = \dfrac{d_o}{d_i}$ (tensile)
- Longitudinal stress, $\sigma_l = \dfrac{p_i d_i^2}{(d_o^2 - d_i^2)} = \dfrac{p_i}{(k^2-1)}$ (tensile)

Given : A cylinder $d_i = 600$ mm; $p_i = 30$ MPa; $\sigma_c = 180$ MPa; $v = 0.25$

(i) Maximum Principal Stress Theory : Failure occurs when the hoop stress exceeds the allowable tensile stress for the material. Thus, for safe design,

$$\frac{k^2+1}{k^2-1} \cdot p_i \leq \sigma$$

or $$\frac{k^2+1}{k^2-1} \times 30 = 180 \text{ or } k = 1.183$$

∴ $$d_o = 600 \times 1.183 = 710 \text{ mm}$$

and thus $$t = \frac{710 - 600}{2} = 55 \text{ mm}$$

(ii) Maximum Shear Stress Theory : Maximum shear stress at inner radius

$$= \frac{1}{2}(\sigma_c + p_i) = \frac{1}{2}\left(\frac{k^2+1}{k^2-1} \cdot p_i + p_i\right)$$

∴ for safe design, $\dfrac{2k^2}{k^2-1} \cdot p_i \leq \sigma$

or $$\frac{2k^2}{k^2-1} \times 30 = 180$$

or $$k = 1.225 \text{ or } d_o = 735 \text{ mm}$$

and $$t = \frac{735 - 600}{2} = 67.5 \text{ mm}$$

(iii) Maximum Strain Energy Theory : When the strain energy per unit volume of a body reaches the value of strain energy at elastic limit in simple tension.

$$\frac{1}{2E}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2v(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = \frac{\sigma^2}{2E}$$

or $$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2v(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = \sigma^2$$

Therefore, for safe design

$$\left(\frac{k^2+1}{k^2-1}\cdot p_i\right)^2 + (-p_i)^2 + \left(\frac{p_i}{k^2-1}\right)^2 -$$

$$2vp_i^2\left[\left(\frac{k^2+1}{k^2-1}\right)(-1) + (-1)\left(\frac{p_i}{k^2-1}\right) + \left(\frac{p_i}{k^2-1}\right)\left(\frac{k^2+1}{k^2-1}\right)\right] \le \sigma^2$$

or $$\frac{p_i^2}{(k^2-1)^2}[(2k^4+3) - 2v(-k^4+1-k^2+1+k^2+1)] \le \sigma^2$$

or $$\frac{p_i^2}{(k^2-1)^2}[2k^4(1+v) + 3(1-2v)] \le \sigma^2$$

or $$\frac{30^2}{(k^2-1)^2}[2k^4(1+0.25) + 3(1-2\times 0.25)] = 180^2$$

$$\frac{1}{(k^2-1)^2}[2.5k^4 + 1.5] = 36$$

On solving $k = 1.195$ or $d_o = 717$ mm

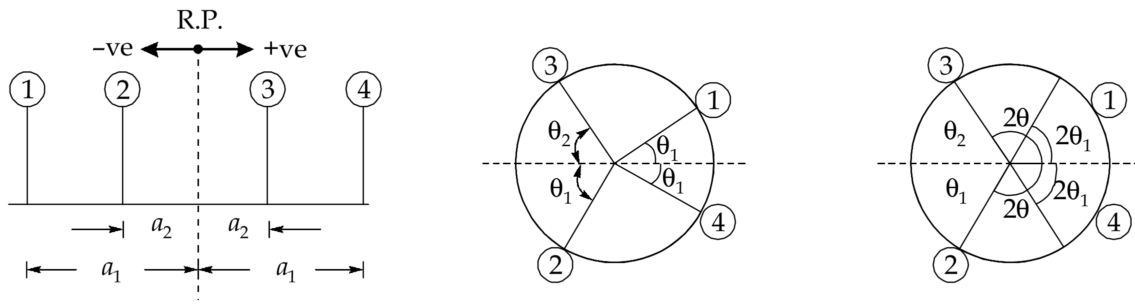
and $t = 58.5$ mm

3. (b) Solution:

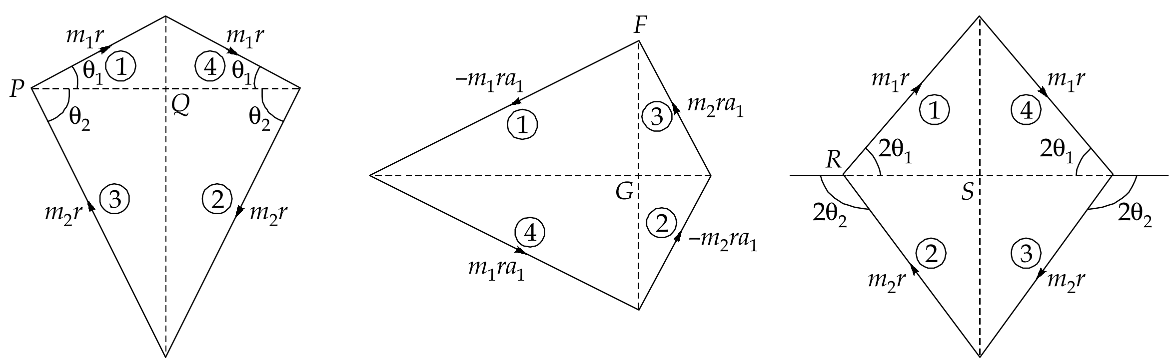
Given: Mass of reciprocating parts at cranks 1 and 4 = m_1; Mass of the reciprocating parts at cranks 2 and 3 = m_2.

The position of planes and primary and secondary crank positions are shown in figure (a), (b) and (c) respectively. Assuming the reference plane midway between the planes of rotation of cranks 2 and 3, the data may be tabulated as below:

Plane (1)	Mass (m) (2)	Radius (r) (3)	Cent. force $\div \omega^2$ ($m.r.$) (4)	Distance from ref. plane (l) (5)	Couple $\div \omega^2$ ($m.r.l$) (6)
1	m_1	r	$m_1 \cdot r$	$-a_1$	$-m_1 \cdot r \cdot a_1$
2	m_2	r	$m_2 \cdot r$	$-a_2$	$-m_2 \cdot r \cdot a_2$
3	m_2	r	$m_2 \cdot r$	$+a_2$	$+m_2 \cdot r \cdot a_2$
4	m_1	r	$m_1 \cdot r$	$+a_1$	$+m_1 \cdot r \cdot a_1$



(a) Position of planes **(b) Primary crank positions** **(c) Secondary crank positions**



(d) Primary force polygon **(e) Primary couple polygon** **(f) Secondary force polygon**

In order to balance the arrangement for primary forces and couples, the primary force an couple polygons must close. Fig. (d) and (e) show the primary force and couple polygons, which are closed figures. From fig. (d),

$$PQ = m_1 \cdot r\cos\theta_1 = m_2 \cdot r\cos\theta_2 \quad \text{or} \quad \frac{m_1}{m_2} = \frac{\cos\theta_2}{\cos\theta_1}$$

From fig. (e), $\quad FG = m_1 \cdot r \cdot a_1 \sin\theta_1 = m_2 \cdot r \cdot a_2 \sin\theta_2$

or $\quad m_1 \cdot a_1 \sin\theta_1 = m_2 \cdot a_2 \sin\theta_2$

$$\frac{m_1}{m_2} \times \frac{a_1}{a_2} = \frac{\sin\theta_2}{\sin\theta_1} \quad \text{or} \quad \frac{\cos\theta_2}{\cos\theta_1} \times \frac{a_1}{a_2} = \frac{\sin\theta_2}{\sin\theta_1} \quad \ldots\left(\because \frac{m_1}{m_2} = \frac{\cos\theta_2}{\cos\theta_1}\right)$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{\sin\theta_2}{\sin\theta_1} \times \frac{\cos\theta_1}{\cos\theta_2} = \frac{\tan\theta_2}{\tan\theta_1}$$

In order to balance the arrangement for secondary forces, the secondary force polygon must close. The position of the secondary cranks is shown in fig. (c) and the secondary force polygon is shown in fig. (f).

Now from fig. (f), $\quad RS = m_1 \cdot r\cos 2\theta_1 = m_2 \cdot r\cos(180° - 2\theta_2)$

or $\quad m_1 \cdot \cos 2\theta_2 = -m_2 \cdot \cos 2\theta_2$

$$\therefore \qquad \frac{m_1}{m_2} = \frac{-\cos 2\theta_2}{\cos 2\theta_1} = \frac{-(2\cos^2\theta_2 - 1)}{2\cos^2\theta_1 - 1} \qquad ...(\because \cos 2\theta = 2\cos^2\theta - 1)$$

$$\frac{\cos\theta_2}{\cos\theta_1} = \frac{(1-2\cos^2\theta_2)}{2\cos^2\theta_1 - 1} \qquad ...\left[\because \frac{m_1}{m_2} = \frac{\cos\theta_2}{\cos\theta_1}\right]$$

$$2\cos^2\theta_1 \times \cos\theta_2 - \cos\theta_2 = \cos\theta_1 - 2\cos^2\theta_2 \cdot \cos\theta_1$$

$$2\cos\theta_1 \cdot \cos\theta_2 (\cos\theta_2 + \cos\theta_2) = \cos\theta_1 + \cos\theta_2$$

$$2\cos\theta_1 \cdot \cos\theta_2 = 1 \quad \text{or} \quad \cos\theta_1 \cdot \cos\theta_2 = \frac{1}{2}$$

3. (c) Solution:

(i) Given: $\sigma_x = 80$ MN/m^2 (tensile); $\sigma_y = 40$ MN/m^2 (tensile); $\theta = 30°$

1. Normal stress, σ_n:

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2}\cos 2\theta$$

$$= \frac{80+40}{2} + \frac{80-40}{2}\cos 60°$$

$$= 60 + 10 = 70 \text{ MN/m}^2$$

2. Shear stress, τ:

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta = \frac{80-40}{2}\sin 60°$$

$$= 17.32 \text{ MN/m}^2$$

3. Resultant stress, σ_r, ϕ:

$$\sigma_r = \sqrt{\sigma_n^2 + \tau^2} = \sqrt{70^2 + 17.32^2}$$

i.e., $\sigma_r = 72.11$ MN/m^2

If ϕ is the angle that the resultant makes with the normal to the plane, then

$$\tan\phi = \frac{\tau}{\sigma_n} = \frac{17.32}{70} = 0.2474$$

$$\phi = 13°54'$$

(ii) Given: $m = 2.5$ kg; $s = 3$ N/mm = 3000 N/m; $x_6 = 0.25\, x_1$

We know that natural circular frequency of vibration.

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{3000}{2.5}} = 34.64 \text{ rad/s}$$

Let c = Damping coefficient of the damper in N/m/s,

x_1 = Initial amplitude, and

x_6 = Final amplitude after five consecutive cycles = $0.25x_1$ ... (Given)

We know that, $$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \frac{x_4}{x_5} = \frac{x_5}{x_6}$$

or $$\frac{x_1}{x_6} = \frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} \times \frac{x_4}{x_5} \times \frac{x_5}{x_6} = \left(\frac{x_1}{x_2}\right)^5$$

$\therefore$ $$\frac{x_1}{x_2} = \left(\frac{x_1}{x_6}\right)^{1/5} = \left(\frac{x_1}{0.25x_1}\right)^{1/5} = (4)^{1/5} = 1.32$$

We know that, $$\log_e\left(\frac{x_1}{x_2}\right) = a \times \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}} \qquad [\text{where } a = \omega_n \cdot \xi]$$

$$\log_e(1.32) = a \times \frac{2\pi}{\sqrt{(34.64)^2 - a^2}} \quad \text{or} \quad 0.2776 = \frac{a \times 2\pi}{\sqrt{1200 - a^2}}$$

Squaring both sides,

$$0.077 = \frac{39.5a^2}{1200 - a^2} \quad \text{or} \quad 92.4 - 0.077a^2 = 39.5a^2$$

$\therefore$ $a^2 = 2.335$ or $a = 1.53$

We know that, $a = c/2m$ or $c = a \times 2m = 1.53 \times 2 \times 2.5 = 7.65$ N/m/s

4. (a) Solution:

Given: $m = 300$ kg; $\delta = 2$ mm $= 2 \times 10^{-3}$ m; $m^2 = 20$ kg; $l = 150$ mm = 0.15 m; $c = 1.5$ kN/m/s = 1500 N/m/s; $N = 480$ r.p.m. or $\omega = 2\pi \times 480/60 = 50.3$ rad/s.

1. Amplitude of the forced vibrations

We know that stiffness of the frame,

$$s = m.g/\delta = 300 \times 9.81/2 \times 10^{-3} = 1.47 \times 10^6 \text{ N/m}$$

Since the length of stroke (l) = 150 mm = 0.15 m, therefore radius of crank.

$$r = l/2 = 0.15/2 = 0.075 \text{ m}$$

We know that the centrifugal force due to the reciprocating parts or the static force,

$$F = m_1 \cdot \omega^2 \cdot r = 20(50.3)^2\, 0.075 = 3795 \text{ N}$$

∴ Amplitude of the forced vibrations (maximum),

$$x_{max} = \frac{F}{\sqrt{c^2 \cdot \omega^2 + (s - m \cdot \omega^2)^2}}$$

$$= \frac{3795}{\sqrt{(1500)^2 (50.3)^2 + [1.47 \times 10^6 - 300(50.3)^2]^2}}$$

$$= \frac{3795}{\sqrt{5.7 \times 10^9 + 500 \times 10^9}} = \frac{3795}{710 \times 10^3} = 5.3 \times 10^{-3} \text{ m}$$

$$= 5.3 \text{ mm}$$

2. Speed of the driving shaft at which the resonance occurs

Let, N = Speed of the driving shaft at which the resonance occurs in r.p.m.

We know that the angular speed at which the resonance occurs,

$$\omega = \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{1.47 \times 10^6}{300}} = 70 \text{ rad/s}$$

∴ $$N = \omega \times 60/2\pi = 70 \times 60/2\pi = 668.4 \text{ r.p.m.}$$

4. (b) Solution:

Given : W = 40 kN = 40 × 10^3 N; σ_t = 70 MPa = 70 N/mm^2; τ = 50 MPa = 50 N/mm^2; σ_s = 105 MPa = 105 N/mm^2

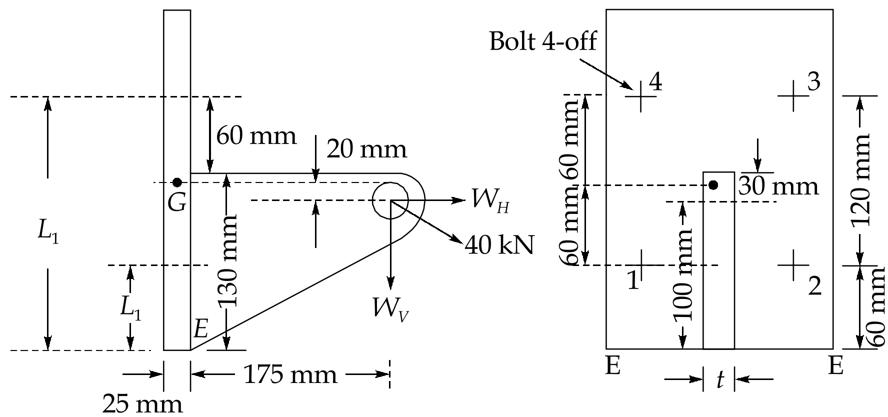
Since the load W = 40 kN is inclined at an angle of 60° to the vertical, therefore resolving it into horizontal and vertical components. We know that horizontal component of 40 kN.

$$W_H = 40 \times \sin 60° = 40 \times 0.866 = 34.64 \text{ kN} = 34640 \text{ N}$$

and vertical component of 40 kN.

$$W_V = 40 \times \cos 60° = 40 \times 0.5 = 20 \text{ kN} = 20000 \text{ N}$$

Due to the horizontal component (W_H), which acts parallel to the axis of the bolts as shown in figure below, the following two effects are produced.



1. A direct tensile load equally shared by all the four bolts, and
2. A turning moment about the centre of gravity of the bolts, in the anticlockwise direction.

∴ Direct tensile load on each bolt,

$$W_{t1} = \frac{W_H}{4} = \frac{34640}{4} = 8660 \text{ N}$$

Since the centre of gravity of all the four bolts lies in the centre at *G* (because of symmetrical bolts), therefore the turning moment is in the anticlockwise direction. From the geometry of the figure, we find that the distance of horizontal component from the centre of gravity (G) of the bolts = 60 + 60 – 100 = 20 mm.

∴ Turning moment due to W_H about *G*.

$$T_H = W_H \times 20 = 34640 \times 20$$

$$= 692.8 \times 10^3 \text{ Nmm} \qquad \text{...(Anticlockwise)}$$

Due to the vertical component W_V, which acts perpendicular to the axis of the bolts as shown in figure, the following two effects are produced.

1. A direct shear load equally shared by all the four bolts, and
2. A turning moment about the edge of the bracket in the clockwise direction.

∴ Direct shear load on each bolt,

$$W_s = \frac{W_V}{4} = \frac{20000}{4} = 5000 \text{ N}$$

Distance of vertical component from the edge E of the bracket = 175 mm

∴ Turning moment due to W_V about the edge of the bracket,

$$T_V = W_V \times 175 = 20000 \times 175$$

$$= 3500 \times 10^3 \text{ Nmm} \qquad \text{...(Clockwise)}$$

From above, we see that the clockwise moment is greater than the anticlockwise moment, therefore,

$$\text{Net turning moment} = 3500 \times 10^3 - 692.8 \times 10^3$$
$$= 2807.2 \times 10^3 \text{ Nmm} \qquad \text{...(Clockwise)} \qquad \text{...(i)}$$

Due to this clockwise moment, the bracket tends to tilt about the lower edge E.

Let w = Load on each bolt per mm distance from the edge E due to the turning effect of the bracket; L_1 = Distance of bolts 1 and 2 from the tilting edge E = 60 mm, and L_2 = Distance of bolt 3 and 4 from the tilting edge E = 60 + 120 = 180 mm

∴ Total moment of the load on the bolts about the tilting edge E,

$$= 2(w \cdot L_1)L_1 + 2(wL_2)L_2$$

...(∵ There are two bolts each at distance L_1 and L_2)

$$= 2w(L_1)^2 + 2w(L_2)^2 = 2w(60)^2 + 2w(180)^2$$

$$= 72000\, w \text{ Nmm} \qquad \text{...(ii)}$$

From equations (i) and (ii),

$$w = 2807.2 \times \frac{10^3}{72000} = 39 \text{ N/mm}$$

Since the heavily loaded bolts are those which lie at a distance from the tilting edge, therefore the upper bolts 3 and 4 will be heavily loaded. Thus the diameter of the bolt should be based on the load on the upper bolts. We know that the maximum tensile load on each upper bolt.

$$W_{t_2} = wL_2 = 39 \times 180 = 7020 \text{ N}$$

∴ Total tensile load on each of the upper bolt,

$$W_t = W_{t_1} + W_{t_2} = 8660 + 7020 = 15680 \text{ N}$$

Since each upper bolt is subjected to a tensile load (W_t = 15680 N) and a shear load (W_s = 5000 N), therefore equivalent tensile load,

$$W_{te} = \frac{1}{2}\left[W_t + \sqrt{(W_t)^2 + 4(W_s)^2}\right]$$

$$= \frac{1}{2}\left[15680 + \sqrt{(15680)^2 + 4(5000)^2}\right] \text{N}$$

$$= \frac{1}{2}[15680 + 18600] = 17140 \text{N}$$

Size of the bolts

Let d_c = Core diameter of the bolts

We know that the tensile load on each bolt = $\frac{\pi}{2}(d_c)^2\sigma_t = \frac{\pi}{4}(d_c)^2 70 = 55(d_c)^2\text{N}$

From equations (iii) and (iv), we get

$$(d_c)^2 = \frac{17140}{55} = 311.64$$

$$d_c = 17.65 \text{ mm}$$

4. (c) Solution:

Given : P = 20 kW = 20 × 10^3 W; N = 200 rpm; W = 900 N; L = 2.5;

τ = 42 MPa = 42 N/mm^2; σ_b = 56 MPa = 56 N/mm^2

Size of the shaft

Let, d = Diameter of the shaft, in mm

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ Nm}$$

$$= 955 \times 10^3 \text{ Nmm}$$

and maximum bending moment of a simply supported shaft carrying a central load,

$$M = \frac{W \times L}{4} = \frac{900 \times 2.5}{4} = 562.5 \text{ Nm}$$

$$= 562.5 \times 10^3 \text{ Nmm}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(562.5 \times 10^3)^2 + (955 \times 10^3)^2}$$

$$= 1108 \times 10^3 \text{ Nmm}$$

We also know that equivalent twisting moment (T_e)

$$1108 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25 d^3$$

$$\therefore \quad d^3 = 1108 \times \frac{10^3}{8.25} = 134.3 \times 10^3$$

$$d = 51.2 \text{ mm}$$

We know that the equivalent bending moment,

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}\left(562.5 \times 10^3 + 1108 \times 10^3\right)$$

$$= 835.25 \times 10^3 \text{ Nmm}$$

We also know that equivalent bending moment (M_e)

$$835.25 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5d^3$$

$$\therefore \qquad d^3 = 835.25 \times \frac{10^3}{5.5} = 152 \times 10^3$$

or $$d = 53.4 \text{ mm}$$

Taking the larger of the two values, we have

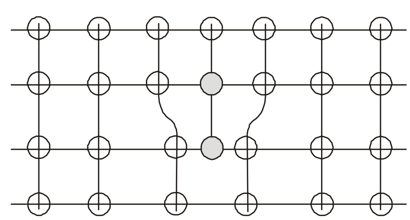
$$d = 53.4 \text{ say } 55 \text{ mm}$$

5. (a) Solution:

(i)

Different between point defects and line defects in a crystalline solid:

1. **Point defects:** These are zero dimensions. These are localized disruptions in otherwise perfect atomic or ionic arrangements in a crystal structure. The disruption affects a region involving several atoms or pair of atoms or ions only. These imperfections are introduced by the movement of atoms or ions when they gain energy. Several point defects are impurity (interstitial and substitutional), vacancy, Schottky and Frenkel defects.

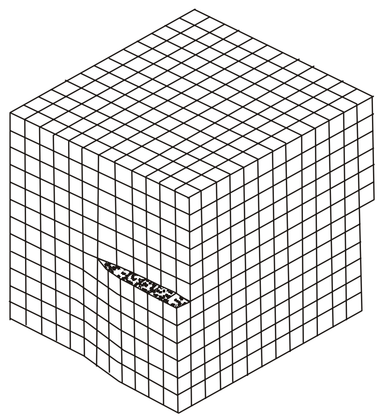


2. **Line defects:** Line defects or dislocations are lines along which whole rows of atoms in a solid are arranged anomalously. The resulting irregularity in spacing is most severe along a line called line of dislocation. Line defects are one-dimensional in nature.

(ii)

The two types of line defects are:

1. **Edge dislocations:** An edge dislocation is a defect where an extra half-plane of atoms is introduced mid way through the crystal, distorting nearby planes of atoms. When enough force is applied from one side of the crystal structure, this extra plane passes through planes of atoms breaking and joining bonds with them until it reaches the grain boundary. Burger's vector of edge dislocation is perpendicular to dislocation line.



2. **Screw dislocations:** The screw dislocations is slightly more

   difficult to visualize. The motion of screw dislocations is also a result of shear stress but the defect line movement is perpendicular to direction of stress and the atom displacement, rather than normal. Burger's vector of screw dislocation is parallel to dislocation line. Screw dislocations cannot move by climb process, whereas edge dislocations cannot cross-slip.

5. (b) Solution:

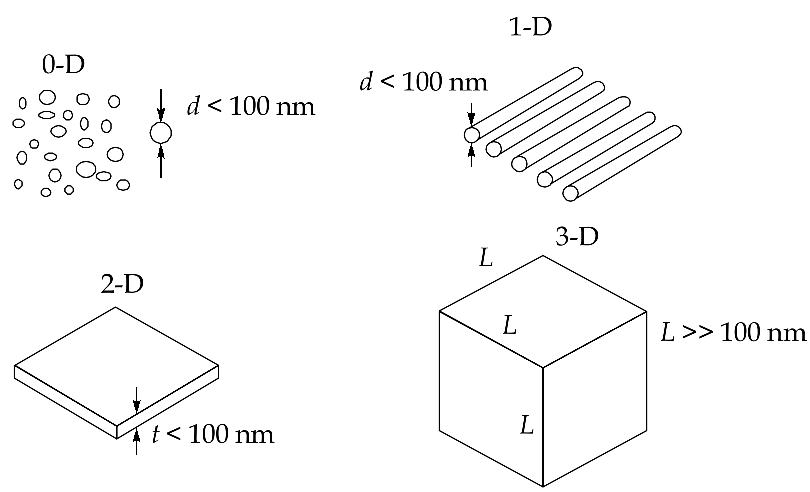
Depending on the number of dimensions in the nanorange, materials can be classified as follows:

(i) Zero-dimensional (called a nanoparticle), where all three dimensions of the particle are in the nanorange. Note that the term 'zero-dimensional' is applied to a particle, which has all the three dimensions in the nanorange and none in the larger-than nanorange. The submicroscopic particles of $CuAl_2$ that precipitate during ageing of a duralumin alloy fall in the nanorange, as also the carbide particles that form during the early stages of tempering of martensite in steels. These examples, however, refer to nanoparticles embedded in a bulk material.

(ii) One-dimensional, where two dimensions are in the nanorange and the third dimension is much larger, e.g., nanorods, nanowires and nanotubes. Carbon nanotubes are typical examples. They are cylindrical tubes of carbon atoms with diameter in the range of 1-2 nm and a much larger length reaching up to a mm.

(iii) Two-dimensional, where one dimension is in the nanorange and the other two are much larger, e.g., nanofilms, nanosheets and nanocoatings. Two-dimensional crystalline nanosheets have thickness in the nanoscale. In addition, the internal structure of the sheet can be nanosized grains.

(iv) Three-dimensional, where all three dimensions of a particle are much larger than the nanorange. This category, sometimes called bulk nanoparticle, forms parts of this classification, because the bulk solid itself may be composed of nanoparticles, e.g. nanosized crystals in a bulk polycystalline material.



Geometry of 0-D, 1-d, 2-d and 3-D nanomaterials

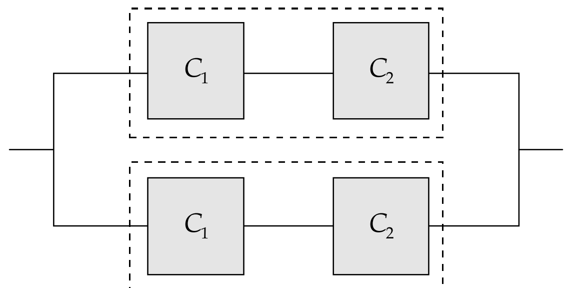
5. (c) Solution:

(i)

Redundancy is the existence of more than one means for performing a required function. Redundancy does not mean to have duplicate hardware. This involves deliberate creation of new parallel paths in the system. In parallel configuration, satisfactory functioning of any one of the elements leads the successful operation of the System. Therefore redundancy is a simple method for improving the reliability of a system when the element reliability cannot be increased. Though successful operation of one of the elements is required for the success of the system, deliberate use of both elements increases the probability of success of the system causing the system to be redundant (surplus).

Few areas where redundancy is extensively used are : aircraft propulsion system, satellite communication system, temperature control system for space vehicle, etc. Various approaches for introducing redundancy are unit redundancy and component redundancy.

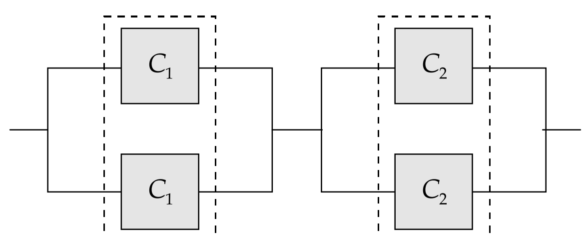
In unit redundancy, additional path for the entire system itself is provided.



Unit redundancy

In the figure shown, C_1 and C_2 are components of a system and two such components are provided in parallel resulting in improving the reliability of the entire system.

In component redundancy, additional path for each component of the system is provided. In the figure shown, additional components of C_1 and C_2 which are the individual components of the system are provided resulting in improving the reliability of each component thereby the entire system.



Component redundancy

(ii)

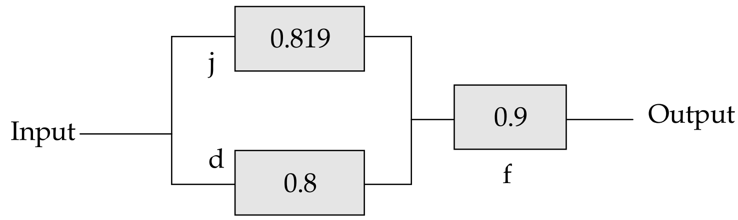
Sub-system containing parallel elements ‘a’ and ‘b’ are reduced to one element named ‘g’.

$$P(g) = P(a) + P(b) - P(a) \times P(b)$$
$$= 0.7 + 0.7 - 0.7 \times 0.7 = 0.91$$

Further elements ‘g’ and ‘c’ are reduced to element ‘j’

$$P(j) = P(g) \times P(c) = 0.91 \times 0.9 = 0.819$$

The reduced system is shown below



System reliability

Further elements 'j' and 'd' are reduced to element 'k'

$$P(k) = P(j) + P(d) - P(j) \times P(d)$$
$$= 0.819 + 0.8 - (0.819 \times 0.8)$$
$$= 0.9638$$

Finally, system reliability is computed as follows

$$P(s) = P(k) \times P(f) = 0.9638 \times 0.9 = 0.86742$$

5. (d) Solution:

The new location of the frame relative to the fixed reference frame can be found by adding the translation vector to the vector representing the original location of the origin of the frame. In matrix form, the new frame representation may be found by pre-multiplying the frame with a matrix representing the transformation. The new location of the frame is

$$F_{\text{new}} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & p_x + d_x \\ n_y & o_y & a_y & p_y + d_y \\ n_z & o_z & a_z & p_z + d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This equation is also symbolically written as

$$F_{\text{new}} = \text{Trans}(d_x, d_y, d_z) \times F_{\text{old}}$$
$$= \text{Trans}(3, 0, 2) \times F_{\text{old}}$$
$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.527 & -0.574 & 0.628 & 8 \\ 0.369 & 0.819 & 0.439 & 10 \\ -0.766 & 0 & 0.643 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0.527 & -0.574 & 0.628 & 11 \\ 0.369 & 0.819 & 0.439 & 10 \\ -0.766 & 0 & 0.643 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

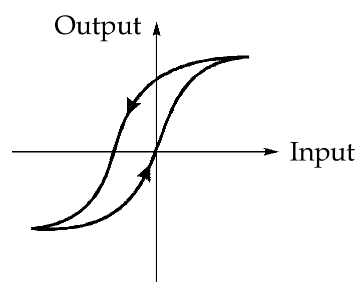
5. (e) Solution:

- **Linearity :** Perfect linearity would allow output versus input to be plotted as a straight line on a graph paper. Linearity is a measure of the consistency of the ratio of output to input. In the form of an equation, it is

$$y = mx$$

Where x is input and y is output, and m is a constant. If m is a variable, the relationship is not linear. For example, m may be a function of x, such as $m = a + bx$ where the value of b would introduce a non-linearity. A measure of the non-linearly could be given as the value of b.

- **Response time :** Response time is the time required for a sensor to respond completely to a change in input. The response time of a system with sensors is the combination of the responses of all individual components, including the sensor. An important aspect in selecting an appropriate sensor is to match its time response to that of the complete system.
- **Bandwidth :** It determines the maximum speed or frequency at which an instrument associated with a sensor or otherwise is capable of operating. High bandwidth implies faster speed of response. Instrument bandwidth should be several times greater than the maximum frequency of interest in the input signals.
- **Hysteresis :** It is defined as the change in the input/output curve when the direction of motion changes, as indicated in figure below. This behaviour is common in loose components such as gears, which have backlash, in magnetic devices with ferromagnetic media, and others.



Hysteresis

6. (a) Solution:

(i)

Salt nitriding : Salt nitriding is usually performed in proprietary salts (trade names of process are tuffriding and melanite).

Tuffriding produces a nitride case on most ferrous metals, which is free from white layer that is common in gas nitriding. The alloys that are difficult to gas nitriding generally respond to this process. Process is useful for thin nitride cases on tool steels with improved scuffing resistance.

Melanite process eliminates the environmental problems, which exist in cyanide salts. In this process, parts are preheated to about 575°C in a furnace with neutral atmosphere. Then, the parts are transferred to the nitriding salt bath, where soaking time is reduced because of preheating. Then, there is salt quench at about 370°C, and finally oil or air quench at room temperature. Salt quench removes cyanide salt remaining on the parts after removal from nitriding salt. This process produces a black surface that has a property of rust resistance.

Cementation : Introduction of one or more elements into the surface of the metal by high-temperature diffusion is called 'cementation'.

Chromium is driven into the surface of the steel from a chromium-rich gas at a temperature of 1100°C, which is called 'chromizing'. If chromizing is performed on low-carbon steel, the surface of steel can acquire corrosion-resistant properties, and acts as ferritic stainless steel. If high-carbon steel is chromized, then carbon combines with chromium to form hard carbides and performs as hard wear-resistance surface. Case depth of the order of 0.05-0.15 mm can be a obtained by cementation.

(ii)

Eutectoid is made at 0.8%C in Fe-Fe_3C diagram,

$$\text{Proeuctectoid cementite} = \frac{1.4-0.8}{6.67-0.8} = 0.1022$$

(iii)

Note that eutectoid reaction takes place at 0.8 percent carbon,

$$\text{Proeutectoid ferrite} = \frac{0.8-0.18}{0.8-0.025} = 0.8$$

$$\text{Eutectoid ferrite (in 6.67 percent C steel)} = \frac{6.67-0.8}{6.67-0.025} = \frac{5.87}{6.645} = 0.883$$

$$\begin{aligned}\text{Total ferrite} &= \text{Eutectoid ferrite} - \text{Proeutectoid ferrite}\\ &= 0.8833 - 0.1022\\ &= 0.7812\end{aligned}$$

6. (b) Solution:

Let x_1 and x_2 be the number of dresses of types A and B respectively. Using the given information, the linear programmiing problem may be stated as follows:

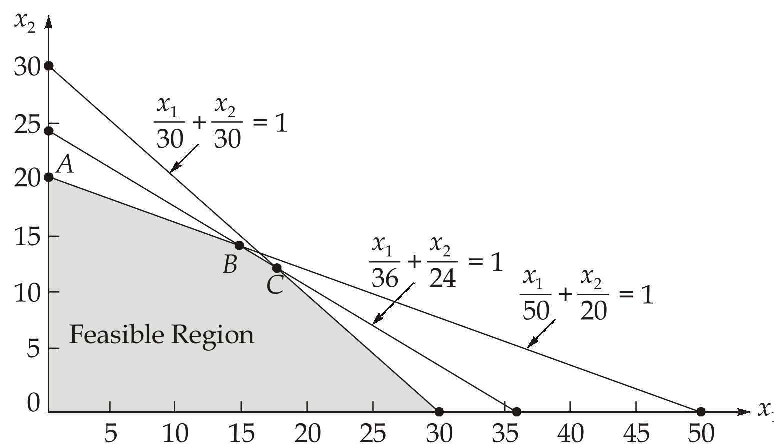
Maximise, $Z = 160x_1 + 180x_2$ Total profit

Subject to $\frac{x_1}{50} + \frac{x_2}{20} \le 1$ Man hours

$\frac{x_1}{36} + \frac{x_2}{24} \le 1$ Machine hours

$\frac{x_1}{30} + \frac{x_2}{30} \le 1$ Cloth material

The problem may be solved graphically now. The constraints are shown plotted in figure. The feasible solution is shown shaded and its vertices are evaluated below.



Graphic determination of Optimal Product Mix

Point	x_1	x_2	Z
0	0	0	0
A	0	20	3600
B	15	14	4920
C	18	12	5040
D	30	0	4800

Thus, optimal solution calls for producing 18 units of type A dress and 12 units of type B dress. total profit = Rs. 5040.

6. (c) Solution:

Basic components of FMS. The basic components of a FMS are given below:

1. **Machine tools and the related equipment:** It is clear from the definition of FMS, that the machines must be automated and reprogrammable to accommodate a large variety of products. For this reason, the majority of equipment consists of standard CNC machines (CNC turning centres and CNC machining centres), special purpose

machine tools, tolling for these machine, inspection stations or special inspection probes used with these machine tools. These machines are capable of accommodating a variety of tooling via an automatic tool changer and tool storage system.

2. **Material handling equipment:** The material handling equipment used in a FMS serves two functions: to move parts between machines and to orient and locate these parts for processing at the machines, automatically. The workpieces mounted on pallets or fixtures move through the system by means of powerful handling system such as towlines combined with shuttle system, roller conveyors, drag chain or automatically guided vehicles, AGVS. Combined with shuttle system, industrial robots, located suitably with respect to a group of machine tools, transfer workpieces from one machine to the next. A robot is normally only capable of addressing one or two machines and a load-and-unload station. Thus, a FMS may consists of a number of robotic workstations.
3. **Computer control system:** An FMS is a complex network of equipment and process that must be controlled via a computer and network of computers. It consists of control of machines, control of material handling system, to monitor the performance of the system and to schedule production.
4. **Human labour:** Even though FMS is a highly automated manufacturing system, involvement of human labour is needed to run the system. The various human labour may include: system manager, tool setter, load/unload man, fixture set up and lead man, electrical technician, mechanical/hydraulic technician and robot operator.

Types of Flexible manufacturing system: There is a large range of definitions which he people use in the flexible, manufacturing technology. Thus the name FMS has been equally applied to a single, relatively simple computer controlled machining centre and a 30-machine tool factory. In this connection the following definitions would be appropriate:

1. **Flexible Manufacturing Unit (FMU):** An FMu consists of a single, multifunction CNC machine tool and is the most simple flexible manufacturing system that can be constructed. It consists of a processing machine (CNC machine tool), a load/unload area and a material handler (a robot). The parts that move done a conveyor are loaded into the machine by a robot. After that, the robot is retracted and the processing begins. After the machining has been completed, the robot takes the part off the machine and moves it to the output bin.
2. **Flexible Manufacturing Cell (FMC):** This flexible system consists of two or more CNC machine tools alongwith one or more robot work stations, but not under DNC-linked control.

3. **Flexible Manufacturing System (FMS):** This system consists of a number of CNC machine tools under supervisory computer control via some form of DNC linkage.
4. **Flexible Manufacturing Transfer Line (FML or FTL):** It consists of a multimachine layout including several CNC machine tools and other specialist pieces of equipment all under supervisory computer control. This system is an alternative to a dedicated transfer line and is used for high volume production.

7. (a) Solution:

(i)

- Cutting velocity, $V = \dfrac{\pi DN}{1000 \times 60} = \dfrac{\pi \times 12.7 \times 400}{1000 \times 60} = 0.26\ \text{m/s}$

  Depth of cut, $d = \dfrac{12.70 - 12.19}{2} = 0.255\ \text{mm}$

  Feed, $f = \dfrac{203.20}{400} = 0.508\ \text{mm/rev}$

- Metal removal rate, MRR = Area of cut × V

  Now area of cut $= b.t. = d.f.$

  $\therefore$ MRR $= d.f.\ \pi.D_{ave.}N$

  $D_{ave.} = 12.445$ m

  $\therefore$ MRR $= 0.255 \times 0.508 \times \pi \times 12.445 \times 400$

  $= 2025.86\ \text{mm}^3/\text{min}$

- Machining time, $T = \dfrac{L}{fN}$, L is tool travel

  $= \dfrac{150}{0.508 \times 400} = 0.738$ min

- Power $= 56\,dfV$, Watts

  d and f are in mm, V is in m/min

  $\therefore$ Power $= 0.255 \times 0.508 \times 0.26 \times 60 \times 56 = 113.17$ watts

- Now, $P = T.\omega$

  $\therefore$ Torque, $T = \dfrac{113.17 \times 60}{2\pi \times 400} = 2.7$ Nm

  $\therefore$ Cutting force, $F_c = \dfrac{2T}{D_{ave.}} = \dfrac{2 \times 2.7 \times 1000}{12.445} = 434.17$ N

(ii) Classification of Industrial controllers: Most industrial controllers may be classified according to their control actions as:

1. Two-position or on-off controllers
2. Proportional controllers
3. Integral controllers
4. Proportional-plus-integral controllers
5. Proportional-plus-derivative controllers
6. Proportional-plus-integral-plus-derivative controllers

Most industrial controllers use electricity or pressurized fluid such as oil or air as power sources. Consequently, controllers may also be classified according to the kind of power employed in the operation, such as pneumatic controllers, hydraulic controllers, or electronic controllers. What kind of controller to use must be decided based on the nature of the plant and the operating conditions, including such considerations as safety, cost, availability, reliability, accuracy, weight, and size.

Proportional Control Action: For a controller with proportional control action, the relationship between the output of the controller $u(t)$ and the actuating error signal $e(t)$ is

$$u(t) = K_p e(t)$$

or, in Laplace-transferred quantities,

$$\frac{U(s)}{E(s)} = K_p$$

where K_p is termed the proportional gain.

Whatever the actual mechanism may be and whatever the form of the operating power, the proportional controller is essentially an amplifier with an adjustable gain.

Integral Control Action: In a controller with integral control action, the value of the controller output $u(t)$ is changed at a rate proportional to the actuating error signal $e(t)$. That is,

$$\frac{du(t)}{dt} = K_i e(t)$$

or,

$$u(t) = K_i \int_0^t e(t)\,dt$$

where K_i is an adjustable constant. The transfer function of the integral controller is

$$\frac{U(s)}{E(s)} = \frac{K_i}{s}$$

7. (b) Solution:

(i)

$$\sigma_d = \sigma_0 \frac{1+B}{B}\left[1-\left(\frac{h_1}{h_0}\right)^B\right]$$

Here,

$$B = \frac{\mu_1+\mu_2}{\tan\alpha - \tan\beta},\ \alpha = 15°,\ \beta = 0$$

$$B = 0.2/\tan 15° = 0.747$$

$$\therefore \quad \frac{\sigma_d}{\sigma_0} = \frac{1.747}{0.747}\left[1-\left(\frac{1.75}{2.5}\right)^{0.747}\right] = 0.547$$

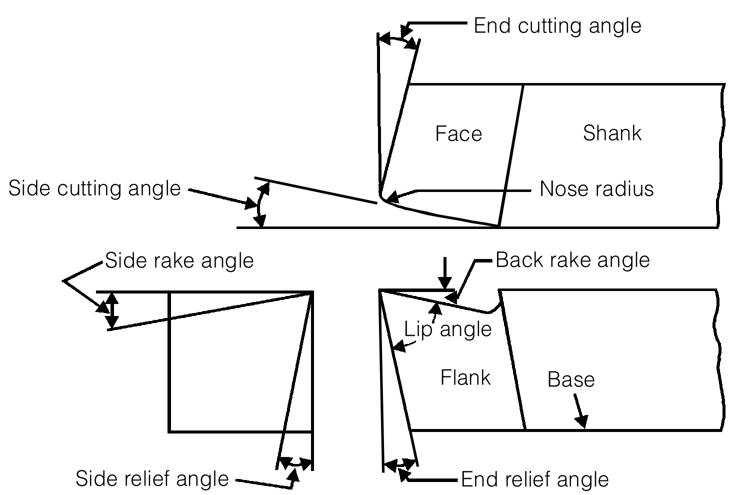
For movable mandrel, μ_1 being = μ_2

$$B = \frac{\mu_1-\mu_2}{\tan\alpha - \tan\beta} = 0$$

$$\therefore \quad \frac{\sigma_d}{\sigma_0} = \ln\left(\frac{h_0}{h_1}\right) = \ln\left(\frac{2.5}{1.75}\right) = 0.357$$

$\therefore$ Use of movable mandrel substantially reduces the drawing force.

(ii)



Rake: The rake is the slope of the top away from the cutting edge. The larger the rake angle, the larger the shear angle, and thereby the cutting force and power reduce. Large rake gives good surface finish.

Back rake angle: It is the angle between the line parallel to the tool axis passing through the tip and the rake face and angle is measured in a plane perpendicular to the base.

Side rake angle: It is the angle between the rake face and the line passing through the tip perpendicular to the tool axis and the angle is measured in a plane perpendicular to the base.

Nose: The nose of a tool in the conjunction of the side-and end-cutting edges. A nose radius increases the tool life and improves surface finish.

Flank: The flank of a cutting tool is that surface which face the workpiece.

Shank: The shank is that portion of the tool bit which is not ground to form cutting edges and is rectangular in cross-section.

Face: The face of the cutting-tool is that surface against which the chip slides forward.

End relief or clearance angle: It is the angle between the end flank and the line passing through the tip perpendicular to the base and angle is measured in plane parallel to the tool axis.

Side relief or clearance: It is the angle between the side flank and the line passing through the tip perpendicular to the base and the angle is measured in a plane perpendicular to the tool axis.

End cutting edge angle: It is the angle between the end cutting edge and the line passing through the tip perpendicular to the tool axis and the angle is measured in a plane parallel to base.

Side cutting edge angle: It is the angle between the side cutting edge and the line extending the shank. The angle is measured in a plane parallel to base.

7. (c) Solution:

Given,

Initial height of cylinder (h_i) = 60 mm

Initial diameter of cylinder (D_i) = 100 mm

Final height of cylinder after forging (h_f) = 30 mm

Final diameter of cylinder after forging (D_f) = ?

Coefficient of friction (μ) = 0.05

Yield strength (σ_0) = 120 N/mm^2

Sticking is not present (given)

Step I : Calculation of final diameter of cylinder after forging

as (volume of cylinder before forging) = (volume of cylinder after forging)

$$\frac{\pi}{4} D_i^2 \times h_i = \frac{\pi}{4} \times D_f^2 \times h_f$$

$$\Rightarrow \quad D_f^2 = D_i^2 \times \frac{h_i}{h_f} = 100^2 \times \frac{60}{30}$$

$$D_f = 141.42 \text{ mm}$$

$$\text{Radius, } R_f = 70.71 \text{ mm}$$

As we know all the forging calculations are done on the final dimensions of the billet.

Step II : Calculation of forging stress

(No sticking, only sliding) forging stress if sticking is not present is given by

$$p = (2K)e^{\frac{2\mu}{h}(R-r)}$$

As we know forging of circular billet is a plane stress case

So, $2K = \sigma_0$

where σ_0 = mean flow stress

$\sigma_0 = 120 \text{ N/mm}^2$ (given)

$$p = (120)e^{\left(\frac{70.71-r}{300}\right)}$$

So, Forging load is given by $= \int_0^R p \times 2\pi r\, dr = (2\pi)\times(120)\int_0^{70.71} r \cdot e^{\left(\frac{70.71-r}{300}\right)} dr$

$$= (240\times\pi)\left[(-300)r\,e^{\frac{(70.71-r)}{300}} - \int(-300)e^{\frac{(70.71-r)}{300}} \cdot dr\right]$$

$$= (240\pi)\left[-300r\,e^{\frac{(70.71-r)}{300}} - (300)^2 e^{\frac{70.71-r}{300}}\right]_0^{70.71}$$

$$= (240\pi)\left[(-300\times 70.71 - 300^2) - \left(0 - 300^2\, e^{\frac{70.71}{300}}\right)\right]$$

$$\text{Forging load} = 2.04 \text{ MN}$$

$$\text{Mean die pressure} = \frac{Load}{Total\ area} = \frac{2.04\times 10^6}{\pi R^2} = \frac{2.04\times 10^6}{\pi\times(70.71)^2} = 130 \text{ MPa}$$

8. (a) Solution:

Geometric mean diameter, $D = \sqrt{80\times 120} = 98$ mm

$\therefore$ Tolerance grade, $i = 0.45(D)^{1/3} + 0.001D$, microns

$= 0.45(98)^{1/3} + 0.001 \times 98$

$= 2.079 + 0.098 = 2.177$ microns

Now, for hole G, tolerance $= 16i = 34.832$ microns

$= 0.035$ mm (rationalized)

For shaft e_g, tolerance $= 25i = 54.425$ microns

$= 0.054$ mm (rationalized)

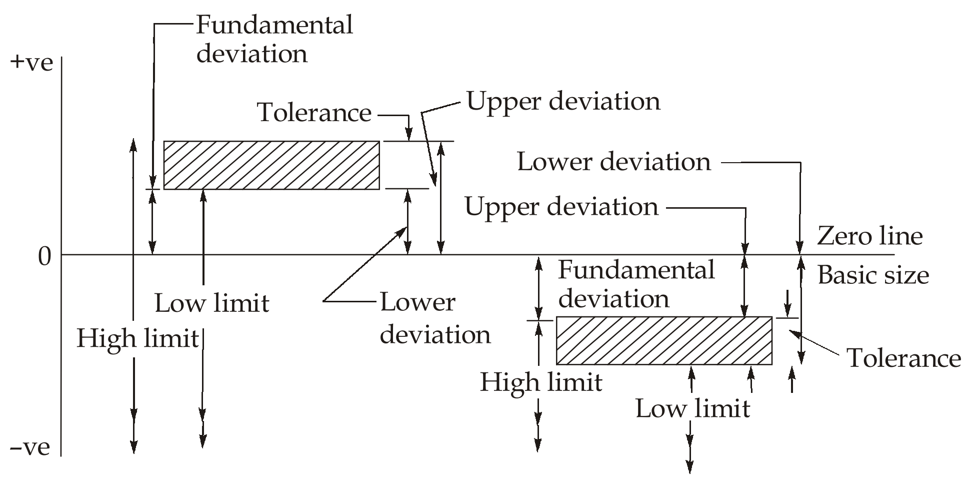
Now, F.D. for hole $G = +2.5D^{0.34}$, microns

$= 2.5(98)^{0.34} = 0.012$ mm (rationalized)

F.D. for shaft $e = -11D^{0.11}$, microns

$= -11(98)^{0.11}$, microns

$= -0.018$ mm (rationalized)



(i) Hole L.L. of Hole = Basic Size + F.D.

$= 100 + 0.012$ mm $= 100.012$ mm

H.L. of Hole = L.L. + Tolerance

$= 100.012 + 0.035 = 100.047$ m

or $100^{+0.047}_{+0.012}$ mm

(ii) Shaft U.L. or H.L. Shaft = Basic Size – F.D.

$= 100 - 0.018 = 99.982$ mm

L.L. of shaft = H.L. – Tolerance

$= 99.982 - 0.054$

$= 99.928$ mm

or $100^{-0.018}_{-0.072}$ mm

8. (b) Solution:

From the data of the problem, we have

$$D = 2000 \text{ units/year}, \; r = 0.25, \; C = \text{Rs } 10 \text{ per item}$$

$$C_0 = \text{Rs } 50/\text{order}; \qquad C_h = C \times r = 10 \times 0.25 = \text{Rs } 2.50$$

When no discount is offered, the optimal order quantity is given by:

$$Q^* = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2 \times 2,000 \times 50}{2.5}} = 283 \text{ units (approx.)}$$

Also, the number of orders per year is given by:

$$N = \frac{D}{D^*} = \frac{2,000}{283} = 7 \text{ orders}$$

The total inventory cost for $Q^* = 283$ becomes

$$TC = DC + \frac{D}{Q^*}C_0 + \frac{Q^*}{2}C_h$$

$$= 2,000 \times 10 + \frac{2,000}{283} \times 50 + \frac{283}{2} \times 2.5 = \text{Rs. } 20,707.10$$

When quantity discounts are offered, the following information is available:

Quantity	Price per unit (Rs)
$0 < Q_1 < 399$	10
$400 \le Q_2 < 699$	9(10% discount)
$700 \le Q_3$	8(20% discount)

The optimal order quantity Q_3^* based on price C_3 = Rs 8 is given by:

$$Q_3 = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2 \times 2,000 \times 50}{8 \times 0.25}} = 316 \text{ units (approx.)}$$

The value $Q_3^* = 316$ lies in the first range, $0 < Q_1 < 399$. Thus computing and then comparing $TC(Q_1^*)$.

$TC(b_1 = 400)$ and $TC\ (b_2 = 700)$ with each other.

$$TC(Q_1^*) = DC_1 + \frac{D}{Q_1^*}C_0 + \frac{Q_1^*}{2}(C_1 \times r)$$

$$= 2{,}000 \times 10 + \frac{2{,}000}{283} \times 50 + \frac{283}{2} \times 2.50 = \text{Rs. } 20{,}707.10$$

$$TC(b_1) = DC_2 + \frac{D}{b_1}C_0 + \frac{b_1}{2}(C_2 \times r)$$

$$= 2{,}000 \times 9 + \frac{2{,}000}{400} \times 50 + \frac{400}{2} \times 2.50 = \text{Rs. } 18{,}700.00$$

$$TC(b_2) = DC_3 + \frac{D}{b_2}C_0 + \frac{b_2}{2}(C_2 \times r)$$

$$= 2{,}000 \times 8 + \frac{2{,}000}{700} \times 50 + \frac{700}{2} \times 2.50 = \text{Rs. } 17{,}017.85$$

Since $TC(b_2 = 700)$ is the lowest cost, therefore the optimal order quantity is $Q^* = b_2 = 700$ units.

Hence, the shopkeeper should accept the offer of 10 per cent discount only because in this case his net saving per year would be Rs (20,707.10 – 17,017.85) = Rs 3,689.25.

8. (c) Solution:

(a) Calculations for expected completion time (t_e) of an activity and variance (σ^2), using following formulae are shown in Table.

$$t_e = \frac{1}{6}\left(t_o + 4t_m + t_p\right) \text{ and } \sigma_i^2 = \frac{1}{6}\left(t_p - t_o\right)^2$$

Activity	t_0	t_p	t_m	$t_e = \frac{1}{6}\left(t_0 + 4t_m + t_p\right)$	$\sigma^2 = \left[\frac{1}{6}\left(t_p - t_0\right)\right]^2$
1 – 2	5	10	8	7.8	0.696
1 – 3	18	22	20	20.0	0.444
1 – 4	26	40	33	33.0	5.429
2 – 5	16	20	18	18.0	0.443
2 – 6	15	25	20	20.0	2.780
3 – 6	6	12	9	9.0	1.000
4 – 7	7	12	10	9.8	0.64
5 – 7	7	9	8	8.0	0.111
6 – 7	3	5	4	4.0	0.111

(b) The earliest and latest expected completion time for all events considering the expected completion time of each activity are shown in figure below:

Forward pass method:

$E_1 = 0$ $\quad$ $E_2 = E_1 + t_{1,2} = 0 + 7.8 = 7.8$

$E_3 = E_1 + t_{1,3} = 0 + 20 = 20$ $\quad$ $E_4 = E_1 + t_{1,4} = 0 + 33 = 33$

$E_5 = E_2 + t_{2,5} + 7.8 + 18 = 25.8$

$E_6 = \max\{E_i + t_{i,6}\} = \max\{E_2 + t_{2,6}; E_3 + t_{3,6}\} = \max\{7.8 + 20; 20 + 9\} = 29$

$E_7 = \max\{E_i + t_{iT}\} = \max\{E_4 + t_{4,7}; E_5 + t_{5,7}; E_6 + t_{6,7}\} = \max\{33 + 9.8; 25.8 + 9; 29 + 4\} = 42.8$

Backward pass method:

$L_7 = E_7 = 42.8$ $\quad$ $L_6 = L_1 - t_{6,7} = 42.8 - 4 = 38.8$

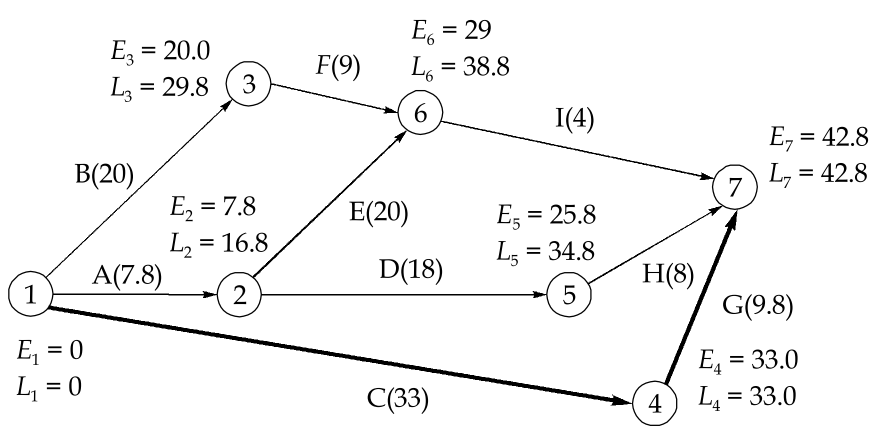
$L_5 = L_1 - t_{5,7} = 42.8 - 8 = 34.8$ $\quad$ $L_4 = L_7 - t_{4,7} = 42.8 - 9.8 = 33$

$L_3 = E_6 - t_{3,6} + 38.8 - 9 = 29.8$

$L_1 = \min\{L_i - t_{1,6}\} = \max\{L_2 - t_{1,2}; L_3 - t_{1,4}\} = \max\{16.8 - 7.8; 29.8 - 20; 33 + 33\} = 0$

$L_2 = \min\{L_i + t_{2,j}\} = \min\{L_5 - t_{2,5}; L_6 - t_{2,6}\} = \min\{34.8 - 18; 38.8 - 20\} = 16.8$

The E-value and L-values are shown in figure.



(c) The critical path is shown by thick line in figure. Where E-values and L-values are the same. The critical is: 1 - 4 - 7 and the expected completion time for the project is 42.8 weeks.

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