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Important Questions for GATE 2022

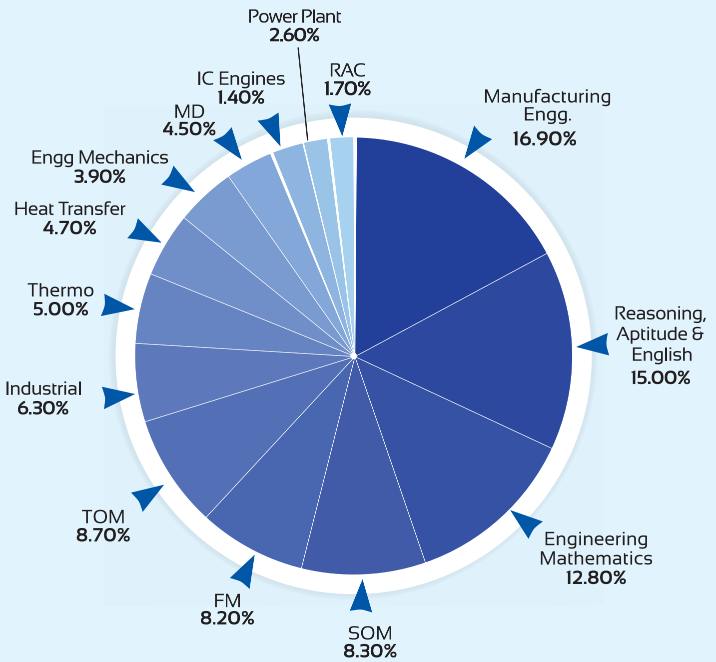
MECHANICAL ENGINEERING

Day 4 of 8

Q.76 - Q.100 (Out of 200 Questions)

Engineering Mathematics

SUBJECT-WISE WEIGHTAGE ANALYSIS OF GATE SYLLABUS



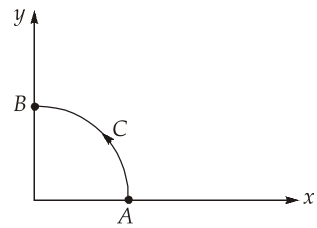
Subject	Average % (last 5 yrs)
Manufacturing Engineering	16.90%
Reasoning, Aptitude & English	15.00%
Engineering Mathematics	12.80%
Strength of Materials	8.30%
Theory of Machines	8.70%
Fluid Mechanics & Hydraulic Machines	8.20%
Industrial Engineering	6.30%
Thermodynamics	5.00%
Heat Transfer	4.70%
Engineering Mechanics	3.90%
Machine Design	4.50%
Internal Combustion Engines	1.40%
Power Plant Engineering	2.60%
Refrigeration & Air Conditioning	1.70%
Total	**100%**

Engineering Mathematics

Q.76 A rectangular sheet of metal of length 6 metres and width 2 metres is given. Four equal square are removed from the corners. The sides of this sheet are now turned up to form an open rectangular box. The height of the box (in cm), such that the volume of the box is maximum

(a) 22
(b) 25
(c) 35
(d) 45

Q.77 The value of line integral, when $F(r) = [-y, -xy] = -y\hat{i} - xy\hat{j}$ and C is the circular arc on the unit circle from A to B is ___________ . (Answer upto 2 decimal places)



Q.78 For the function $f(y) = y^2e^{-y}$, the maximum occurs when y is equal to _______ .

Q.79 Consider the differential equation $4y''(x) + 64\ y(x) = 0$ with initial conditions $y(0) = 0$ and $y'(0) = 1024$. The value of y at $x = 1$ is ________ . (Answer upto 2 decimal places)

Q.80 At the point $x = 2$, the function

$$f(x) = \begin{cases} x^3 - 8 & 2 < x < \infty \\ x - 2 & -\infty < x \le 2 \end{cases} \text{ is}$$

(a) continuous and differentiable
(b) continuous and not differentiable
(c) discontinuous and differentiable
(d) discontinuous and not differentiable

Q.81 The parabolic arc $y = 2\sqrt{x}, 2 \le x \le 3$ is revolved around the x-axis. The volume of the solid of revolution is

(a) 10π
(b) 8π
(c) 6π
(d) 4π

Q.82 The type of partial differential equation,

$$\frac{\partial^2 P}{\partial x^2} + \frac{1}{2}\frac{\partial^2 P}{\partial x \partial y} - \frac{5\partial P}{\partial x} + \frac{2\partial P}{\partial y} = 0 \text{ is}$$

(a) elliptical
(b) parabolic
(c) hyperbolic
(d) none

Q.83 The value of $\int_0^{\pi} \frac{dx}{c + d\cos x}$ (when $c > 0$, $|d| < c$) is

(a) $\frac{\pi}{\sqrt{c^2 - d^2}}$
(b) $\frac{\pi}{2\sqrt{c^2 - d^2}}$
(c) $\frac{2\pi}{\sqrt{d^2 - c^2}}$
(d) $\frac{\pi}{\sqrt{d^2 - c^2}}$

Q.84 A solution of the ordinary differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0$ is such that $y(0) = 2$ and $y(1) = -\frac{1-3e}{e^3}$. The value of $\frac{dy}{dt}(0)$ is _______.

Q.85 A random variable x has the following probability distribution.

x	0	1	2	3	4
$P(x)$	c	$2c$	$2c$	c^2	$5c^2$

The mean and variance of x is

(a) 1.638, 1.45
(b) 1.638, 1.204
(c) 1.204, 1.45
(d) 1.45, 1.638

Q.86 The solution of $x\frac{dy}{dx} + y = x^4$ with the condition $y(2) = \frac{21}{5}$ is

(a) $y = \frac{4x^4}{5} + \frac{2}{5x}$
(b) $y = \frac{x^5}{5} + 2x$
(c) $y = \frac{x^4}{5} + 2$
(d) $y = \frac{x^4}{5} + \frac{2}{x}$

Q.87 A manufacturer makes condensers which on an average are 1% defective. He packs them in boxes of 100. The probability that a box picked at random will contain 3 or more faulty condensers is ________ .

Q.88 A triangle in the xy plane is bounded by the straight lines $3x = 4y$, $y = 0$ and $x = 4$. The volume above the triangle and under the plane $x + y + z = 8$ is ________ .

Q.89 A function $f(x)$ is defined as

$$f(x) = \begin{cases} 3x + 4 & x < 0 \\ -4 & x = 0 \\ 4 - 6x & x > 0 \end{cases}$$

then at $x = 0$ for $f(x)$

(a) Limit does not exists
(b) Limit exists but discontinous
(c) Limit exists & is continuous
(d) Is not defined

Q.90 The volume V of the tetrahedron bounded by the coordinate planes and the plane $z = 12 - 3x + 4y$ is __________ .

Q.91 A tank initially contain 50 gallons of fresh water. Brine, containing 2 pounds per gallons of salt, flows into the tank at the rate of 2 gallons per minute and the mixture is kept uniform by stirring, flows out at the same rate. The time required for the quantity of salt in the tank to increase from 40 to 80 pound is _______ (in minutes).

Q.92 The surface integral $\iint_S \frac{2}{\pi}(12xi - 8yj)\, n\, dS$ over the sphere given by $x^2 + y^2 + z^2 = 16$ is

(a) $\frac{1024}{3}$
(b) $\frac{512}{5}$
(c) $\frac{4096}{11}$
(d) $\frac{2048}{3}$

Q.93 Number of tigers in a reserve is normally distributed with mean and variance respectively as 1200 and 9×10^4. The probability of finding more than 1800 tigers approximately is

(a) 0.0125
(b) 0.025
(c) 0.05
(d) None of these

Q.94 An exam can be given maximum four times. The probability of selection in first attempt is $\frac{1}{24}$ and it increases by 50% for every next attempt. The probability of selection is _______ . (Given that individual can give only one successful attempt)

(a) 0.2
(b) 0.3
(c) 0.4
(d) 0.5

Q.95 In a certain college, 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student body. If a student is selected at random from student body and is found to be studying mathematics, then the probability that the student is a girl is _______ .

Q.96 If $f(x) = 12x^{4/3} - 6x^{1/3}$, $x \in [-1, 1]$ then the absolute minimum value of the function is ______ .

Q.97 The directional derivative of $\phi = 3x^2y - 4yz^2 + 6z^2x$ at point (1, 1, 1) in the direction of line $\frac{x-1}{2} = \frac{y-4}{2} = \frac{z}{3}$ is

(a) $2\sqrt{17}$
(b) $\frac{2}{\sqrt{17}}$
(c) $\frac{29}{\sqrt{17}}$
(d) $\frac{40}{\sqrt{17}}$

Q.98 The non zero value of K such that the system of equations,

$$x + Ky + 3z = 0,\ 4x + 3y + Kz = 0,\ 2x + y + 2z = 0$$

has non-trival solution is

(a) 4
(b) 2.5
(c) 3.5
(d) None

Q.99 The matrix A is defined as $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$, the determinant of matrix $B = 3A^3 + 5A^2 - 6A + 2I$ is

(a) 110
(b) 1100
(c) 4400
(d) 440

Multiple Select Question (MSQ)

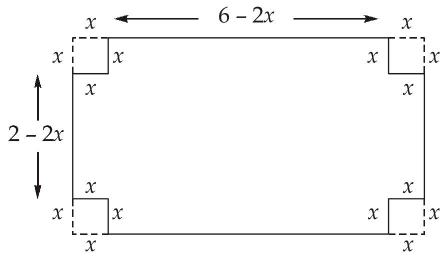
Q.100 The eigen vectors of matrix $\begin{bmatrix} 4 & 6 \\ 2 & 8 \end{bmatrix}$ is/are

(a) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$
(b) $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
(d) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

■■■■

Detailed Explanations

76. (d)



Let the side of each of the square cut off be x m and the sides of the base are $6 - 2x$, $2 - 2x$ m.

$\therefore$ Volume V of the box $= x(6 - 2x)(2 - 2x)$

$= 4(x^3 - 4x^2 + 3x)$

Then, $\dfrac{dV}{dx} = 4(3x^2 - 8x + 3)$

For volume to be maximum

$$\frac{dV}{dx} = 0 \text{ and } \frac{d^2V}{dx^2} < 0$$

$$3x^2 - 8x + 3 = 0$$

$$\therefore \quad x = \frac{8 \pm \sqrt{64 - 36}}{6} = \frac{8 \pm \sqrt{28}}{6} = 0.45 \text{ m or } 2.2 \text{ m}$$

2.2 m is not possible

For $x = 0.45$ m.

$$\frac{d^2V}{dx^2} = 4(6x - 8)$$

$$= 4(6 \times 0.45 - 8) = -21.2 < 0$$

Hence the volume of the box is maximum when its height is 45 cm.

77. 0.45 (0.43 to 0.47)

We may represent C by $r(t) = [\cos t, \sin t]$

$$= \cos t\,\hat{i} + \sin t\,\hat{j}$$

Where, $0 \le t \le \dfrac{\pi}{2}$

Then $x(t) = \cos t$, $y(t) = \sin t$

and $F(r(t)) = -y(t)\hat{i} - x(t)y(t)\hat{j}$

$= -\sin t\,\hat{i} - \cos t \sin t\,\hat{j}$

By differentiation, $r'(t) = [-\sin t, \cos t]$

$= -\sin t\,\hat{i} + \cos t\,\hat{j}$

We know that,
A line integral of a vector function $F(r)$ over a curve,

$$\int_c F(r)\cdot dr = \int_a^b F(r(t))\cdot r'(t)dt$$

$$= \int_0^{\pi/2}\left[-\sin t\,\hat{i} - \cos t\sin t\,\hat{j}\right]\cdot\left[-\sin t\,\hat{i} + \cos t\,\hat{j}\right]dt$$

$$= \int_0^{\pi/2}\left[\sin^2 t - \cos^2 t\sin t\right]dt$$

$$= \int_0^{\pi/2}\frac{1}{2}(1-\cos 2t)dt - \int_0^{\pi/2}\cos^2 t\sin t\,dt \qquad \text{put } \cos t = u$$

$$= \frac{1}{2}\left[\frac{\pi}{2}-0\right] - 0 - \int_1^0 u^2(-du)$$

$$= \frac{\pi}{4} - 0 - \frac{1}{3} = 0.4521$$

78. (2)

$$f(y) = y^2e^{-y}$$

$$f'(y) = y^2\left(-e^{-y}\right) + e^{-y}\times 2y$$

$$= e^{-y}\left(2y - y^2\right)$$

Putting $f'(y) = 0$

$$e^{-y}\left(2y-y^2\right) = 0$$

$$e^{-y}y(2-y) = 0$$

$y = 0$ or $y = 2$ are the stationary points

Now, $$f''(y) = e^{-y}(2-2y) + \left(2y-y^2\right)\left(-e^{-y}\right)$$

$$= e^{-y}\left(2-2y-2y+y^2\right)$$

$$= e^{-y}\left(y^2-4y+2\right)$$

At $y = 0$, $$f''(0) = e^{-0}(0-0+2) = 2$$

Since $f''(0) = 2$ is > 0 at $y = 0$ we have a minima

Now, at $y = 2$ $$f''(2) = e^{-2}\left(2^2 - 4\times 2 + 2\right)$$

$$= e^{-2}(4-8+2)$$

$$= -2e^{-2} < 0$$

$\therefore$ At $y = 2$ we have a maxima.

79. **–193.74 (–203 to –185)**

The equation is $4y''(x) + 64\,y(x) = 0$

The auxiliary equation is

$$4m^2 + 64 = 0$$
$$m^2 + 16 = 0$$
$$m = \pm 4i$$

Solution is $\quad y = C_1\cos 4x + C_2\sin 4x$

Given that $\quad y(0) = 0$

$\therefore \quad 0 = C_1$

$$y' = -4C_1\sin 4x + 4C_2\cos 4x$$
$$y'(0) = 1024$$
$$1024 = 4C_2$$

$\Rightarrow \quad C_2 = 256$

$\therefore$ Solution is $y = 256\sin 4x$

At $x = 1$, $y = 256\sin 4 = -193.74$

80. **(b)**

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(x-2) = 0$$
$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x^3-8) = 0$$

Also $\quad f(2) = 0$

Thus $\quad \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\therefore \quad f$ is continuous at $x = 2$

and $\quad Lf'(2) = 1$ and $Rf'(2) = 12$

$\therefore \quad f$ is not differentiable at $x = 2$.

81. **(a)**

The volume of a solid generated by revolution about the x-axis, of the area bounded by curve $y = f(x)$, the x-axis and the ordinates $x = a$, $y = b$ is

Volume $\int_a^b \pi y^2\,dx$

Here, $a = 2$, $b = 3$ and $y = 2\sqrt{x} \Rightarrow y^2 = 4x$

$\therefore \quad$ Volume $= \int_2^3 \pi 4x\,dx = 4\pi\left[\frac{x^2}{2}\right]_2^3 = 2\pi[x^2]_2^3$

$$= 2\pi\,[9-4] = 10\pi$$

82. **(c)**

Comparing the given equation with general form of second order partial differential equation,

$$A = 1,$$
$$B = \frac{1}{2},$$

$$C = 0$$

$$\Rightarrow \quad B^2 - 4AC = \frac{1}{4} > 0$$

$\therefore$ PDE is hyperbolic.

83. (a)

$$\int_0^{\pi} \frac{dx}{c\left(\cos^2\frac{x}{2} + \sin^2\frac{x}{2}\right) + d\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)} = \int_0^{\pi} \frac{dx}{(c+d)\cos^2\frac{x}{2} + (c-d)\sin^2\frac{x}{2}}$$

$$= \int_0^{\pi} \frac{\sec^2\frac{x}{2}dx}{(c+d) + (c-d)\tan^2\frac{x}{2}} = \frac{1}{(c-d)}\int_0^{\pi} \frac{\sec^2\frac{x}{2}dx}{\frac{(c+d)}{(c-d)} + \tan^2\frac{x}{2}}$$

$$= \frac{2}{c-d}\sqrt{\frac{c-d}{c+d}}\left[\tan^{-1}\left\{\tan\frac{x}{2}\sqrt{\frac{c-d}{c+d}}\right\}\right]_0^{\pi}$$

$$= \frac{2}{c-d}\sqrt{\frac{c-d}{c+d}}\left[\tan^{-1}\infty - \tan^{-1}0\right]$$

$$= \frac{2}{\sqrt{(c-d)(c+d)}} \times \frac{\pi}{2} = \frac{\pi}{\sqrt{c^2 - d^2}}$$

Tric ; put $d = 0$ and check

84. (–3)

$$D^2 + 5D + 6 = 0$$

$$D = -2,\ -3$$

$$y(t) = c_1 e^{-2t} + c_2 e^{-3t}$$

Given, $\quad y(0) = 2$

$$\Rightarrow \quad c_1 + c_2 = 2 \quad \text{...(i)}$$

$$y(1) = -\left(\frac{1-3e}{e^3}\right)$$

$$\Rightarrow \quad \frac{c_1}{e^2} + \frac{c_2}{e^3} = -\left(\frac{1-3e}{e^3}\right)$$

$$\Rightarrow \quad ec_1 + c_2 = 3e - 1 \quad \text{...(ii)}$$

Now solving equation (i) and (ii), we get

$$c_1 = 3$$

$$c_2 = -1$$

Substituting in $y(t)$, we get

$$y(t) = 3e^{-2t} - e^{-3t}$$

Now, $$\frac{dy}{dt} = -6e^{-2t} + 3e^{-3t}$$

$$\left(\frac{dy}{dt}\right)_{t=0} = -6 + 3 = -3$$

85. (a)

Since $$\sum_{x=0}^{4} P(x) = 1$$

$$c + 2c + 2c + c^2 + 5c^2 = 1$$

$$6c^2 + 5c - 1 = 0$$

$$c = \frac{1}{6}, -1$$

Since $P(x) \geq 0$, the possible value of

$$c = \frac{1}{6}$$

x	0	1	2	3	4
$P(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{36}$	$\frac{5}{36}$
$xP(x)$	0	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{3}{36}$	$\frac{20}{36}$

$$\text{Mean} = \sum_{x=0}^{4} xP(x) = 0 + \frac{2}{6} + \frac{4}{6} + \frac{3}{36} + \frac{20}{36}$$

$$= \frac{59}{36} = 1.638$$

$$\text{Variance} = \sigma^2 = E(x^2) - [E(x)]^2$$

$$= \left[0\left(\frac{1}{6}\right) + 1\left(\frac{2}{6}\right) + 4\left(\frac{2}{6}\right) + 9\left(\frac{1}{36}\right) + 16\left(\frac{5}{36}\right) - \left(\frac{59}{36}\right)^2\right]$$

$$= 1.45$$

86. (d)

Given differential equation is

$$x\frac{dy}{dx} + y = x^4$$

$$\Rightarrow \quad \frac{dy}{dx} + \left(\frac{y}{x}\right) = x^3 \quad \text{... (i)}$$

Standard form of liebnitz linear equation is

$$\frac{dy}{dx} + Py = Q \quad \text{... (ii)}$$

where P and Q function of x only and solution is given by

$$y(IF) = \int Q(IF)dx + c$$

where, integrating factor (IF) = $e^{\int P dx}$

here in equation (ii),

$$P = \frac{1}{x}, \text{ and } Q = x^3$$

$$IF = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$$

Solution $$y(x) = \int x^3 \cdot x dx + c = \frac{x^5}{5} + c$$

Given condition, $$y(2) = \frac{21}{5}$$

$$\therefore \quad \frac{21}{5}\times 2 = \frac{2^5}{5}+c$$

$$\Rightarrow \quad c = \frac{42-32}{5} = 2$$

$$\therefore \quad yx = \frac{x^5}{5}+2$$

$$\Rightarrow \quad y = \frac{x^4}{5}+\frac{2}{x}$$

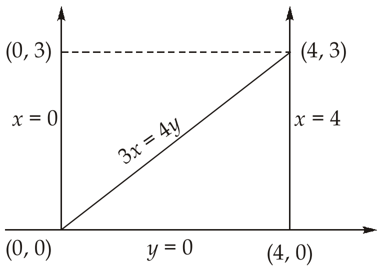
87. (0.08) (0.075 to 0.085)

$$P = 1\% = 0.01$$
$$n = 100$$
$$m = nP = 100 \times 0.01 = 1$$
$$P(r) = \frac{e^{-m}m^r}{r!}$$

P(3 or more faulty condensers)

$$= P(3) + P(4) + ... + P(100)$$
$$= 1 - [P(0) + P(1) + P(2)]$$
$$= 1-\left[\frac{e^{-1}1^0}{0!}+\frac{e^{-1}1^1}{1!}+\frac{e^{-1}1^2}{2!}\right] = 1-\left[e^{-1}+e^{-1}+\frac{e^{-1}}{2}\right]$$
$$= 1-e^{-1}\left[\frac{5}{2}\right] = 0.0803$$

88. (26)



$$\text{Volume} = \iiint dzdxdy = \iint zdydx$$

$$= \int_0^4 \int_0^{\frac{3x}{4}} (8-x-y)dydx = \int_0^4 \left(8y - xy - \frac{y^2}{2}\right)\Bigg|_0^{\frac{3x}{4}} dx$$

$$= \int_0^4 \left(6x - \frac{33x^2}{32}\right)dx = \left(\frac{6x^2}{2} - \frac{33}{32}\times\frac{x^3}{3}\right)\Bigg|_0^4$$

$$= \left(3x^2 - \frac{11x^3}{32}\right)\Bigg|_0^4 = 48 - 22 = 26$$

89. **(b)**

For limit to exist, LHL = RHL

Now,

$$\text{RHL} \quad \lim_{x\to 0^+} f(x) = 4 - 6x = 4$$

$$\text{LHL} \quad \lim_{x\to 0^-} f(x) = 3x + 4 = 4$$

$\Rightarrow$ Limit exists

Now for continuity,

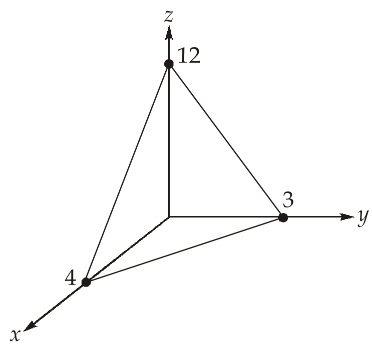
LHL = RHL = Functional value

At $x = 0$

$f(x) = -4 \neq \text{RHL}$

$\Rightarrow$ Discontinous.

90. **24 (23 to 25)**



As shown in figure above, the solid lies above the triangle in the xy-plane bounded by $3x + 4y = 12$ and the x and y axes.

$$V = \int_0^4 \int_0^{3-3x/4} (12 - 3x - 4y)\,dy\,dx$$

$$= \int_0^4 \left[12y - 3xy - 2y^2\right]_0^{3-3x/4} dx = \int_0^4 \left(3 - \frac{3x}{4}\right)\left(12 - 3x - 6 + \frac{3x}{2}\right) dx$$

$$= \int_0^4 3\left(\frac{4-x}{4}\right) 3\left(\frac{4-x}{2}\right) dx = \int_0^4 \frac{9}{8}(4-x)^2\,dx$$

$$= \left[\frac{9}{8}\left(-\frac{1}{3}\right)(4-x)^3\right]_0^4 = \frac{-3(-4^3)}{8} = \frac{192}{8} = 24$$

91. **27.46 (27.00 to 28.00)**

Let the salt content at time t be u lb, so that its rate of change is $\frac{du}{dt}$.

$$\frac{du}{dt} = 2 \text{ gal} \times 2 \text{ lb} = 4 \text{ lb/min}$$

If C is concentration of brine at time t, the rate at which salt content decrease due to out flow

$$= 2 \text{ gal} \times C\text{lb} = 2\,C \text{ lb/min}$$

$$\therefore \quad \frac{du}{dt} = 4 - 2C \qquad \text{...(i)}$$

Also, since there is no increase in volume of liquid the concentration,

$$C = \frac{u}{50}$$

$$\therefore \text{ Eq. (i) becomes} \quad \frac{du}{dt} = 4 - \frac{2u}{50}$$

Separating the variables and integrating,

$$\int dt = 25\int \frac{du}{100-u} + K$$

$$t = -25 \log_e (100 - u) + K \qquad \text{...(ii)}$$

Putting $t = 0$ for $u = 0$,

$$\Rightarrow \quad 0 = -25 \log_e 100 + K \qquad \text{...(iii)}$$

$$\therefore \quad t = 25\log_e \frac{100}{100-u}$$

If $u = 40$ at $= t = t_1$

and $u = 80$ at $t = t_2$,

$$t_1 = 25\log \frac{100}{60}$$

and
$$t_2 = 25\log \frac{100}{20}$$

$$\therefore \quad t_2 - t_1 = 25\log_e \frac{100}{20} \times \frac{60}{100}$$

$$= 25 \times \log_e 3 = 27.46 \text{ minutes}$$

92. (d)

According to Gauss divergence theorem

$$\iint_S \frac{2}{\pi}(12xi - 8yj)\,n\,dS = \frac{2}{\pi}\int \text{divergence } (12xi - 8yj)\,dv$$

$$= \frac{2}{\pi}[12-8] \times \frac{4}{3} \times \pi\left[r^3\right] \qquad [r = 4 \text{ (given)}]$$

$$= \frac{2}{\pi} \times 4 \times \frac{4}{3}\pi \times 64 = \frac{2048}{3}$$

93. (b)

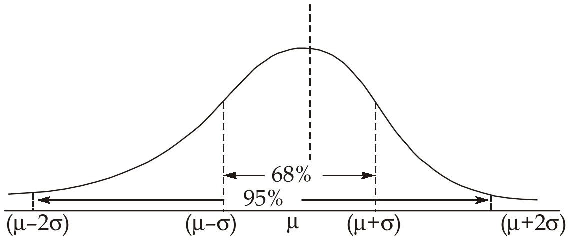
Given,

Mean, $\mu = 1200$

Variance, $\sigma^2 = 9 \times 10^4$

$\Rightarrow$ Standard deviation, $\sigma = \sqrt{9 \times 10^4} = 300$

Using Standard normal curve,



Probability of finding tigers between

$$(\mu - 2\sigma) \ \& \ (\mu + 2\sigma) = 0.95$$

$$\mu - 2\sigma = 1200 - 2 \times 300 = 600$$

$$\mu + 2\sigma = 1200 + 2 \times 300 = 1800$$

i.e. $P(600 \le X \le 1800) = 0.95$

$\Rightarrow P(X \le 600) + P(X \ge 1800) = 0.05$

Since normal curve is symmetric wrt mean value,

So, $P(X \le 600) = P(X \ge 1800)$

$\Rightarrow \quad 2P(X \ge 1800) = 0.05$

$\Rightarrow \quad P(X \ge 1800) = 0.025$

94. (b)

Let, P_1, P_2, P_3, P_4 be probability of selection in 1st, 2nd, 3rd & 4th attempt respectively,

Now,

$$P_1 = \frac{1}{24}; \ P_2 = \frac{1}{24}[1+0.5]$$

$$P_2 = \frac{1}{24} \times \frac{3}{2}$$

$$P_3 = \frac{1}{24} \times \frac{3}{2}[1+0.5] = \frac{1}{24} \times \left(\frac{3}{2}\right)^2$$

$$P_4 = \frac{1}{24} \times \left(\frac{3}{2}\right)^3$$

Now let A_i be selection in i^{th} attempt & $\overline{A_i}$ be unsuccessful attempt,

So,

$$P_{selection} = A_1 + \overline{A_1}A_2 + \overline{A_1}\,\overline{A_2}\,A_3 + \overline{A_1}\,\overline{A_2}\,\overline{A_3}\,A_4$$

$$= \frac{1}{24} + \frac{23}{24} \times \frac{1}{24} \times \frac{3}{2} + \frac{23}{24}\left(1 - \frac{3}{48}\right) \times \frac{1}{24} \times \left(\frac{3}{2}\right)^2 + \frac{23}{24}\left(1 - \frac{3}{48}\right)$$

$$\left(1 - \frac{9}{96}\right) \cdot \frac{1}{24} \times \left(\frac{3}{2}\right)^3 = 0.3$$

95. 0.375 (0.37 to 0.38)

$$P(\text{Girl}) = P(G) = \frac{60}{100} = \frac{3}{5}$$

$$P(\text{Boy}) = P(B) = \frac{40}{100} = \frac{2}{5}$$

Probability that mathematics is studied given that the student is a girl

$$= P\left(\frac{M}{G}\right) = \frac{10}{100} = \frac{1}{10}$$

By Baye's theorem, probability of maths student is a girl

$$= P\left(\frac{G}{M}\right) = \frac{P(G)P\left(\frac{M}{G}\right)}{P(M)} = \frac{P(G)P\left(\frac{M}{G}\right)}{P(G)P\left(\frac{M}{G}\right) + P(B)P\left(\frac{M}{B}\right)}$$

$$= \frac{\frac{3}{5}\times\frac{1}{10}}{\frac{4}{25}} = \frac{3}{8} = 0.375$$

96. (–2.25) (–2.30 to –2.20)

Given, $f(x) = 12x^{4/3} - 6x^{1/3}$, $x \in [-1, 1]$

Differentiating w.r.t. x, we get,

$$f'(x) = 12\cdot\frac{4}{3}x^{1/3} - 6\cdot\frac{1}{3}x^{-2/3}$$

$$= 16x^{1/3} - \frac{2}{x^{2/3}} = \frac{2(8x-1)}{x^{2/3}}$$

Now, $f'(x) = 0$

$$2(8x - 1) = 0$$

$$x = \frac{1}{8}$$

$$f\left(\frac{1}{8}\right) = 12\left(\frac{1}{8}\right)^{4/3} - 6\left(\frac{1}{8}\right)^{1/3}$$

$$= 12\left(\frac{1}{2}\right)^{4} - 6\left(\frac{1}{2}\right)$$

$$= \frac{12}{16} - 3 = -\frac{9}{4}$$

$$f(-1) = 12(-1)^{4/3} - 6(-1)^{1/3} = 12(1) - 6(-1) = 18$$

$$f(1) = 12(1)^{4/3} - 6(1)^{1/3} = 12 - 6 = 6$$

$\therefore$ Absolute minimum value $= \frac{-9}{4} = -2.25$

97. **(a)**

$$\vec{\nabla}\phi = \left(\frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}\right)\left[3x^2y - 4yz^2 + 6z^2x\right]$$

$$\Rightarrow \qquad \vec{\nabla}\phi = (6xy + 6z^2)\,\hat{i} + (3x^2 - 4z^2)\,\hat{j} + (-8yz + 12\,zx)\,\hat{k}$$

Now at (1, 1, 1)

$$\vec{\nabla}\phi = 12\hat{i} - \hat{j} + 4\hat{k} \qquad .....(1)$$

Also direction of line is, $\hat{A} = \dfrac{2\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{17}}$ .....(2)

$\Rightarrow$ Directional derivative using (1) & (2)

$$\vec{\nabla}\phi.\hat{A} = \left(12\hat{i} - \hat{j} + 4\hat{k}\right)\left(\frac{2\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{17}}\right)$$

$$= \frac{24 - 2 + 12}{\sqrt{17}} = \frac{34}{\sqrt{17}} = 2\sqrt{17}$$

98. **(d)**

The set of equations is written in the form of matrix

$$\begin{bmatrix} 1 & K & 3 \\ 4 & 3 & K \\ 2 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$AX = B,$$

$$C = [A : B] = \left[\begin{array}{ccc|c} 1 & K & 3 & 0 \\ 4 & 3 & K & 0 \\ 2 & 1 & 2 & 0 \end{array}\right]$$

On interchanging first and third rows, we have,

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 0 \\ 4 & 3 & K & 0 \\ 1 & K & 3 & 0 \end{array}\right]$$

Applying, $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - \dfrac{1}{2}R_1$

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 0 \\ 0 & 1 & K-4 & 0 \\ 0 & K-\frac{1}{2} & 2 & 0 \end{array}\right]$$

Applying $R_3 \to R_3 - \left(K - \dfrac{1}{2}\right)R_2$

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 0 \\ 0 & 1 & K-4 & 0 \\ 0 & 0 & 2-\left(K-\frac{1}{2}\right)(K-4) & 0 \end{array}\right]$$

For a nontrivial solution

$$\rho(A) = \rho(C) = 2$$

So,
$$2-\left(K-\frac{1}{2}\right)(K-4) = 0$$

$$2-K^2+\frac{1}{2}K+4K-2 = 0$$

$$-K^2+\frac{9}{2}K = 0$$

$$K\left(-K+\frac{9}{2}\right) = 0$$

$$K = \frac{9}{2} \text{ and } 0$$

Hence non-zero value of $K = 4.5$

Alternative Solution :

$$Ax = 0 \text{ has non-trivial solution}$$

$$|A| = 0$$

$$\begin{vmatrix} 1 & K & 3 \\ 4 & 3 & K \\ 2 & 1 & 2 \end{vmatrix} = 0$$

$$= 1(6-K) - K(8-2K) + 3(4-6) = 0$$

$$6 - K - 8K + 2K^2 - 6 = 0$$

$$2K^2 - 9K = 0$$

$$K(2K-9) = 0$$

$$K = 0, K = 4.5$$

99. (c)

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & -3 \\ 0 & 3-\lambda & 2 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(3-\lambda)(-2-\lambda) = 0$$

$$\lambda = 1, 3, -2$$

Eigen values of A^3 = 1, 27, –8

Eigen values of A^2 = 1, 9, 4

Eigen values of A = 1, 3, –2

Eigen values of I = 1, 1, 1

Eigen values of matrix B,

$$\text{First eigen value} = 3(1)^3 + 5(1)^2 - 6(1) + 2(1) = 4$$
$$\text{Second eigen value} = 3(27) + 5(9) - 6(3) + 2(1) = 110$$
$$\text{Third eigen value} = 3(-8) + 5(4) - 6(-2) + 2(1) = 10$$

Hence, required eigen values are, 4, 110 and 10

$$\text{Determinant of matrix } B = \lambda_1\lambda_2\lambda_3 = 4400$$

100. **(b, c)**

The characteristic equation $[A - \lambda I] = 0$

i.e. $$\begin{bmatrix} 4-\lambda & 6 \\ 2 & 8-\lambda \end{bmatrix} = 0$$

or $$(4 - \lambda)(8 - \lambda) - 12 = 0$$
or $$32 - 8\lambda - 4\lambda + \lambda^2 - 12 = 0$$
$$\Rightarrow \lambda^2 - 12\lambda + 20 = 0$$
$$\Rightarrow \lambda^2 - 10\lambda - 2\lambda + 20 = 0$$
$$\Rightarrow (\lambda - 10)(\lambda - 2) = 0$$
$$\Rightarrow \lambda = 10, 2$$

Corresponding to $\lambda = 10$, we have

$$[A - \lambda I]x = \begin{bmatrix} -6 & 6 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Which gives, $-6x + 6y = 0$
$$\Rightarrow x = y$$
$$2x - 2y = 0$$
$$\Rightarrow x = y$$

i.e. eigen vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Corresponding to $\lambda = 2$, we have

$$[A - \lambda I]x = \begin{bmatrix} 2 & 6 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Which gives, $2x + 6y = 0$i.e. eigen vector $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

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