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Important Questions for GATE 2022

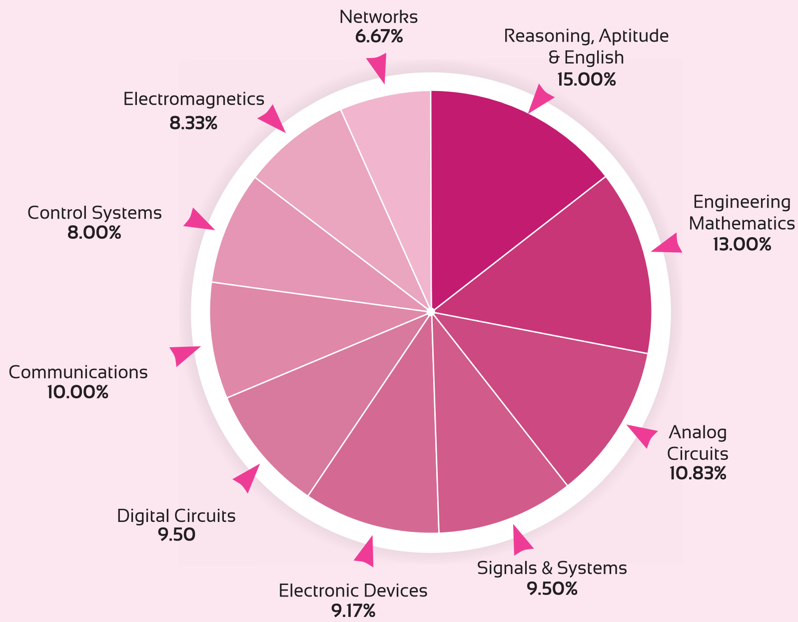
ELECTRONICS ENGINEERING

Day 7 of 8

Q.151 - Q.175 (Out of 200 Questions)

Signals & Systems and Communications

SUBJECT-WISE WEIGHTAGE ANALYSIS OF GATE SYLLABUS



Subject	Average % (last 5 yrs)*
Reasoning, Aptitude & English	15.00%
Engineering Mathematics	13.00%
Analog Circuits	10.83%
Signals & Systems	9.50%
Electronic Devices	9.17%
Digital Circuits	9.50%
Communications	10.00%
Control Systems	8.00%
Electromagnetics	8.33%
Networks	6.67%
Total	**100%**

Signals & Systems and Communications

Q.151 The Fourier Transform of a discrete time signal $x(n)$ is given by $X(e^{j\omega})$. If $X(e^{j\omega}) = -2j\,(2\sin 2\omega + \sin\omega)$, then the signal $x(n)$ is

(a) $\{2,1,\underset{\uparrow}{0},-1,-2\}$
(b) $\{-2,-1,\underset{\uparrow}{0},1,2\}$
(c) $\{-4,2,\underset{\uparrow}{0},-2,-4\}$
(d) $\{2,-1,\underset{\uparrow}{0},1,2\}$

Q.152 Let $x(t)$ be a signal whose Fourier transform is $X(j\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5)$, and let $h(t) = u(t) - u(t - 2)$. Then the period of the signal $y(t) = x(t) * h(t)$ is __________ rad/sec.

Q.153 Consider the system described by the difference equation $y(n) = ay(n - 1) + bx(n)$; and $s(n)$ is the zero-state step response of the system, then express coefficient 'b' interms of coefficient 'a' so that $s(\infty) = 1$. [Assume $a < 1$]

(a) $b = a$
(b) $b = 1 + a$
(c) $b = 1 - a$
(d) $b = -a$

Q.154 The z-transform of $y[n] = x(n + 2)u[n]$ is

(a) $z^2X(z) - z^2x(0) - zx(1)$
(b) $z^2X(z) + z^2x(0) - zx(1)$
(c) $z^2X(z) - z^2x(0) + zx(1)$
(d) $z^2X(z) + z^2x(0) + zx(1)$

Q.155 Let $x(t)$ be a continuous time signal $x(t) = \delta(t) + \delta(t - 1) + \delta(t - 2)$. Signal $x(t)$ is applied as input to a system whose impulse response is $h(t) = 2u(t) - u(t - 1) - u(t - 2)$. The output of the system is $y(t)$, then the value of $y(t)$ at $t = 2$ is ______.

Q.156 The z-transform of a sequence $x[n]$ is

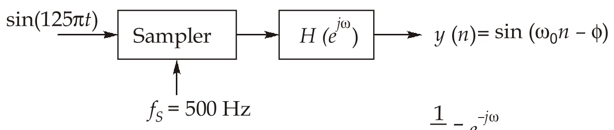
$$X(z) = \frac{z + 2z^{-2} + 4z^{-3}}{3 - z^{-4} + 6z^{-5}}$$

If the region of convergence includes the unit circle, the DTFT of $x[n]$ at $\omega = \pi$ is _________.

Q.157 Let $x[n]$ be a signal whose 4-point DFT is represented as $X[k]$. If $x[n] = [4, 12, 4, 12]$, then the value of $\frac{1}{N^2}\left[DFT\{X^*[k]\}\right]^*$, where $*$ denotes the complex conjugate.

(a) [4, 9, 4, 9]
(b) $[2 - 3j, 3, 2 + 3j, 3]$
(c) [1, 3, 1, 3]
(d) [4, 12, 4, 12]

Q.158 For digital signal processing the following system is implemented with input and outputs as shown below.

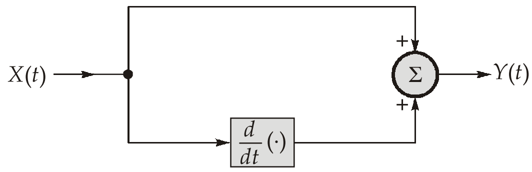


$$\text{Where } H(e^{j\omega}) = \frac{\frac{1}{2} - e^{-j\omega}}{1 - \frac{1}{2}e^{-j\omega}}$$

the value of the phase shift ϕ is ______.
(where ϕ is in degrees)

Q.159 Consider a discrete time signal $x[n] = 0.2x_1[n - n_0] - 2$. A discrete transformed signal of $x[n]$ is given as $x_1[n] = 5x[n + n_0] + k$. Then the value of k is ________.

Q.160 For the system shown in the figure below, the input $X(t)$ is a stationary random process with an auto-correlation function $R_X(\tau) = e^{-\pi\tau^2}$.



The average power of the output process $Y(t)$ is ________ W.

Q.161 A continuous time signal $x(t)$ has fourier transform

$$X(j\omega) = \frac{\omega^3}{1+\omega^2}$$

Then the fourier transform of the signal $y(t) = x(3t - 6)$ is

(a) $\dfrac{e^{j2\omega}\,\omega^3}{9(9+\omega^2)}$ (b) $\dfrac{e^{-j2\omega}\,\omega^3}{9(9-\omega^2)}$

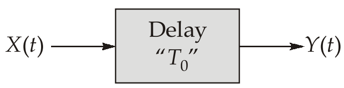
(c) $\dfrac{e^{j2\omega}\omega^3}{9(9-\omega^2)}$ (d) $\dfrac{e^{-j2\omega}\omega^3}{9(9+\omega^2)}$

Q.162 Let the continuous time signal defined as $y(t) = e^{-t}\,u(t) * \sum_{k=-\infty}^{\infty} \delta(t-3k)$, it is found that $y(t) = Ae^{-t}$ for $0 \le t < 3$,, then the coefficient 'A' can be written as

(a) $\dfrac{1}{1-e^3}$ (b) $\dfrac{1}{1+e^{-3}}$

(c) $\dfrac{1}{1-e^{-3}}$ (d) $\dfrac{1}{1+e^3}$

Q.163 $X(t)$ is a zero mean WSS process with auto-correlation function $R_X(\tau)$. It is applied as input to a delay element as shown below.



If the delay produced is T_0 and, $X(t)$ and $Y(t)$ are uncorrelated, then which of the following relations is true?

(a) $R_X(0) = 0$ (b) $R_X(T_0) = 0$

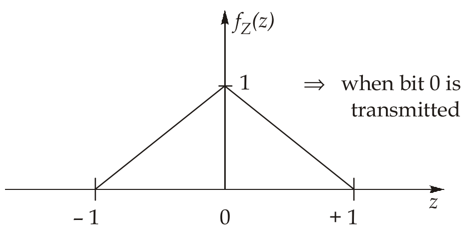
(c) $R_X(2T_0) = 0$ (d) $R_X(\tau)$ = constant

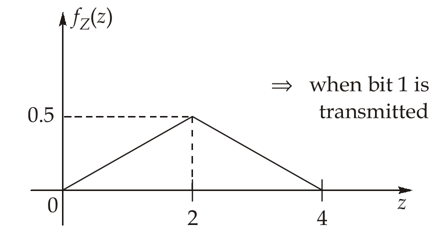
Q.164 Given the following periodic function, $f(t)$

$$f(t) = \begin{cases} t^2 & \text{for} \quad 0 \le t \le 2 \\ -t+6 & \text{for} \quad 2 \le t \le 6 \end{cases}$$

The co–efficient a_0 of the continuous Fourier series associated with $f(t)$ is given as $\dfrac{X}{9}$, the value of X is ________.

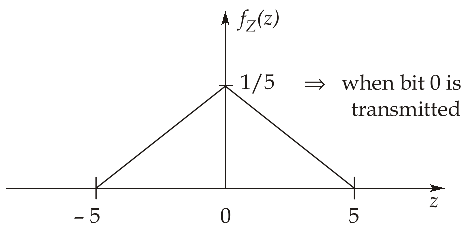
Q.165 In a communication system bit 0 and bit 1 are transmitted with probabilities 0.2 and 0.8 respectively. At the receiver, the PDFs of the respective received signals for both bits are shown below.

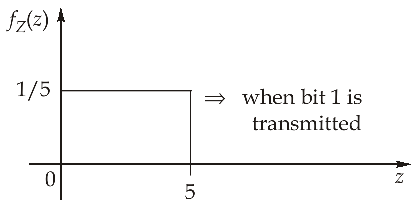




If the optimum threshold is selected using maximum aposteriori (MAP) criteria, then the threshold value will be ________.

Q.166 In a communication system bit 0 and bit 1 are transmitted with probabilities 0.4 and 0.6 respectively. At the receiver, the PDFs of the respective received signals for both bits are shown below.



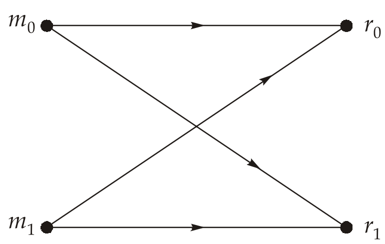


If the optimum threshold is selected using maximum aposteriori (MAP) criteria, then the probability of error will be

(a) 0.5 (b) 0.4
(c) 0.3 (d) 0.2

Q.167 In a QPSK modulator with an input data rate of R_b = 10 Mbps, the sinusoidal carrier frequency of 70 MHz is used. The minimum channel bandwidth required to transmit the modulated signal with Nyquist baseband pulse shaping is ________ MHz.

Q.168 Consider the lossless discrete memoryless channel shown below.



$P(m_0) = 0.5$, $P\left(\frac{r_1}{m_0}\right) = 0.1$ and $P\left(\frac{r_0}{m_1}\right) = 0.2$. If the optimum detection is used at the receiving end, then the probability of error will be

(a) 0.10 (b) 0.15
(c) 0.05 (d) 0.50

Q.169 The capacity of a discrete lossless channel, with 3 input symbols and 5 output symbols, is _________ bits/symbol.

Q.170 Let X and Y are two continuous random variables with probability density functions $f_X(x)$ and $f_Y(y)$ respectively. If the joint probability density function of X and Y is $f_{XY}(x, y)$, then the mutual information of X and Y [i.e., $I(X ; Y)$] can be expressed as

Note that, $E[\cdot]$ represents the expectation operator.

(a) $E\left[\ln\left(\dfrac{f_X(x)\,f_Y(y)}{f_{XY}(x,y)}\right)\right]$

(b) $E\left[\ln\left(f_{XY}(x,y)\right)\right]$

(c) $E\left[\ln\left(\dfrac{f_{XY}(x,y)}{f_X(x)\,f_Y(y)}\right)\right]$

(d) $E\left[\left(f_X(x)+f_Y(y)\right)\ln\left(f_{XY}(x,y)\right)\right]$

Q.171 Let A, B and C are three discrete random variables. If $I(X ; Y)$ represents the mutual information between two variables X and Y, then consider the following relations:

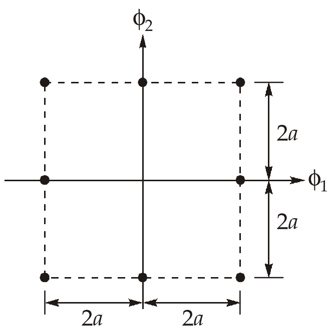
R1 : $I(A ; B \mid C) = H(A \mid C) + H(B \mid C) - H(A \cdot B \mid C)$

R2 : $I(A ; B \mid C) = H(A \cdot B \mid C) - H(A \mid B \cdot C) - H(B \mid A \cdot C)$

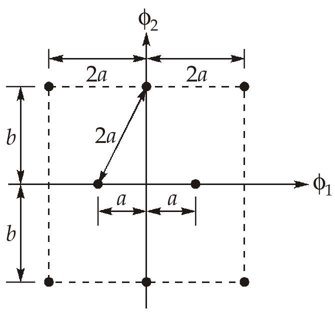
Select the correct relation(s) using the codes given below.

(a) R1 only
(b) R2 only
(c) Both R1 and R2
(d) Neither R1 nor R2

Q.172 Consider the two 8-point QAM signal constellations shown in the figure below:



Constellation of system - 1



Constellation of system - 2

In both the systems, the signal points are equally probable. The minimum distance between adjacent points in both the systems is same (i.e., $d_{min} = 2a$). If the average symbol energy transmitted in system-1 is E_1 and that of in system-2 is E_2, then select the correct one of the following statements.

(a) E_1 is larger than E_2 by 1.33 dB
(b) E_1 is larger than E_2 by 1.25 dB
(c) E_2 is larger than E_1 by 1.33 dB
(d) E_2 is larger than E_1 by 1.25 dB

Q.173 A binary data is transmitted through an ideal AWGN channel with infinite bandwidth. The two-sided power spectral density of the noise is $N_0/2$. If the average energy transmitted per bit is E_b, then the minimum value of (E_b/N_0) required, for error free transmission, will be _______ dB.

Multiple Select Questions (MSQ)

Q.174 If DFT of signal $x(n)$ is defined as $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk}$, $k = 0, 1, 2, N - 1$. If signal $y(n)$ is defined as $y(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{-j\frac{2\pi}{N}nk}$; $n = 0, N - 1$, then which of the following is/are not true.

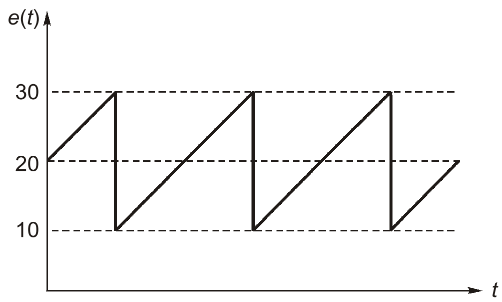
(a) $\sum_{n=0}^{N-1} x(n) \cdot \sum_{n=0}^{N-1} y(n) = 0$

(b) $\sum_{n=0}^{N-1} x(n) + \sum_{n=0}^{N-1} y(n) = 0$

(c) $\sum_{n=0}^{N-1} x(n) - \sum_{n=0}^{N-1} y(n) = 0$

(d) $\sum_{n=0}^{N-1} x(n) = \frac{1}{N}\sum_{n=0}^{N-1} y(n) = 0$

Q.175 The figure below shows the positive portion of the envelope of an AM signal.



If the message signal has zero DC value, then which of the below is/are correct?

(a) The modulation index is 0.5.

(b) Modulation efficiency is 11.11%.

(c) The carrier power is 200 W.

(d) The sideband power is 16.66 W.

■■■■

Detailed Explanations

151. (b)

$$X(e^{j\omega}) = -2j\left(\frac{2(e^{j2\omega}-e^{-j2\omega})}{2j}+\frac{(e^{j\omega}-e^{-j\omega})}{2j}\right)$$

$$X(e^{j\omega}) = 2e^{-j2\omega} - 2e^{j2\omega} + e^{-j\omega} - e^{j\omega}$$

$$\delta(n) \leftrightarrow 1$$

$$\delta(n-2) \leftrightarrow e^{-j2\omega}$$

$$\delta(n+2) \leftrightarrow e^{j2\omega}$$

$$\delta(n-1) \leftrightarrow e^{-j\omega}$$

$$\delta(n+1) \leftrightarrow e^{j\omega}$$

so, $$x(n) = 2\delta(n-2) - 2\delta(n+2) + \delta(n-1) - \delta(n+1)$$

$$x(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$$

152. (1.26) (1.15 to 1.50)

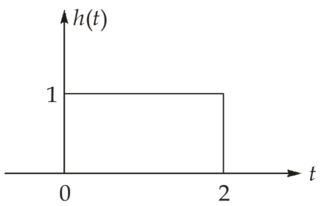
Given, $$X(j\omega) = \delta(\omega) + \delta(\omega-\pi) + \delta(\omega-5)$$

$$h(t) = u(t) - u(t-2)$$

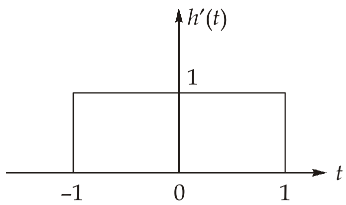
From the convolution property of the Fourier transform,

$$Y(j\omega) = X(j\omega)H(j\omega)$$

where $H(j\omega)$ is Fourier transform of $h(t)$.



Let,



$$\xleftrightarrow{\text{FT}} \frac{2\sin\omega}{\omega}$$

then we may write, $$h(t) = h'(t-1)$$

$\therefore$ $$H(j\omega) = e^{-j\omega}H'(j\omega)$$

$$H(j\omega) = e^{-j\omega}\frac{2\sin\omega}{\omega}$$

where as the above $H(j\omega) = 0$ when $\omega = K\pi$

K is the non-zero integer.

$\therefore$ $$Y(j\omega) = X(j\omega)H(j\omega)$$

$$= [\delta(\omega)+\delta(\omega-\pi)+\delta(\omega-5)]\left[e^{-j\omega}\frac{2\sin\omega}{\omega}\right]$$

$$Y(j\omega) = \delta(\omega) + \delta(\omega-5)$$

by taking inverse Fourier transform,

$$y(t) = \frac{1}{2\pi} + \frac{1}{2\pi}e^{-j5t}$$

∴ $y(t)$ is a complex exponential summed with a constant. We know that complex exponential is periodic. Adding constant to a complex exponential does not affect its periodicity.

∴ The fundamental frequency of $y(t)$ is $\frac{2\pi}{5}$ rad/sec (or) 1.26 rad/sec.

153. (c)

Given, $y(n) = ay(n-1) + bx(n)$

by taking z-transform,

$$Y(z) = az^{-1}Y(z) + bX(z)$$

$$(1 - az^{-1})Y(z) = bX(z)$$

∴ $$H(z) = \frac{Y(z)}{X(z)} = \frac{b}{1 - az^{-1}}$$

∴ $$h(n) = ba^n u(n)$$

We know that, step response $s(n) = \sum_{k=0}^{n} h(n-k)$

$$s(n) = \sum_{k=0}^{n} ba^{n-k}u(n-k)$$

$$= b\left[\frac{1 - a^{n+1}}{1 - a}\right]u(n)$$

Given, $s(\infty) = 1$

∴ $$s(\infty) = \frac{b}{1-a} = 1$$

∴ $$b = 1 - a$$

154. (a)

Given, $y[n] = x(n+2)u(n)$

from the definition of z-transform,

$$Y(z) = z\{x(n+2)u[n]\}$$

$$= \sum_{n=0}^{\infty} x(n+2)z^{-n}$$

Let $n + 2 = P$

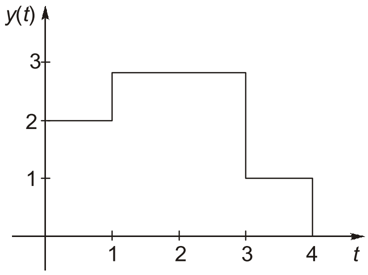
$$n = P - 2$$

$$= \sum_{P=2}^{\infty} x(P)z^{-(P-2)} = z^2 \sum_{P=2}^{\infty} x(P)z^{-P}$$

$$= z^2\left[\sum_{P=0}^{\infty} x(P)z^{-P} - x(0) - x(1)z^{-1}\right] = z^2X(z) - z^2x(0) - zx(1)$$

155. (3)

$$y(t) = x(t) * h(t)$$
$$= [\delta(t) + \delta(t-1) + \delta(t-2)] * [2u(t) - u(t-1) - u(t-2)]$$
$$= 2u(t) - u(t-1) - u(t-2) + 2u(t-1) - u(t-2) - u(t-3) + 2u(t-2) - u(t-3) - u(t-4)$$
$$y(t) = 2u(t) + u(t-1) - 2u(t-3) - u(t-4)$$



thus, $y(2) = 3$

156. 0.75 (0.70 to 0.80)

If $X(z)$ is the z-transform of $x[n]$ and the unit circle is within the region of convergence, the DTFT of $x[n]$ may be found by evaluating $X(z)$ around the unit circle.

$$X(e^{j\omega}) = X(z)\big|_{z=e^{j\omega}} \qquad \because z = e^{j\omega} = e^{j\pi} = -1$$

Therefore, the DTFT at $\omega = \pi$ is

$$X(e^{j\omega})\big|_{\omega=\pi} = X(z)\big|_{z=e^{j\pi}} = X(z)\big|_{z=-1}$$

and we have

$$X(e^{j\omega})\big|_{\omega=\pi} = \frac{z+2z^{-2}+4z^{-3}}{3-z^{-4}+6z^{-5}}\bigg|_{z=e^{j\pi}=-1}$$
$$= \frac{-1+2-4}{3-1-6} = \frac{-3}{-4}$$
$$= \frac{3}{4} = 0.75$$

157. (c)

$$x[n] = \frac{1}{N}\left[\sum_{n=0}^{N-1} X[k]e^{j(2\pi/N)nk}\right] = \frac{1}{N}\left[\sum_{n=0}^{N-1} X^*[k]e^{-j(2\pi/N)nk}\right]^*$$

$$x[n] = \frac{1}{N}\left[DFT\left\{X^*[k]\right\}\right]^*$$

$$\Rightarrow \quad \frac{1}{N^2}\left[DFT\left\{X^*[k]\right\}\right]^* = \frac{x[n]}{N} = \frac{1}{4}[4,12,4,12] = [1, 3, 1, 3]$$

158. –77.64 (–79.00 to –76.00)

now $$x[n] = x[nT_s]$$

$$= \sin\left(125\pi n \times \frac{1}{500}\right)$$

$$= \sin\left(\frac{n\pi}{4}\right)$$

and $$= H(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{\frac{1}{2} - e^{-j\omega}}{1 - \frac{1}{2}e^{-j\omega}} = H(e^{j\omega})$$

Output will be $\sin\left[\frac{n\pi}{4}\right] \cdot H\left(e^{j\pi/4}\right)$

$$\Rightarrow \quad \sin\left[\frac{n\pi}{4}\right] \cdot (1\angle 77.64)$$

$$\Rightarrow \quad \sin\left[\frac{n\pi}{4} + 77.64°\right]$$

159. (10)

Given that, $x_1[n] = 5x[n + n_0] + k$

Substituting $n \to n - n_0$, we get,

$$x_1[n - n_0] = 5x[n - n_0 + n_0] + k$$
$$x_1[n - n_0] = 5x[n] + k$$
$$\therefore \quad x[n] = 0.2[5x[n] + k] - 2$$
$$x[n] = x[n] + 0.2k - 2$$
$$\therefore \quad 0.2k = 2$$
$$k = 10$$

160. 7.283 (7.10 to 7.50)

$$H(\omega) = 1 + j\omega$$
$$|H(\omega)|^2 = 1 + \omega^2$$
$$S_X(f) = \text{CTFT}\{R_X(\tau)\} = e^{-\pi f^2}$$

Note : $e^{-\pi\tau^2} \xleftrightarrow{\text{CTFT}} e^{-\pi f^2}$

$$S_Y(f) = |H(\omega)|^2 S_X(f) = (1 + 4\pi^2 f^2) e^{-\pi f^2}$$

$$P_Y = \int_{-\infty}^{\infty} S_Y(f)df = \int_{-\infty}^{\infty} e^{-\pi f^2} df + 4\pi^2 \int_{-\infty}^{\infty} f^2 e^{-\pi f^2} df \text{ W}$$

Let, $f_Z(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{z^2}{2\sigma^2}}$ $\Rightarrow$ Zero mean Gaussian PDF with variance σ^2

When $2\sigma^2 = \frac{1}{\pi}$, $f_Z(z) = e^{-\pi z^2}$

$$\int_{-\infty}^{\infty} f_Z(z)\,dz = 1 \text{ and } \int_{-\infty}^{\infty} z^2 f_Z(z)\,dz = \sigma^2 \text{ (as zero mean)}$$

By using the above standard results,

$$\int_{-\infty}^{\infty} e^{-\pi f^2}\,df = 1$$

$$\int_{-\infty}^{\infty} f^2 e^{-\pi f^2}\,df = \frac{1}{2\pi}$$

So, $$P_Y = 1 + 4\pi^2\left(\frac{1}{2\pi}\right) = 1 + 2\pi = 7.283 \text{ W}$$

161. (d)

Given, $$X(\omega) = \frac{\omega^3}{1+\omega^2}$$

$$x(3t) \xleftrightarrow{FT} \frac{1}{3}X\left(\frac{\omega}{3}\right) \quad (\because \text{Time scaling})$$

$$x(3t-6) \xleftrightarrow{FT} e^{-j2\omega}\cdot\frac{1}{3}X\left(\frac{\omega}{3}\right) \quad (\because \text{Time shifting property})$$

$$\xleftrightarrow{FT} e^{-j2\omega}\cdot\frac{1}{3}\left[\frac{\left(\frac{\omega}{3}\right)^3}{1+\left(\frac{\omega}{3}\right)^2}\right]$$

$$x(3t-6) \xleftrightarrow{FT} \frac{e^{-j2\omega}\cdot\omega^3}{9(9+\omega^2)}$$

162. (c)

Given, $$y(t) = e^{-t}u(t) * \sum_{k=-\infty}^{\infty}\delta(t-3k)$$

Above signal $y(t)$ may be written as,

$$y(t) = \ldots + e^{-(t+6)}u(t+6) + e^{-(t+3)}u(t+3) + e^{-t}u(t) + e^{-(t-3)}u(t-3) + e^{-(t-6)}u(t-6) + \ldots$$

In the range $0 \le t < 3$, we may write $y(t)$ as

$$y(t) = \ldots + e^{-(t+6)}u(t+6) + e^{-(t+3)}u(t+3) + e^{-t}u(t)$$

$$= e^{-t} + e^{-(t+3)} + e^{-(t+6)} + \ldots..$$

$$= e^{-t}\left[1 + e^{-3} + e^{-6} + \ldots..\right]$$

$$y(t) = \frac{e^{-t}}{1-e^{-3}}$$

By comparing with given $y(t) = Ae^{-t}$; $0 \le t < 3$

$$\therefore \quad A = \frac{1}{1-e^{-3}}$$

163. (b)

$$Y(t) = X(t - T_0)$$

$X(t)$ and $Y(t)$ are uncorrelated.

So, $$C[X(t)Y(t)] = 0 \quad \text{...(i)}$$

$$C[X(t)Y(t)] = E[X(t)Y(t)] - E[X(t)]E[Y(t)]$$

$X(t)$ is a zero mean process.

So, $$E[X(t)] = 0$$

and $$C[X(t)Y(t)] = E[X(t)X(t - T_0)] = R_X(T_0) \quad \text{...(ii)}$$

From equations (i) and (ii),

$$R_X(T_0) = 0$$

164. (16)

The coefficient a_0 of the continuous Fourier series associated with the given function $f(t)$ can be computed as

$$a_0 = \left(\frac{1}{T}\right)\left[f(t)dt\right]$$

$$a_0 = \frac{1}{6}\left[\int_0^2 t^2 dt + \int_2^6 (-t+6)\,dt\right]$$

$$a_0 = \frac{1}{6}\left[\left(\frac{t^3}{3}\right)_0^2 + \left(\frac{-t^2}{2} + 6\times t\right)_2^6\right]$$

$$a_0 = 1.78 = \frac{16}{9}$$

$$\Rightarrow \quad X = 16$$

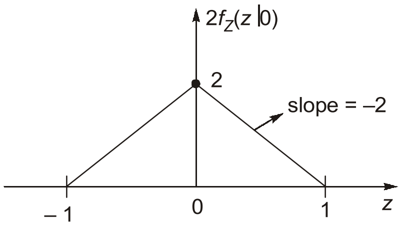
165. 0.5 (0.40 to 0.60)

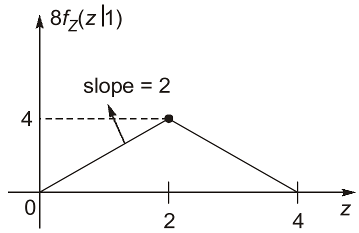
As per MAP criteria,

$$f_Z(z\,|\,0)\;P(0) \underset{H_1}{\overset{H_0}{\gtrless}} f_Z(z\,|\,1)\;P(1)$$

$$\frac{2}{10} f_Z(z\,|\,0) \underset{H_1}{\overset{H_0}{\gtrless}} \frac{8}{10} f_Z(z\,|\,1)$$

$$2f_Z(z\,|\,0) \underset{H_1}{\overset{H_0}{\gtrless}} 8f_Z(z\,|\,1)$$





at $z = (z_{th})_{\text{optimum}}$, $2f_Z(z|0) = 8f_Z(z|1)$

i.e. optimum threshold exists at intersection of $2f_Z(z|0)$ and $8f_Z(z|1)$.

$$-2z_{th} + 2 = 2z_{th}$$
$$4z_{th} = 2$$
$$z_{th} = 0.5$$

166. (d)

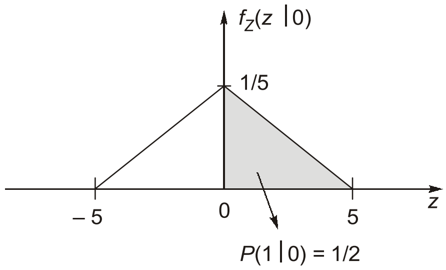
To get optimum threshold:

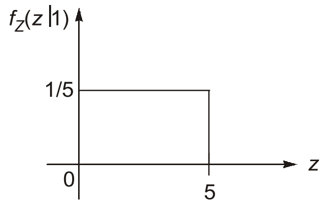
$$f_Z(z|0)\,P(0) \underset{H_1}{\overset{H_0}{\gtrless}} f_Z(z|1)\,P(1)$$

given $P(0) = 0.4$, $P(1) = 0.6$

- for $z < 0 \Rightarrow f_Z(z|1) = 0$. So, $f_Z(z|0)\,P(0) > f_Z(z|1)\,P(1)$
- for $z > 0 \Rightarrow f_Z(z|1) = \frac{1}{5}$ and $f_Z(z|0) < \frac{1}{5}$ and given that, $P(0) < P(1)$.
  So, $f_Z(z|0)\,P(0) < f_Z(z|1)\,P(1)$.
- So, optimum threshold exists at $z = 0$.
- then, probability of error is,

$$P_e = P(1|0)\,P(0) + P(0|1)\,P(1)$$





$$P(1|0) = \frac{1}{2} \text{ and } P(0|1) = 0;\ \ P(0) = 0.4 \text{ and } P(1) = 0.6$$

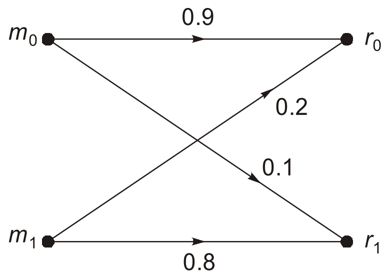
so,
$$P_e = \frac{0.4}{2} = 0.2$$

167. (5)

Minimum BW required to transmit a QPSK signal with Nyquist baseband pulse shaping is

$$(BW)_{min} = \frac{R_b}{\log_2 4} \qquad \because M = 4 \text{ for QPSK}$$
$$= \frac{10}{2}\text{MHz} = 5\text{ MHz}$$

168. (b)



$$P(m_0) = P(m_1) = 0.5$$
$$P(r_0|m_0)P(m_0) = (0.9)\ (0.5)$$
$$P(r_0|m_1)P(m_1) = (0.2)\ (0.5)$$

So, when r_0 is received decision must made in favour of m_0.

$$P(r_1|m_0)P(m_0) = (0.1)\ (0.5)$$
$$P(r_1|m_1)P(m_1) = (0.8)\ (0.5)$$

So, when r_1 is received decision must made in favour of m_1.

$$P_e = P(r_0|m_1)P(m_1) + P(r_1|m_0)P(m_0)$$
$$= (0.2)\ (0.5) + (0.1)\ (0.5)$$
$$= 0.15$$

169. 1.585 (1.57 to 1.60)

For a lossless channel,

$$H(X\,|\,Y) = 0$$

So, $$I(X\,;\,Y) = H(X) - H(X\,|\,Y) = H(X)$$

Channel capacity, $$C_S = \max_{\{P(X)\}} I(X\,;\,Y) = \max_{\{P(X)\}} H(X) = \log_2(m) \text{ bits/symbol}$$

Where, m = number of input symbols.

Given that, $m = 3$.

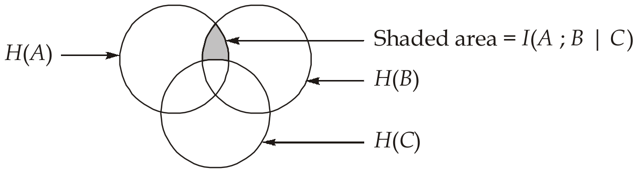
So, $$C_S = \log_2(3) = 1.585 \text{ bits/symbol}$$

170. (c)

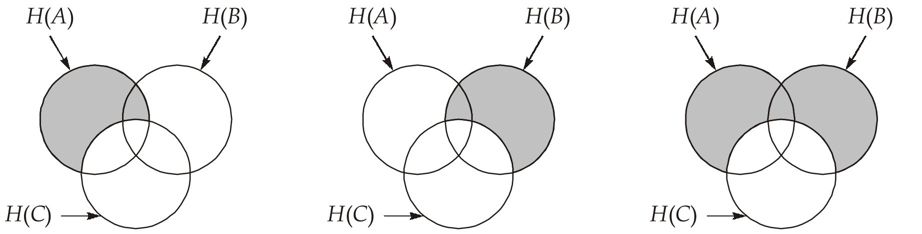
$$I(X\,;\,Y) = H(X) + H(Y) - H(X, Y)$$
$$= -\int_{-\infty}^{\infty} f_X(x)\ln(f_X(x)) - \int_{-\infty}^{\infty} f_Y(y)\ln(f_Y(y)) + \int_{-\infty}^{\infty} f_{XY}(x,y)\ln(f_{XY}(x,y))$$
$$= -E[\ln(f_X(x))] - E[\ln(f_Y(y))] + E[\ln(f_{XY}(x,y))]$$
$$= E[\ln(f_{XY}(x,y)) - \ln(f_X(x)) - \ln(f_Y(y))]$$
$$= E\left[\ln\left(\frac{f_{XY}(x,y)}{f_X(x)\,f_Y(y)}\right)\right]$$

171. (c)

Using the set theory,



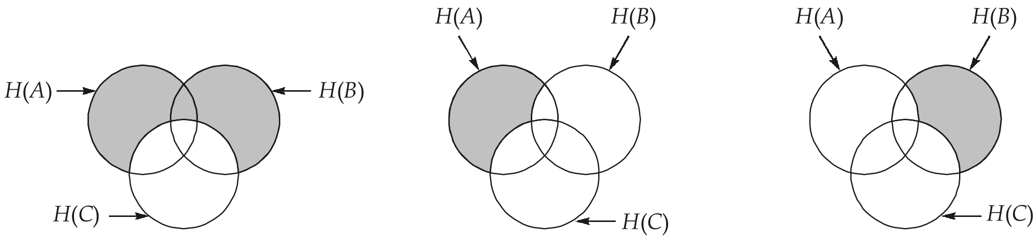
From the given relation R1:



Shaded area = $H(A\,|\,C)$ Shaded area = $H(B\,|\,C)$ Shaded area = $H(A \cdot B\,|\,C)$

$H(A\,|\,C) + H(B\,|\,C) - H(A \cdot B\,|\,C) = I(A\,;B\,|\,C)$ $\Rightarrow$ Relation R1 is correct.

From the given relation R2:



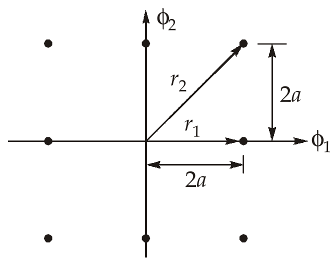
Shaded area = $H(A \cdot B\,|\,C)$ Shaded area = $H(A\,|\,B \cdot C)$ Shaded area = $H(B\,|\,A \cdot C)$

$H(A \cdot B\,|\,C) - H(A\,|\,B \cdot C) - H(B\,|\,A \cdot C) = I(A\,;B\,|\,C)$ $\Rightarrow$ Relation R2 is correct.

So, both the given relations R1 and R2 are correct.

172. (b)

For System-1:



$$r_1 = 2a$$

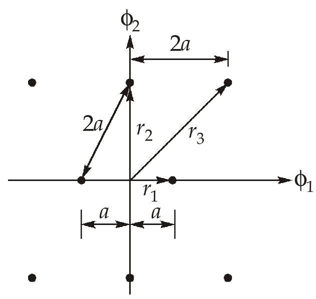
$$r_2 = \sqrt{(2a)^2 + (2a)^2} = 2\sqrt{2}\,a$$

In this system, 4 points are at a radial distance of "$2a$"

and 4 points are at a radial distance "$2\sqrt{2}\,a$", from the origin.

So, $$E_1 = \frac{4(2a)^2 + 4(2\sqrt{2}\,a)^2}{8} = 6a^2$$

For System-2:



$$r_1 = a$$

$$(2a)^2 = r_2^2 + a^2 \Rightarrow r_2 = \sqrt{3}\,a$$

$$r_3 = \sqrt{r_2^2 + (2a)^2} = \sqrt{7}\,a$$

In this system, 2 points are at a radial distance of "a", 2 points are at a radial distance of "$\sqrt{3}\,a$" and 4 points are at a radial distance of "$\sqrt{7}\,a$", from the origin.

So,
$$E_2 = \frac{2(a)^2 + 2(\sqrt{3}a)^2 + 4(\sqrt{7}a)^2}{8} = 4.5a^2$$

$$\frac{E_1}{E_2} = \frac{6a^2}{4.5a^2} = \frac{4}{3} = 1.33$$

$$E_1(\text{dB}) - E_2(\text{dB}) = 10\log_{10}\left(\frac{4}{3}\right) \approx 1.25 \text{ dB}$$

So, E_1 is larger than E_2 by 1.25 dB.

173. –1.60 (–1.65 to –1.55)

From the Shannon's channel capacity theorem,

$$C = B\log_2\left(1+\frac{S}{N}\right); \qquad S = E_bR_b \text{ and } N = N_0B$$

When the channel is ideal with infinite BW,

$$C = \frac{S}{N_0}\log_2(e)$$

For error free transmission,

$$R_b \leq C$$

$$R_b \leq \frac{S}{N_0}\log_2(e) = \frac{E_bR_b}{N_0}\log_2(e)$$

$$\frac{E_b}{N_0} \geq \log_e(2)$$

$$\left(\frac{E_b}{N_0}\right)_{\min} = \log_e(2) = 0.693$$

In decilogs, $$\left[\left(\frac{E_b}{N_0}\right)_{\min}\right]_{\text{dB}} = 10\log_{10}(0.693) \text{ dB} \approx -1.60 \text{ dB}$$

174. (a, b, d)

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}\cdot nk}$$

Considering a four point DFT

Let $x(n) = \{a, b, c, d\}$

$N = 4$

The twiddle matrix is given as

$$[X(k)] = \left[W_4^{nk}\right][x(n)]$$

Similarly again DFT of $X(k)$, will

$$[y(n)] = \frac{1}{4}\left[W_4^{nk}\right][X(k)]$$

$$[y(n)] = \frac{1}{4}\left[W_4^{nk}\right]\left[W_4^{nk}\right][x(n)]$$

$$[y(n)] = \frac{1}{4}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -j & -1 & j\\ 1 & -1 & 1 & -1\\ 1 & j & -1 & -j\end{bmatrix}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -j & -1 & j\\ 1 & -1 & 1 & -1\\ 1 & j & -1 & -j\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}$$

$$[y(n)] = \frac{1}{4}\begin{bmatrix}4 & 0 & 0 & 0\\ 0 & 0 & 0 & 4\\ 0 & 0 & 4 & 0\\ 0 & 4 & 0 & 0\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}$$

$$[y(n)] = \{a, d, c, b\}$$

$$\sum_{n=0}^{3} x(n) = a + b + c + d$$

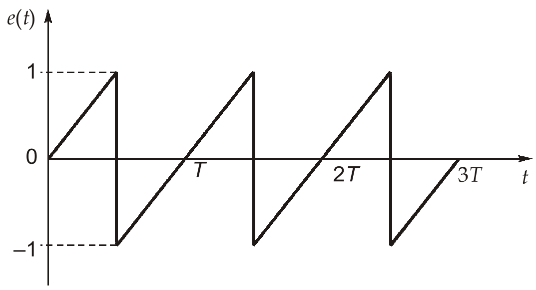
$$\sum_{n=0}^{3} y(n) = a + d + c + b$$

$$\sum_{n=0}^{3} x(n) - \sum_{n=0}^{3} y(n) = 0$$

175. (a, c, d)

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} = \frac{30-10}{30+10} = 0.5$$

Since, the message signal has DC value. Therefore, the normalized message signal is as below:



$$m(t) = \frac{2t}{T}, 0 \le t \le \frac{T}{2}$$

$$\overline{m^2(t)} = \frac{2}{T}\int_0^{T/2}\left(\frac{2t}{T}\right)^2 \cdot dt = \frac{1}{3}$$

Hence, $$\%\eta = \frac{\mu^2 \overline{m^2(t)}}{1+\mu^2 \overline{m^2(t)}} \times 100$$

$$= \frac{(0.5)^2 \times 1/3 \times 100}{1+(0.5)^2 \times 1/3} = 7.69\%$$

Here, $A_C(1 + \mu) = 30 \Rightarrow A_C = \frac{30}{1.5} = 20 \text{ V}$

Carrier power, $P_C = \frac{A_C^2}{2R} = 200 \text{ W}$

$$\eta = \frac{P_{SB}}{P_{SB} + P_C} = 0.0769$$

$$\Rightarrow \quad P_{SB} = \frac{0.0769 \times P_C}{0.9231} = 16.66 \text{ W}$$

■■■■