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Important Questions *for* GATE 2022

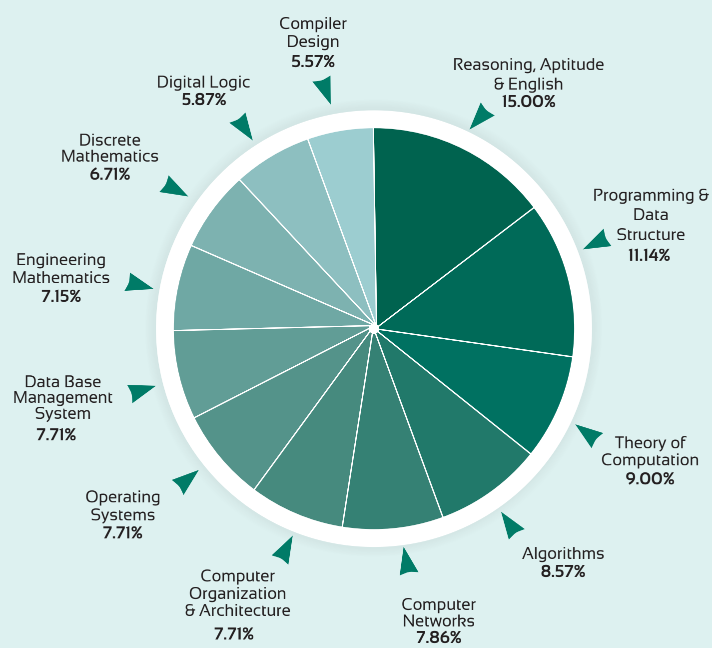
COMPUTER SCIENCE & IT

Day 8 of 8

Q.176 - Q.200 (Out of 200 Questions)

Databases + Engineering Mathematics

SUBJECT-WISE WEIGHTAGE ANALYSIS OF GATE SYLLABUS



Subject	**Average % (last 5 yrs)**
Reasoning, Aptitude & English	15.00%
Programming & Data Structure	11.14%
Theory of Computation	9.00%
Algorithms	8.57%
Computer Networks	7.86%
Operating Systems	7.71%
Computer Organization & Architecture	7.71%
Data Base Management System	7.71%
Engineering Mathematics	7.15%
Discrete Mathematics	6.71%
Digital Logic	5.87%
Compiler Design	5.57%
Total	**100%**

Databases + Engineering Mathematics

Q.176 Consider the following schedule:

$$r_1(A)\ r_2(B)\ w_1(C)\ w_3(B)\ r_3(C)\ w_2(B)\ w_3(A)$$

Which of the following time stamp ordering allows to execute schedule using Thomas write rule time stamp ordering protocol?

(a) $(T_1, T_2, T_3) = (10, 30, 20)$
(b) $(T_1, T_2, T_3) = (20, 30, 10)$
(c) $(T_1, T_2, T_3) = (30, 20, 10)$
(d) $(T_1, T_2, T_3) = (10, 20, 30)$

Q.177 Consider a table T with a key field K. A B tree of order P where P denotes the maximum number of record pointers in a B tree node. Assume K is 10 bytes long, disk block size is 512 bytes, record pointer is 8 bytes and block pointer size is 5 bytes long. In order for each B tree node to fit in a single disk block the maximum value of P.

(a) 20
(b) 22
(c) 23
(d) 32

Q.178 How many view equal serial schedules are possible for the following schedule?

$$S: w_1(A)\ r_2(A)\ w_3(A)\ r_4(A)\ w_5(A)\ r_6(A)\ w_7(A)\ r_8(A)$$

Q.179 How many minimum relations are required for the following relation $R(A, B, C, D, E,)$ with $FD\{A \to BC, CD \to E, B \to D, E \to A\}$ to convert into BCNF without violation of lossless and dependency preserving decomposition?

Q.180 Assume $R(A, B)$ and $S(C, D)$ relations have the following instances:

R

A	B
1	2
2	1
3	3

S

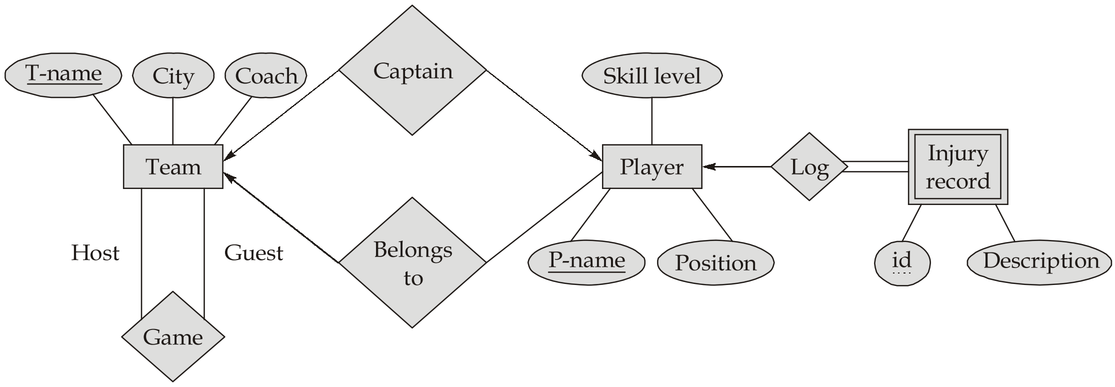
C	D
1	2
3	4
3	5

Find number of tuples returned by the following query (ρ is used to rename the attribute)

$$\pi_{AD}(R \times S) - \rho_{A \leftarrow B}\left(\pi_{BD}\left(R \bowtie_{B=C} S\right)\right)$$

Q.181 In a database file, the search key field is 9 bytes long the block size is 512 bytes, a record pointer is 6 bytes and block pointer is 7 bytes. Find the largest possible order of a non leaf node in B^+ tree implementing this file structure {order defines maximum number of keys present}

Q.182 Consider the following ER-diagram for simple database for the National Hockey League given below:



The minimum number of table required for database to be in 2NF is ________.

Q.183 Consider two relations R and S have n and m tupples respectively. Match the following expression with maximum and minimum number of tupples as result:

Expression	**Tupples**
A. $R \cup S$	**1.** max = $n \times m$, min = 0
B. $R \bowtie S$	**2.** max = $n + m$, min = max (m, n)
C. $\sigma_C(R) \times S$	**3.** max = $m \times n$, min = n
D. $\pi_A(R) - S$	**4.** max = n, min = 0

	A	B	C	D
(a)	1	2	3	4
(b)	2	1	3	4
(c)	2	1	1	4
(d)	3	2	1	4

Q.184 Consider the schedules S_1, S_2 and S_3 given below:

$S_1 : r_1(X); r_2(Z); r_1(Z); r_3(X); r_3(Y); w_1(X); c_1; w_3(Y); c_3; r_2(Y); w_2(Z); w_2(Y); c_2;$

$S_2 : r_1(X); r_2(Z); r_1(Z); r_3(X); r_3(Y); w_1(X); w_3(Y); r_2(Y); w_2(Z); w_2(Y); c_1; c_2; c_3;$

$S_3 : r_1(X); r_2(Z); r_3(X); r_1(Z); r_2(Y); r_3(Y); w_1(X); c_1; w_2(Z); w_3(Y); w_2(Y); c_3; c_2;$

Which of the following is true about above schedules?

(a) S_1 is cascadeless and strict, S_2 is not recoverable and S_3 is cascadeless but not strict.
(b) S_1 is cascadeless but not strict, S_2 and S_3 is strict.
(c) S_1 is cascadeless but not strict, S_2 is not recoverable and S_3 is cascadeless but not strict.
(d) S_1, S_2 and S_3 is strict.

Q.185 Consider the following relational schema:

Student (Sid: integer, Sname: string, address: string)
Course (Cid: integers, Cname: string, branch: string)
Enrols (Sid: integers, Cid: integer, employee: integer)

Which of the following queries are equivalent to this query in English? "Find the Sid of students who are enrolled in some courses of 'CS' branch and some courses of 'IT' branch".

1. $\rho(R_1, \pi_{sid} (\pi_{cid} (\sigma_{branch = 'CS'} (\text{Course})) \bowtie \text{Enrols}))$
$\rho(R_2, \pi_{sid} (\pi_{cid} (\sigma_{branch = 'IT'} (\text{Course})) \bowtie \text{Enrols}))$
$R_1 \cap R_2$

2. $\{T \mid \exists T_1 \in \text{enrols} (\exists x \in \text{courses} (x.\text{branch} = \text{'CS'} \wedge x.\text{cid} = T_1.\text{cid}) \wedge \exists T_2 \in \text{Enrols} (\exists y \in \text{courses}$ $(y.\text{branch} = \text{'IT'} \wedge y.\text{cid} = T_2.\text{cid}) \wedge T_2.\text{sid} = T_1.\text{sid}) \wedge T.\text{sid} = T_1.\text{sid})\}$

3.
```
Select Sid
From courses P, Enrols C
where P.branch = 'CS' AND P.cid = C.cid AND EXISTS (Select Sid
                                 From courses P2, Enrol C2
                                 where P2.branch = 'IT' AND C2.sid = C.sid
                                 AND P2.cid = C2.cid)
```

(a) Only 1 and 2
(b) Only 3 and 4
(c) Only 2 and 3
(d) All of the above

Q.186 Consider the following database table:

Emp (<u>Eid</u>, Ename, age)

Project (<u>Pid</u>, Pname, budget)

Works for (<u>Eid Pid</u>)

```
Select Eid
From Emp E
where age > 30 and not Exists   (select Pid
                From project P
                where Pname = 'database' and not exist (select Pid
                                    from works W
                                    where W.Eid = E.Eid
                                    and W.Pid = P.Pid))
```

Which of the following sets is computed by the above query retrieves employees whose

(a) Age more than 30 and works for every project with project name database.
(b) Age more than 30 and works for some project with project name database.
(c) Age more than 30 and not works for every project with project name database.
(d) Age more than 30 and not works for any project with project name database.

Q.187 Consider a disk with block size 512 bytes, pointer is P = 6 bytes long. A file has R = 300000 EMPLOYEE records of fixed-length. Each record has the following fields:

Field Name	Size (in Bytes)
NAME	30
SSN	9
DEPARTMENT CODE	9
ADDRESS	40
PHONE	9
BIRTHDATE	8
SEX	1
JOBCODE	4
SALARY	4

An additional byte is used as a deletion marker. Suppose the file is ordered by the key field SSN and we want to construct a primary index on SSN. The number of levels needed if we make it into a multi-level index is ________.

Q.188 The following key values are inserted into a B^+- tree in which order of the internal nodes is 4, and that of the leaf nodes is 3, in the sequence given below. The order of internal nodes is the maximum number of tree pointers in each node and the order of leaf nodes is the maximum number of data items that can be stored in it. The B^+-tree is initially empty.

50, 15, 30, 40, 35, 20, 8, 10, 5

The maximum number of times nodes would get split up as a result of these insertions is ________.

Q.189 Consider the transaction T_1 and T_2 given below:

T_1 : $R_1(A)\ W_1(A)\ R_1(B)\ W_1(B)$

T_2 : $R_2(A)\ W_2(A)\ R_2(B)\ W_2(B)$

Where Ri(Z) represent the read operation by transaction Ti on variable Z and Wi(Z) represent the write operation by transaction Ti on variable Z. The total number of possible conflict serializable schedules formed by T_1 and T_2 are ________.

Q.190 Consider the following factorization of a matrix A.

$$A = LU, \text{ where } L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix}_{3\times3}, \; U = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}_{3\times3} \text{ and } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$$

If $i = 1$ then $a_{ij} = j$ otherwise $a_{ij} = 3 + a_{(i-1)j}$. Find the matrix U.

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & -6 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & -6 \\ 0 & -1 & -6 \\ 0 & 0 & 0 \end{bmatrix}$

Q.191 Find an eigen vector corresponding to largest eigen value of matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

(a) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix}$

Q.192 If a fair coin is tossed until the same result turns up in succession (both head or both tail) then find the probability when the number of tosses are even. (Round upto two decimal places)

Q.193 What is the result of $\int_0^1 \log(1+x)dx$?

(a) $\log\left(\frac{2}{e}\right)$

(b) $\log\left(\frac{4}{e}\right)$

(c) $\log\left(\frac{e}{2}\right)$

(d) $\log\left(\frac{e}{4}\right)$

Q.194 What is the value of $\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$?

(a) abc
(b) $\sqrt[2]{abc}$
(c) $\sqrt[3]{abc}$
(d) $(abc)^3$

Q.195 An artillery target may be either at point 1 with probability $\frac{8}{9}$ or at point 2 with probability $\frac{1}{9}$. We have 21 shells, each of which can be fired at point 1 or point 2. Each shell may hit the target, independently of other shells, with probability $\frac{1}{2}$. If 12 shells are fired at point 1 and 9 shells are fired at point 2, what is the probability that the target is hit?

(a) $\frac{8}{9}2^{12} + \frac{1}{9}2^9$
(b) $\frac{8}{9}\left(\frac{1}{2^{12}}\right) + \frac{1}{9}\left(\frac{1}{2^9}\right)$
(c) $\frac{8}{9}\left(1 - \frac{1}{2^{12}}\right) + \frac{1}{9}\left(1 - \frac{1}{2^9}\right)$
(d) None of these

Q.196 A function $y = 7x^2 + 12x$ is defined over an open interval $x = (1, 3)$. At least at one point is this interval, $\frac{dy}{dx}$ is exactly

(a) 26
(b) 40
(c) 62
(d) 54

Q.197 Consider the following linear equations of system:

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 5z &= 10 \\ 2x + 3y + \lambda z &= \mu \end{aligned}$$

Which of the following is correct about system?

(a) System has unique solution for $\lambda \neq 6$ and $\mu \neq 16$
(b) System has no solution for $\lambda = 6$ and $\mu \neq 16$
(c) System has infinite solution for $\lambda = 6$ and $\mu = 16$
(d) All of the above

Q.198 Probability density function of a random variable X is distributed uniformly between 0 and 10. The probability that X lies between 2.5 to 7.5 and the mean square value of X are respectively

(a) $\frac{1}{2}$ and $\frac{100}{3}$
(b) 5 and 100
(c) 5 and $\frac{100}{3}$
(d) $\frac{1}{2}$ and 100

Q.199 Assume A and B are matrix of size $n \times n$, which of the following is true?
(a) If A is invertible, the $ABA^{-1} = B$.
(b) If A is an idempotent non-singular matrix, then A must be the identity matrix.
(c) If the coefficient matrix A of the system $Ax = b$ is invertible, then the system has infinitely many solution.
(d) If $AB = B$ then B is identity matrix.

Multiple Select Question (MSQ)

Q.200 Which of the following statement(s) is/are true from the following statements about Normal Forms:
(a) Lossless join and dependency preserving decomposition into 3NF is always possible.
(b) Lossless join and dependency preserving decomposition into BCNF is always possible.
(c) Any Relation with two attributes is in BCNF.
(d) BCNF is stronger than 3NF.

■■■■

Detailed Explanations

176. (d)

T_1 (10)	T_2 (20)	T_3 (30)
$r(A)$		
	$r(B)$	
$w(C)$		$w(B)$
		$r(C)$
	$w(B)$	
		$w(A)$

$W_3(B)$, $W_2(B)$ is allowed in TWR.

177. (b)

$$
\begin{aligned}
(10+8)P + (P+1)\cdot 5 &\le 512 \\
18P + 5P + 5 &\le 512 \\
23P &\le 507 \\
P &\le 22.04 \\
P &= 22
\end{aligned}
$$

178. (6)

$$\underbrace{\boxed{W_1(A)\ r_2(A)}\ \boxed{W_3(A)\ r_4(A)}\ \boxed{W_5(A)\ r_6(A)}}\ \boxed{W_7(A)\ r_8(A)}$$

These three blocks can execute in any order.

179. (3)

$R(A, B, C, D, E)$

$R_1(B, D)$	$R_2(A, B, C, E)$	$R_3(C, D, E)$
$B \to D$	$A \to BCE$	$CD \to E$
	$E \to ABC$	$E \to CD$
	$BC \to AE$	

Lossloss de composite dependency presenting BCNF

180. (6)

$\pi_{AD}(R \times S)$

A	D	
1	2	✓
1	4	
1	5	
2	2	
2	4	
2	5	
3	2	
3	4	✓
3	5	✓

$- \rho_{A \leftarrow B}$

B	D
1	2
3	4
3	5

=

A	D
1	4
1	5
2	2
2	4
2	5
3	2

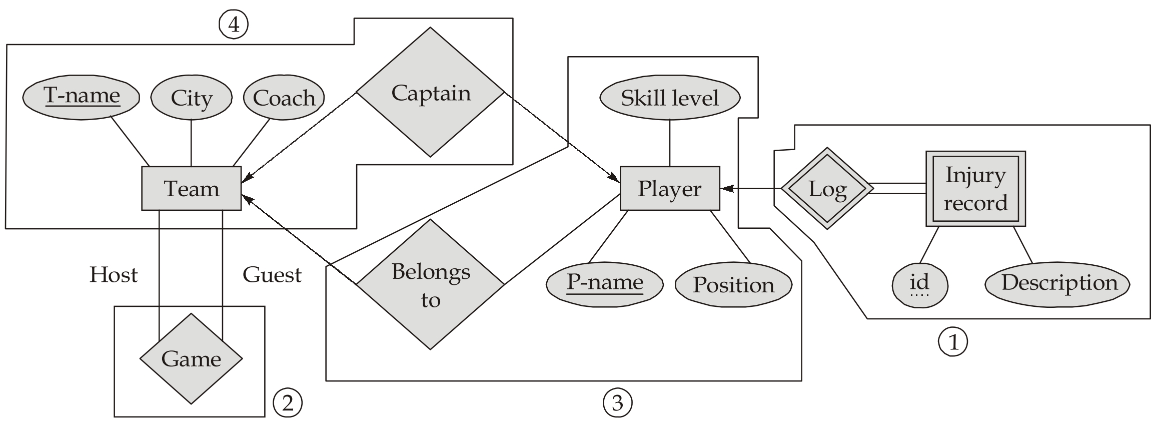
6 tuples are returned by the query.

181. (31)

Internal node in B^+ contains tree pointer and search key.

$$\begin{aligned} (P+1)\cdot 7 + P \cdot 9 &\le 512 \\ 7P + 7 + 9P &\le 512 \\ 16P &\le 505 \\ P &\le 31.5 \\ \text{order } (P) &= 31 \end{aligned}$$

182. (4)



So, minimum 4 tables are required.

183. (c)

A. $R \cup S$: maximum = $n + m$, minimum = max (m, n)

B. $R \bowtie S$: maximum = $m \times n$, minimum = 0

Maximum will be when both the tables have same attribute value then it will give $n \times m$ tuples.

C. $\sigma_C(R) \times S$: maximum = $n \times m$, minimum = 0

D. $\pi_A(R) - S$: maximum = n, minimum = 0

184. (a)

In S_1, every transaction commits right after it writes some items. There is no write to or read from an item before the last transaction that wrote that item has committed. So S_1 is strict. And cascadeless too.

In S_2, T_2 reads item Y from T_3 but T_2 commits before T_3 commits. So S_2 is non-recoverable.

S_3 is not strict because T_2 writes Y before T_3 commits. But S_3 is cascadeless because there is no transaction reads items that were written by an uncommitted transaction.

185. (d)

1. $\rho(R_1, \pi_{sid}(\pi_{cid}(\sigma_{branch = 'CS'}(\text{Course})) \bowtie \text{Enrols}))$

$\rho(R_2, \pi_{sid}(\pi_{cid}(\sigma_{branch = 'IT'}(\text{Course})) \bowtie \text{Enrols}))$

$R_1 \cap R_2$

Find the Sid who enrolled atleast one course of CS branch then find the Sid who enrolled atleast one course of IT branch. Then take inter-section both Sid.

2. $\{T \mid \exists T_1 \in \text{enrols}\ (\exists x \in \text{courses}\ (x.\text{branch} = \text{'CS'} \wedge x.\text{cid} = T_1.\text{cid}) \wedge \exists T_2 \in \text{Enrols}\ (\exists y \in \text{courses}\ (y.\text{branch} = \text{'IT'} \wedge y.\text{cid} = T_2.\text{cid}) \wedge T_2.\text{sid} = T_1.\text{sid}) \wedge T.\text{sid} = T_1.\text{sid})\}$

Find the Sid who enrolled atleast one course of CS branch then find the Sid who enrolled atleast one course of IT branch with same Sid. Then return Sid.

3.
```
Select Sid
From courses P, Enrols C
where P.branch = 'CS' AND P.cid = C.cid AND EXISTS (Select Sid
                                          From courses P2, Enrol C2
                                          where P2.branch = 'IT' AND C2.sid = C.sid
                                          AND P2.cid = C2.cid)
```

Find the Sid who enrolled atleast one course of CS branch then find the same Sid enrolled for atleast one course of IT branch and return it.

186. (a)

```
Select Eid     ⇐ employee age max than 30
From Emp E
where age > 30 and
      not Exists  (select Pid     ⇐ all the project id whose project name is database
                   From project P
                   where Pname = 'database' and
                            not exist (select Pid ⇐ the P.id where Eid is not in work relation
                                       from works W
                                       where W.Eid = E.Eid
                                       and W.Pid = P.Pid))
```


187. (3)

$$\text{Record length R} = (30 + 9 + 9 + 40 + 9 + 8 + 1 + 4 + 4) + 1 = 115 \text{ bytes}$$

$$\text{Blocking factor bf} = \text{Floor (B/R)} = \text{Floor}\left(\frac{512}{115}\right) = 4 \text{ records per block}$$

$$\text{Number of blocks needed for file} = \text{Ceiling(r/bf)} = \text{Ceiling}\left(\frac{30000}{4}\right) = 7500$$

$$\text{Index entry size} = (\text{VSSN} + \text{P})$$
$$= (9 + 6) = 15 \text{ bytes}$$

$$\text{Index blocking factor} = \text{floor (B/Index record size)} = \text{floor}\left(\frac{512}{15}\right) = 34$$

Number of first-level index entries R1 = Number of file blocks b = 7500 entries
Number of first-level index blocks B1 = Ceiling(R1/Index blocking factor)

$$= \text{Ceiling}\left(\frac{7500}{34}\right) = 221 \text{ blocks}$$

Number of second-level index entries R2 = Number of first-level blocks B1 = 221 entries
Number of second-level index blocks B2 = Ceiling(R2/Index blocking factor)

$$= \text{Ceiling}\left(\frac{221}{34}\right) = 7 \text{ blocks}$$

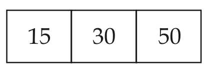
Number of third-level index entries R3 = Number of second-level index blocks B2 = 7 entries
Number of third-level index blocks B3 = Ceiling(R3/Index blocking factor)

$$= \text{Ceiling}\left(\frac{7}{34}\right) = 1$$

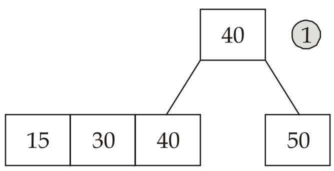
Since the third level has only one block, it is the top index level.
So 3 levels are required.

188. (8)

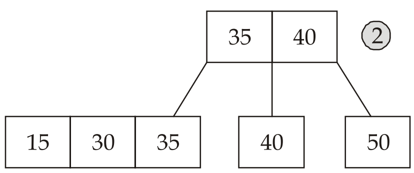
1. On insertion of 50, 15, 30



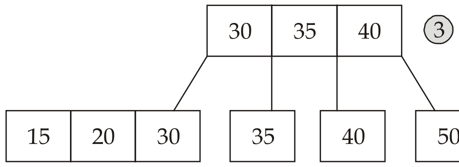
2. On insertion 40



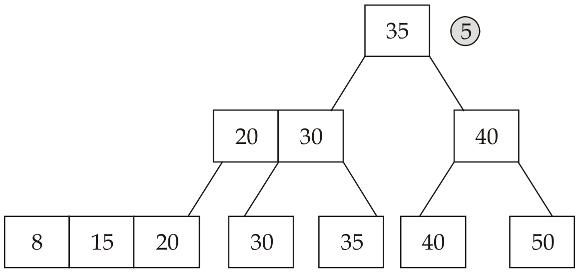
3. On insertion 35



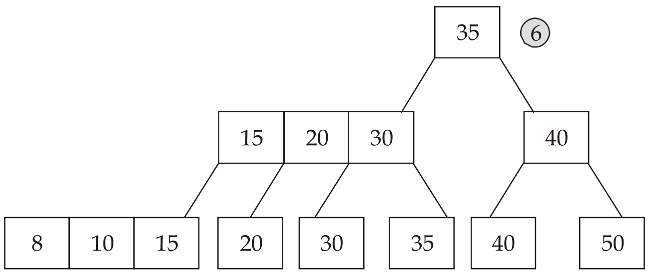
4. On insertion 20



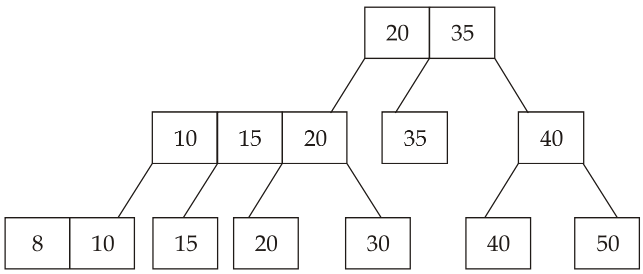
5. On insertion 8



6. On insertion 10



7. On insertion 5
Two more times nodes going to split.
So $6 + 2 = 8$ times



189. **(12)**

Conflict-equivalent to $T_1 \rightarrow T_2$:

T_1	T_2
R(A) W(A)	
	R(B) W(B)

Remaining 4 transactions can be arranged in any possible order.

Number of possibilities: $\dfrac{4!}{2! \times 2!} = 6$

Conflict-equivalent to $T_2 \rightarrow T_1$:

T_1	T_2
	R(A) W(A)
R(B) W(B)	

Remaining 4 transactions can be arranged in any possible order.

Number of possibilities: $\dfrac{4!}{2! \times 2!} = 6$

Total number of possibilities 6 + 6 = 12

190. (c)

$$A = LU$$

Given $$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix}$$

$$\therefore \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$a_{ij} = j, \text{ if } i = 1$$
$$= 3 + a_{(i-1)j}; \text{ otherwise}$$

$$\therefore \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$A = LU$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

(*i*) $a = 1$, (*ii*) $b = 2$ (*iii*) $c = 3$

(*iv*) $4b + d = 5 \Rightarrow 4 \times 2 + d = 5 \Rightarrow d = -3$

(*v*) $4c + e = 6 \Rightarrow 4 \times 3 + e = 6 \Rightarrow e = -6$

(*vi*) $7c + 2e + f = 9 \Rightarrow 7 \times 3 + 2 \times (-6) + f = 9 \Rightarrow f = 0$

$$\therefore \quad U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

191. (a)

$$|\lambda - AI| = (1 - \lambda)(\lambda^2 - 2) + (2 - \lambda) - \lambda$$
$$= -\lambda^3 + \lambda^2$$
$$\Rightarrow \quad -\lambda^3 + \lambda^2 = 0$$
$$\Rightarrow \quad -\lambda^2(\lambda - 1) = 0$$
$$\lambda = 0, \lambda = 1$$

The largest eigen value is 1

$$A - I = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_1 \leftrightarrow R_2}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_3 \leftarrow R_2 + R_1}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}_{R_3 \leftarrow R_3 - R_2}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}_{R_1 \leftarrow R_1 - 2R_2}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[A - I]\vec{x} = 0$$

$$x_1 - x_3 = 0 \Rightarrow x_1 = x_3$$

$$-x_2 + x_3 = 0 \Rightarrow x_2 = x_3$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x^3$$

$\therefore$ $x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigen vector

192. (0.66) [0.65 to 0.67]

The probability computed for the last two tosses are either head or tail

$$P(n = \text{even}) = 2[P(n = 2) + P(n = 4) + P(n = 6) + \ldots]$$

$$= 2\left[\frac{1}{2}\cdot\frac{1}{2} + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^6 + \ldots\right]$$

$$= 2\left[\frac{1}{2^2}\left[1 + \frac{1}{4} + \frac{1}{4^2} + \ldots\right]\right]$$

$$= \frac{1}{2}\left[\frac{1}{1 - \frac{1}{4}}\right] \qquad (\because \text{ Use geometric progression})$$

$$= \frac{1}{2}\cdot\left(\frac{4}{3}\right) = \frac{2}{3}$$

193. (b)

$$\int_0^1 \log(1+x)dx$$

Using integration by parts:

$$= \int_0^1 \log(1+x)\cdot 1\,dx$$

$$= \left|\log(1+x)\cdot x\right|_0^1 - \int_0^1 \frac{1}{1+x}\cdot x\,dx$$

$$= \log(2) - \int_0^1 \frac{x}{1+x}dx$$

$$= \log(2) - \int_0^1 \frac{1+x-1}{1+x}dx$$

$$= \log(2) - \int_0^1 1dx + \int_0^1 \frac{dx}{1+x}$$

$$= \log(2) - \left|x\right|_0^1 + \left|\log(1+x)\right|_0^1$$

$$= \log(2) - 1 + \log(2)$$

$$= \log(2^2) - 1$$

$$= \log(2^2) - \log_e(e)$$

$$= \log\left(\frac{4}{e}\right)$$

194. (c)

$$\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$$

$$= \lim_{x\to 0}\left(\frac{3+a^x+b^x+c^x-3}{3}\right)^{1/x}$$

$$= \lim_{x\to 0}\left(1+\frac{a^x+b^x+c^x-3}{3}\right)^{1/x}$$

$$= \lim_{x\to 0}\left(1+\frac{(a^x-1)+(b^x-1)+(c^x-1)}{3}\right)^{1/x}$$

We know that:

$$\lim_{x\to 0}(1+\lambda x)^{1/x} = e^{\lambda}$$

$$= e^{\lim\limits_{x\to 0} \frac{(a^x-1)}{3x}+\frac{(b^x-1)}{3x}+\frac{(c^x-1)}{3x}}$$

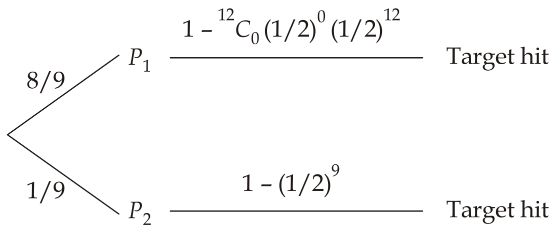
$$= e^{\lim\limits_{x\to 0} \frac{1}{3}\left(\frac{a^x-1}{x}+\frac{b^x-1}{x}+\frac{c^x-1}{x}\right)}$$

$$= e^{1/3\ (\log a\ +\ \log b\ +\ \log c)} \qquad \left[\because\ \lim_{x\to 0}\frac{a^x-1}{x} = \log a\right]$$

$$= e^{1/3 \log (abc)} = e^{\log (abc)^{1/3}} = (abc)^{1/3}$$

$$= \sqrt[3]{abc}$$

195. (c)



$$P(\text{Target hit}) = \frac{8}{9}\left(1-\frac{1}{2^{12}}\right)+\frac{1}{9}\left(1-\frac{1}{2^9}\right)$$

So option (c) is correct answer.

196. (b)

$$y = 7x^2 + 12x$$

Using Lagrange's mean value theorem:

At $x = 1, y = 7 + 12 = 19$

$x = 3, y = 63 + 36 = 99$

$$f'(x) = \frac{f(b)-f(a)}{b-a} = \frac{99-19}{3-1} = 40$$

So option (b) is correct answer.

197. (d)

$$AX = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 2 & 3 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$C = (A, B) = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 5 & : & 10 \\ 2 & 3 & \lambda & : & \mu \end{bmatrix}$$

After performing $R_2 \leftarrow R_2 - R_1$ and $R_3 \leftarrow R_3 - 2R_1$

$$C = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 4 & : & 4 \\ 0 & 1 & \lambda-2 & : & \mu-12 \end{bmatrix}$$

After performing $R_3 \leftarrow R_3 - R_2$

$$C = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 4 & : & 4 \\ 0 & 0 & \lambda-6 & : & \mu-16 \end{bmatrix}$$

Since, $n = R(A) = R(C)$ for unique solution

So $\lambda - 6 \neq 0$, $\lambda \neq 6$, $\mu - 16 \neq 0$, $\mu \neq 16$

For no solution $R(A) \neq R(C)$ then $R(A) = 2$ and $R(C) = 3$

$$\lambda - 6 = 0$$

$\Rightarrow$ $\lambda = 6$ and $\mu - 16 \neq 0 \Rightarrow \mu \neq 16$

For infinite solution $R(A) = R(C) = 2$

then $\lambda - 6 = 0$ and $\mu - 16 = 0$

$\lambda = 6$ and $\mu = 16$

So all of options are true.

198. (a)

In uniform distribution $[a, b]$

$$k = \frac{1}{b-a}$$

$$= \frac{1}{10-0} = \frac{1}{10}$$

$$P(2.5 \le X \le 7.5) = \int_{2.5}^{7.5} \frac{1}{10} dx = \frac{1}{10} x \Big|_{2.5}^{7.5} = \frac{1}{10}(7.5-2.5) = \frac{1}{2}$$

$$E(x^2) = \text{Mean square value} = \int_0^{10} x^2 f(x) dx$$

$$\int_0^{10} \frac{1}{10} x^2 dx = \frac{1}{10}\frac{x^3}{3}\Big|_0^{10} = \frac{10^3 - 0^3}{30} = \frac{1000}{30} = \frac{100}{3}$$

199. **(b)**

- $ABA^{-1} = B$ given,

$\Rightarrow$ $AB = BA$ since matrix multiplication is not commutative. So false even if A is invertible.

- A is idempotent, so $A^2 = A$, since A is non-singular, so it is invertible i.e. A^{-1} exist.

$$I = A^{-1} . A = A^{-1} . A^2 = IA = A$$

So A must be identity matrix. So true.

- If coefficient matrix A is invertible for $Ax = b$ then $x = A^{-1}$ unique solution exist. So false
- If B is zero matrix, then also $AB = B$ = zero matrix. So false

200. **(a, c, d)**

- Lossless join and dependency preserving decomposition into 3NF is always possible. **True**
- Lossless join and dependency preserving decomposition into BCNF is always possible. **False** Not always possible.
- Any Relation with two attributes is in BCNF. **True**
- BCNF is stronger than 3NF. **True**

■■■■