UPPSC-AE 2020

Uttar Pradesh
Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Electrical Engineering

Networks and Systems

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

MADE EASY Publications

Networks and Systems

Contents

CHAPTER 1

Basic Concepts

1.1 Introduction

Electric circuit theory is the most fundamental branch of electrical engineering. Other branches such as power systems, electric machines, communications, control system and instrumentation are based on electric circuit theory. Thus it is very essential to go through the basic concepts of circuit theory to understand the electrical engineering.

We commence our study by knowing some basic concepts include charge, current, voltage, circuit elements, power and energy.

1.2 Basic Terminology

1.2.1 Charge

- Charge is an electrical property of the atomic particles of which matter consists, measured in Coulombs (C).
- According to experimental observations, the only charges that occur in nature are integral multiple of the electronic charge ($e = -1.602 \times 10^{-19}$ C).
- The Coulomb is a large unit for charges. In 1C of charge, therefore

$$\frac{1}{(1.602 \times 10^{-19})} = 6.24 \times 10^{18} \text{ electrons}$$

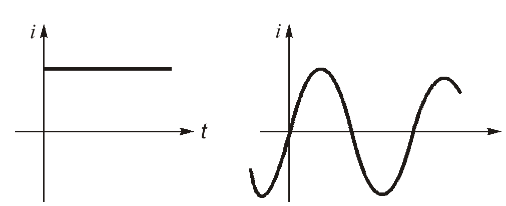
- The law of conservation of charge states that a charge can neither be created nor be destroyed, can be only transferred.

1.2.2 Current

- The phenomenon of transferring charge from one point in a circuit to another is described by the term electric current. An electric current may be defined as the time rate of net motion of electric charge across a cross-sectional boundary. A random motion of electrons in a metal does not constitute a current unless there is a net transfer of charge with time. In equation form, the current is,

$$\boxed{i = \frac{dq}{dt}}$$

- If the charge q is given in coulombs and the time 't' is measured in seconds, then current is measured in Amperes.
- A current that is constant in time is termed as direct current, or simply dc, and is shown in Fig. (a).
- A current that vary sinusoidally with time is often referred as alternating current as shown in Fig. (b).



(a) (b)

Example - 1.1 The charge in a capacitor is given by

$$q = \left(v + \frac{1}{3}v^3\right)$$

If the voltage across this capacitor be $v(t)$ = sin$t$, the current $i(t)$ through the capacitor is

(a) $(1 + \sin^2 t)\cos t$ (b) $(1 + \sin^2 t)$

(c) $(1 + \cos^2 t)\sin t$ (d) $\sin^2 t \cos t$

Solution : (a)

The current through the capacitor is, $$i = \frac{dq}{dt} = \frac{dq}{dv}\cdot\frac{dv}{dt}$$

Now, $$\frac{dq}{dv} = (1 + v^2) \text{ and } \frac{dv}{dt} = \cos t$$

$\therefore$ $$i = (1 + v^2)\cdot\cos t$$
$$= (1 + \sin^2 t)\cdot\cos t$$

1.2.3 Voltage

- To move the electrons from one point to other point in particular direction external force is required. In analytical circuit external force is provided by emf and it is given by,

$$\boxed{v = \frac{dw}{dq}}$$

where a differential amount of charge dq is given with a differential increase in energy dw. The quantity "energy per unit charge" or identically, "work per unit charge", is given the name voltage. Thus, the voltage across a terminal pair is a measure of the work required to move the charge through the element.

- A voltage can exist between a pair of electrical terminals whether a current is flowing or not. An automobile battery, for example, has a voltage of 12 V across its terminals even if nothing whatsoever is connected to the terminals.

1.2.4 Power

- If potential is multiplied by the current, dq/dt, as

$$\boxed{\frac{dw}{dq}\times\frac{dq}{dt} = \frac{dw}{dt} = p}$$

the result is seen as to be a time rate of change of energy, which is power 'p'. Thus power is the product of potential and current,

$$\boxed{p = vi = \frac{v^2}{R} = i^2R}, \quad \text{where } R \rightarrow \text{resistance}$$

- It is measured in Watt (W).

1.2.5 Energy

- The capacity to do the work is called as energy. Energy as a function of power is found by integrating equation (A). Thus total energy at time 't' is the integral

$$\boxed{w = \int_{-\infty}^{t} p\,dt}$$

- The change in energy from time t_1 to time t_2 may similarly be found by integrating from t_1 to t_2.
- It is measured in Joules or Watt-hours (Wh)

$$1 \text{ Wh} = 3600 \text{ J}$$

1.2.6 Electric Circuits and Network

- An electric circuit is an interconnection of electrical elements.
- Network is any possible inter-connection of circuit elements or branches. Circuit is a closed energized network.

1.3 Circuit Elements

- Any individual circuit component (inductor, resistor, capacitor, generator etc.) with two terminals, by which it can be connected to other electrical components.
- If the voltage across the element is linearly proportional to the current through it, then element is called as a **resistor**.
- If the terminal voltage is proportional to *derivative of current* with respect to time, then element is called as a **inductor**.
- If the terminal voltage is proportional to *integral of current* with respect to time, then element is called as a **capacitor**.
- If the terminal voltage is completely independent of current, or the current is completely independent of voltage, then element is called as an **independent source**.
- The element for which either the voltage or current depend upon a current or voltage elsewhere in the circuit; such elements are called as **dependent source**.

1.4 Classification of Circuit Elements

1.4.1 Active and Passive Elements

- If we have a network element that is absorbing power i.e. energy delivered to the element $\left(\int_{-\infty}^{t} v(t)i(t)\,dt\right)$ is positive, then the element is *passive element*. Eg: resistor, inductor, diode and capacitor.
- If we have a network element that is delivering power i.e. energy delivered to the element $\left(\int_{-\infty}^{t} v(t)i(t)\,dt\right)$ is negative, then the element is *active element*. Eg: Independent sources, transistor and op-amp.

NOTE: An active element can provide power or power gain to the circuit for infinite duration of time, that is why charged capacitor or inductor are not active elements.

1.4.2 Bilateral and Unilateral Elements

- For a *bilateral element*, the voltage current relationship is the same for current flowing in either direction. Eg: resistor, inductor and capacitor.
- For a *unilateral element*, the voltage current relationship is different for two directions of current flow. Eg: diode.

1.4.3 Lumped and Distributed Elements

- *Lumped elements* are considered as the separate elements which are very small in size. For Eg: resistor, inductor and capacitor.
- *Distributed elements* are not electrically separable. These are distributed over the entire length of the circuit. Eg: Transmission lines.

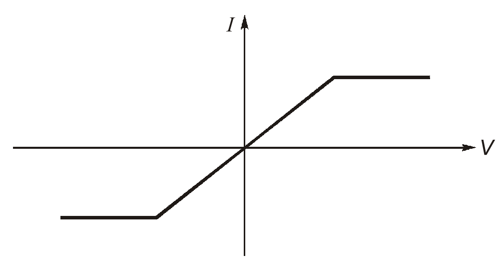
1.4.4 Linear and Non-Linear Elements

- An element that follows *additivity* and *homogeneity property* for relationship between excitation and response is called a *linear element*.
- An element that does not follow *additivity* and *homogeneity property* for relationship between excitation and response is called a *non-linear element*.

NOTE

- The *homogeneity property* requires that if excitation is multiplied by a constant, then the response gets multiplied by the same constant.
- The *additivity property* requires that the response to a sum of inputs is the sum of the responses to each input applied separately.

Example - 1.2 The following v-i characteristic of an element is shown below. The element is



(a) Non-linear, unidirectional, passive
(b) Linear, bidirectional, active
(c) Non-linear, bidirectional, passive
(d) Non-linear, unidirectional, active

Solution : (c)

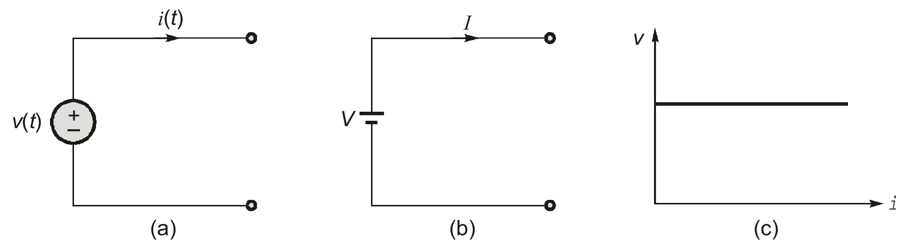
- As characteristics is similar in opposite quadrants, then the element is bidirectional.
- Element is passive as ratio of V/I is not negative at any point on characteristics curve.

1.5 Sources

- Sources are classified as voltage sources and current sources, and further as independent and dependent sources.

1.5.1 Independent Voltage Source

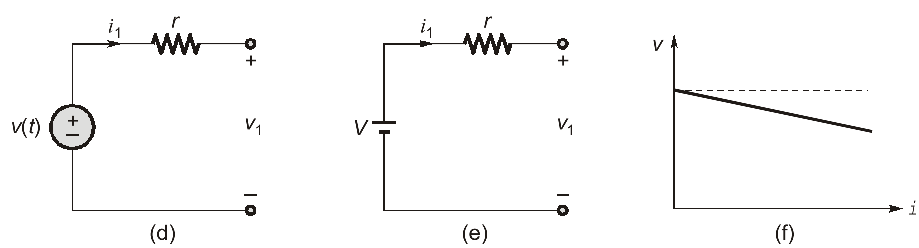
- An ideal voltage source is two-terminal element which maintains a terminal voltage v(t) regardless of the value of the current through its terminals; as shown in figure (a) and (b). Internal resistance of an ideal voltage source is equal to zero.



(Ideal voltage source and v-i characteristics)

- In a practical voltage source, the voltage across the terminals of the source keeps falling as the current through it increases, as shown in figure. This behaviour can be explained by putting a resistance in series with an ideal voltage source, as in figure (d). Then we have the terminal voltage v_1 as

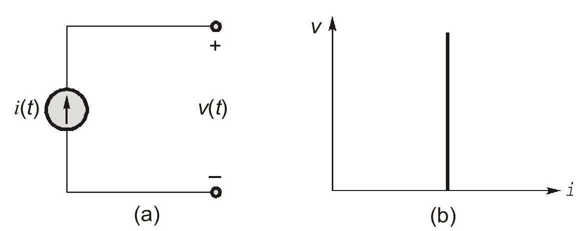
$$\boxed{v_1 = v - i_1 r}$$



(Practical voltage source and v-i characteristics)

1.5.2 Independent Current Source

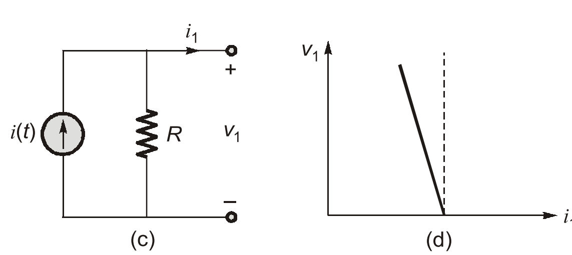
- An ideal current source is a two-terminal element which maintains a current $i(t)$ flowing through its terminals regardless of the value of the terminal voltage as shown in figure (a). The internal resistance of ideal current source is infinite.



(Ideal current source and v-i characteristics)

- In a practical current source, the current through the source decreases as the voltage across it increases, as shown in figure (d). This behaviour can be explained by putting a resistance across the terminals of the source, as in figure (c). Then the terminal current is given by,

$$\boxed{i_1 = i - \frac{v_1}{R}}$$



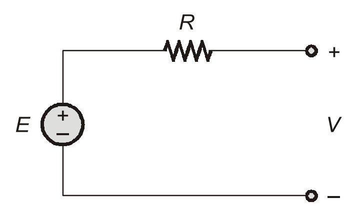
(Practical current source and v-i characteristics)

Example - 1.3 The internal voltage drop of a voltage source

(a) is the highest when no load is applied
(b) does not influence the terminal voltage
(c) depends upon the internal resistance of the source
(d) decreases with increase load current

Solution: (c)

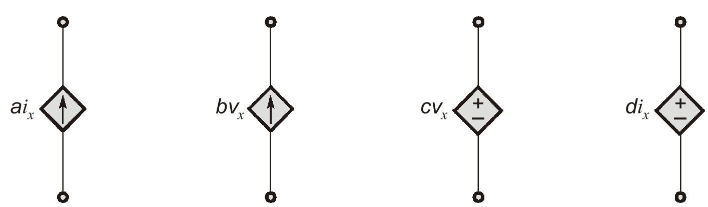
Internal voltage drop depends upon internal resistance,



Internal voltage drop, $\Delta V = E - V$ and $\Delta V = iR$, if load is present.

1.5.3 Dependent Sources

- The two types of sources that we have discussed up to now are called independent sources because the value of the source quantity is not affected in any way by activities in the remainder of the circuit.



(a) Current-controlled current source (b) Voltage-controlled current source (c) Voltage-controlled voltage source (d) Current-controlled voltage source

(The four different types of dependent sources)

- This is in contrast with yet another kind of ideal source, the dependent, or controlled source, in which the source quantity is determined by a voltage or current existing at some other location in the system being analyzed.

NOTE

- Diamond shape is used for symbol to represent dependent sources.
- A dependent source may absorb or supply power.

1.6 Standard Input Signals

- Some of the standard signals used as excitation in electrical networks. These are:

1.6.1 Step Signal

- The step is a signal whose value changes from one level (usually zero) to another level A in zero time.

- The mathematical representation of the step function is

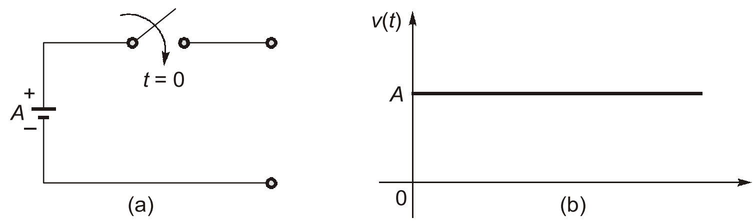
$$v(t) = A\,u(t)$$

where, $u(t) = 1\ ;\quad t > 0$

$\qquad\quad = 0\ ;\quad t < 0$

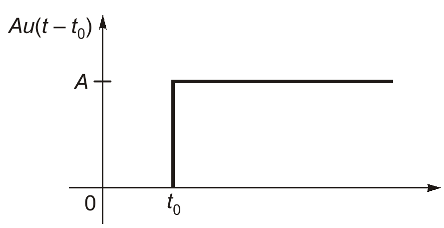
$Au(t)$ is a step of magnitude A.

- Realization of step voltage [in figure (a)] and the graphical representation of a step signal [in figure (b)] are shown below:



- A shifted or delayed step function as shown below can be expressed as:

$$Au(t - t_0) = 0\ ;\quad t < t_0$$
$$= A\ ;\quad t > t_0$$

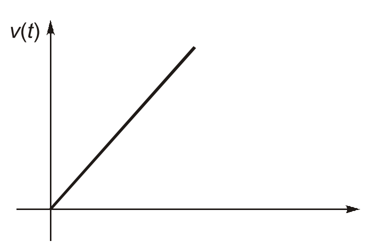


1.6.2 Ramp Signal

- The ramp is a signal which starts at a value of zero and increases linearly with time. Mathematically,

$$v(t) = At\ ;\quad t > 0$$
$$= 0\ ;\quad t < 0$$

where A is the slope of ramp function. The graphical representation of a ramp signal is shown below in the figure.



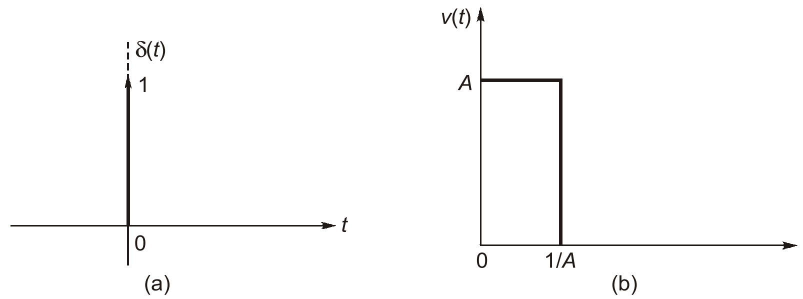
1.6.3 Impulse Signal

- A unit-impulse is defined as a signal which has zero value everywhere except at $t = 0$, where its magnitude is infinite. It is generally called the δ-function or the dirac-delta function and has the following property

$$\delta(t) = 0\ ;\ t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

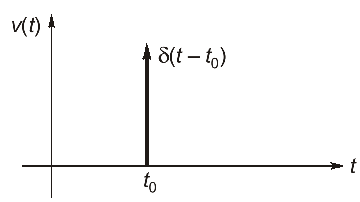
- Since a perfect impulse can not be achieved in practice, it is usually approximated by a pulse of small width but unit area as shown in the figure (b).



(Unit impulse signal)

- A unit impulse function occurring at $t = t_0$ is defined as

$$\delta(t-t_0) = \begin{cases} 0, & t \neq t_0 \\ \infty, & t = t_0 \end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(t-t_0)\, dt = 1$$



(Delayed unit impulse function)

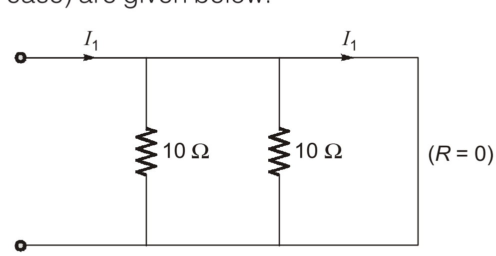
1.7 Resistance

- The physical property of a material by virtue of which it opposes the flow of electrons through the material is known as resistance. Resistance is denoted by 'R' or 'r' and unit is ohm(Ω). Resistance is given as

$$\boxed{R = \frac{\rho l}{a}}$$

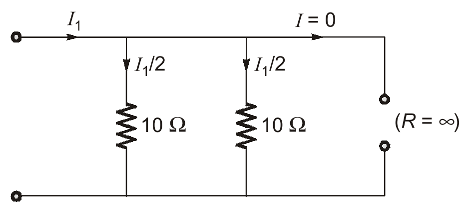
where, ρ........is resistivity of material (reciprocal of conductivity)
l.........length of material
a........area of cross-section of material

- Resistance converts electrical energy into heat energy.
- When circuit resistance is approaching to zero then the circuit is called as short circuit. Properties of short circuit (ideal case) are given below:



$$\boxed{\begin{matrix} R = 0 \\ V = 0 \end{matrix}}$$

- When circuit resistance is approaching to infinite then the circuit is called as open circuit. Properties of open circuit (ideal case) are given below:



$$\boxed{\begin{matrix} I = 0 \\ R = \infty \end{matrix}}$$

- Resistivity depends on
  (a) Composition of the material of the conductor.
  (b) Temperature.

NOTE

- Resistivity of an alloy is generally higher than that of its constituent metals.
- Resistivity increases with increase in temperature of the metal.
- Resistivity is independent of the conductor dimension.

Example - 1.4 For a fixed supply voltage, the current flowing through a conductor will increase when its
(a) area of cross-section is reduced
(b) length is reduced
(c) length is increased
(d) length is increased and area of cross-section is reduced

Solution : (b)

$$I \propto \frac{1}{R} \propto \frac{A}{\rho l} \propto \frac{A}{l}$$

When l is reduced. I will be increased and vice-versa.

1.8 Inductance

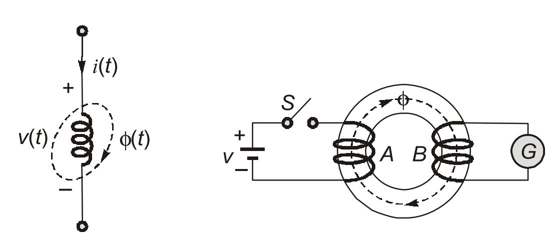
- If the current 'i' flowing in an element of figure (a) changes with time, the magnetic flux 'ϕ' produced by the current also changes, which causes a voltage to be induced in the circuit, equal to the rate of flux linkages. That is,

$$v = \frac{d\phi}{dt}$$

Now, $$v \propto \frac{di}{dt}$$

i.e. $$\boxed{v = L\frac{di}{dt}}$$

where, L is constant of proportionality and is called self inductance.



(a) (b)

(a) Inductive circuit, (b) Mutually coupled circuit

- In the circuit of figure (b),

  N = number of turns in coil B

  e = induced emf in coil B

  ϕ = magnetic flux in Webers

  The changing magnetic flux ϕ in the core (ϕ changes whenever current i changes through the coil A) passes through (cuts) coil B. Therefore, there is an induced emf in coil B, which is equal to

$$e = N\left(\frac{d\phi}{dt}\right)$$

  Again, the induced emf is proportional to di/dt i.e.

$$e = L\left(\frac{di}{dt}\right)$$

  as (di/dt) is directly proportional to $(d\phi/dt)$.

  Comparison of these two equations gives,

$$\boxed{N\phi = Li}$$

- In this context, it may be noted that an inductor is a device, while inductance is the quantity L.

Now,
$$\boxed{L = \frac{N\phi}{i}}$$

The magnetic flux,
$$\phi = \frac{\text{MMF}}{\text{Magnetic reluctance}}$$

(where MMF, the magnetomotive force = Ni)

Magnetic reluctance $\propto$ length of the coil

$$\propto \left(\frac{1}{\text{Cross-sectional area A}}\right)$$

So, Magnetic reluctance $= \dfrac{l}{\mu A}$

μ is the magnetic permeability and is a property of the material of the core.

Now,
$$\phi = \frac{Ni}{l/\mu A} = \frac{Ni\mu A}{l}$$

and
$$\boxed{L = \frac{N\phi}{i} = \frac{N^2\mu A}{l}} \text{(Henry)}$$

NOTE

- The unit of magnetic permeability (μ) is Henry per meter.
- $\mu = \mu_o \mu_r$, where μ_r is relative permeability of material.
- $\mu_o = 4\pi \times 10^{-7}$ Henry/meter.

Example - 1.5 A voltage waveform $v(t) = 12t^2$ is applied across 1 H inductor for $t \geq 0$, with initial current through it being zero. The current through inductor for $t = 0$ will be

(a) $4t^3$ (b) $12t^3$

(c) $24t$ (d) $12t$ [UPPSC]

Solution : (a)

Given,

$$v(t) = 12t^2\text{ V}; \quad L = 1\text{ H}$$

$$i_L(0^-) = 0\text{ A}$$

$\because$

$$i_L(t) = \frac{1}{L}\int_0^t v(t)\,dt + i_L(0^-) = \frac{1}{1}\int_0^t (12t^2)\,dt = \frac{12}{3}[t^3]_0^t = 4t^3$$

1.8.1 Voltage Current Relationship

- We have defined inductance by a simple differential equation,

$$v = L\frac{di}{dt}$$

or

$$di = \frac{1}{L}v\,dt$$

- If we desire the current i at time t and merely assume that the current is $i(t_0)$ at time t_0,

$$\int_{i(t_0)}^{i(t)} di' = \frac{1}{L}\int_{t_0}^{t} v(t')\,dt'$$

$\Rightarrow$

$$i(t) - i(t_0) = \frac{1}{L}\int_{t_0}^{t} v\,dt'$$

$\therefore$

$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt' + i(t_0)}$$

1.8.2 Energy Storage

- The absorbed power is given by the current voltage product

$$p = vi = Li\frac{di}{dt}$$

- The energy W_L accepted by the inductor is stored in the magnetic field around the coil. The change in this energy is expressed by the integral of the power over the desired time interval:

$$\int_{t_0}^{t} p\,dt' = L\int_{t_0}^{t} i\frac{di}{dt'}\,dt' = L\int_{i(t_0)}^{i(t)} i'\,dt'$$

$$= \frac{1}{2}L\{[i(t)]^2 - [i(t_0)]^2\}$$

Thus,

$$W_L(t) - W_L(t_0) = \frac{1}{2}L\{[i(t)]^2 - [i(t_0)]^2\}$$

where, we have again assumed that the current is $i(t_0)$ at time t_0. In using the energy expression, it is customary to assume that a value of t_0 is selected at which the current is zero; it is also customary to assume that the energy is zero at this time. We then have simply,

$$\boxed{W_L(t) = \frac{1}{2}L\,i^2}$$

Example - 1.6 At a particular instant an inductance of 1 H carries a current of 2 A while the voltage across it is 1 V. The energy stored in the inductance in Joules is

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{3}{2}$ (d) 2 [UPPSC]

Solution : (d)

Given, $L = 1\text{ H}, \quad I = 2\text{ A}$

Energy stored in an inductor is, $E = \frac{1}{2}L(I)^2 = \frac{1}{2}\times(1)(2)^2 = 2$ Joules

1.8.3 Characteristics of an Ideal Inductor

- There is no voltage across an inductor if the current through it is not changing with time. An inductor is therefore a short circuit to dc.
- A finite amount of energy can be stored in an inductor even if the voltage across the inductor is zero, such as when the current through it is constant.
- It is impossible to change the current through an inductor by a finite amount in zero time, for this requires an infinite voltage across the inductor. An inductor resists an abrupt change in the current through it in a manner analogous to the way a mass resists an abrupt change in its velocity.
- The inductor never dissipates energy, but only stores it. Although this is true for the mathematical model, it is not true for a physical inductor due to series resistances.

1.9 Capacitance

- It is the capability of an element to store electric charge within it. A capacitor stores electric energy in the form of electric field being established by the two polarities of charges on the two electrodes of a capacitor. Quantitatively capacitance is measure of charge per unit voltage that can be stored in an element. The unit of capacitance (C) is farad (F). The element, which has capacitance, is called capacitor.
- q being the amount of charge that can be stored in a capacitor of capacitance C against a potential difference of v volts, we can write,

$$\boxed{C = \frac{q}{v}}$$

i.e. $$\boxed{i = C\frac{dv}{dt}} \qquad \left[\because\ i = \frac{dq}{dt}\right]$$

or $$dv = \frac{1}{C}i\,dt$$

or $$\int_{v_0}^{v_t} dv = \frac{1}{C}\int_0^t i\,dt$$

where, v_0 = initial voltage of capacitor and v_t = final voltage of capacitor.

$$v_t - v_0 = \frac{1}{C}\int_0^t i\,dt$$

i.e. $$\boxed{v_t = \frac{1}{C}\int_0^t i\,dt + v_0}$$

If initial voltage across the capacitor is assumed to be zero, then

$$\boxed{v_t = \frac{1}{C}\int_0^t i\,dt}$$

1.9.1 Energy Storage

The power absorbed by the capacitor is given by,

$$p = vi = vC\frac{dv}{dt}$$

and the energy stored by the capacitor is

$$W = \int_0^t p\,dt = \int_0^t vC\frac{dv}{dt}dt$$

$\Rightarrow$ $$\boxed{W = \frac{1}{2}Cv^2}$$

1.9.2 Important Characteristics of an Ideal Capacitor

- There is no current through a capacitor if the voltage across it is not changing with time. A capacitor is therefore an **open circuit to dc.**
- A finite amount of energy can be stored in a capacitor even if the current through the capacitor is zero, such as when the voltage across it is constant.
- It is impossible to change the voltage across a capacitor by a finite amount in zero time, for this requires an infinite current through the capacitor. A capacitor resists an abrupt change in the voltage across it in a manner analogous to the way a spring resists an abrupt change in its displacement.
- A capacitor never dissipates energy, but only stores it. Although this is true for the mathematical model, it is not true for a physical capacitor due to finite resistances associated with the dielectric as well as the packaging.

Example - 1.7 If the voltage applied across a capacitor is triangular in waveform, the waveform of the current is

(a) Triangular (b) Rectangular
(c) Sinusoidal (d) Trapezoidal [UPPSC]

Solution : (b)

We know, current through capacitor is given as,

$$i = C\frac{dV(t)}{dt}$$

Given, $V(t) \rightarrow$ a triangular wave

$\frac{dV(t)}{dt} \rightarrow$ a rectangular wave

i.e., $i \rightarrow$ a rectangular wave