Electronics Engineering

Electronic Measurements and Instrumentation

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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Electronic Measurements and Instrumentation

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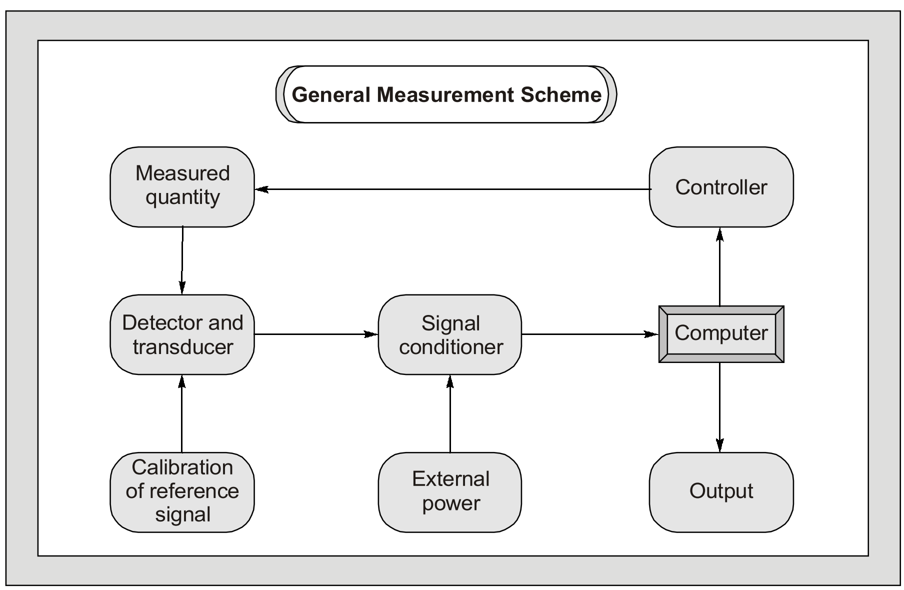
Introduction to Electronic Measurements and Instrumentation

Measurement and instrumentation systems have wide applications such as measurement of electrical and physical quantities like current, voltage, power, temperature, pressure, displacement etc.

The reason for measurement arises when one wants to generate data for design or when one wants to propose a theory based on a set of measurement and instrumentation for commerce.

The measurement and instrumentation systems can also be used to locate things or events. Like employees present in a building, the epicenter of an earthquake. Sometimes, measurement systems are made a part of control system. One can observe the change in the field of measurement and instrumentation due to the introduction of new standards, and sensors.



This course on instrumentation and measurement is intended to make the engineers familiar about the art of modern instrumentation and measurement systems. It is well suited for classroom courses of engineering as well as for various competitive examinations.

Equal importance has been provided to both theory as well as problems with illustrative examples after every topic. It has been tried to cover every topic so that even a beginner understands it easily to excel in the subject of measurement and instrumentation.

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CHAPTER 1

Introduction to Measurements

1.1 Measurements and it's Significance

The measurement of a given quantity is essentially an act or the result of comparison between the quantity (whose magnitude is unknown) and a predefined standard. Measurement is the process by which one can convert physical parameters to meaningful numbers. The measuring process is one in which the property of an object or system under consideration is compared to an accepted standard unit, a standard defined for that particular property. For the result of the measurement to be meaningful, the standard used for comparison purposes must be accurately defined and should be commonly accepted. Also, the apparatus used and the method adopted must be provable. The importance of measurement is simply expressed in the following statement of the famous physicist "Lord Kelvin":

"I often say that when you can measure what you are speaking about and can express it in numbers, you know something about it; when you can't express it in numbers your knowledge is of a meager and unsatisfactory kind."

1.2 Basic Block Diagram of Measurement System

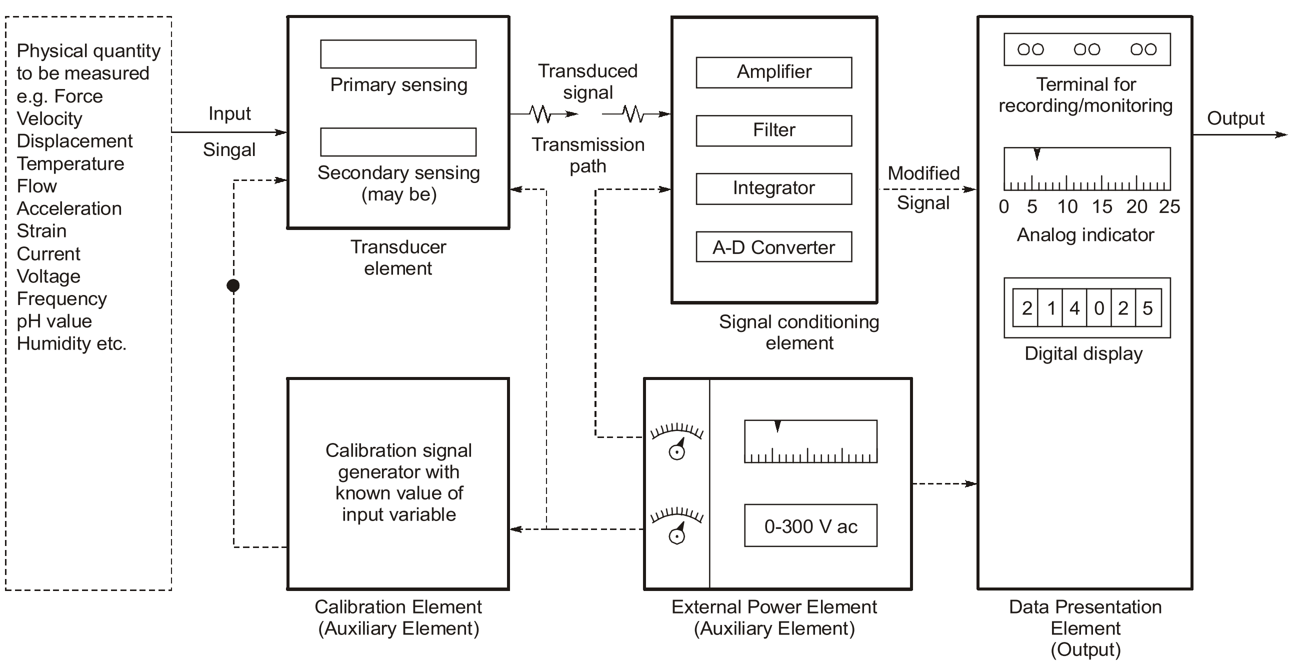


Figure-1.1

Method of Measurement

Direct Measurement

- In this method, the measured or the unknown quantity is directly compared against a standard.
- This method of measurement sometimes produces human errors and hence gives inaccurate results.

Indirect Measurement

- This method of measurement is more accurate and more sensitive.
- These are more preferred over direct measurement.

Calibration

The calibration of all instruments is important since it affords the opportunity to check the instrument against a known standard and subsequently to find errors and accuracy. Calibration procedures involve a comparison of the particular instrument with a primary standard or, a secondary standard or, an instrument of known accuracy.

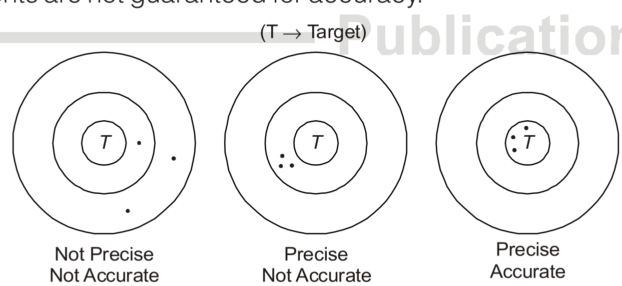
1.3 Static Characteristics of Instrument and Measurement Systems

Accuracy

- It is the closeness with which an instrument reading approaches the true value of the quantity being measured.
- The accuracy can be specified in terms of inaccuracy or limits of error.
- The best way to conceive the idea of accuracy is to specify it in terms of the true value of the quantity being measured.
- The accuracy of a measurement means conformity to truth.

Precision

- It is a measure of the reproducibility of the measurements i.e. given a fixed value of a variable, precision is a measure of the degree to which successive measurements differ from one another.
- The term "Precise" means clearly or sharply defined.
- Precision is used in measurements to describe the consistency or the reproducibility of results.
- Precision instruments are not guaranteed for accuracy.



- Precision depends upon number of significant figures
- The more significant figures, then the precision is more
- Significant figures convey actual information regarding the magnitude and the measurement precision of a quantity.

Example: 302 A (Number of significant figures = 3)
302.10 V (Number of significant figures = 5)
0.000030 Ω (Number of significant figures = 6)

Example - 1.1 In calculating voltage drop, a current of 4.37 A is recorded in a resistance of 31.27 Ω. Calculate the voltage drop across the resistor to the appropriate number of significant figures.

Solution:

Current, $I = 4.37A$ (3 significant figures)

Resistance, $R = 31.27\Omega$ (4 significant figures)

Voltage drop, $V = IR = 4.37 \times 31.27 = 136.6499$ volt (7 significant figures)

Since number of significant figures used in multiplication is 3.

So answer can be written only to a maximum of three significant figures i.e. $V = 137$

NOTE: 248 volt; 248.0 volt ⇒ More precised than other two.

⇒ 0.00248 MV

Example - 1.2 A reading is recorded as 23.90°C. The reading has

(a) three significant figures
(b) five significant figures
(c) four significant figures
(d) none of these

Solution: (c)

Example - 1.3 Assertion (A): A precision instrument is always accurate.

Reason (R): A precision instrument is one where the degree of reproducibility of the measurements is very good.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Solution: (d)

Precision instruments are not guaranteed for accuracy. Refer to definition of precision.

Linearity

- If the output is proportional to input then, it is called linear.
- Non-linear behaviour of an instrument doesn't essentially lead to inaccuracy.
- Most of the time it is necessary that measurement system component should have linear characteristics. For example, the resistance used in a potentiometer should vary linearly with displacement of the sliding contact in order that the displacement is directly proportional to the sliding contact voltage. Any departure from linearity result in error in the read out system.

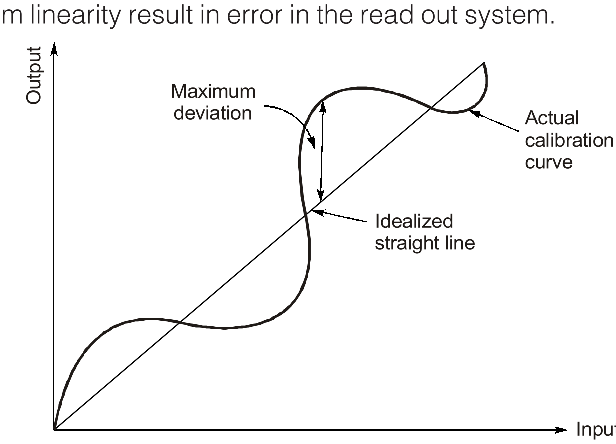


Figure-1.2 : *Linearity w.r.t. actual calibration curve and idealized straight line*

$$\text{N.L.} = \frac{\text{Maximum deviation of output from the idealized straight line}}{\text{Actual reading}} \times 100 \; ; \; \text{N.L.} = \frac{\text{Maximum deviation of output from the idealized straight line}}{\text{Full scale deflection}} \times 100$$

Reproducibility

It is the degree of closeness with which a given value may be repeatedly measured. It may be specified in terms of units for a given period of time.

Static Sensitivity

- The "static sensitivity" of an instrument is the ratio of the magnitude of the output signal or response to the magnitude of input signal or the quantity being measured.
  It's units are mm/μA; per volts etc. depending upon type of input and output.
- Sometimes the static sensitivity is expressed as the ratio of the magnitude of the measured quantity to the magnitude of the response.

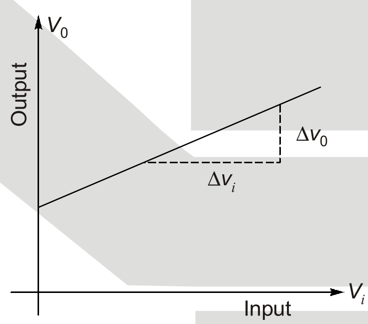


***Figure*-1.3 :** *Sensitivity*

$$\text{Static Sensitivity} = \frac{\text{Small change in output}}{\text{Small change in input}} = \frac{\Delta V_0}{\Delta V_i}$$

- The sensitivity an instrument should be high and therefore, instrument should not have a range greatly exceeding the value to be measured.

$$\text{Deflection Factor} = \frac{1}{(\text{Static Sensitivity})}$$

Resolution or Discrimination

The small measurable input change that can be measured by the instrument is called resolution or discrimination.

Threshold

If the input is slowly increased from some arbitrary (non-zero) input value, it will again be found that output doesn't change at all until a certain increment is exceeded. i.e. threshold is defined as smallest measruable input.

Example - 1.4 A digital voltmeter has a read-out range from 0 to 9,999 counts. Determine the resolution of the instrument in volt when the full scale reading is 9.999 V.

Solution:

Resolution of instrument = 1 count in 9,999

$$\text{Resolution} = \frac{1}{9999}\text{ count} = \frac{1}{9999} \times 9.999 = 10^{-3}\text{ volt} = 1\text{ mV}$$

Dead Time

Dead time is defined as the time required by a measurement system to begin to respond to a change in the measurand. Figure 1.4 shows the measured quantity (input) and its value as indicated by an instrument (output). Dead time infact, is the time before the instrument begins to responds after the measured quantity has been changed.

Dead Zone: Dead zone is the largest change of input quantity for which there is no output of the instrument.

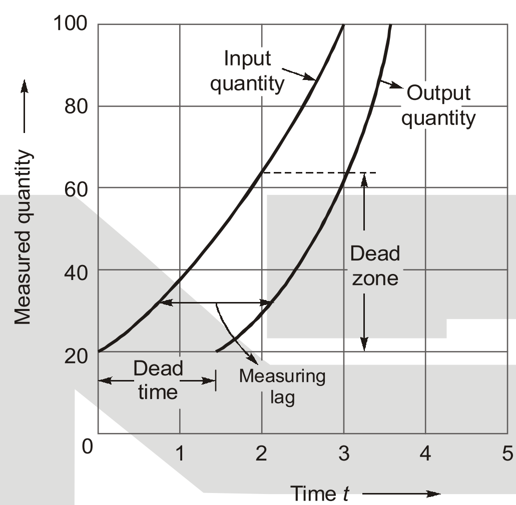


***Figure*-1.4:** *Dead Zone and Dead Time*

Signal to Noise Ratio (S/N)

- Noise is an unwanted signal superimposed upon the signal of interest thereby causing a deviation of the output from it's expected value.
- The ratio of desired to the unwanted noise is called signal to noise ratio and is expressed as

$$\frac{S}{N} = \frac{\text{Signal Power}}{\text{Noise Power}}$$

- In any measurement system, it is desired to have a large signal-to-noise ratio. This can be achieved by increasing the signal level without increasing the noise level or decreasing the noise level with some suitable technique.

Repeatability

It is the repetition of reading of an instrument from a given set of reading.

1.4 Dynamic Characteristics of Instrument and Measurement Systems

The dynamic characteristics of an instrument are

(i) **Speed of response:** It is the rapidity with which an instrument responds changes in the measured quantity.

(ii) **Fidelity:** It is the degree to which an instrument indicates the changes in the measured variable without dynamic error (faithful reproduction).

(iii) **Lag:** It is the retardation or delay in the response of an instrument to changes in the measured variable

(iv) **Dynamic Error:** It is the difference between the true value of a quantity changing with time and the value indicated by the instrument, if no static error is assumed.

1.5 Errors in Measurements and their Analysis

Measurements done in a laboratory or at some other place always involve errors. No measurement is free from errors. If the precision of the equipment is adequate, no matter what it's accuracy is, a discrepancy will always be observed between two measured results.

True Value

The true value of quantity to be measured may be defined as the average of an infinite number of measured values when the average deviation due to various contributing factors tends to zero.

Limiting Errors (Guarantee Errors)

The accuracy and precision of an instrument depends upon it's design, the material used and the work manship that goes into making the instrument. Components are guaranteed to be within a certain percentage of the rated value. Thus, the manufacturer has to specify the deviations from the "nominal value" of a particular quantity. The limits of these deviations from the specified value are defined as ***"Limiting Errors"*** or ***"Guarantee Errors"***.

For example, the magnitude of a resistor is 200 Ω with a limiting error of ±10 Ω. The magnitude of the resistance will be between the limits

$$R = 200 \pm 10\ \Omega$$

or $$R \geq 190\ \Omega$$

and $$R \leq 210\ \Omega$$

Hence, the manufacturer guarantees that the value of the resistor lies between 190 Ω and 210 Ω.

Absolute (Relative) Limiting Error

The relative (fractional) error is defined as the ratio of the error to the specified (nominal) magnitude of a quantity.

$$\boxed{\text{Relative limiting error, } \varepsilon_r = \left(\frac{\text{Measured value} - \text{True value}}{\text{True value}}\right) \times 100}$$

or, $$\% \varepsilon_r = \left(\frac{\text{Actual value} - \text{True value}}{\text{True Value}}\right) \times 100$$

or, $$\boxed{\% \varepsilon_r = \left(\frac{A_m - A_T}{A_T}\right) \times 100} \qquad \left\{\begin{array}{l} A_m = \text{Measured value} \\ A_T = \text{True value} \end{array}\right.$$

Now, $$\% \varepsilon_r = \frac{A_m - A_T}{A_T} \quad \text{or} \quad \frac{A_m}{A_T} = 1 + \varepsilon_r \quad \text{or} \quad \boxed{\frac{A_T}{A_m} = \frac{1}{1+\varepsilon_r}}$$

$$\boxed{A_T = \left(\frac{1}{1+\varepsilon_r}\right) A_m}$$

Here, $\boxed{\dfrac{1}{1+\varepsilon_r} = \text{Correction factor}}$

NOTE: Nominal value = True value and Actual value = Measured value

Example - 1.5 A resistance has nominal value of 50 Ω. When it is measured it's actual value is found to be 60 Ω. Find the percentage limiting error.

Solution:

$$\% \text{ error}, \varepsilon_r = \left(\frac{A_m - A_T}{A_T}\right)\times 100 = \left(\frac{60-50}{50}\right)\times 100 = 20\%$$

$$\boxed{\% \text{ error} = 20\%}$$

Example - 1.6 The measured value of a resistor is 100 Ω and it's relative error is ±10% then, it's true value and the range is

Solution:

$$\varepsilon_r = \pm 10\% \text{ of } 100 = \pm 10\ \Omega$$

Range, $A_T = (100 - 10) \text{ to } (100 + 10) = 90\ \Omega \text{ to } 110\ \Omega$

Example - 1.7 The dead zone in a certain pyrometer is 0.125 percent of span. The calibration is 400°C to 1000°C. What temperature change might occur before it is detected?

(a) 0.25°C
(b) –0.50°
(c) 1.25°C
(d) 0.75°C

Solution: (d)

$$\text{Span} = 4000 - 400 = 600°\text{ C}$$

∴ $\text{Dead zone} = 0.125\% \text{ of span} = \dfrac{0.125}{100}\times 600 = 0.75°\text{C}$

Hence, a change of 0.75°C must occur before it is detected.

Combination of Quantities with Limiting Errors

When two or more quantities, each having a limiting error, are combined, it is advantageous to be able to compute the limiting error of the combination.

1. Sum or Difference of Two or more quantities

Let, $x_1 = a \pm \varepsilon_{r1}$

$x_2 = b \pm \varepsilon_{r2}$

$x_3 = c \pm \varepsilon_{r3}$

∴ $x = x_1 + x_2 + x_3$

or, $x = -x_1 - x_2 - x_3$

So, $x = \pm (x_1 + x_2 + x_3)$

Relative limiting error in x is given by

$$\boxed{\varepsilon_x = \pm\left(\frac{a}{a+b+c}\cdot\varepsilon_{r1} + \frac{b}{a+b+c}\cdot\varepsilon_{r2} + \frac{c}{a+b+c}\varepsilon_{r3}\right)}$$

(ε_x = worst possible error)

Example - 1.8 Three resistances $R_1 = 10 \pm 2\%$, $R_2 = 20 \pm 5\%$, $R_3 = 50 \pm 3\%$ are connected in series. Find the % limiting error for the series combination.

Solution:

$$\varepsilon_R = \pm\left(\frac{10}{10+20+50}\times 2 + \frac{20}{10+20+50}\times 5 + \frac{50}{10+20+50}\times 3\right)$$

or, $$\varepsilon_R = \pm 3.375\%$$

$$\boxed{\%\ \text{Limiting error} = \pm 3.375\%}$$

Given, $$R_T = 10 + 20 + 50 = 80\Omega$$

$$\boxed{R_{\text{measured}} = 80 \pm 3.375\%}$$

2. Multiplication or Division Terms

Let, $$x = \frac{x_1\, x_2}{x_3} \text{ or } \frac{x_2\, x_3}{x_1} \text{ or } x_1 x_2 x_3 \text{ or } \frac{x_1}{x_1 x_3}$$

Then, relative limiting error is

$$\boxed{\varepsilon_x = \pm(\varepsilon_{r1} + \varepsilon_{r1} + \varepsilon_{r3})}$$

NOTE

When, $$x = \frac{x_1\, x_2}{x_2 + x_3} \text{ or } \frac{x_1}{x_2 + x_3} \text{ or } \frac{x_1\, x_2}{x_2 - x_1}$$

Then, multiplication or division form is not applicable for finding relative limiting error.

Example - 1.9 In the measurement of unknown resistance by using a wheat stone bridge if $P = 20 \pm 5\%$, $Q = 50 \pm 3\%$ and $S = 30 \pm 2\%$. Find the value of the unknown resistance R and it's limiting error.

Solution:

Limiting error, $$\varepsilon_R = \pm(5 + 3 + 2) = \pm 10\%$$

$$R = \frac{P}{Q}\cdot S = \frac{20}{50}\times 30 = 12\,\Omega$$

So, unknown resistance, $$R = 12 \pm 10\%$$

3. Power of a Factor

Let, $$x = x_1^m \cdot x_2^n \cdot x_3^p \text{ or } \frac{x_1^m x_2^n}{x_3^p} \text{ or } \frac{x_1^m}{x_1^n x_3^p}$$

Then, Relative limiting error is $\varepsilon_r = \pm(m\,\varepsilon_{r1} + n\,\varepsilon_{r2} + p\,\varepsilon_{r3})$

NOTE

When x is of the form $\dfrac{x_1^m}{x_2^n + x_3^p}$ or $\dfrac{x_1^m + x_2^n}{x_3^p}$

then, above method is not applicable for finding relative limiting error.

Example - 1.10 The power is measured in a resistor by passing current through the ammeter and ammeter measures I = (5 ± 4%) A across the resistance of R = (10 ± 2%) Ω. Find the power consumed by the resistor and it's limiting error.

Solution:

Power consumed, $P = I^2R = 5^2 \times 10 = 250$ watts

and limiting error, $\varepsilon_p = \pm(2\varepsilon_I + \varepsilon_R) = \pm(2 \times 4 + 2) = 10\%$

∴ $P = (250 \pm 10\%)$ watt

4. Special Case

Resistance in parallel:

Let, $R_1 = 10 \pm 10\%$ (Range = 9 Ω to 11 Ω)

and $R_2 = 20 \pm 5\%$ (Range = 19 Ω to 21 Ω)

Equivalent resistance of parallel combination is $R = \dfrac{R_1 R_2}{R_1 + R_2}$

True value; $R = \dfrac{10 \times 20}{10 + 20} = 6.66\,\Omega = R_T$

Resistance in lower range; $R_L = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{9 \times 19}{9 + 19} = 6.107\,\Omega$ Measured value in low range = L_m

Resistance in higher range; $R_H = \dfrac{11 \times 21}{11 + 21} = 7.21875\,\Omega$ Measured value for high range = H_m

Error in low range (for low value) = $\%\,\varepsilon_r = \left(\dfrac{L_m - R_T}{R_T}\right) \times 100$

Error in high range (for high value) = $\%\,\varepsilon_r = \left(\dfrac{H_m - R_T}{R_T}\right) \times 100$

For present case,

Error in low range; $\%\varepsilon_r = \left(\dfrac{6.10 - 6.66}{6.66}\right) \times 100 = -8.4\,\%$

Error in high range; $\%\varepsilon_r = \left(\dfrac{7.2187 - 6.66}{6.66}\right) \times 100 = 8.38\,\%$

Example - 1.11 A 4-dial decade box has

Decade a of 10 × 1000 Ω ± 0.1%,

Decade b of 10 × 100 Ω ± 0.1%

Decade c of 10 × 10 Ω ± 0.5%

Decade d of 10 × 1 Ω ± 1.0%

It is set at 4639 Ω. Find the percentage limiting error and the range of resistance value.

Solution:

Error for decade a = $\pm 4000 \times \dfrac{0.1}{100} = \pm 4\,\Omega$

$$\text{Error for decade } b = \pm 600 \times \frac{0.1}{100} = \pm 0.6\ \Omega$$

$$\text{Error for decade } c = \pm 30 \times \frac{0.5}{100} = \pm 0.15\ \Omega$$

$$\text{Error for decade } d = \pm 9 \times \frac{1}{100} = \pm 0.09\ \Omega$$

$\therefore$ Total error $= \pm(4 + 0.6 + 0.15 + 0.09) = 4.84\ \Omega$

Relative limiting error, $\varepsilon_r = \pm\dfrac{4.84}{4639} = \pm 0.00104$

% limiting error; $\%\ \varepsilon_r = \pm 0.00104 \times 100 = \pm 0.104\%$

Limiting value of resistance $= 4639\,(1 \pm 0.00104) = (4639 \pm 5)\ \Omega$

Example - 1.12 Find the uncertainty in the measurement of power dissipated by resistor if the current flowing through the resistor is 5 A and the voltage across the resistor is 200 V and the uncertainty of the ammeter is 0.2 A and the voltmeter is 1.5 V. Find the uncertainty of the power.

Solution:

We know that, $P = VI$

$\therefore$ $\dfrac{\delta P}{\delta V} = I = 5$ A

$W_V = 1.5$ V (given)

$\dfrac{\delta P}{\delta I} = V = 200$ V,

$W_I = 0.2$ A

$\therefore$ Uncertainity in the measurement of power is given by

$$W_P = \sqrt{\left(\frac{\delta P}{\delta V}\right)^2 \cdot W_V^2 + \left(\frac{\delta P}{\delta I}\right)^2 \cdot W_I^2} = \sqrt{(200)^2 \times 0.2^2 + (5)^2 \times 1.5^2} = 40.69 \text{ watt}$$

$\therefore$ Uncertainity in the measurement of power = 40.69 watt.

1.6 Types of Errors

Error: Deviation of the measured value from the true value of the quantity being measured is called an error.

A study of errors is a first step in finding ways to reduce them. Errors may arise from different sources and are classified as under:

1. Gross error
2. Systematic error
3. Random error

Gross Error	Systematic Error	Random Errors
1. These types of error mainly comprises of human mistakes in reading instruments and recording and calculating measurement results. 2. The experimenter is mainly responsible for these errors. 3. Some gross errors are easily detected while some are difficult to detect. 4. These errors can be avoided by taking great care in reading and recording the data. Also, two or three or even more readings should be taken for the quantity under measurement. 5. Computational mistakes, incorrect adjustment and improper application of instruments can lead to gross errors.	1. Systematic errors are classified into three types: (i) **Instrument Errors:** • Occurs due to short coming in the instrument • Misuse of the instrument • Loading effect of the instrument (ii) **Environmental Errors:** • These errors occurs due to external environment factors like humidity, dust, vibrations or external magnetic field etc. (iii) **Observation Errors:** • Different experimenters may produce different results, when sound and light measurements are involved since no two observers possess the same physical response.	1. Random errors are those errors whose causes can't be established because of random variations in the parameters or the system of measurement. 2. The happenings or disturbances about which we are unaware are lumped together and called "Random" or "Residual" and error caused due to these happenings are called "Random" error.

1.7 Statistical Treatment of Data

1.7.1 Histogram

When a number of multisample observations are taken experimentally there is a scatter of the data about some central value. One method presenting test results in the form of a Histogram. The technique is illustrated in Figure 1.5 representing the data given in table.

Length (mm)	Number of Readings
99.7	1
99.8	4
99.9	12
100.0	19
100.1	10
100.2	3
100.3	1

Total number of readings = 50.

This histogram of Figure 1.5 represents these data where the ordinate indicates the number of observed readings (frequency or occurrence) of a particular value. A histogram is als called a **frequency distribution curve.**

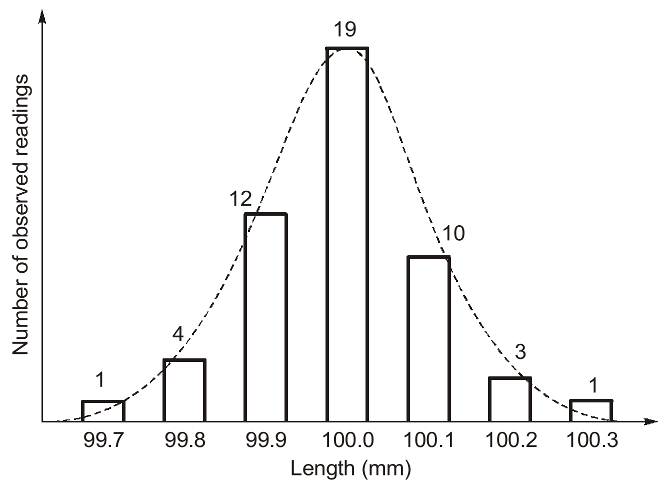


***Figure*-1.5**

With more and more data taken at smaller and smaller increments the histogram would finally change into a smooth curve, as indicated by the dashed in Figure 1.5.

The smooth curve is symmetrical with respect to the central value. Many physical cases have been found which give experimental data agreeing fairly well with the smooth symmetrical curve.

1.7.2 Arithmetic Mean

The arithmetic mean is given by

$$\bar{X} = \frac{x_1 + x_2 + x_3 + x_4 + \ldots + x_n}{n} = \frac{\Sigma x}{n}$$

where, $\bar{X}$ = Arithmetic mean ; $x_1, x_2, \cdots x_n$ = Readings or variates or samples ; n = Number of readings

1.7.3 Deviation

Deviation is departure of the observed reading from the arithmetic mean of the group of readings.

Then, $d_n = x_n - \bar{X}$

1.7.4 Average Deviation

$$\bar{D} = \frac{|-d_1| + |d_2| + |-d_3| + \ldots + |-d_n|}{n} = \frac{\Sigma |d|}{n}$$

1.7.5 Standard Deviation (S.D.)

Standard deviation or the root mean square deviation is an important term in the analysis of random errors. The standard deviation of an infinite number of data is defined as the square root of the sum of the individual deviation squared, divided by the number of readings.

Let $x = f(x_1, x_2, \ldots, x_n)$

Average, $\bar{x} = \left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right)$

Deviations are:

$$|d_1| = x_1 - \bar{x}$$
$$|d_2| = x_2 - \bar{x}$$
$$\vdots \quad \vdots \quad \vdots$$
$$|d_n| = x_n - \bar{x}$$

$$\text{Average deviation} = \frac{|d_1 + d_2 + ... + d_n|}{n}$$

Standard deviation,

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + ... + d_n^2}{(n-1)}} \quad (\text{for } n \le 20)$$

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + ... + d_n^2}{n}} \quad (\text{for } n > 20)$$

(n = No. of observations)

Variance, $V = \sigma^2 = (\text{standard deviation})^2$

When standard deviation of $x_1, x_2, ..., x_n$ are $\sigma_{x_1}, \sigma_{x_2}, ..., \sigma_{x_n}$ then standard deviation of x is given by:

$$\sigma_x = \sqrt{\left(\frac{dx}{dx_1}\right)^2 \cdot \sigma_{x_1}^2 + \left(\frac{dx}{dx_2}\right)^2 \cdot \sigma_{x_2}^2 + ... + \left(\frac{dx}{dx_n}\right)^2 \cdot \sigma_{x_n}^2}$$

1.7.6 Variance

The variance is the mean square deviation, which is the same as S.D., except that square root is not extracted.

$$\text{Variance } V = (\text{Standard Deviation})^2$$

$$= (\text{S.D.})^2 = \frac{\Sigma d^2}{n}$$

But when the number of observations is less than 20.

$$\text{Variance } V = s^2 = \frac{\Sigma d^2}{n-1}$$

1.7.7 Uncertainity Error

The uncertainity analysis in measurements when many variables are involved is done on the same basis as is done for error analysis when the result are expressed as standard deviations or probable errors.

Let $x = f(x_1, x_2, ..., x_n)$

$W_{x_1}, W_{x_2}, ..., W_{x_n}$ be the uncertainties of $x_1, x_2, ... x_n$ respectively.

Then, uncertainity of x is given by

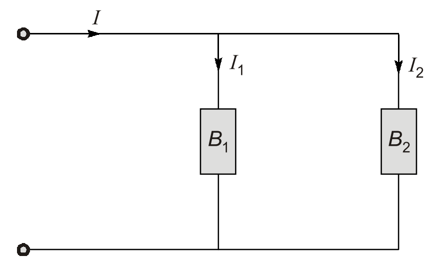
$$W_x = \sqrt{\left(\frac{dx}{dx_1}\right)^2 \cdot W_{x_1}^2 + \left(\frac{dx}{dx_2}\right)^2 \cdot W_{x_2}^2 + \left(\frac{dx}{dx_3}\right)^2 \cdot W_{x_3}^2 + ... + \left(\frac{dx}{dx_n}\right)^2 \cdot W_{x_n}^2}$$

Example - 1.13 A parallel circuit has two branches carries currents of $I_1 = (100 \pm 2)$A and $I_2 = (200 \pm 5)$ A. Find the standard deviation in the measurement of total current if the errors in the currents I_1 and I_2 are due to standard deviation.

Solution:

We have

$$I = I_1 + I_2$$



$\therefore$ $$\frac{\delta I}{\delta I_1} = 1 \quad \text{and} \quad \frac{\delta I}{\delta I_2} = 1$$

$$\delta I_1 = 2$$

$$\delta I_2 = 5 \quad \text{(given)}$$

$\therefore$ Standard deviation in the measurement of total current is:

$$\sigma_I = \sqrt{\left(\frac{\delta I}{\delta I_1}\right)^2 \cdot \sigma_{I_1}^2 + \left(\frac{\delta I}{\delta I_2}\right)^2 \cdot \sigma_{I_2}^2} = \sqrt{1^2 \times 2^2 + 1^2 \times 5^2} = 5.38\text{ A}$$

$$\boxed{\sigma_I = 5.38\text{ A}}$$

So, $I = [(200 + 100) \pm 5.38]\text{ A} = [(I_1 + I_2) \pm \sigma_I]$

or $I = (300 \pm 5.38)\text{ A}$

Example - 1.14 A circuit was tuned for resonance by eight different students, and the values of resonant frequency in kHz were recorded as 532, 548, 543, 535, 546, 531, 543 and 536.

Calculate:

(a) the arithmetic mean

(b) deviations from mean

(c) the average deviation

(d) the standard deviation, and

(e) variance

Solution:

(a) The arithmetic mean of readings is given by

$$\bar{X} = \frac{\Sigma x}{n} = \left(\frac{532 + 548 + 543 + 535 + 546 + 531 + 543 + 536}{8}\right) = 539.25\text{ kHz}$$

(b) The deviations are,

$d_1 = x_1 - \bar{X} = 532 - 539.25 = -7.25\text{ kHz}$; $d_2 = x_2 - \bar{X} = 548 - 539.25 = +8.75\text{ kHz}$

$d_3 = x_3 - \bar{X} = 543 - 539.25 = +3.75\text{ kHz}$; $d_4 = x_4 - \bar{X} = 535 - 539.25 = -4.25\text{ kHz}$

$d_5 = x_5 - \bar{X} = 546 - 539.25 = +6.75\text{ kHz}$; $d_6 = x_6 - \bar{X} = 531 - 539.25 = -8.25\text{ kHz}$

$d_7 = x_7 - \bar{X} = 543 - 539.25 = +3.75\text{ kHz}$; $d_8 = x_8 - \bar{X} = 536 - 539.25 = -3.25\text{ kHz}$

(c) Average deviation is given by

$$\bar{D} = \frac{\Sigma |d|}{n} = \left(\frac{7.25 + 8.75 + 3.75 + 4.25 + 6.75 + 8.25 + 3.75 + 3.25}{8}\right)$$

$$= 5.75\text{ kHz}$$

(d) Since the number of observations are 8 which is less than 20, therefore, the standard deviation will be given by

$$S = \sqrt{\frac{\Sigma d^2}{(n-1)}}$$

$$= \sqrt{\frac{(-7.25)^2 + (+8.75)^2 + (3.75)^2 + (-4.25)^2 + (+6.75)^2 + (-8.25)^2 + (3.75)^2 + (3.25)^2}{(8-1)}}$$

$= 6.54$ kHz

(e) Variance $= S^2 = 42.77\ (\text{kHz})^2$

1.7.8 Error at Desired Scale

Error at any desired scale is given by:

$$\%\,\varepsilon_r = \frac{\%\text{ full scale error} \times \text{Full scale value}}{\text{Desired value}}$$

Example - 1.15 An ammeter measures a full scale current of 100 *A* produces a full scale error of 5%. Find the error if the ammeter reads.

(a) 50 A (b) 25 A (c) 10 A

Solution:

(a) Desired ammeter reading = 50 A

$\therefore$ $\%\text{ error, } \varepsilon_r = \dfrac{5 \times 100}{50} = 10\%$

(b) Desired ammeter reading = 25 A

$\therefore$ $\%\text{ error, } \varepsilon_r = \dfrac{5 \times 100}{25} = 20\%$

(c) Desired ammeter reading = 50 A

$\therefore$ $\%\text{ error, } \varepsilon_r = \dfrac{5 \times 100}{10} = 50\%$

NOTE: As the desired instrument reading approaches the full scale value of measurement of the unknown quantity, error is reduced in the measurement.

Example - 1.16 A 160 ± 10% pF capacitor, an inductor of 160 ± 10% µH and a resistor of 1200 ± 11 Ω are connected in series.

(a) If all the three components are ±0% and resonant frequency is $f_r = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}}$, compute the resonant frequency of the combination.

(b) If all the three components are +10%, compute the expected resonant frequency of the combination and the percentage error when compared to the result of part (a).

(c) When all the three components are –10%, compute the expected resonant frequency and the percentage error when compared to the result of part (a).

Solution:

(a) When all the components have zero error,

$$L = 160\ \mu\text{H} = 160 \times 10^{-6}\ \text{H and } C = 160\ \text{pF} = 160 \times 10^{-12}\ \text{F}$$

$\therefore$ Resonant frequency, $f_r = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}} = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{160 \times 10^{-6} \times 160 \times 10^{-12}}} = 1\ \text{MHz}$

(b) When the components are +10%,

$$C = 160 + 0.1 \times 160 = 176 \text{ pF},$$
$$L = 160 + 0.1 \times 160 = 176 \text{ }\mu\text{H}$$

$$\therefore \quad f_r = \frac{1}{2\pi}\sqrt{\frac{1}{176\times10^{-6}\times176\times10^{-12}}} = 0.9 \text{ MHz}$$

Hence, $$\text{error} = \left(\frac{0.9-1.0}{1.0}\right) = -10\%$$

OR

This is a case of known errors and can be solved by using the equation

$$f_r = \frac{1}{2\pi}L^{-1/2}C^{-1/2}$$

$\therefore$ Relative error in f_r is,

$$\frac{\delta f_r}{f_r} = \left(-\frac{1}{2}\frac{\delta L}{L}-\frac{1}{2}\frac{\delta C}{C}\right) = -\frac{1}{2}(0.1+0.1)$$
$$= -0.1 = -10\%$$

(c) When the components are –10%,

$$C = 160 - 0.1 \times 160 = 144 \text{ pF}$$
$$L = 160 - 0.1 \times 160 = 144 \text{ }\mu\text{H}$$

$$\therefore \quad f_r = \frac{1}{2\pi}\sqrt{\left(\frac{1}{144\times10^{-6}\times144\times10^{-12}}\right)} = 1.1 \text{ MHz}$$

$$\therefore \quad \text{Error} = \left(\frac{1.1-1.0}{1.0}\right)\times100 = +10\%$$

Example - 1.17 The errors introduced by an instrument fall in which category?

(a) Systematic errors
(b) Random errors
(c) Gross errors
(d) Environmental errors

Solution: (a)

Systematic errors occur due to the short coming of the instrument.

Unit, Dimensions and Standards

Unit: The standard measure of each kind of physical quantity is called a "unit". Measurements mean comparison with a standard value.

Magnitude of a physical quantity = (Numerical ratio) × (Unit)

Absolute Unit: An absolute system of units is defined as a system in which the various units are all expressed in terms of a small number of fundamental units. Absolute unit compare the measured quantity with arbitrary units of the same type.

Dimensions: Dimension is the quantity of every quantity which distinguishes it from all other quantities. It is written in a characteristics notation, [ ].