Electronics Engineering

Control Systems

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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Control Systems

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CHAPTER 1

Introduction

Control System:
Control system is a means by which any quantity of interest in a machine, mechanism or other equipment is maintained or altered in accordance with a desired manner.
Control system can also be defined as the combination of elements arranged in a planned manner wherein each element causes an effect to produce a desired output.

Control systems are classified into two general categories as Open-loop and close-loop systems.

1.1 Open Loop Control Systems

An open loop control system is one in which the control action is independent of the output.

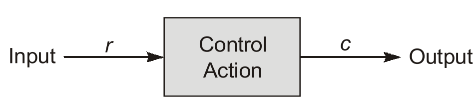


Figure-1.1 : *Open-loop control system*

This is the simplest and most economical type of control system and does not have any feedback arrangement.

Some common examples of open-loop control systems are
(a) Traffic light controller
(b) Electric washing machine
(c) Automatic coffee server
(d) Bread toaster

Advantages of Open Loop Control Systems

(a) Simple and economic
(b) No stability problem

Disadvantages of Open Loop Control Systems

(a) Inaccurate
(b) Unrealisable
(c) The effect of parameter variation and external noise is more

Note: Open loop control systems does not require performance analysis.

1.2 Closed Loop Control Systems

A *closed loop control system* is one in which the control action is some how dependent on the output.

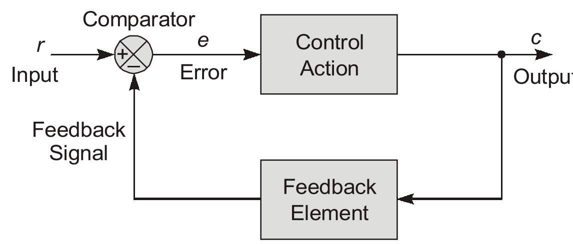


Figure-1.2 : *Closed loop control system*

The closed loop system has same basic features as of open loop system with an additional feedback feature. The actual output is measured and a signal corresponding to this measurement is fedback to the input section, where it is with the input to obtain the desired output.

Some common examples of closed loop control systems are:

(a) Electric iron
(b) DC motor speed control
(c) A missile launching system (direction of missile changes with the location of moving target)
(d) Radar tracking system
(e) Human respiratory system
(f) Autopilot system
(g) Economic inflation

Advantages of Closed Loop Control Systems

(a) Accurate and reliable
(b) Reduced effect of parameter variation
(c) Bandwidth of the system can be increased with negative feedback
(d) Reduced effect of non-linearities

Disadvantages of Closed Loop Control Systems

(a) The system is complex and costly
(b) System may become unstable
(c) Gain of the system reduces with negative feedback

Remember

- Feedback is not used for improving stability
- An open loop stable system may also become unstable when negative feedback is applied
- Except oscillators, in positive feedback, we have always unstable systems.

1.3 Comparison Between Open Loop and Closed Loop Control Systems

Open Loop System	Closed Loop System
1. So long as the calibration is good, open-loop system will be accurate	1. Due to feedback, the close-loop system is more accurate
2. Organization is simple and easy to construct	2. Complicated and difficult
3. Generally stable in operation	3. Stability depends on system components
4. If non-linearity is present, system operation degenerates	4. Comparatively, the performance is better than open-loop system if non-linearity is present

***Example*-1.1** Match List-I (Physical action or activity) with List-II (Category of system) and select the correct code:

List-I

A. Human respiration system
B. Pointing of an object with a finger
C. A man driving a car
D. A thermostatically controlled room heater

List II

1. Man-made control system
2. Natural including biological control system
3. Control system whose components are both man-made and natural

Codes:

	A	B	C	D
(a)	2	2	3	1
(b)	3	1	2	1
(c)	3	2	2	3
(d)	2	1	3	3

***Solution:* (*a*)**

1.4 Laplace Transformation

In order to transform a given function of time $f(t)$ into its corresponding Laplace transform first multiply $f(t)$ by e^{-st}, s being a complex number ($s = \sigma + j\omega$). Integrate this product with respect to time with limits from zero to ∞. This integration results in Laplace transform of f(t), which is denoted by $F(s)$ or $\mathcal{L} f[(t)]$.

The mathematical expression for Laplace transform is,

$$\mathcal{L} f[(t)] = F(s),\ t \geq 0$$

where,

$$F(s) = \int_0^{\infty} f(t).e^{-st} dt$$

The original time function f(t) is obtained back from the Laplace transform by a process called inverse Laplace transformation and denoted as $\mathcal{L}^{-1}$

Thus, $$\mathcal{L}^{-1}[Lf(t)] = \mathcal{L}^{-1}[F(s)] = f(t)$$

The time function $f(t)$ and its Laplace transform $F(s)$ form a transform pair.

S.No.	$f(t)$	$F(s) = L[f(t)]$
1.	$\delta(t)$ unit impulse at $t = 0$	1
2.	$u(t)$ unit step at $t = 0$	$\frac{1}{s}$
3.	$u(t-T)$ unit step at $t = T$	$\frac{1}{s}e^{-sT}$
4.	t	$\frac{1}{s^2}$
5.	$\frac{t^2}{2}$	$\frac{1}{s^3}$
6.	t^n	$\frac{n!}{s^{n+1}}$
7.	e^{at}	$\frac{1}{s-a}$
8.	e^{-at}	$\frac{1}{s+a}$
9.	$t\,e^{at}$	$\frac{1}{(s-a)^2}$
10.	$t\,e^{-at}$	$\frac{1}{(s+a)^2}$
11.	$t^n\,e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
12.	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
13.	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$

Table-1.1 *Table of Laplace Transform Pairs*

Basic Laplace Transform Theorems

Basic theorems of Laplace transform are given below:

(*a*) Laplace Transform of Linear Combination:

$$\mathcal{L}[af_1(t) + bf_2(t)] = aF_1(s) + bF_2(s)$$

where $f_1(t)$, $f_2(t)$ are functions of time and a, b are constants.

(*b*) If the Laplace Transform of $f(t)$ is $F(s)$, then:

(*i*) $$\mathcal{L}\left[\frac{df(t)}{dt}\right] = [sF(s) - f(0^+)]$$

(*ii*) $$\mathcal{L}\left[\frac{d^2f(t)}{dt^2}\right] = [s^2F(s) - sf(0^+) - f'(0^+)]$$

(*iii*) $$\mathcal{L}\left[\frac{d^3f(t)}{dt^3}\right] = [s^3F(s) - s^2 f(0^+) - sf'(0^+) - f''(0^+)]$$

where $f(0^+)$, $f'(0^+)$, $f''(0^+)$ ... are the values of $f(t)$, $\frac{df(t)}{dt}$, $\frac{d^2f(t)}{dt^2}$ ... at $t = (0^+)$.

(*c*) If the Laplace Transform of *f*(*t*) is *F*(*s*), then:

(*i*) $$\mathcal{L}\left[\int f(t)\right] = \left[\frac{F(s)}{s} + \frac{f^{-1}(0^+)}{s}\right]$$

(*ii*) $$\mathcal{L}\left[\iint f(t)\right] = \left[\frac{F(s)}{s^2} + \frac{f^{-1}(0^+)}{s^2} + \frac{f^{-2}(0^+)}{s}\right]$$

(*iii*) $$\mathcal{L}\left[\iiint f(t)\right] = \left[\frac{F(s)}{s^3} + \frac{f^{-1}(0^+)}{s^3} + \frac{f^{-2}(0^+)}{s^2} + \frac{f^{-3}(0^+)}{s}\right]$$

where $f^{-1}(0^+)$, $f^{-2}(0^+)$, $f^{-3}(0^+)$ ... are the values of $\int f(t)$, $\iint f(t)$, $\iiint f(t)$ ... at $t = (0^+)$.

(*d*) If the Laplace Transform of f(t) is *F*(*s*), then:

$$\mathcal{L}[e^{\pm at} f(t)] = F(s \mp a)$$

(*e*) If the Laplace Transform of *f*(*t*) is *F*(*s*), then:

$$\mathcal{L}[t\, f(t)] = -\frac{d}{ds}F(s)$$

(*f*) Initial Value Theorem:

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} s\,\mathcal{L}[f(t)]$$

or

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$$

(*g*) Final Value Theorem:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s\,\mathcal{L}[f(t)]$$

or

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

The final value theorem gives the final value ($t \to \infty$) of a time function using its Laplace transform and as such very useful in the analysis of control systems. However, if the denominator of $sF(s)$ has any root having real part as zero or positive, then the final value theorem is not valid.

Example-1.2 Laplace transform of sin(ωt + α) is

(a) $\dfrac{s\cos\alpha + \omega \sin\alpha}{s^2 + \omega^2}$

(b) $\dfrac{\omega}{s^2 + \omega^2}\cos\alpha$

(c) $\dfrac{s}{s^2 + \omega^2}\sin\alpha$

(d) $\dfrac{s\sin\alpha + \omega\cos\alpha}{s^2 + \omega^2}$

Solution: **(d)**

$$\sin(\omega t + \alpha) = \sin\omega t\cos\alpha + \cos\omega t\sin\alpha$$

$$\mathcal{L}\{\sin(\omega t + \alpha)\} = \frac{\omega\cos\alpha}{s^2 + \omega^2} + \frac{s\sin\alpha}{s^2 + \omega^2}$$

$$= \frac{s\sin\alpha + \omega\cos\alpha}{s^2 + \omega^2}$$

CHAPTER 2

Transfer Function

2.1 Transfer Function and Impulse Response Function

In control theory, transfer functions are commonly used to characterise the input-output relationships of components or systems that can be described by linear, time-invariant differential equations.

Transfer Function

The transfer function of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

Linear Systems

A system is called linear if the principle of superposition and principle of homogeneity apply. The principle of superposition states that the response produced by the simultaneous application of two different forcing functions is the sum of the two individual responses. Hence, for the linear system, the response to several inputs can be calculated by transferring one input at a time and adding the results. It is the principle that allows one to build up complicated solutions to the linear differential equations from simple solutions.

In an experimental investigation of a dynamic system, if cause and effect are proportional, thus implying that the principle of superposition holds, then the system can be considered as linear.

Linear Time-Invariant Systems and Linear-Time Varying Systems

A differential equation is linear if the coefficients are constants or functions only of the independent variable. Dynamic systems that are composed of linear time-invariant lumped-parameter components may be described by linear time-invariant differential equations i.e. constant-coefficient differential equations. Such systems are called linear time-invariant (or linear constant-coefficient) systems. Systems that are represented by differential equations whose coefficients are function of time are called linear time varying systems. An example of a time-varying control system is a space craft control system (the mass of a space craft changes due to fuel consumption).

The definition of transfer function is easily extended to a system with multiple inputs and outputs (i.e. a multivariable system). In a multivariable system, a linear differential equation may be used to describe the relationship between a pair of input and output variables, when all other inputs are set to zero. Since the principle of superposition is valid for linear systems, the total effect (on any output) due to all the inputs acting simultaneously is obtained by adding up the outputs due to each input acting alone.

Example-2.1 When deriving the transfer function of a linear element

(a) both initial conditions and loading are taken into account

(b) initial conditions are taken into account but the element is assumed to be not loaded.

(c) initial conditions are assumed to be zero but loading is taken into account

(d) initial conditions are assumed to be zero and the element is assumed to be not loaded.

***Solution:* (c)**

While deriving the transfer function of a linear element only initial conditions are assumed to be zero, loading (or input) can't assume to be zero.

Example-2.2 If the initial conditions for a system are inherently zero, what does it physically mean?

(a) The system is at rest but stores energy

(b) The system is working but does not store energy

(c) The system is at rest or no energy is stored in any of its part

(d) The system is working with zero reference input

***Solution:* (c)**

A system with zero initial conditions is said to be at rest since there is no stored energy.

Example-2.3 What are the properties of linear systems not valid for non-linear systems? Explain each briefly?

Solution:

- Linear systems satisfy properties of superposition and homogeneity. Any system that does not satisfy these properties is non-linear.

  Property of superposition: When the output corresponding to v_{in_1} is v_{out_1} and the output corresponding to v_{in_2} is v_{out_2} then the output corresponding to $av_{\text{in}_1} + bv_{\text{in}_2}$ is $av_{\text{out}_1} + bv_{\text{out}_2}$.

  Property of homogeneity: It states that for a given input x in the domain of the function *f* and for any real number *k*

  $$f(kx) = kf(x)$$

- Linear systems have one equilibrium point at the origin. Non-linear systems may have many equilibrium points.

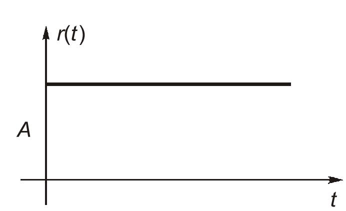
2.2 Standard Test Signals

1. Step Signal

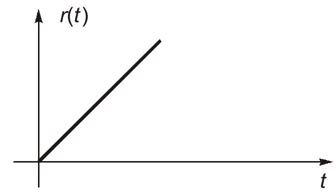
$$r(t) = A\,u(t)$$

where, $$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

Laplace transform, $$R(s) = A/s$$



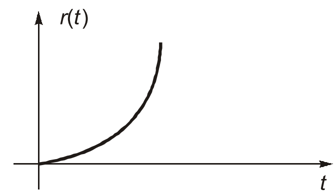
2. Ramp Signal



$$r(t) = \begin{cases} A\,t & t > 0 \\ 0 & t < 0 \end{cases}$$

Laplace transform, $R(s) = A/s^2$

3. Parabolic Signal

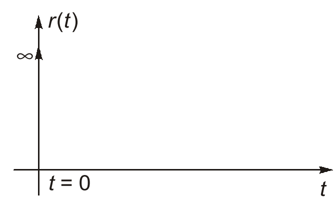


$$r(t) = \begin{cases} A\,t^2/2 & , \; t > 0 \\ 0 & , \; t < 0 \end{cases}$$

Laplace transform, $R(s) = A/s^3$

4. Impulse Signal

$$r(t) = \begin{cases} \infty & , \; t = 0 \\ 0 & , \; t \neq 0 \end{cases} \quad ; \quad \int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$



Laplace transform, $R(s) = 1$

Transfer function, $G(s) = \dfrac{C(s)}{R(s)}$

$$C(s) = F(s)\,R(s)$$

Let, $R(s) = \text{Impulse signal} = 1$

$$C(s) = \text{Impulse response} = G(s) \times 1 = \text{T.F.}$$

$$\mathcal{L}\{\text{Impulse Response}\} = \text{Transfer function} = \left[\frac{C(s)}{R(s)}\right]$$

NOTE

- d/dt (Parabolic Response) = Ramp Response
- d/dt (Ramp Response) = Step Response
- d/dt (Step Response) = Impulse Response

Consider, a linear time-invariant system has the input $u(t)$ and output $y(t)$. The system can be characterized by its impulse response $g(t)$, which is defined as the output when the input is a unit-impulse function $\delta(t)$. Once the impulse response of a linear system is known, the output of the system $y(t)$, with any input $u(t)$, can be found by using the transfer function.

Let $G(s)$ denotes the transfer function of a system with input $u(t)$, output $y(t)$, and impulse response $g(t)$. The transfer function $G(s)$ is defined as

$$G(s) = \mathcal{L}[g(t)] = \left.\frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]}\right|_{\text{initial conditions} \to 0} = \frac{Y(s)}{U(s)}$$

Remember

Sometimes, students do a common mistake, they first find $y(t)/u(t)$ and then take its Laplace transform to determine the transfer function which is absolutely wrong. Because,

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]} \neq \mathcal{L}\left[\frac{y(t)}{u(t)}\right]$$

2.3 Poles and Zeros of a Transfer Function

The transfer function of a linear control system can be expressed as

$$G(s) = \frac{A(s)}{B(s)} = \frac{K(s-s_1)(s-s_2)\ldots(s-s_n)}{(s-s_a)(s-s_b)\ldots(s-s_m)}$$

where K is known as gain factor of the transfer function $G(s)$.

In the transfer function expression, if s is put equal to s_a, s_b ... s_m then it is noted that the value of the transfer function is infinite. These s_a, s_b, ... s_m are called the poles of the transfer function.

In the transfer function expression, if s is put equal to s_1, s_2 ... s_n then it is noted that the value of the transfer function is zero. These s_1, s_2 ... s_n are called the zeros of the transfer function.

Multiple Poles and Multiple Zeros

The poles s_a, s_b ... s_m or the zeros s_1, s_2 ... s_n are either real or complex and the complex poles or zeros always appear in conjugate pairs.

It is possible that either poles or zeros may coincide; such poles or zeros are called multiple poles or multiple zeros.

Simple Poles and Simple Zeros

Non-coinciding poles or zeros are called simple poles or simple zeros. From the transfer function expression, it is observed that

- If $n > m$, then the value of transfer function is found to be infinity for $s = \infty$, Hence, it is concluded that there exists a pole of the transfer function at infinity (∞) and the multiplicity (order) of such a pole being $(n-m)$.
- If $n < m$, then the value of transfer function is found to be zero for $s = \infty$. Hence, it is concluded that there exists a zero of the transfer function at infinity (∞) and the multiplicity (order) of such a zero being $(m-n)$.

  Therefore, for a rational transfer function the total number of zeros is equal to the total number of poles.

  The transfer function of a system is completely specified in terms of its poles, zeros and the gain factor.

Consider the following transfer function:

$$G(s) = \frac{s+3}{(s+2)(s+1+3j)(s+1-3j)}$$

For the above transfer function, the poles are at

(a) $s_a = -2$ (b) $s_b = -1 - 3j$ and (c) $s_c = -1 + 3j$

The zeros are at $s_1 = -3$.

As the number of zeros should be equal to number of poles, the remaining two zeros are located at $s = \infty$. The pole-zero plot is plotted as shown:

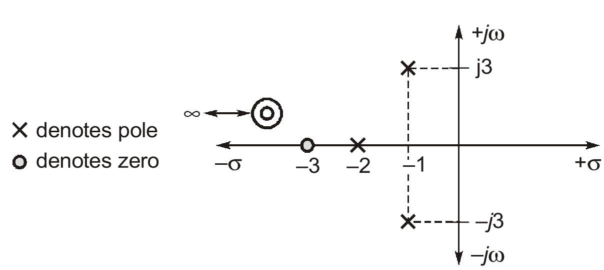


Figure-2.1 : *Pole-zero plot*

Poles and zero are those complex/critical frequencies which make the transfer function infinity or zero.

Proper Transfer Functions

The transfer functions are said to be strictly proper if the order of the denominator polynomial is greater than that of the numerator polynomial (i.e. $m > n$). If $m = n$, the transfer function is called proper. The transfer function is improper if $n > m$.

In the transfer function expression of a control system, the highest power of s in the numerator is generally either equal to or less than that of the denominator.

Example-2.4 A transfer function has two zeros at infinity. Then the relation between the numerator degree (N) and the denominator degree (M) of the transfer function is

(a) $N = M + 2$ (b) $N = M - 2$
(c) $N = M + 1$ (d) $N = M - 1$

Solution: (b)

For a rational transfer function the total number of zeros are equal to total number of poles.

Therefore, Number of poles $= M$; Number of zeros $= N + 2$

For a rational transfer function: $M = N + 2$ or $N = M - 2$

2.4 Properties of Transfer Function

The properties of the transfer function are summarized as follows:

1. The transfer function is defined only for a linear time-invariant system. It is not defined for non-linear or time variant systems.
2. The transfer function between an input variable and an output variable of a system is defined as the Laplace transform of the impulse response. Alternately, the transfer function between a pair of input and output variables is the ratio of the Laplace transform of the output to the Laplace transform of the input.
3. All initial conditions of the system are set to zero.
4. Transfer function is independent of the input of the system.
5. The transfer function of a continuous-data system is expressed only as a function of the complex variables. It is not a function of the real variable, time, or any other variable that is used as the independent variable or discrete-data system modelled by difference equations, the transfer function is a function of Z, when the Z-transform is used.
6. If the system transfer function has no poles or zeros with positive real parts, the system is a minimum phase system.
   Non-minimum phase functions are the functions which have poles or zeros on right hand side of s-plane.
7. The stability of a time-invariant linear system can be determined from its characteristic equation.
   Characteristic equation: The characteristic equation of a linear system is defined as the equation obtained by setting the denominator polynomial of the closed loop transfer function to zero.

Example-2.5 State and explain minimum phase and non-minimum phase transfer functions with examples.

Solution:

Minimum phase transfer function:

⇒ Transfer functions which have all poles and zeros in the left half of the s-plane i.e. system having no poles and zeros in the RHS of the s-plane are minimum phase transfer functions.

⇒ On the otherhand, a transfer function which has one or more zeros in the right half of s-plane is known as "non-minimum phase transfer function".

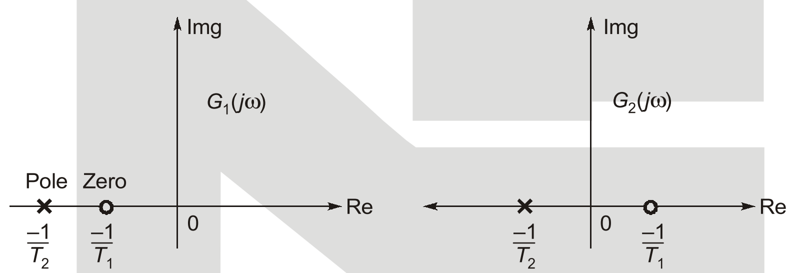
Let $$G_1(s) = \frac{1+sT_1}{1+sT_2}$$

⇒ $$G_1(j\omega) = \frac{1+j\omega T_1}{1+j\omega T_2} \quad \text{...(i)}$$

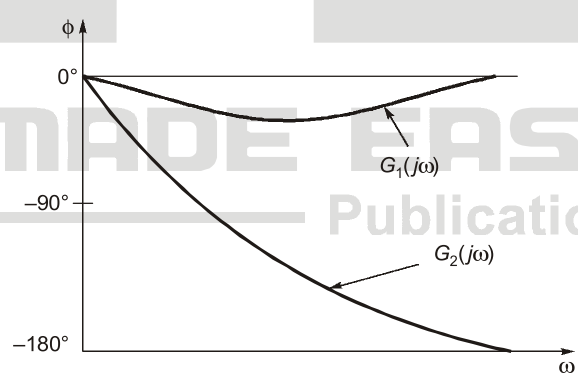
and $$G_2(j\omega) = \frac{1-j\omega T_1}{1+j\omega T_2} \quad \text{...(ii)}$$

The transfer function given by equation (i) represents the minimum-phase transfer function and equation (ii) represents the non-minimum phase transfer function .

⇒ The pole-zero configuration of above transfer function as given by equation (i) and (ii) may be drawn as:



⇒ The ***minimum phase function*** has unique relationship between its phase and magnitude curves. Typical phase angle characteristics are shown below:



⇒ It will be seen that larger the phase lags present in a system, the more complex are its stabilization problems. Therefore in control systems, elements with non minimum phase transfer function are avoided as far as possible.

⇒ A common example of a non-minimum phase system is "***transportation lag***" which has the transfer function,

$$G(j\omega) = e^{-j\omega T} = 1\angle -\omega T \text{ Radian}$$
$$= 1\angle -57.3\,\omega T \text{ degree}$$