

UPPSC-AE

2020

Uttar Pradesh Public Service Commission

Combined State Engineering Services Examination
Assistant Engineer

Electrical Engineering

Elements of Electrical Machines

Well Illustrated **Theory** with
Solved Examples and **Practice Questions**



MADE EASY
Publications

Note: This book contains copyright subject matter to MADE EASY Publications, New Delhi. No part of this book may be reproduced, stored in a retrieval system or transmitted in any form or by any means. Violators are liable to be legally prosecuted.

Electrical Machines

Contents

UNIT	TOPIC	PAGE NO.
1.	Electromagnetic System and General Concepts of Rotating Machines	1 - 25
2.	Transformers	26 - 68
3.	DC Machines	69 - 120
4.	Synchronous Machines	121 - 164
5.	3-Phase Induction Machines	165 - 207
6.	Special Machines	208 - 229



Electromagnetic System and General Concepts of Rotating Machines

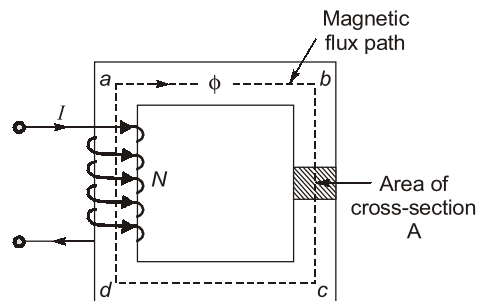
1.1 Introduction

The electromagnetic system is an essential element of all rotating electric machinery and electromechanical device and static devices like the transformer. Electromechanical energy conversion takes place via the medium of a magnetic field or electrical field, but most practical converters use magnetic field as the coupling medium between electrical and mechanical systems.

This chapter deals with the general concepts of mmf, emf and torque in rotating electrical machines. The basic emf and torque expressions are applicable to both DC and AC machines as the fundamental principles underlying their operation are same, but final forms of expressions differ only due to difference in constructional features.

1.2 Magnetic Circuits and Related Terminology

- **Magnetic circuits:** It is a complete closed path containing lines of magnetic flux which is usually generated either by permanent magnet or electromagnets.



(Magnetic circuit with N -turns and current I)

Magnetomotive Force (MMF)

- In a magnetic circuit, the magnetic flux is due to the presence of a magnetomotive force same as in an electric circuit, the current is due to the presence of an electromotive force.
- The mmf is created by a current flowing through one or more turns.

$$\text{MMF} = \text{Current} \times \text{Number of turns in the coil}$$

$$\text{i.e., } I = \text{MMF} = NI \text{ (ampere-turns) or (ATs)}$$

Reluctance (R_l)

Opposition offered by the magnetic circuit to magnetic flux is called **reluctance**.

$$R_l = \frac{l}{\mu A} \text{ AT/Wb}$$

where, l = length of the magnetic path
 A = area of cross-section normal to flux path, m^2
 $\mu = \mu_0 \mu_r$ = permeability of the magnetic material
 μ_r = relative permeability of the magnetic material
 μ_0 = permeability of free space = $4\pi \times 10^{-7}$ H/m.

Magnetic Flux (ϕ)

- The magnetic flux may be defined as the magnetomotive force per unit reluctance.

$$\phi = \frac{\text{MMF}}{\text{Reluctance}} \quad \text{or} \quad \phi = \frac{N \cdot I \cdot \mu_0 \mu_r \cdot A}{l} \text{ Wb}$$

- Direction of magnetic flux produced by coil can be found by right-hand grip rule.

Permeance (P)

Reciprocal of reluctance is called permeance.

$$\text{Permeance} = \frac{1}{\text{Reluctance}} = \frac{\mu_0 \mu_r A}{l} \text{ Wb/AT}$$

Magnetic Field Density (B)

It is defined as the magnetic flux per unit cross-sectional area of the core.

$$B = \frac{\text{Magnetic flux, } \phi}{\text{core area, } A} = \frac{NI \cdot \mu_0 \mu_r}{l} \text{ Wb/m}^2$$

Magnetic Field Intensity (H)

The magnetomotive force per unit length of magnetic circuit is termed as the magnetic field intensity.

$$H = \frac{\text{MMF}}{\text{mean length of magnetic circuits}} \quad \text{or} \quad H = \frac{NI}{l} \text{ ATs/m}$$



NOTE

- In magnetic system there are no magnetic insulators. Even in the best known magnetic insulator air, the flux can be established.
- Energy is needed for establishing the required flux once the requisite flux is created then no more energy is needed in maintaining it.



Example - 1.1 A magnetic circuit has 180 turns coil and cross sectional area $5 \times 10^{-4} \text{ m}^2$ and length of magnetic circuit is 25 cm. The value of relative permeability for current 2A and flux of 0.35 mWb is:

- | | |
|------------|-------------|
| (a) 401.88 | (b) 1215.27 |
| (c) 386.83 | (d) None |

Solution : (c)

Given,

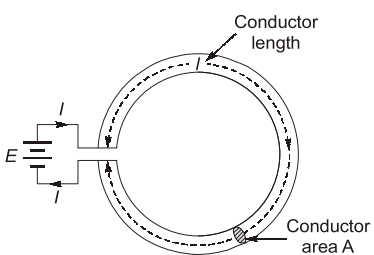
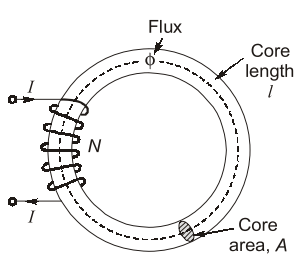
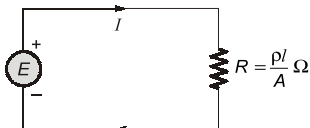
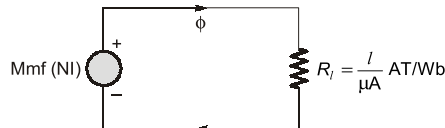
$$N = 180, A = 5 \times 10^{-4} \text{ m}^2, l = 25 \text{ cm} = 25 \times 10^{-2} \text{ m}, I = 2\text{A}, \phi = 0.35 \times 10^{-3} \text{ Wb}$$

$$\therefore \phi = \frac{\text{mmf}}{\text{Reluctance}} = \frac{NI \mu_0 \cdot \mu_r \cdot A}{l}$$

$$\Rightarrow 0.35 \times 10^{-3} = \frac{180 \times 2 \times 4\pi \times 10^{-7} \times \mu_r \times 5 \times 10^{-4}}{25 \times 10^{-2}}$$

$$\Rightarrow \mu_r = 386.83$$

1.3 Comparison Between Magnetic and Electric Circuits

Comparison between Magnetic and Electric Circuits	
Electrical Circuit	Magnetic Circuit
 <p>A toroidal copper ring of length l, cross-sectional area A is connected to emf E so that current I flows.</p>	 <p>A toroidal iron ring of length l, cross area A is excited by a coil of N turns carrying I amperes so that flux ϕ is produced.</p>
<p>Similarities:</p> <ol style="list-style-type: none"> 1. Closed path for electrical current is called an electric circuit. 2. Driving force is emf E, volts 3. Resistance, $R = \frac{\rho \cdot l}{A}$ V/A or Ω 4. Equivalent circuit,  <ol style="list-style-type: none"> 5. Circuit, $I = \frac{\text{Driving force}}{\text{Resistance}} = \frac{E}{R} A$ 6. Current density, $J = \frac{I}{A}$ A/m² 7. Electric field intensity, $\epsilon = \frac{E}{l}$ V/m Also, $\epsilon = \frac{E}{l} = \frac{IR}{l} = \frac{I}{l} \cdot \frac{\rho l}{A} = \rho \cdot \frac{I}{A} = \rho \cdot J$ V/m 8. Conductivity of current path, $\sigma = 1/\rho$ So that, $J = \sigma \epsilon A/m^2$ <p>Dissimilarities:</p> <ol style="list-style-type: none"> 1. The electrical current actually flows in an electric circuit. For the existence of this current, energy is drawn from the source continuously. This energy gets dissipated in resistance in the form of heat. 2. Electrical insulator confine the current to well defined paths. 	<p>Similarities:</p> <ol style="list-style-type: none"> 1. Closed path for the magnetic flux is called a magnetic circuit. 2. Driving force is Mmf = NI ATs 3. Reluctance, $R_l = \frac{l}{\mu A}$ AT/Wb 4. Equivalent circuit,  <ol style="list-style-type: none"> 5. Magnetic flux, $\phi = \frac{\text{Driving force}}{\text{Reluctance}} = \frac{\text{Mmf}}{R_l}$ Wb 6. Magnetic flux density, $B = \frac{\phi}{A}$ T (or Wb/m²) 7. Magnetic field intensity, $H = \frac{NI}{l}$ AT/m Also, $H = \frac{NI}{l} = \frac{\text{Mmf}}{l} = \frac{\phi \cdot R_l}{l} = \frac{\phi}{l} \cdot \frac{l}{\mu A} = \frac{1}{\mu} \cdot \frac{\phi}{A} = \frac{1}{\mu} \cdot B$ AT/m 8. Conductivity of current path, μ So that, $B = \mu H$ Wb/m² or T <p>Dissimilarities:</p> <ol style="list-style-type: none"> 1. Strictly speaking, magnetic flux does not flow. Energy is needed for establishing the required flux. Once the requisite flux is created, no more energy is needed in maintaining it. 2. There are no magnetic insulators. Even in the best known magnetic insulator air, the flux can be established.



Example - 1.2 Find the strength of magnetic field in a conductor 0.5 m long moving with velocity of 10 m/s and inducing an emf of 25 V and also magnetic field, velocity and length of conductor are mutually perpendicular to each other

- (a) 0 T (b) 5 T
 (c) 2 T (d) 10 T

Solution: (b)

For B , l , v to be perpendicular to each other, induced emf is given as

$$e = B l v$$

$$\therefore 25 = B \times 0.5 \times 10$$

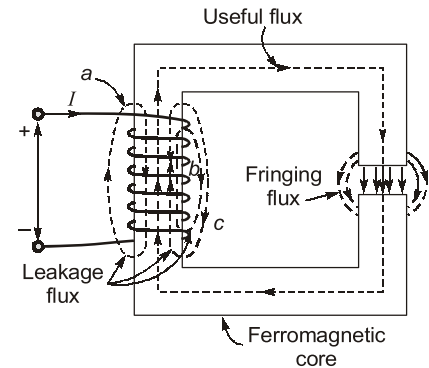
$$\Rightarrow B = 5 \text{ T}$$

1.4 Concept of Leakage Flux and Fringing

Leakage Flux

In Ideal magnetic circuits, all the flux produced by an exciting coil is confined to the desired magnetic path of negligible reluctance. But in practical magnetic circuits, a small amount of flux does follow a path through the surrounding air. Therefore, leakage flux may be defined as that flux which does not follow the intended path in a magnetic circuit.

Leakage flux does exist in all practical ferromagnetic device. Its effect on the analysis of electrical machinery is carried out by replacing it by an equivalent leakage reactance.



Fringing

At an air-gap in a magnetic core, the flux fringes out into neighboring air path as shown in the given. Longer the air gap, more is the flux fringing.

The effect of fringing flux is to increase the effective cross-sectional area of the air gap. As a result, flux density in the air gap is not uniform and average flux density gets reduced,

$$\therefore B = \frac{\phi}{A}$$

If area of air gap increases then total area of core with consideration of air gap increases. Then average flux density gets reduced.



NOTE

- Longer the air gap, more is the flux fringing.
- Fringing flux reduces the average flux density.
- Leakage factor = $\frac{\text{Total flux handled by exciting winding}}{\text{Useful flux}}$

1.5 Induced emf

- Faraday's law of electromagnetic induction states that an e.m.f is induced in a coil when the magnetic flux linking this coil change with time.

- It is expressed as, $e = \frac{d\Psi}{dt} = N \frac{d\phi}{dt}$
 where, $e =$ e.m.f. induced in volts
 $N =$ Number of turns in the coil
 $\Psi = N\phi =$ Flux linkages with the coil, Wb-turns
 $t =$ time (seconds)

Lenz’s Law

- When the coil circuit is closed, a current begins to flow in the coil. The direction of this induced emf, or induced current, is governed by Lenz’s law. According to this law; the induced current develops a flux which always opposes the change responsible for inducing this current.

$$e = -\frac{d\Psi}{dt} = -N \frac{d\phi}{dt}$$

- Lenz’s law is based on the law: For every action, there is always an equal and opposite reaction. Thus in short, Lenz’s law is effect opposes the cause.

Motional emf

- Flux cutting action occurs due to relative movement of coil and flux and the emf so induced is known as motional emf or speed emf.
- This principle is used in DC machines and synchronous machines.
- It is also called dynamically induced emf.

The e.m.f. induced ‘e’ by a conductor of length ‘l’ metres cutting a flux of density ‘B’ webers per square metre at a velocity ‘v’ metres per second is given by

$$e = B l v \text{ volts}$$

Provided that B, l and v are mutually perpendicular. If not, B l v must be multiplied by the sine of the angle between any two of the three quantities, B, l and v.



Example - 1.3 The emf induced in a conductor of machine driven at 800 rpm for the parameters given below is

$B_{\text{peak}} = 2 \text{ Wb/m}^2$, diameter of machine = 2.5 m, length of machine = 35 cm

- (a) 64.85 V
- (b) 73.33 V
- (c) 68.51 V
- (d) 90.5 V

Solution : (b)

$$\text{Area, } A = 2\pi r l = 2\pi \times \frac{D}{2} \times l = \pi D l$$

$$A = 3.14 \times 2.5 \times 0.35 = 2.75 \text{ m}^2$$

Flux, $\phi = B \cdot A = 2 \times 2.75 = 5.5 \text{ Wb}$

induced emf, $e = \frac{N\phi}{T}$

$$= 5.5 \times \frac{800}{60} = 73.33 \text{ V}$$

1.6 Principle of Energy Conversion

- According to this principle, energy can neither be created nor destroyed, it can merely be converted from one form into another.
- In an energy conversion device, some energy is converted into the required form, some energy is stored and the rest is dissipated.
- Energy conversion process is basically a reversible process.
- Operating principles of energy conversion devices are similar, but their structural details differ depending upon their function.
- Coupling between the electrical and mechanical systems of devices is through magnetic or electric field.

Energy Balance Equation

- **For a electrical motor:**

(Total electrical energy Input) = (Mechanical energy output) + (Total energy stored) + (Total energy dissipated)

$$W_{ei} = W_{mo} + (W_{es} + W_{ms}) + (\text{ohmic energy losses} + \text{coupling field energy losses}) + (\text{Energy losses in mechanical system})$$

where, W_{ei} = Total electrical energy input from the supply mains.

W_{mo} = The mechanical energy output

$W_{es} + W_{em}$ = Energy stored in magnetic field + Energy stored in mechanical system

= Total energy stored in any device and Total energy dissipated

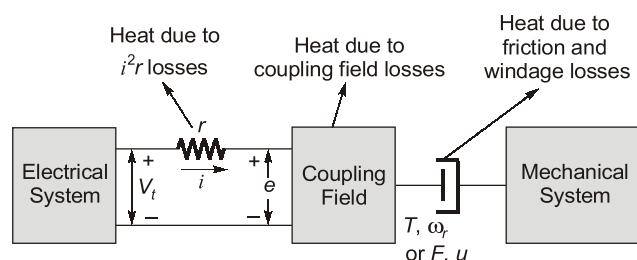
= Energy dissipated in electric circuit as ohmic loss + Energy dissipated as magnetic core loss (hysteresis and eddy-current losses) + Energy dissipated in mechanical system (Friction and windage losses etc.)

- **For generator action:**

Total Mechanical energy input = (Electrical energy output) + (Total energy stored) + (Total energy dissipated)

- $(W_{ei} - \text{ohmic energy losses}) = (W_{mo} + W_{ms} + \text{Mechanical energy losses}) + (W_{es} + \text{Coupling field energy losses})$

- $W_{elec} = W_{mech} + W_{fld}$



(General representation of electromechanical energy conversion device)



Example-1.4 A physical system of electromechanical energy conversion, consists of a stationary part creating a magnetic field with electrical energy input and a moving part is kept fixed, the entire electrical energy input will be;

- (a) stored in the magnetic field
- (b) stored in the electric field
- (c) divided equally between both the fields
- (d) zero

Solution:(a)

∴

where,

$$W_{elec} = W_{mf} + W_{mech}$$

$$W_{mech} = \text{mechanical energy output}$$

$$W_{elec} = \text{electrical energy input}$$

$$W_{mf} = \text{energy stored in magnetic field}$$

As movable part is fixed, hence

$$W_{mech} = 0$$

Thus

$$W_{elec} = W_{mf}$$

1.7 General Terms in Rotating Machines

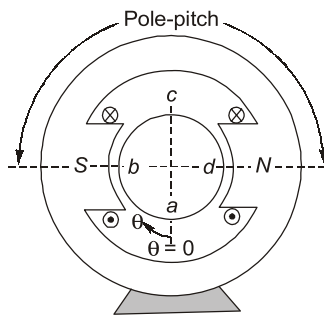


Fig. (a): Elementary two pole machine

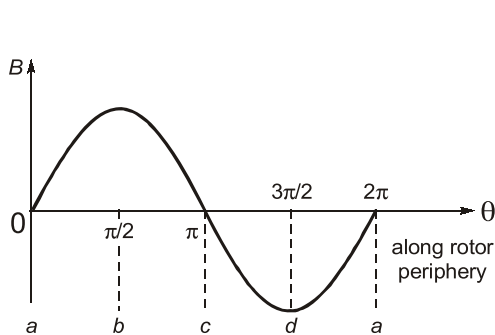


Fig. (b): Flux density variation along air-gap periphery

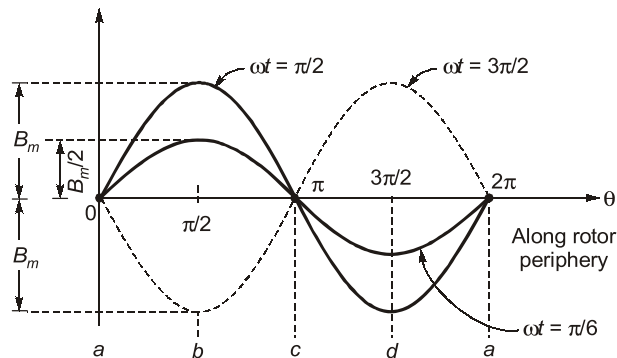


Fig. (c): Pulsating flux

Consider the figure (a), it shows an elementary two-pole machine, with its 2 field coils excited by direct current. The flux density at the point a in between the 2 poles will be zero. Under the centre of the pole indicated by point b, the flux density would be maximum positive; at c it is zero and at point 'd' it is again maximum but negative.

South pole on the stator or north pole on the rotor, produces positive flux density.

1.7.1 Electrical And Mechanical Degree

- For a P -pole machine, $\frac{P}{2}$ cycles of emf will be generated in one revolution.

$$\theta_{\text{elect}} = \frac{P}{2} \theta_{\text{mech}}$$

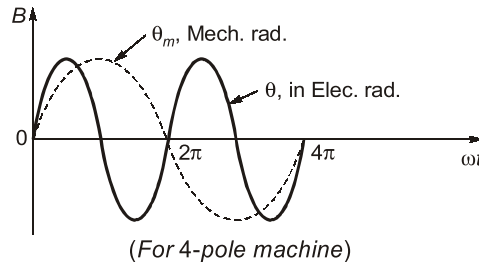
or

$$\omega_e = \frac{P}{2} \omega_m$$

where ω_e is the angular speed in electrical radians per second and ω_m is the angular speed in mechanical radians per second.

- If the speed N is in rpm, then

$$f = \frac{PN}{120} \text{ Hz}$$



Example - 1.5 If the torque angle of a 4-pole synchronous motor is 8 degree electrical, its value in mechanical angle is

- 4 degree
- 2 degree
- 0.5 degree
- 0.25 degree

[UPPSC]

Solution: (a)

Given,

$$P = 4$$

$$\theta_{\text{elec}} = 8 \text{ degree}$$

\therefore

$$\theta_{\text{elec}} = \frac{P}{2} \theta_{\text{mech}}$$

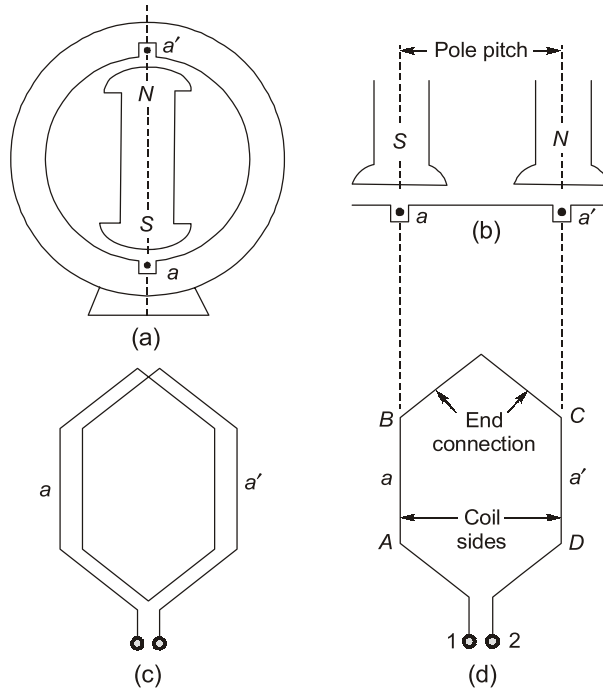
\Rightarrow

$$\theta_{\text{mech}} = \frac{2\theta_{\text{elec}}}{P} = \frac{2 \times 8^\circ}{4} = 4^\circ$$

1.7.2 Coil Pitch and Pole Pitch

Coil

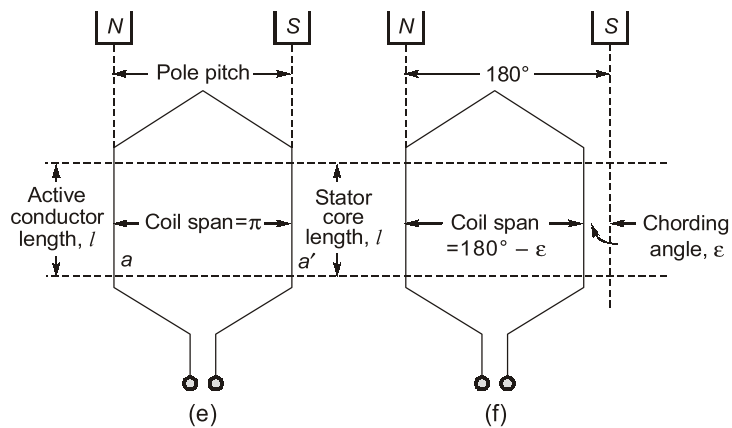
- Consider a 2 pole machine with one coil $a - a'$. The emf is generated in active lengths AB and CD only. These active lengths are called the two coil-sides of a coil.
- One turn consists of two conductors and one coil is made up of two coil sides.



Pole Pitch

- The peripheral distance between two adjacent poles is called pole pitch. It is always expressed in electrical degrees and pole pitch is always equal to 180 electrical degree or π -electrical radians.
- A coil with two coil sides 180° (electrical) space degree apart (or one pole pitch apart) is called a full pitch coil.

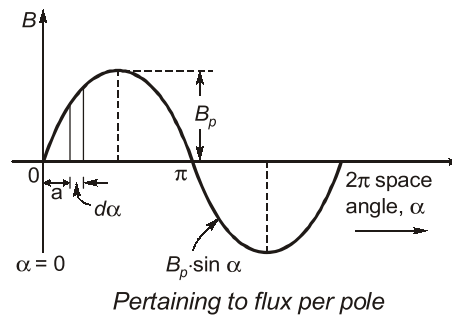
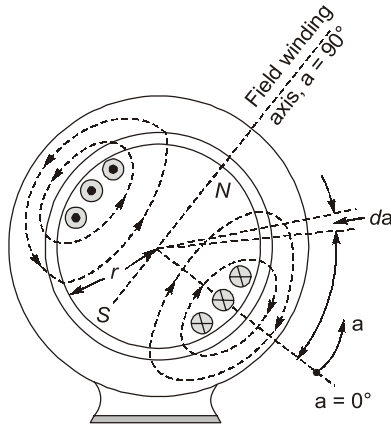
Coil Pitch



- Coil-span (or coil-pitch) is defined as the distance between the two coil-sides of one coil. Coil-span is measured in terms of electrical degrees, coil-sides or slots.
- Coil-span is less than 180°; this coil is, therefore, called a short-pitch, or chorded coil.
- Chording angle (ϵ) is defined as the angle by which coil-span departs from 180° electrical space degrees.

**NOTE**

- If chording angle (ϵ), then coil span = $180^\circ - \epsilon$
- For full pole pitch, coil span = 180°

1.7.3 Flux Per Pole

- Consider figure shown above, where the field windings are taken on the rotor.
- When $\alpha = 0^\circ$, the flux density B is zero, when $\alpha = 90^\circ$, B is maximum say B_p , when $\alpha = 180^\circ$, B is again zero.
- The flux density B can be expressed as,

$$B = B_p \cdot \sin \alpha$$

- For a 2-pole machine, flux per pole = $2 \cdot B_p \cdot l \cdot r$
where,
 l = axial length of armature core
 r = radius of armature core

- For a P -pole machine, flux per pole is given by $\frac{4}{P} B_p \cdot l \cdot r$

- Flux per pole (ϕ) can be written as

$\phi = (\text{Average value of constant-amplitude flux density wave under one pole}) \times (\text{Area pertaining to one pole of the flux density wave})$

$$= (B_{av}) \left(\frac{2\pi r l}{P} \right)$$

$$\therefore B_{av} = \frac{2}{\pi} B_p$$

$$\text{Total flux per pole} = \left(\frac{2}{\pi} B_p \right) \cdot \left(\frac{2\pi r l}{P} \right) = \frac{4}{P} B_p \cdot r l$$



Example-1.6 In a 4 pole machine, what is the flux per pole produced, if the armature length is l and radius is r and B_p is peak value of sinusoidal flux?

- | | |
|-----------------------------|-------------------------------------|
| (a) $4 B_p \cdot l \cdot r$ | (b) $2 B_p \cdot l \cdot r$ |
| (c) $B_p \cdot l \cdot r$ | (d) $\frac{B_p \cdot l \cdot r}{2}$ |

Solution : (c)

$$\therefore \text{Total flux per pole} = \frac{4}{P} B_p \cdot r \cdot l$$

Here $P = 4$

Hence $\phi_p = \frac{4}{4} \cdot B_p \cdot l \cdot r = B_p \cdot l \cdot r$



Example - 1.7 A pole pitch in electrical machines is:

- (a) = 180° electrical
- (b) > 180° electrical
- (c) = 180° mechanical
- (d) < 180° electrical

[UPPSC]

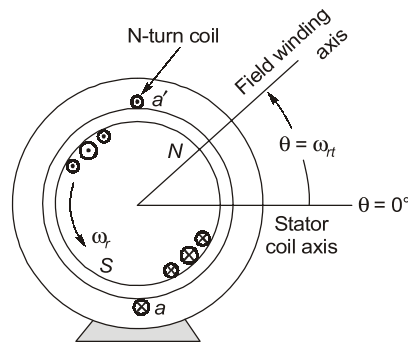
Solution : (a)

Pole pitch is always equal to 180 electrical degree. It's always expressed in electrical degrees.

1.8 Generated EMF

In rotating electrical machines, emfs can be generated in armature windings:

- By rotating these winding through a magnetic field
- By rotating the magnetic field with respect to these windings
- By designing the magnetic circuit to have variable reluctances with rotor rotation.



(Field poles on cylindrical rotor)

1.8.1 Generated emf in Full Pitched Coil

- Let the variation of flux passing through the coil be written as $\phi \cos \omega_r t$.
- The flux linkages ψ with the full-pitch N-turn armature coil, at any time t .

$$\begin{aligned} \psi &= N (\text{flux passing through the coil at any time } t) \\ &= N \phi \cos \omega_r t \end{aligned}$$

- By Faraday's law, the emf induced in N-turn armature coil.

$$e = N \omega_r \phi \sin \omega_r t$$

- If the single N-turn coil belongs to AC machines, then the maximum value of the generated emf.

$$E_{\max} = \omega_r N \phi = 2\pi f_r N \phi$$

- The rms value of the generated emf in a full pitched coil.

$$E = \frac{E_{\max}}{\sqrt{2}} = \sqrt{2} \pi f_r N \phi = 4.44 f_r N \phi$$

Here f_r may be called the rotational or speed frequency.

- The magnitude of rotational frequency,

$$f_r = \frac{P n_r}{2} \text{ Hz}$$

where n_r is the relative speed in rps between the armature coil and flux-density wave.

- Induced emf in armature coil

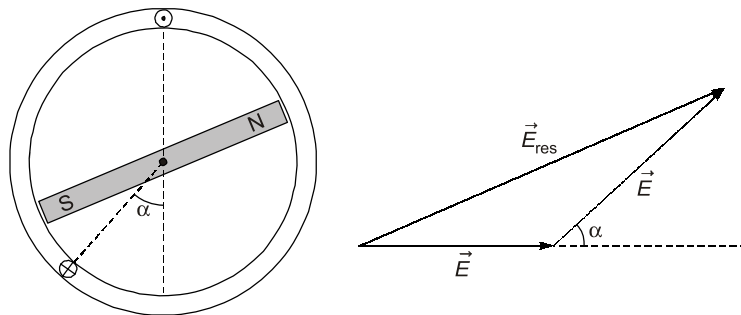
$$e = N \omega_r \phi \cos\left(\omega_r t - \frac{\pi}{2}\right)$$

or,

$$e = E_{\max} \cos\left(\omega_r t - \frac{\pi}{2}\right)$$

1.8.2 Generated emf in Short Pitched Coil

- The distance between the two sides of a coil is called the coil span or coil pitch. The angular distance between the central line of one pole to the central line of the next pole is called pole pitch. A pole pitch is always 180 electrical degrees regardless of the number of poles on the machine. A coil having a span equal to 180° electrical is called a full-pitch coil.
- A coil having a span less than 180° electrical is called a short-pitch coil, or fractional-pitch coil. It is also called a chorded coil. A stator winding using fractional-pitch coils is called a chorded winding. If the span of the coil is reduced by an angle α electrical degrees, the coil span will be $(180 - \alpha)$ electrical degrees.



- Effect of short pitched coil is to reduce the generated emf by a factor $\cos\left(\frac{\epsilon}{2}\right)$, which is referred to as coil pitch factor or coil span factor (K_p).
- Pitch factor, $K_p = \cos\left(\frac{\epsilon}{2}\right)$.
- Hence rms value of generated emf in a short pitched N -turn armature coil of an AC machine is

$$E = \sqrt{2} \pi f K_p \cdot N \phi$$

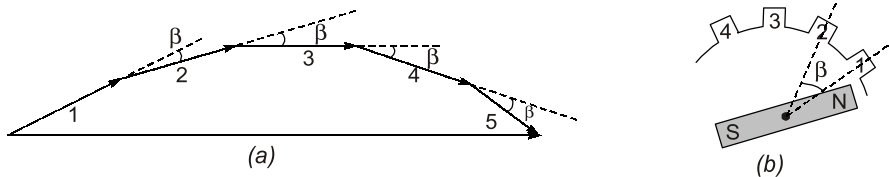


NOTE

- Output voltage is reduced by $\cos \epsilon/2$, when using short-pitch coil. In order to compensate this reduction in output voltage, more turns and therefore more copper is essential.
- Since the advantage of reducing the higher order harmonics in the e.m.f wave is much more important than the disadvantage of higher initial cost, the use of chorded coils is justified.
- Advantages of fractional pitch winding are:
 - (i) Reduction in copper for overhang and thereby reduces cost.
 - (ii) Reduction of harmonics in generated emf wave.

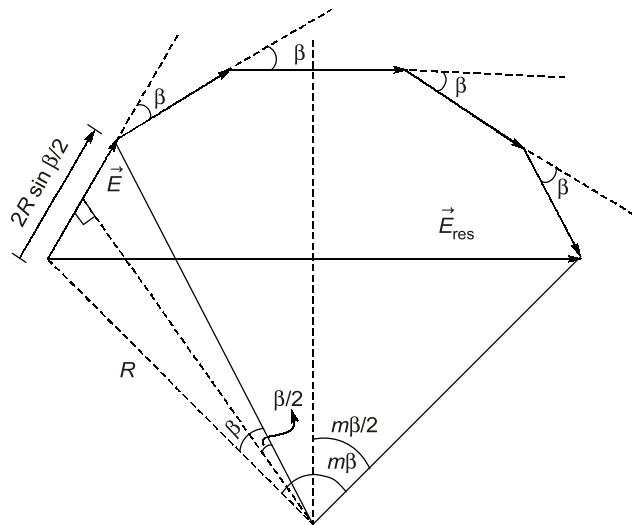
1.8.3 Distributed or Spread Winding

- In a concentrated winding, the coil sides of a given phase are concentrated in a single slot under a given pole. The individual coil voltage induced are in phase with each other.
- These voltages may be added arithmetically. In order to determine the induced voltage per phase, a given coil voltage is multiplied by the number of series-connected coils per phase, In actual practice, in each phase, t concentrated in a single slot, but are distributed in a number of slots in space to form a polar group under each pole. The voltages induced in coil sides constituting a polar group are not in phase but differ by an angle equal to the angular displacement β of the slots. The total voltage induced in any phase will be the phasor sum of the individual coil voltages.



$$\beta = \text{angle between adjacent slots} = \frac{360^\circ}{\text{Total number of slots}} \text{ mechanical}$$

$$\beta = \frac{P}{2} \times \frac{360^\circ}{\text{Number of slots}} \text{ electrical} = \frac{180^\circ}{\text{Number of slots/pole}}$$



$$m = (\text{Number of slots /pole})/\text{phase}$$

$$\left| \vec{E}_{\text{res}} \right|_{\text{distributed winding}} = 2R \sin \frac{m\beta}{2}$$

$$\left| \vec{E}_{\text{res}} \right|_{\text{concentrated winding}} = m \times 2R \sin \frac{\beta}{2}$$

∴ Distribution/spread/ breath/ belt factor (K_d),

$$K_d = \frac{\left| \vec{E}_{\text{res}} \right|_{\text{distributed winding}}}{\left| \vec{E}_{\text{res}} \right|_{\text{concentrated winding}}} = \frac{\text{Phasor sum of coil voltages per phase}}{\text{Arithmetic sum of coil voltages per phase}}$$

From (i) and (ii),

$$K_d = \frac{2R \sin \frac{m\beta}{2}}{m \times 2R \sin \frac{\beta}{2}}$$

$$K_d = \frac{\sin \frac{m\beta}{2}}{m \sin \frac{\beta}{2}}$$

For n^{th} harmonic,

$$K_d = \frac{\sin \frac{nm\beta}{2}}{m \sin \frac{n\beta}{2}}$$

So, $E_{ph} = K_c K_d \sqrt{2} \pi f \phi N_{ph}$ V/phase

Actual voltage generated, $E_{ph} = K_w \sqrt{2} \pi f \phi N_{ph}$ V/phase, $K_w =$ winding factor $= K_c K_d$

Approximate K_d :

$$K_d = \frac{\sin \frac{m\beta}{2}}{m \sin \frac{\beta}{2}}$$

For uniformly distributed winding,

∴ $\frac{\beta}{2}$ is very small $\rightarrow \sin \frac{\beta}{2} \simeq \frac{\beta}{2}$ electric radian, ($\beta < 15^\circ$).

So, $K_d \simeq \frac{\sin \frac{m\beta}{2}}{\frac{m\beta}{2}}$, $m\beta =$ phase spread

- **Phase Spread:** The peripheral angular distance of a phase belt is called its phase spread $\alpha (= m\beta)$ in electrical space degrees.
- **Winding Factor:** The product of distribution factor k_d and the pitch factor k_p is referred as the winding factor.

$$k_w = k_d \times k_p$$

where,

- $k_w =$ winding factor,
- $k_d =$ distribution factor
- $k_p =$ pitch factor



NOTE

Advantages of distributing the windings in slots are:

- Full utilization of armature iron and copper.
- Reduction of harmonics in the generated emf wave.
- Adding rigidity and mechanical strength to the winding.

1.8.4 EMF Polygon

- Angle γ is called angular slot pitch and is given by

$$\gamma = \frac{\pi P}{S} \text{ electrical (radians)}$$

or

$$\gamma = \frac{180P}{S} \text{ electrical (degrees)}$$

where P is number of poles and S is total number of slots.

1.8.5 Elimination of Harmonics from emf Waveforms

Although harmonics emfs are always generated in an alternator and these can be however eliminated or suppressed as follows:

- **Distribution:** The distribution of the armature winding along the air-gap periphery tends to make the e.m.f waveform sinusoidal.
- **Chording:** with coil-span less than pole pitch, the harmonics can be eliminated.
- **Skewing.** By skewing the armature slots, only tooth harmonics or slot harmonics can be eliminated.
- **Fractional slot winding.** In fractional-slot windings, the space relation between teeth and slots under a given face is not the same as under the next and the succeeding pole faces.
- Star or delta connections of alternators suppress triplen harmonics from appearing across the lines.



NOTE

- Line emf in star connection does not include triplen harmonics.
- In an alternator, the output emf wave can't contain even harmonics.



Example - 1.8 EMF equation, $e = N\omega_r \phi \sin \omega_r \cdot t$ is applicable to

- AC systems with time variant field flux
- DC systems with time variant field flux
- Both AC and DC systems with time invariant field flux
- Both AC and DC systems with time variant field flux

Solution : (c)

$$\therefore e = \frac{Nd\phi(t)}{dt} \text{ where } \phi(t) = \phi \sin \omega_r t$$

$$\therefore e = N\omega_r \phi \sin \omega_r t = N \left(\frac{d\phi}{dt} \right) (\cos(\omega_r t))$$

For time invariant flux, $\frac{d\phi}{dt} = 0$

$\Rightarrow e = N\omega_r \phi \sin (\omega_r t)$ is general equation and is applicable to both AC and DC systems.