Production & Industrial Engineering

Operations Research and Operations Management

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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Operations Research and Operations Management

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INTRODUCTION

One of the major issues in the analysis of any traffic system is the analysis of delay. Delay is a more subtle concept. It may be defined as the difference between the actual travel time on a given segment and some ideal travel time of that segment. This raises the question as to what is the ideal travel time. In practice, the ideal travel time chosen will depend on the situation; in general, however, there are two particular travel times that seem best suited as benchmarks for comparison with the actual performance of the system. These are the travel time under free flow conditions and travel time at capacity.

Most recent research has found that for highway systems, there is comparatively little difference between these two speeds. That being the case, the analysis of delay normally focuses on delay that results when demand exceeds its capacity; such delay is known as queuing delay, and may be studied by means of queuing theory. This theory involves the analysis of what is known as a queuing system, which is composed of a server; a stream of customers, who demand service; and a queue, or line of customers waiting to be served.

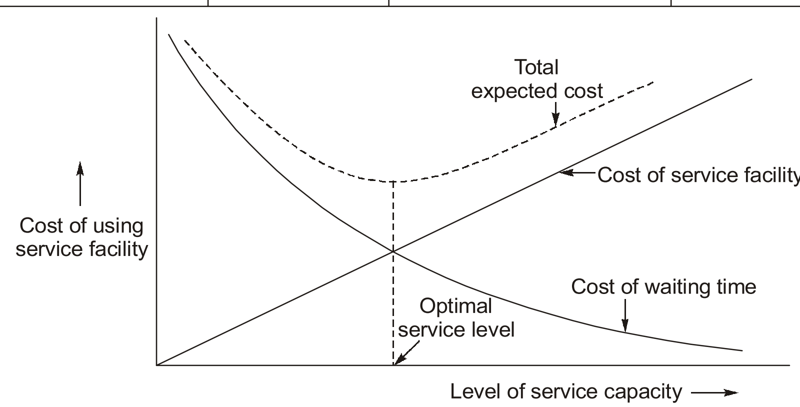
2.1 Issues involved in Waiting Line

The most important issue in waiting line problem is to decide the best level of service that the organisation should provide. For example, to cope up with the railway reservation queue, how many counters must be opened? If the counters are too less, there will be very long queue, resulting in long waiting time. This results in dissatification among the customers. However, if the service counters are too many the counters may remain unoccupied for quite some time. This would result in loss to the service organisation. An important issue to understand in queuing problem is about arrival pattern. Generally, the arrival of customers is random, which may be governed by a probability distribution. Besides this, the arrival may also be governed by hours of a day, season, days of month, etc. For example, the arrival at railway reservation booth is more at morning hours as compared to afternoon. Similarly, longer queues may be observed at rail reservation counters when summer vacation are or puja vacation are due. The management may open more or less service counters, depending upon the arrival; but each extra counter means additional cost in running this service. Use of computers in processing data and updating files at the service counter is now very common. The reason is simple. Computer is very fast, accurate, programmable, good in storage and retrieval features for data, and consistent. A reduced and dependable service time is what customer and management want.

The key issue in waiting line is to provide a compromise between good service (by less service time) and less cost in running the service points. Figure 2.1 illustrates that the total expected cost of service, which is the sum of providing service and cost of waiting time, is minimum at certain service level. This service level should be optional service level.

Table. Some Applications of Waiting Line Problem

S. No.	Application Area	Arrival	Waiting Line	Service Facility
1	Factory	Material/tools	In-process inventory (WIP)	Work stations
2	Assembly line	Sub-assemblies	WIP	Employees currently processing the WIP
3	Machine maintenance	Repair tools & equipment	Machines needing repair	Maintenance crew
4	Airport	Aeroplanes	Aeroplanes ready to fly	Runway
5	Bank	Customer	Deposit/withdrawal	Bank employee & computer
6	Walk-in-interview	Job seekers	Applicants	Interviewers
7	Phone exchange	Dialled number	Caller	Switchboard
8	Govt. office	Files	Backlog files	Clerks
9	Post office	Letters	Mailbox	Postal employees
10	Executive note	Dictation note	Letters to be typed	Secretary
11	Grocery shop	Customers	Customer on the counter	Checkout clerks and bag packers
12	Traffic light	Vehicles	Vehicles in line	Traffic signals crossing
13	Car service station	Cars	Unserviced cars	Service facilities
14	Railways	Passengers	Waiting passengers on platform/waiting room	Trains
15	Tool crib	Mechanics	Waiting mechanics	Store keeper
16	Hospital	Patients	Sick people	Doctor & operation facility



Waiting line cost and service levels

2.2 Characteristics of Queuing Process

One thing we have to remember is that when we speak of queue, we have to deal with two elements, i.e. Arrivals and Service facility. Entire queuing system can be completely described by:

(a) The input (Arrival pattern)

(b) The service mechanism or service pattern,

(c) The queue discipline and

(d) Customer behavior.

Components of the queuing system are arrivals, the element waiting in the queue, the unit being served, the service facility and the unit leaving the queue after service. This is shown below in figure.

$$= \frac{\lambda}{\mu} = \frac{7.5}{20} = 0.375$$

(ii) Average length of the queue (L_q)

$$= \frac{\rho^2}{1-\rho} = \frac{(0.375)^2}{1-0.375} = 0.225 = 0.225 \text{ person}$$

(iii) Probability that it will take him more than 12 minutes altogether to wait for the phone to complete his call

$$= e-(\mu-\lambda)\tau = e^{(7.5-20)\times\frac{12}{60}} = 0.082.$$

Student's Assignments 1

Q.1 As per queue discipline the following is not a negative behaviour of a customer:
(a) Balking (b) Reneging
(c) Boarding (d) Collusion.

Q.2 The expediting or follow up function in production control is an example of
(a) LIFO (b) FIFO
(c) SIRO (d) Pre emptive

Q.3 In M/M/S: N/FIFO the following does not apply
(a) Poisson arrival
(b) Limited service
(c) Exponential service
(d) Single server

Q.4 The dead bodies coming to a burial ground is an example of :
(a) Pure Birth Process
(b) Pure Death Process
(c) Birth and Death Process
(d) Constant rate of arrival

Q.5 The system of loading and unloading of goods usually follows:
(a) LIFO (b) FIFO
(c) SIRO (d) SBP

Q.6 A steady state exist in a queue if :
(a) $\lambda > \mu$ (b) $\lambda < \mu$
(c) $\lambda \le \mu$ (d) $\lambda \ge \mu$

Q.7 If the operating characteristics of a queue are dependent on time, then is said to be:
(a) Transient state (b) Busy state
(c) Steady state (d) Explosive state.

Q.8 A person who leaves the queue by losing his patience to wait is said to be:
(a) Reneging (b) Balking
(c) Jockeying (d) Collusion

Q.9 The characteristics of a queuing model is independent of :
(a) Number of service stations
(b) Limit of length of queue
(c) Service Pattern
(d) Queue discipline.

Q.10 The unit of traffic intensity is :
(a) Poisson (b) Markow
(c) Erlang (d) Kendall

Q.11 In (M/M/1) : (“/FCFS) model, the length of the system L_s is given by:
(a) $\rho^2/1-\rho$ (b) $\rho/1-\rho$
(c) $\lambda^2/(\mu-\lambda)$ (d) $\lambda^2/\mu(\mu-\lambda)$

Q.12 The queue discipline in stack of plates is :
(a) SIRO (b) Non-Pre-Emptive
(c) FIFO (d) LIFO

Q.13 SIRO discipline is generally found in :
(a) Loading and unloading
(b) Office filing
(c) Lottery draw
(d) Train arrivals at platform

Q.14 For a simple queue (M/M/1), $\rho = \lambda/\mu$ is known as:
(a) Poisson busy period
(b) Random factor
(c) Traffic intensity
(d) Exponential service factor

Q.15 If the number of arrivals during a given time period is independent of the number of arrivals that have already occurred prior to the beginning of time interval, then the new arrivals follow __________ distribution.

(a) Erlang (b) Poisson
(c) Exponential (d) Normal

Q.16 Arrival → Service → Service → Service → Out →
The figure given represents:

(a) Single Channel Single Phase system
(b) Multi channel single-phase system
(c) Single channel multi phase system
(d) Multi channel multi phase system

Q.17 When the operating characteristics of the queue system dependent on time, then it is said to be:

(a) Steady state (b) Explosive state
(c) Transient state (d) Any one of the above

Q.18 Traffic intensity is given by :

(a) Mean arrival rate/Mean service rate,
(b) $\lambda\mu$
(c) μ/λ
(d) Number present in the queue / Number served

Q.19 In Kenedall Notation - What does fourth turn represent?

(a) Number of server
(b) Queue discipline
(c) Queue length
(d) Arrival probability distribution

Q.20 For a telephone booth it is given that arrival rate is 6 person per hour with Poisson's distribution and it takes on an average 6 minute to leave a customer from telephone booth with exponential distribution. The expected number of customer in the queue is ______.

Q.21 The length of non-empty queue with Poisson's arrival rate, λ = 8 customer/hour and exponential service rate, μ = 10 customer/hour will be_____.

Q.22 For an arrival rate, λ = 6 customer per hour with Poisson's distribution and service time of 6 min per customer with exponential distribution, what is the probability that system is idle ________.

Student's Assignments 2

Q.23 Arrival rate for telephone calls at a telephone booth is according to Poisson distribution, with an average time of 8 minutes between two arrivals. The length of telephone call is assumed to be exponentially distributed with mean of 3.5 minutes. The probability that an arrival will have to wait for more than 12 minutes before the phone is free will be ________%.

Q.24 The number of person arriving at a mobile store is 15 customer/hour, probability of at least three customers waiting in the queue is 40.96%. The service rate for the store is ______ customer/hour.

Q.25 In a queuing system, arrival rate is 6/hr and mean service time is 6 min. What is the probability that arrival has to wait less than 9 min in the system?

(a) 63.2% (b) 45.2%
(c) 58.2% (d) 54.8%

Q.26 In a queuing model - arrival rate 3/hr, mean service time 15 min, wage rate ₹4/hr, idle cost ₹6/hr. What will be the total cost of queuing model for one hour?

(a) ₹26 (b) ₹16
(c) ₹22 (d) ₹18

Q.27 In a Queuing system, mean arrival time is 15 minute and mean service time is 10 min, then the number of jobs ahead will be ________.

Q.28 On a railway station reservation counter average 10 customers per hour visit for reservation. If the probability that serving a customer takes more than 12 minute is 8%. What will be the service rate?

(a) 26.5 customers/hour
(b) 24 customers/hour
(c) 29 customers/hour
(d) 21.5 customers/hour

Q.29 On an average 120 patients per 24 hour require the service of an emergency ward. On an average a patient requires 10 minutes of active attention. There is only one emergency ward which can handle only one patient at a time. If it costs ₹100 per patient treated for clinic to obtain an average servicing time of 10 minutes, and that each minute of decrease in this average time would cost `10 per patient treated. If average size of queue is reduced from $\left(\frac{25}{6}\right)$ to $\left(\frac{3}{2}\right)$ a patient then the budget per patient of clinic will become ₹______.

Q.30 American bank has a single automated teller machine (ATM) located in a shopping centre. Data were collected during a period of peak usage on Saturday afternoon and it was found that the average time between customer arrivals is 2.1 minutes with a standard deviation of 0.8 minute. It was also found that it takes on an average 1.9 minutes for a customer to complete a transition with a standard deviation of 2 minutes. Expected server utilization in (%) is _______.

Q.31 American Vending Inc (AVI) supplies vended food to a large university. Because students of kick the machines out of anger and frustration, management has a constant repair problem. The machine breakdown on an average of three per hour and the breakdowns are distributed in a Poisson's manner. Downtime costs the company ₹25 per hour per machine, and each maintenance worker gets ₹4 per hour. One worker can service machine at an average rate of five per hour, distributed exponentially, two workers working together can service 7 per hour, distributed exponentially. The total cost per hour (in ₹) for two workers is _______.

Q.32 On an average 5 customers reach a barber's shop every hour. The probability that exactly 3 customer will reach in a 30 minute period, assuming that arrival follows Poisson distribution is ________. (Write upto 3 decimal points)

ANSWERS

1. (c)	2. (d)	3. (d)	4. (a)
5. (a)	6. (c)	7. (a)	8. (a)
9. (d)	10. (c)	11. (b)	12. (d)
13. (c)	14. (c)	15. (b)	16. (c)
17. (c)	18. (a)	19. (b)	20. (0.9)
21. (5)	22. (0.4)	23. (6.36)	24. (18.75)
25. (b)	26. (c)	27. (2)	28. (a)
29. (117.66)	30. (90.5)	31. (26.75)	32. (0.214)

HINTS

19. (b)

a/*b*/*c* : : *d*/*e*/*f*

$$d = \text{Queue discipline}$$

20. (0.9)

$$\lambda = 6 \text{ customer per hour}$$

$$\mu = \frac{60}{6} = 10 \text{ customer per hour}$$

$$\rho = \frac{\lambda}{\mu} = \frac{6}{10} = 0.6$$

$$L_q = \frac{\rho^2}{1-\rho}$$

$$= \frac{0.6^2}{1-0.6} = \frac{0.36}{0.4} = 0.9$$

21. (5)

$$L_q' = \frac{1}{1-\rho}$$

$$\rho = \frac{\lambda}{\mu} = \frac{8}{10} = 0.8$$

Length of non-empty queue,

$$L_q' = \frac{1}{1-0.8} = \frac{1}{0.2} = 5$$

22. (0.4)

$$\lambda = 6 \text{ customer/hour}$$

$$\mu = \frac{1}{6}\frac{\text{customer}}{\text{min}}$$

$$= \frac{60}{6}\frac{\text{customer}}{\text{hour}}$$

$$= 10 \text{ customer/hr}$$

$$P_0 = 1-\frac{\lambda}{\mu} = 1-\frac{6}{10} = 0.4$$

23. (6.36)(6.30 to 6.40)

$$P(W_q \geq 12 \text{ min}) = \rho e^{-t/W_s}$$

where, $$\rho = \frac{\lambda}{\mu} = \frac{1/8}{1/3.5} = \frac{3.5}{8} = 0.4375$$

$$W_s = \frac{L_s}{\lambda} = \frac{\lambda}{\lambda(\mu-\lambda)} = \frac{1}{(\mu-\lambda)}$$