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Director's Message

B. Singh (Ex. IES)

During the last few decades of engineering academics, India has witnessed geometric growth in engineering graduates. It is noticeable that the level of engineering knowledge has degraded gradually, while on the other hand competition has increased in each competitive examination including GATE and UPSC examinations. Under such scenario higher level efforts are required to take an edge over other competitors.

The objective of MADE EASY books is to introduce a simplified approach to the overall concepts of related stream in a single book with specific presentation. The topic-wise presentation will help the readers to study & practice the concepts and questions simultaneously.

The efforts have been made to provide close and illustrative solutions in lucid style to facilitate understanding and quick tricks are introduced to save time.

Following tips during the study may increase efficiency and may help in order to achieve success.

- Thorough coverage of syllabus of all subjects
- Adopting right source of knowledge, i.e. standard reading text materials
- Develop speed and accuracy in solving questions
- Balanced preparation of Paper-I and Paper-II subjects with focus on key subjects
- Practice online and offline modes of tests
- Appear on self assessment tests
- Good examination management
- Maintain self motivation
- Avoid jumbo and vague approach, which is time consuming in solving the questions
- Good planning and time management of daily routine
- Group study and discussions on a regular basis
- Extra emphasis on solving the questions
- Self introspection to find your weaknesses and strengths
- Analyze the exam pattern to understand the level of questions
- Apply shortcuts and learn standard results and formulae to save time

B. Singh (Ex. IES)
CMD, MADE EASY Group

ESE 2019 : Main Examination

Electrical Engineering: Paper-I

Conventional Solved Questions of UPSC Engineering Services Examination

Contents

■■■

1 Electrical Materials

Revised Syllabus of ESE: *Electrical engineering materials, crystal structures and defects, ceramic materials, insulating materials, magnetic materials-basics, properties and applications, ferrities, ferro-magentic materials and components, basics of solid state physics, conductors, photo-conductivity, basics of nano materials and superconductors.*

1. Introduction to Engineering Materials

Q.1 What is the basis for classifying a material as a conductor, semi-conductor or a dielectric? What is the conductivity of a perfect dielectric?

[12 marks : 2002]

Solution:

Classification of insulators, metals and semiconductors are mainly by its conductivity and its temperature coefficient of resistivity.

Conductor:

In which conduction band and valence band are overlapping and there is no band gap, between them ($\Delta E_g = 0$). Due to this large number of electrons available for electrical conduction and therefore its resistivity is low and temperature coefficient of resistivity is positive ($\rho = 10^{-2}$ Ωm to 10^{-8} Ωm).

Insulator:

In which energy gap is large (E_g > 3eV). So in conduction band there are no electrons and so no electrical conduction is possible. Here energy gap is so large that electrons cannot be easily excited from the valence band to conduction band by any external energy.

$$\rho = 10^8\ \Omega\text{-m}$$

Temperature coefficient of resistivity is negative

Semiconductor:

In which a finite but small band gap (E_g < 3eV) exists. Due to this small band gap some electrons can be thermally excited to "conduction band". These thermally excited electrons can move in conduction band and can conduct current, their resistivity is in medium range. Its $\rho = 10^5 - 10^{10}$ Ωm. temperature coefficient of resistance is negative. Conductivity of a perfect dielectric is zero.

Q.2 Write the notable shortcoming of the free electron model. How it is overcome by band-theory of solids? Sketch the energy bands of diamond. With the help of energy band diagram, explain an insulator.

[12 marks : 2003]

Solution:

In free electron theory developed by Arnold Sommerfeld we assume the electron to be completely free. But it fails mainly because for most part electrons are not completely free. They interact with

(i) lattice (ii) with each other (iii) impurities

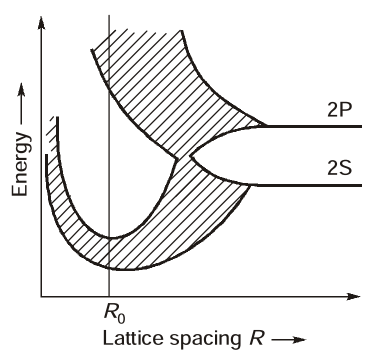
Drude incorporated into his model in the form of a hard sphere interaction but completely neglected the other interactions Sommerfeld made some corrections to this purely classical model by insuring that the electron gas obey fermi statistics. This faced some problems (like the overestimate of the electronic heat capacity but still didn't cover for the missing interaction in the model. Due to its limitations, the free electron model was unable to account for things like magnetoresistance, thermal dependence of conductivity, optical properties and the host of many body phenomenon that result from electron-electron interaction being of importance.

In band theory of e^- s, A solid is assumed to contain many bands in which electrons in it are packed. The most important are valence band and conduction band. The energy of electrons in these bands will be different. The difference in valence band and conduction determines whether solid is a conductor, semi-conductor, or insulator.

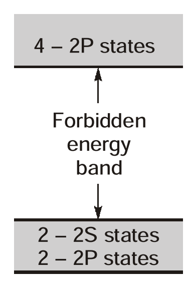
So it clearly explains the dependence of conductivity on temp which can't be explain by free e^- theory. When temp increases the energy of e^- s increases so e^- s started to jump from valence band to conduction so conduction of e^{-s} changes with temperature in *CB* so as conductivity changes with temp.

It also explains many other properties like optical, magneto resistance etc. easily so shortcomings of free e^- theory is overcome by band theory in which we assume there is interaction between e^- s only energy state of different e^- s is different. We also take in account the interaction of e^- with lattice and impurities.

Diamond is the crystalline form of carbon and is of interest because it lies in the same period and has the same type of binding as silicon and germanium. The shape of the energy-band structure of diamond as a function of atomic spacing is shown in Fig. (a), it is shown that there is a crossover point at which two of the 2 P states form a band with the two 2S state, thus making available a band with four available states. The remaining four of the total of six 2 P states off and form a higher band of their own. At normal atomic spacing, the forbidden gap between the higher and lower bands is wide. Since there is a total of four electrons in the 2 S and 2 P states of carbon, all these electrons are used up in filling the lower band. The upper band is left empty, and hence diamond is a very good insulator. Fig. (b) shows the stable energy level of diamond.



(a) The energy levels in the tetrahedral of diamond as a function of lattice spacing.

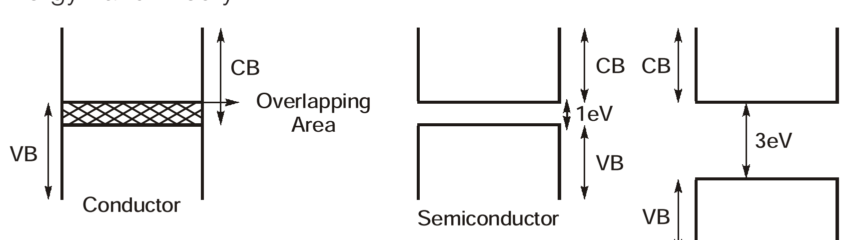


(b) The stable energy level of the diamond crystal.

Q.3 Distinguish between metals, insulators and semiconductors. [12 marks : 2007]

Solution:

According to Energy Band Theory:



e.g. (Cu, Fe, Al, Na etc.) e.g. (Si, Ge, Ga As etc.)

Classification of insulators, metals and semiconductors mainly by its conductivity and its temperature coefficient of resistivity.

Conductor:

In which conduction band and valence band are overlapping and there is no band gap, between them it $\Delta E_g = 0$. Due to this large number of electrons available for electrical conduction and therefore its resistivity is low and temperature coefficient of resistivity is positive (ρ = 10^{-2} to 10^{-8} Ω-m).

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$$\rho = 10^8 \ \Omega\text{-m}$$

Temperature coefficient of resistivity is negative.

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In which a finite but small band gap ($E_g < 3$eV) exists. Due to this small band gap some electrons can be thermally excited to "conduction band". These thermally excited electrons can move in conduction band and can conduct current, their resistivity is in medium range. Its ρ = (10^5 to 10^{10}) Ω-m temperature coefficient of resistance is negative. Conductivity of a perfect dielectric is zero.

Q.4 Explain with the diagram band structure of metals, insulators and semiconductors.

[10 marks : 2013]

Solution:

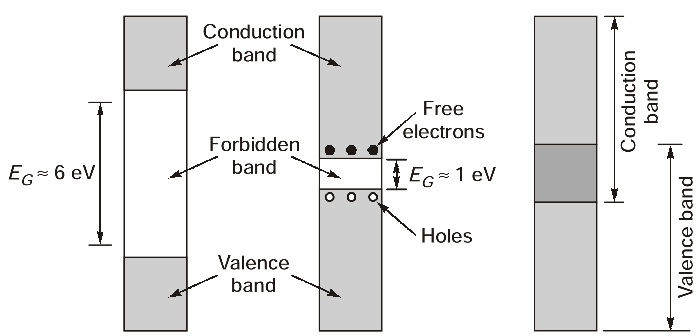


Fig. (a) Fig. (b) Fig. (c)

Insulators:

- An insulating material has an energy band diagram as shown in figure (a).
- It has a very wide forbidden-energy gap ($\simeq$ 6 eV) separating the filled valence region from the vacant conduction band. Because of this, it is practically impossible for an electron in the valence band to jump the gap, reach the conduction band.
- At room temperature, an insulator does not conduct. However it may conduct if its temperature is very high or if a high voltage is applied across it. This is termed as the **breakdown of the insulator.**
- **Example:** diamond.

Semiconductors:

- A semiconductor has an energy-band gap as shown in figure (b).
- At 0°K semiconductor materials have the same structure as insulators except the difference in the size of the band gap E_G, which is much smaller in semiconductors ($E_G \simeq 1$ eV) than in insulators.
- The relatively small band gaps of semiconductors allow for excitation of electrons from the lower (valence) band to the upper (conduction) band by reasonable amount of thermal or optical energy.
- The difference between semiconductors and insulators is that the conductivity of semiconductors can increase greatly by thermal or optical energy.
- **Example:** Ge and Si.

Metals:

- There is no forbidden energy gap between the valence and conduction bands. The two bands actually overlap as shown in figure (c).
- Without supplying any additional energy such as heat or light, a metal already contains a large number of free electrons and that is why it works as a good conductor.
- **Example:** Al, Cu etc.

Q.5 State three effects of moisture on insulation and explain any two methods adopted to protect them against moisture along with salient features.

[12 marks : 2017]

Solution:

The effect of moisture on insulation:

(i) **Change in electrical properties:** Moisture absorbed by the insulation causes a decrease in the volume resistivity, and especially surface resistivity, an increase in the dissipation factor and a certain increase in dielectric constant, a reduction in the dielectric strength due to change in field distribution within the insulating material. Under high humidity and electric tension current, conducting bridge may appear across the surface of the insulating material. In some cases the thin films formed by the moisture on insulating material dries up when the equipment is working. Such places get a carbonized spot. Such carbonized spot may join together with time and build up a conducting bridge, thus a short-circuit may result.

(ii) **Physical and mechanical changes:** Due to high humidity for long time some insulating materials like plastics polymers and material filled with cellulose fillers swell. The swelling of inner and outer layers of insulating material are unequal, so small cracks appear on the surface layer, through which moisture easily penetrates into the material. Mechanical strength of the insulating material looses with moisture.

(iii) **Chemical changes:** High humidity often causes hydrolysis. For example under the effect of moisture, linseed base insulating vanishes soften and even change into liquid. This evolves organic acids which deteriorate other organic materials and bring about intensive corrosion of metals. High humidity favours the growth of fungi in some insulating materials. The products of life activity formed by fungi degrade organic insulating materials.

Protection of insulation against moisture:

(i) **Impregnation of winding:** The windings of all low voltages pieces of equipment are impregnated with baking varnishes and some time with compounds. This treatment solidifies the windings, increases their thermal conductivity, improves their electrical and mechanical strength and heat resistance. Impregnating varnishes and compounds raise the moisture resistance of windings.

(ii) **Rendering insulation hydrophobic:** Sometimes the insulating materials assemblies are rendered hydrophobic (or waterproofed) so as to protect them against moisture. This treatment is particularly effective for polymer containing hydrolysis and for cellulose-base insulating materials. It is a good practice to render some inorganic materials hydrophobic so as to decrease their wettability and to raise the surface resistivity of these materials. In addition to old widely used techniques employing asphalts, bitumens, waxes, waterproofing by means of some hydrophobic silicon compositions free of hydroxyls and carboxyls is finding ever increasing favour. Paper, cotton fabrics are rendered hydrophobic by dipping them in the solution of methyl butoxidiamine silane in carbon terrachloride or methyl triethoxisilane in absolute alcohol.

Q.6 Show that the surface area to volume ratio increases with the reduction in radius of nanoparticle. Plot a graph of surface to volume ratio as a function of radius.

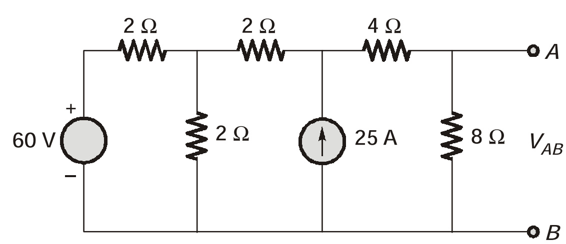
[8 marks : 2017]

2 Electric Circuits

Revised Syllabus of ESE: *Circuit elements, network graph, KCL, KVL, Node and Mesh analysis, ideal current and voltage sources, Thevenin's, Norton's, Superposition and Maximum power transfer theorems, transient response of DC and AC networks, Sinusoidal steady state analysis, Basic filter concepts, Two-port networks, Three-phase circuits, Magnetically coupled circuits.*

1. Circuit Elements, Nodal and Mesh Analysis

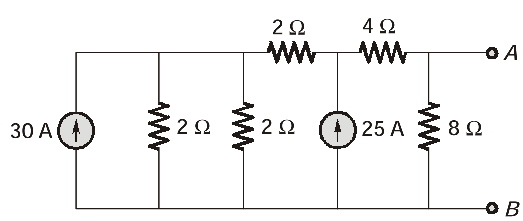
Q.1 Find V_{AB} for the circuit shown below using source transformation.



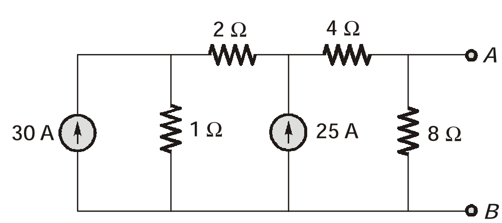
[10 marks : 2002]

Solution:

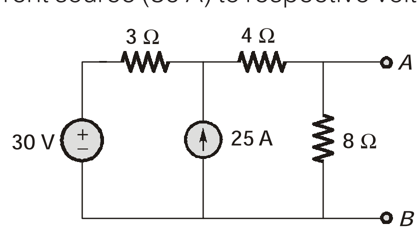
By(60 V) voltage source changes to respective current source.



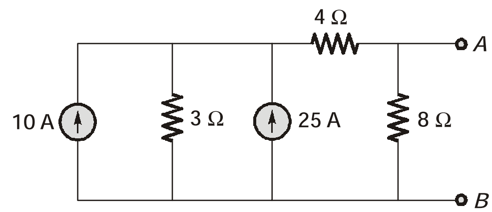
Now we combine the parallel resistance



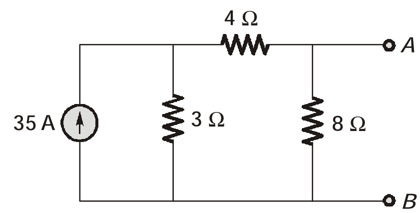
then we are going to change current source (30 A) to respective voltage source



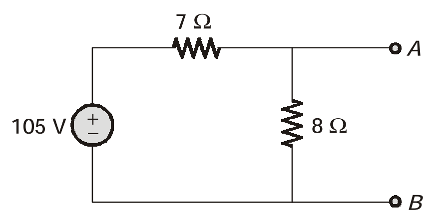
By changing (30 V) voltage source to respective current source



We combining current source



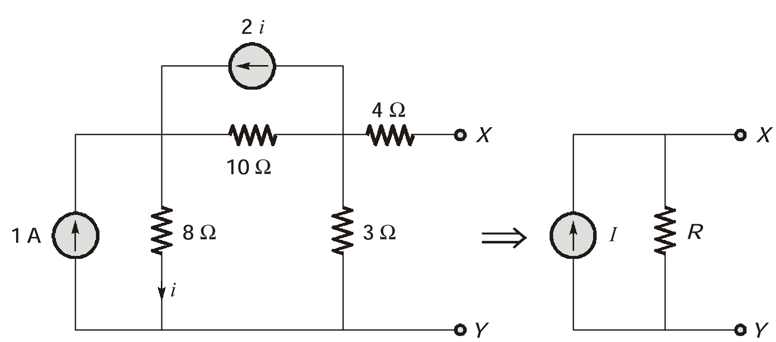
Now we make voltage source 35 A into current source



then by potential divider rule,

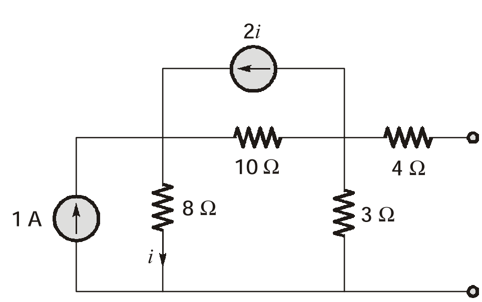
$$V_{AB} = \frac{105}{15} \times 8 = 56 \text{ V}$$

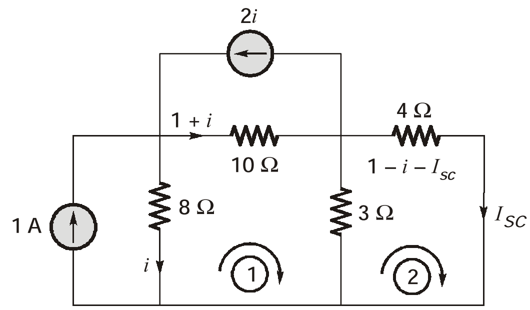
Q.2 Determine the value of I and R in the circuit shown in the figure.



[14 marks : 2003]

Solution:





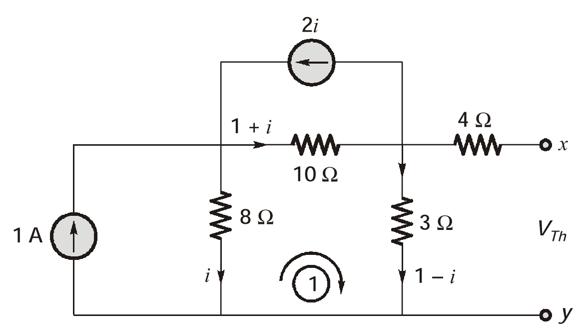
Using KVL in loop (1) we have,

$$-10(1+i) - 3(1 - i - I_{SC}) + 8i = 0$$
$$i + 3I_{SC} = 13$$

Using KVL in loop (2) we have,

$$-4I_{SC} + 3(1 - i - I_{SC}) = 0$$
$$7I_{SC} + 3i = 3$$
$$7I_{SC} + 3(13 - 3I_{SC}) = 3$$
$$2I_{SC} = 36$$
$$I_{SC} = 18 \text{ Amps.}$$

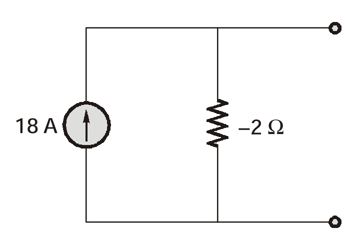
Now opening circuit the terminal xy, we have using KVL in the loop (1) we have



$$-10(1+i) - 3(1-i) + 8i = 0$$
$$13 \text{ A} = i$$
$$V_{th} = (1-i)\cdot 3 = -36 \text{ volts}$$

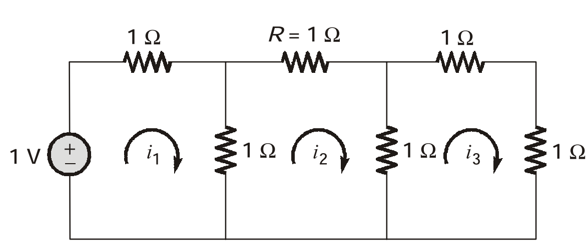
then,
$$R_{th} = \frac{V_{th}}{I_{SC}} = \frac{-36}{18} = -2\ \Omega$$

and
$$I = 18 \text{ Amps.}$$



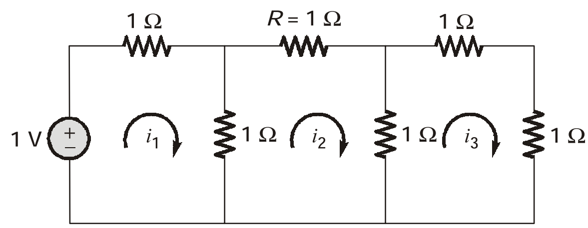
Note: The negative value of the resistance tells us that the circuit in supplying power. Of course, the resistor in the circuit can not supply power (they absorb power). It is the dependent source that supplies the power. This is a circuit how dependent source and resistance could be used to simulate negative resistance.

Q.3 Find the power dissipated in the resistor *R* in the ladder network shown in the figure below.



[12 marks : 2008]

Solution:



Using KVL in loop,

$$1 = 2i_1 - i_2 \quad ...(i)$$

$$0 = 3i_2 - i_1 - i_3 \quad ...(ii)$$

$$0 = 3i_3 - i_2 \quad ...(iii)$$

$$\therefore \quad i_3 = \frac{i_2}{3}$$

By solving the equations, we get,

$$i_2 = \frac{3}{13}$$

$\therefore$ Power dissipated in the resistor $R = i^2 R = \dfrac{9}{169}$ W

Q.4 State 'Voltage to current source transformation' theorem. It is required to replace network *N* in figure by a suitable equivalent network. Which of the networks of figure could be valid equivalent network (s)?

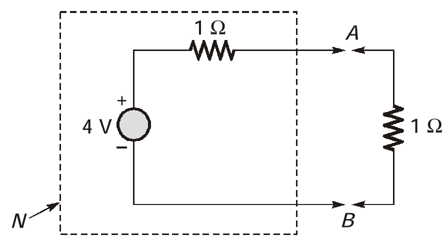


Fig. (a)

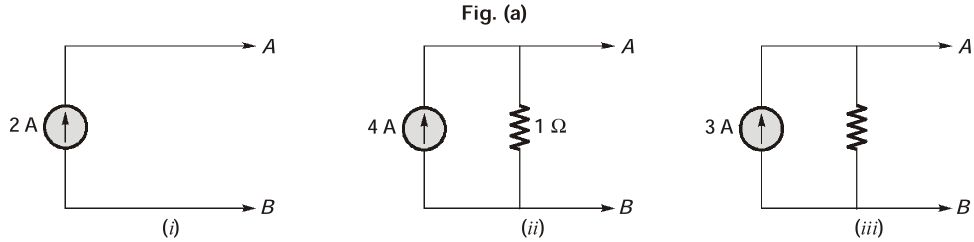


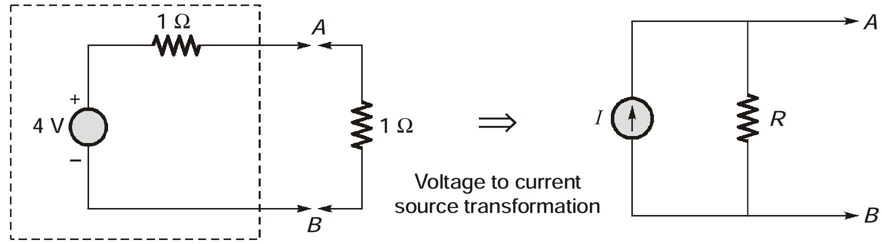
Fig. (b)

[12 marks : 2009]

Solution:

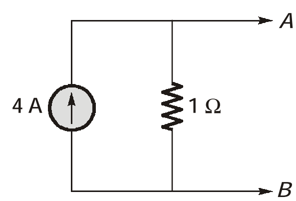
Source Transformation:

- Any voltage source consists of an ideal voltage source in series with an internal impedance (for ideal voltage source, this impedance being zero, the output voltage becomes independent of the load current) while any current source is an ideal current source in parallel with an internal impedance (for ideal current source, this parallel impedance is infinity such that the source current does not face any branching through this internal impedance path.
- The voltage and current sources are mutually transferable.
- For any voltage source, if the ideal voltage be *V* and internal resistance be *R*, the voltage source can be replaced by current source *I* with the internal resistance in parallel to the current source and the value of *I* is given by $I = V/R$.



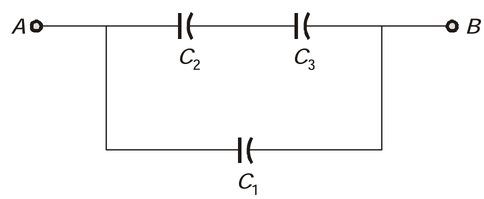
where, $R = 1\ \Omega$ and $I = \frac{V}{R} = \frac{4}{1} = 4\,A$

Hence,



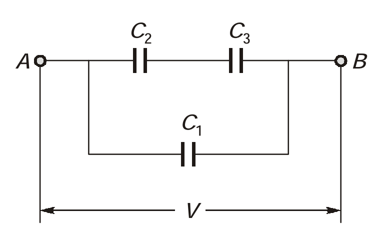
is the valid equivalent network.

Q.5 Three capacitors C_1, C_2 and C_3 whose values are 10 μF, 5 μF and 2 μF respectively, have breakdown voltages of 10 volts, 5 volts and 2 volts respectively. For their interconnection shown below, find the maximum safe voltage that can be applied across the combination and the corresponding total charge stored in the effective capacitance across the terminals *AB*.



[12 marks : 2014]

Solution:



Charge on C_1 = 10 μF × 10 V = 100 μC

Charge on C_2 = 5 μF × 5 = 25 μC

Charge on C_3 = 2 μF × 2 = 4 μC

Since, the capacitors are connected in series = Charge is same (i.e.)

(Minimum charge value of C_2 or C_3) = 4 μC

Effective capacitance of C_2 and $C_3 = \frac{C_2 C_3}{C_2 + C_3} = \frac{5\mu F \times 2\mu F}{5\mu F + 2\mu F} = 1.4285\ \mu F$

$\Rightarrow$ Voltage $(V) = \left(\frac{Q}{C}\right)$

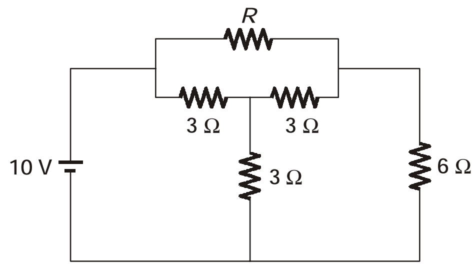
$$V = \frac{4\mu C}{1.4285 \times 10^{-6}} = 2.8\text{ V}$$

New charge on the capacitor (C_1) = 2.8 × 10 μF = 28 μC

The total charge stored in the effective capacitance = 28 μC + 4 μC = 32 μC

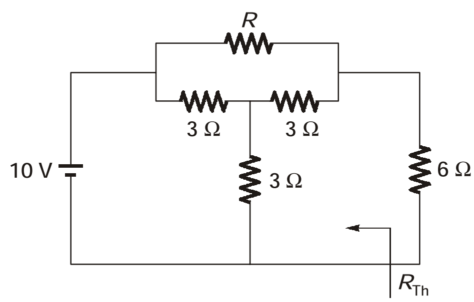
2. Network Theorems

Q.16 Determine the value of R shown in the below figure such that the 6 Ω-resistor consumes the maximum power.

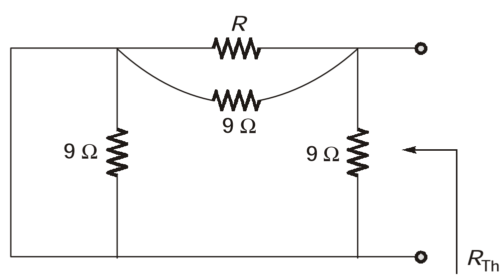


[8 marks : 2002]

Solution:



R_{Th} is calculated by shorting the voltage source and converting star into delta network,



$$R_{th} = \frac{9R}{9+R} \Big\| 9 = \frac{\left(\frac{9R}{9+R}\right) \times (9)}{\frac{9R}{9+R} + 9} = \frac{9R}{2R+9}$$

According to maximum power transfer theorem,

$$\text{Load resistance} = R_{Th}$$

$$R_{th} = \frac{9R}{2R+9} = 6$$

$$9R = 12R + 54$$

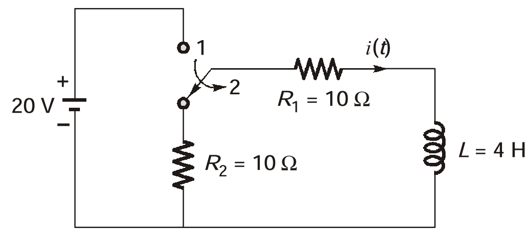
$$R = -\frac{54}{3}\ \Omega$$

Resistance can not be negative.

If $R = 0$, voltage source will be directly connected to the load resistor (6 Ω), all the power will be absorbed by load resistor (6 Ω).

So, $R = 0$ for maximum power transfer.

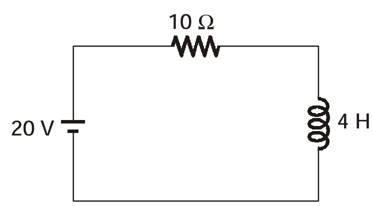
Q.32 The circuit in the figure below is initially under steady-state condition. The switch is moved from position 1 to position 2 at $t = 0$. Find the current after switching.



[12 marks : 2008]

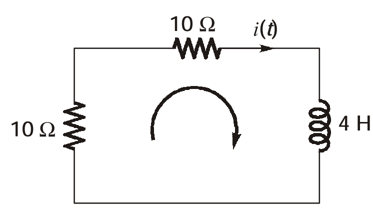
Solution:

For $t < 0$, at steady state the inductor will be short circuited



$$i(0^-) = \frac{20}{10} = 2\text{ A}$$

For $t \geq 0$



Applying KVL,

$$L\frac{di(t)}{dt} + (10+10)i(t) = 0$$

$$\frac{di(t)}{dt} + \frac{20}{4}i(t) = 0$$

Taking Laplace transform

$$[sI(s) - i(0^-)] + 5\,I(s) = 0$$

$$[s+5]I(s) = i(0^-) = 2$$

$$I(s) = \frac{2}{s+5}$$

$$i(t) = 2e^{-5t}\,u(t)$$

Q.33 A sinusoidal voltage $e(t) = 141 \sin 314t$ is applied to an initially de-energized series R-L circuit in which $R = 100\ \Omega$ and $L = 0.5$ H.

(i) If the switch which applied the voltage to R-L circuit is closed at the instant when $e(t)$ passing through zero with a positive slope, determine the initial value of the transient current.

(ii) Write the complete expression for the transient solution.

(iii) Write the expression for the complete solution of the current response.

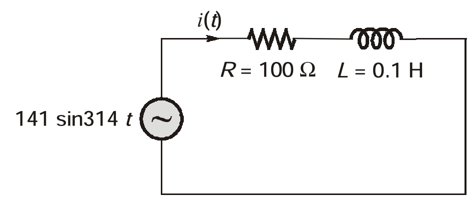
(iv) At what instantaneous value of the applied voltage will the closing of the switch result in no transient component of the current.

[8 + 4 = 12 marks : 2010]

Solution:

(i) As the circuit is de-energized initially, $i(0^-) = 0$ and instantaneous value of the applied voltage is zero at the instant of switching.

Hence, initial value of the transient current is 0.



Applying KVL, $$e(t) = R\,i(t) + \frac{L\,di(t)}{dt}$$

$$\Rightarrow \quad 141\sin 314t = 100\,i(t) + 0.5\frac{di(t)}{dt}$$

$$\Rightarrow \quad 282\sin 314t = 200\,i(t) + \frac{di(t)}{dt}$$

Applying Laplace transform,

$$\frac{282\times 314}{s^2+314^2} = 200\,I(s) + s\,I(s)$$

$$\Rightarrow \quad I(s) = \frac{282\times 314}{(s+200)\,(s^2+314^2)}$$

(ii) Using partial fraction method,

$$\frac{A}{(s+200)} + \frac{Bs+C}{s^2+314^2} = \frac{282\times 314}{(s+200)\,(s^2+314^2)}$$

$A(s^2 + 314^2) + (Bs + C)\,(s + 200) = 282 \times 314$

$$A + B = 0 \quad \ldots(i)$$

$$200\,B + C = 0 \quad \ldots(ii)$$

$$314^2 A + 200\,C = 282\times 314 \quad \ldots(iii)$$

On solving equation (*i*), (*ii*) and (*iii*), we get

$A = 0.6388,\ B = -0.6388,\ C = 127.77$

$$I(s) = \frac{0.6388}{s+200} - \frac{0.6388s}{s^2+314^2} + \frac{127.77}{s^2+314^2}$$

$$= \frac{0.6388}{s+200} - \frac{0.6388s}{s^2+314^2} + 0.4069\times\frac{314}{s^2+314^2}$$

$$i(t) = L^{-1}[I(s)] = 0.6388\,e^{-200t} - 0.6388\cos 314t + 0.4069\sin 314t$$

(iii) Complete solution of current response,

$$i(t) = 0.6388\,e^{-200t} + 0.757\sin(314t - 57.7)$$

where, $i(t)$ = transient response of the current = $0.6388\,e^{-200t}$

$i_{ss}(t)$ = steady state response of the current = $0.757\sin(314t - 57.7)$

(iv) As the circuit is initially deenergized before switching, $i(0^-) = 0$ and inductor is present in the circuit.

so, $i(0^+) = i(0^-) = 0$

Hence, $i(t) = 0$ initial current is zero, at the instant of switching (a)

$$i(t) = i_r(t) + i_{ss}(t) \quad \ldots(iv)$$

For transient free response $i_r(t) = 0$(c)

Using conditions (a) and (c), equation (*iv*) becomes,

$$0 = 0 + i_{ss}(t)$$

$$i_{ss}(t) = 0$$

$$0.757\sin(314t - 57.7) = 0$$

$$\Rightarrow \quad 314t = 57.7$$

so, switch should be closed at $314t = 57.7$ for transient free current.

Instantaneous value of the applied voltage at the instant of switching.

$$e(t) = 141\sin 314t = 141\sin 57.7 = 118.92 \text{ volt}$$

Q.98 A balanced three-phase, 208 Volts generator supplies a unit of 1800 Watts at a line current of 10 Amp. When three identical impedances are arranged in Y-connection across the line terminals of the generator calculate the resistive and reactive components of each phase impedance. Sketch also the phase and magnitude relations between phase and line voltages of a Y-connected three-phase system.

[20 marks : 2017]

Solution:

Given that,

Generator terminal voltage, $V = 208\angle 0°\ \text{V}$

Line current, $I_L = 10\ \text{A}$

Power supplies, $P = 1800\ \text{W}$

$$P = \sqrt{3}V_L I_L \cos\phi$$

$$\cos\phi = \frac{1800}{\sqrt{3}\times 208\times 10} = 0.50$$

Phase angle, $\phi = 60°$

In star connected load, $I_L = I_{ph} = 10\ \text{A}$

and $V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}}\ \text{V}$

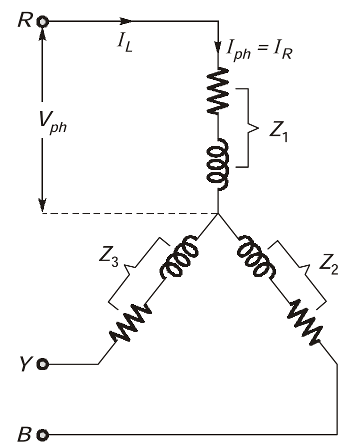
Given, $Z_1 = Z_2 = Z_3 = Z$

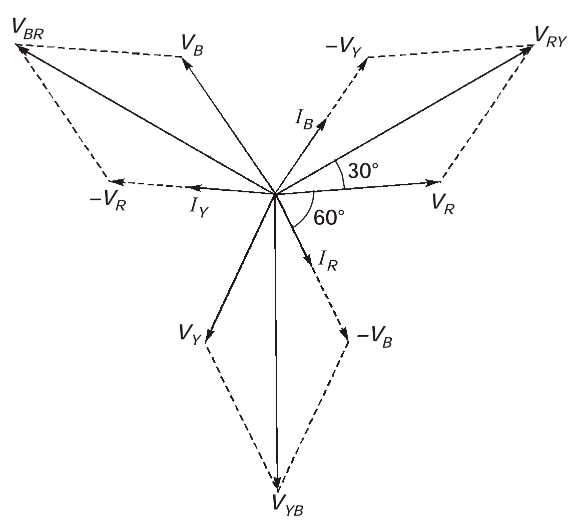
Impedance, $Z = \frac{V_{ph}}{I_{ph}} = \frac{(208/\sqrt{3})\angle 0°}{10\angle -60°} = 12\angle 60°\ \Omega$

$= (6.0 + j10.4)\ \Omega$

Resistive components, $R = 6.0\ \Omega$

Reactive component, $X = 10.4\ \Omega$





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3 Electric Fields

Revised Syllabus of ESE: *Gauss theorem, electric field and potential due to point, line, plane and spherical charge distributions, Ampere's and Biot-Savart's laws; inductance, dielectrics, capacitance; Maxwell's equations.*

1. Coordinate Systems & Vector Calculus

Q.1 "A hand 'curl meter' in the form of a pin wheel is used to indicate curl of a vector field." Justify the statement.

[12 marks : 2001]

Solution:

A pin or paddle wheel as a 'curl meter': The force is exerted on the each blade of the paddle wheel, the force being proportional to the component of the field normal to the surface of that blade. To test and field for curl we dip one paddle wheel into the field with the axis of the paddle wheel lined up with the direction of the component of curl desired. No rotation means no curl, larger angular velocities mean greater values of the curl a reverse in the direction of spin means a reversal in the sign of the curl. In order find the direction of vector curl, we should place one paddle wheel in the field and hunt around for the orientation which produces the greatest torque. The direction of the curl is then along the axis of the paddle wheel.

Q.2 Explain the concept of gradient. Determine the gradient of the given field:

$$V = \rho z \sin\phi + z^2 \cos^2\phi + \rho^2$$

[5 marks : 2013]

Solution:

Gradient of a Scalar:

Gradient V = Grad $V = \nabla V$ ... always a vector quantity.

∇V

Magnitude $|\nabla V|$ — Direction

Magnitude represents the rate of change of scalar quantity V with respect to various co-ordinates.

Direction represents the direction in which this rate of change is maximum.

$$\vec{E} = -\nabla V = -\text{Grad}\,V$$

$$V = -\int_c \vec{E}\cdot d\vec{l}$$

Numerical:

$$V = \rho z \sin\phi + z^2 \cos^2\phi + \rho^2$$

Given system is in cylindrical coordinate system.

Hence, $$\nabla V = \frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z$$

Now, $$\frac{\partial V}{\partial \rho} = \frac{\partial}{\partial \rho}(\rho z \sin\phi + z^2\cos^2\phi + \rho^2) = z\sin\phi + 0 + 2\rho$$

$$\frac{\partial V}{\partial \rho} = z\sin\phi + 2\rho$$

Similarly, $$\frac{\partial V}{\partial \phi} = \left[z(\cos\phi) + z^2(2\cos\phi \times (-\sin\phi))\right] + 0 = z\cos\phi - 2z^2\sin\phi\cos\phi$$

Similarly, $$\frac{\partial V}{\partial z} = \frac{\pi}{\rho}\sin\phi + 2z\cos^2\phi$$

So, $$\nabla V = (z\sin\phi + 2\rho)\hat{a}_\rho + \frac{(z\cos\phi - 2z^2\sin\phi\cos\phi)}{\rho}\hat{a}_\phi + \left(\frac{\pi}{\rho}\sin\phi + 2z\cos^2\phi\right)\hat{a}_z$$

Q.3 State and explain Divergence theorem. Verify Divergence theorem for the vector A in spherical co-ordinates for the volume enclosed between $r = 1$ and $r = 2$, where $A = 5r^2\,\hat{a}_r$.

[10 marks : 2013]

Solution:

Divergence Theorem:

$$\oiint \vec{A}\cdot d\vec{s} = \iiint_V \nabla\cdot\vec{A}\,dV$$

Statement: Closed surface integral of any vector $\vec{A}$ integrated over any closed surface area S is always equal to the volume integral of the divergence of the vector $\vec{A}$ integrated over the volume V which is enclosed by the closed surface area 'S'. The theorem holds good, irrespective of

- the shape of closed surface area 'S'
- the type of the coordinate system
- the type of the vector $\vec{A}$

Numerical:

Given, $\vec{A} = 5r^2\,\hat{a}_r$

From $r =$ 1 to 2

$$\Rightarrow \quad \nabla\cdot\vec{A} = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}(r^2\vec{A}_r\hat{a}_r)$$

$$= \frac{1}{r^2}\times\frac{\partial}{\partial r}(r^2\times 5r^2) = \frac{1}{r^2}[5\times 4r^3] = 20\,r$$

$$dV = r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$\therefore \quad \int_V(\nabla\cdot\vec{A})\,dV = \int_0^{2\pi}\int_0^{\pi}\int_1^{2} r^2\sin\theta\times 20r\,dr\,d\theta\,d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_1^{2} 20r^3\sin\theta\,dr\,d\theta\,d\phi = \int_0^{2\pi}\int_0^{\pi} 20\times\left[\frac{r^4}{4}\right]_1^2\sin\theta\,d\theta\,d\phi$$

$$= \int_0^{2\pi} 20 \times \left(\frac{16-1}{4}\right)[-\cos\theta]_0^{\pi} d\phi$$

$$= 20 \times \frac{15}{4} \times 2 \times 2\pi = 300\pi \qquad \ldots(i)$$

$$\oint_s \vec{A} \cdot ds = \int_0^{2\pi}\int_0^{\pi} Ar^2 \cdot r^2 \sin\theta \, d\theta d\phi$$

$$= Ar^4 \times \sin\theta \, d\theta \, d\phi = 5 \times (15) \times 2 \times 2\pi$$

$$= 300\,\pi \qquad \ldots(ii)$$

From equations (*i*) and (*ii*), $\boxed{\oiint \vec{A} \cdot d\vec{s} = \iiint_V \nabla \cdot \vec{A}\, dV}$

Hence, divergence theorem is verified.

2. Electrostatic Fields

Q.4 State and explain Gauss's law. A spherical volume charge distribution ρ is given by

$$\rho = \rho_0\left(1 - \frac{r^2}{100}\right) \qquad \text{for } r \le 10 \text{ mm}$$

$$= 0 \qquad \text{for } r > 10 \text{ mm}$$

Show that the maximum value of electric field intensity *E* occurs at *r* = 7.45 mm. Obtain the value of *E* at *r* = 7.45 mm.

[12 marks : 2001]

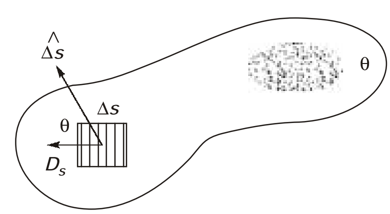
Solution:

Gauss's Law

Surface integral of the normal component of the electric vector *D* over any closed surface is equal to the total charge enclosed by the surface.

The total flux through the closed surface,

$$\Psi = \int d\Psi = \oint \bar{D}_s \cdot \bar{ds} = \int_{Vol} \rho_v \, dv$$



$$\oint \bar{D} \cdot \overline{ds} = \int_{Vol} \rho_v dv$$

Applying Gauss's law to the spherical surface at ($r \le 10$)

$$E'(4\,\pi\, r^2) = \frac{Q'}{\epsilon_0} = \frac{1}{\epsilon_0}\left[4\pi\rho_0 \int\left(1 - \frac{r^2}{100}\right) r^2 dr\right]$$

$$= \frac{4\pi\,\rho_0\, r^2}{\epsilon_0}\left[\frac{r}{3} - \frac{1}{5}\frac{r^3}{100}\right] = \frac{\rho_0}{\epsilon_0}\left[\frac{r}{3} - \frac{1}{5}\frac{r^3}{100}\right]$$

As clearly seen from the above expression, (E') varies with '*r*' and for maximum value of (E')

$$\frac{dE'}{dr} = 0$$

Differentiating with respect to *r* and equating to zero

$$\frac{\rho_0}{\epsilon_0}\left(\frac{1}{3} - \frac{3}{5}\frac{r^2}{10^2}\right) = 0$$

$$\frac{r^2}{10^2} = \frac{5}{3}\times\frac{1}{3}$$

$$\therefore \quad r = 7.45 \text{ mm}$$

$$\frac{d^2E'}{dr^2} = \frac{\rho_0}{\epsilon_0}\left(\frac{-3}{5}\cdot\frac{2r}{10^2}\right)$$

At $r = 7.45$ mm, $\quad \frac{d^2E'}{dr^2} < 0$

Therefore, maximum E occurs at $r = 7.45$ mm

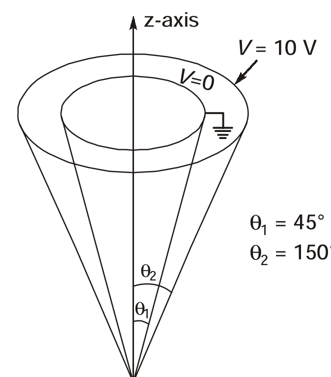
$$E'_{max} = \frac{\rho_0}{\epsilon_0}\left[\frac{7.45}{3} - \frac{1}{5}\times\frac{7.45^3}{10^2}\right] = \frac{\rho_0}{\epsilon_0}\times 1.6563 \text{ V/mm}$$

Q.5 Two infinite and conducting cones both on z-axis, one is $\theta = \theta_1 = 45°$ (constant) cone and the other is $\theta = \theta_2 = 150°$ constant cone. The region between them is characterized by ϵ, $\rho_v = 0$, $V = 0$ at θ_1 and $V = 10$ V at θ_2. Find expression for V between $45° < \theta < 150°$.

[10 marks : 2001]

Solution:

Consider the coaxial cone as shown in figure. Here V depends only on θ, so Laplace's equation on spherical coordinate becomes.



$$\nabla^2 V = \frac{1}{r^2\sin\theta}\frac{d}{d\theta}\left[\sin\theta\frac{dV}{d\theta}\right] = 0$$

or $\quad \frac{d}{d\theta}\left[\sin\theta\frac{dV}{d\theta}\right] = 0$

Integrating once we get

$$\frac{dV}{d\theta} = \frac{A}{\sin\theta}$$

Integrating this result we get

$$V = A\int\frac{d\theta}{\sin\theta} + B = A\int\frac{d\theta}{2\cos\theta/2\sin\theta/2} + B$$

$$V = A\int\frac{1}{2}\frac{\sec^2\theta/2}{\tan\theta/2}d\theta + B = A\int\frac{d(\tan\theta/2)}{\tan\theta/2} + B$$

$$V = A\, ln \tan(\theta/2) + B$$

Now apply the boundary condition to determine the value of A and B

$V(\theta = \theta_1) = 0$

$$O = A\, ln\tan\left(\frac{45°}{2}\right) + B$$

$$B = -A\, ln\tan(\theta_1/2)$$

$$\therefore \quad V = A\, ln\left[\frac{\tan(\theta/2)}{\tan(\theta_1/2)}\right]$$

Now at $\quad \theta = \theta_2,\ V = V_0 = 10$ V

$$\therefore \quad 10 = A\, ln\left[\frac{\tan(\theta_2/2)}{\tan(\theta_1/2)}\right]$$

$$\therefore \qquad V = \frac{10\, ln\left[\dfrac{\tan(\theta/2)}{\tan(\theta_1/2)}\right]}{ln\left[\dfrac{\tan(\theta_2/2)}{\tan(\theta_1/2)}\right]} = \frac{10}{2.198}\, ln\left[\frac{\tan(\theta/2)}{\tan(\theta_1/2)}\right]$$

$$V = 4.548\, ln\left[\frac{\tan(\theta/2)}{\tan(\theta_1/2)}\right] \text{ for } 45° < \theta < 150°$$

Q.6 **A sheet of charge, ρ_s = 2 nc/m^2 is present at the plane x = 3 in free space, and a line charge, ρ_L = 20 nC/m is located at x = 1, z = 4. Find**

(i) the magnitude of the electric field intensity $\bar{E}$ at the origin

(ii) the direction of $\bar{E}$ at (4, 5, 6)

(iii) the force per meter length on the line charge. **[12 marks : 2002]**

Solution:

(i) A sheet charge is a surface charge distribution at a rate ρ_s = 2 nc/m^2 located at the plane (x = 3). Field due to this at origin

$$E_s = \frac{\rho_s}{2\varepsilon_0}(-a_x) = \frac{2\times10^{-9}}{2\times\varepsilon_0}(-a_x) = -113\,\bar{a}_x$$

Field produced by the line charge is (at origin),

$$|E_L| = \frac{\rho_L}{2\pi\varepsilon_0\times r}\hat{n}$$

where, $\rho_L = 20\times10^{-9}$ C/m, $\quad \vec{r} = -\hat{a}_x - 4\hat{a}_z$

$$|\vec{r}| = \sqrt{1^2+4^2} = \sqrt{17}$$

$$E_L = \frac{20\times10^{-9}}{2\pi\varepsilon_0\times\sqrt{17}}\times\frac{-\hat{a}_x-4\hat{a}_z}{\sqrt{17}} = \frac{20\times10^{-9}\times18\times10^9}{17}(-\hat{a}_x-4\hat{a}_z)$$

$$E_{\text{Resultant}} = E_S + E_L = -21.17\ \hat{a}_x - 84.70\hat{a}_z - 113\,\hat{a}_x = 134\,\hat{a}_x - 84.70\,\hat{a}_z$$

$$\text{Magnitude, } |E_{\text{Resultant}}| = |-134\ \hat{a}_x - 84.70\ \hat{a}_z| = 158.425 \text{ V/m}$$

(ii) At point (4, 5 ,6)

Electric field due to surface charge,

$$E_{(s)} = +113\,\hat{a}_x \qquad (\because \text{point is on the right side of sheet of charge})$$

Electric field due to line charge at point (4, 5, 6) due to line charge located at (x = 1 and z = 4).

Vector distance between this the two point

$$\vec{r}_1 = (4-1)\hat{a}_x + 5\hat{a}_y + 2\hat{a}_z \qquad \left(\because \vec{r}_1 = (4-1)\hat{a}_x + 5\hat{a}_y + (6-4)\hat{a}_z\right)$$

$$\vec{E}_l = \frac{\rho_L}{2\pi\varepsilon_0\times|\vec{r}_1|}\hat{n} = \frac{20\times10^{-9}\times18\times10^9}{\sqrt{38}}\times\frac{3\hat{a}_x+5\hat{a}_y+2\hat{a}_z}{\sqrt{38}}$$

$$\vec{E}_l = 9.474(3\hat{a}_x+5\hat{a}_y+2\hat{a}_z)$$

$$E_{\text{Resultant}} = 28.421\ \hat{a}_x + 47.368\hat{a}_y + 18.949\hat{a}_z \qquad (\because \vec{E}_{\text{resultant}} = \vec{E}_s + \vec{E}_l)$$

(iii)

$$\text{Force/Length} = \bar{E}_s \times \rho_L$$

$$= (113\ \hat{a}_x)\times\rho_L = (+2.26\times10^{-6})a_x \text{ N/m}$$

4 Electrical & Electronic Measurements

Revised Syllabus of ESE: *Principles of measurement, accuracy, precision and standards; Bridges and potentiometers; moving coil, moving iron, dynamometer and induction type instruments, measurement of voltage, current, power, energy and power factor, instrument transformers, digital voltmeters and multi-meters, phase, time and frequency measurement, Q-meters, oscilloscopes, potentiometric recorders, error analysis, Basics of sensors, Transducers, basics of data acquisition systems.*

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1. Characteristics of Instruments and Measurement Systems

Q.1 What is the dimension of a quantity? Derive dimensions of capacitance, resistance and inductance in electrostatic and electromagnetic systems. Find a relation between two systems of units.

[8 marks : 2001]

Solution:

Dimensions: Every quantity has a quality which distinguishes it from all other quantities. This unique quality is called Dimension. The dimension is written in a characteristic notation, as for example [L] for length, [T] for time etc.

Dimension in electrostatic system of units:

In electrostatic system of units

$$\text{Dimension of charge } [Q] = [\epsilon^{1/2} M^{1/2} L^{3/2} T^{-1}]$$

$$\text{emf } [E] = [\epsilon^{-1/2} M^{1/2} L^{1/2} T^{-1}]$$

and

$$[I] = [\epsilon^{1/2} M^{1/2} L^{3/2} T^{-2}]$$

Capacitance:

Capacitance, $C = \dfrac{Q}{E}$

Therefore, dimensions of capacitance are,

$$[C] = \frac{[Q]}{[E]} = \frac{[\epsilon^{1/2} M^{1/2} L^{3/2} T^{-1}]}{[\epsilon^{-1/2} M^{1/2} L^{1/2} T^{-1}]} = [\epsilon L]$$

Resistance:

Resistance, $R = \dfrac{E}{I}$

Therefore, dimensions of resistance are,

$$[R] = \frac{[E]}{[I]} = \frac{[\epsilon^{-1/2} M^{1/2} L^{1/2} T^{-1}]}{[\epsilon^{1/2} M^{1/2} L^{3/2} T^{-2}]} = [\epsilon^{-1} L^{-1} T]$$

Inductance:

Inductance, $L = \dfrac{\text{emf}}{\text{rate of change of current}} = \dfrac{E}{dI/dt}$

Therefore, dimensions of inductance are,

$$[L] = \frac{[E]}{[I]/[T]} = \frac{[E][T]}{[I]} = \frac{[\in^{-1/2} M^{1/2} L^{1/2} T^{-1}][T]}{[\in^{1/2} M^{1/2} L^{3/2} T^{-2}]}$$

$$= [\in^{-1} L^{-1} T^2]$$

Dimensions in electromagnetic system:

In electromagnetic system,

Dimension of pole strength

$$[Q] = [\mu^{-1/2} M^{1/2} L^{1/2}]$$

$$[E] = [\mu^{1/2} M^{1/2} L^{3/2} T^{-2}]$$

$$[I] = [\mu^{-1/2} M^{1/2} L^{1/2} T^{-1}]$$

Capacitance:

Capacitance, $C = \frac{Q}{E}$

Therefore, dimensions of capacitance are

$$[C] = \frac{[Q]}{[E]} = \frac{[\mu^{-1/2} M^{1/2} L^{1/2}]}{[\mu^{1/2} M^{1/2} L^{3/2} T^{-2}]} = [\mu^{-1} L^{-1} T^2]$$

Resistance:

Resistance, $R = \frac{E}{I}$

Therefore, dimensions of resistance are,

$$[R] = \frac{[E]}{[I]} = \frac{[\mu^{1/2} M^{3/2} L^{1/2} T^{-2}]}{[\mu^{-1/2} M^{1/2} L^{1/2} T^{-1}]} = [\mu L T^{-1}]$$

Inductance:

Inductance, $L = \frac{\text{emf}}{\text{rate of change of current}} = \frac{E}{dI/dt}$

Therefore, dimensions of inductance are,

$$[L] = \frac{[E]}{[I]/[T]} = \frac{[E][T]}{[I]} = \frac{[\mu^{1/2} M^{1/2} L^{3/2} T^{-2}][T]}{[\mu^{-1/2} M^{1/2} L^{1/2} T^{-1}]} = [\mu L]$$

Relationship between Electrostatic and Electromagnetic System of units: We observe that two different dimensional equations are obtained for the same quantity in e.m. and e.s. systems of units. Any one quantity should have the same dimensions whatever may be the system.

The dimensions of charge in e.m.u. system are:

$$[Q] = [\mu^{-1/2} M^{1/2} L^{1/2}]$$

and in e.s.u. system are:

$$[Q] = [\in^{1/2} M^{1/2} L^{3/2} T^{-1}]$$

Equating the above two dimensions,

$$[\mu^{-1/2} M^{1/2} L^{1/2}] = [\in^{1/2} M^{1/2} L^{3/2} T^{-1}]$$

We have:

$$[\mu^{-1/2} \in^{-1/2}] = [L T^{-1}]$$

Now dimensions $[L T^{-1}]$ are those of velocity,

$\therefore$ Equating the dimensional equations of other quantities, we always get the same result.

Q.2 Describe briefly the primary and secondary standards of mass and length.

[4 marks : 2001]

Solution:

Primary Standard of Mass: The primary unit of mass is a prototype kilogramme kept at National Physical Laboratories of every country. This has an accuracy of 1 part in 10^8 and is occasionally verified against the standard kept at the International Bureau.

Secondary Standard of Mass: The secondary standards of mass are kept by industrial laboratories. These standards have an accuracy of 1 ppm and are checked against the primary standards.

International Units of Length: The international unit of length, the metre, was defined in 1960 in terms of an optical standard. This was the orange-red radiation of krypton atom. The internationally recognised krypton-86 discharge lamp when excited and observed under well defined conditions, emits orange light whose wavelength constitutes the basic unit of length. This can be measured to an accuracy of 1 ppm. The metre is equal to 1650763.73 wavelengths in vacuum of the orange-red light radiation of the krypton-86 atom in its transition between levels 2_{p10} and 5_{d5}. In terms of time the length travelled by light in vacuum in a time interval of 1/299792458 second.

Working Standards of Length: These are usually precision Gauge Blocks made up of steel. These blocks have two parallel surfaces and the distance between the two surfaces is specified. The accuracy tolerance of this distance is usually 0.5 – 0.25 micron. The most outstanding advantages gained by using of gauge blocks are their precision, low cost and accuracy which is 1 part per million (1 ppm).

Q.3 If $\mu = \dfrac{\pi r^4 (P_1 - P_2)}{8Ql}$, determine the dimensions of μ. (r and l are radius and length P_1, P_2 are pressures, Q is flow).

If, $r = (0.5 \pm 0.01)$ mm, $P_1 = (200 \pm 3)$ kPa, $P_2 = (150 \pm 2)$ kPa, $Q = 4 \times 10^{-7}$ m³/sec. Calculate the absolute error in μ.

[12 marks : 2003]

Solution:

Dimensions of $r = [L]$; $Q = [L^3 T^{-1}]$,

$$P_1 \text{ and } P_2 = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}] = [M\,L^{-1}\,T^{-2}]$$

π and 8 are dimensionless. Hence,

$$\text{Dimension of } \mu = \frac{\pi\, r^4\,(P_1 - P_2)}{8Ql} = \frac{[L]^4[ML^{-1}T^{-2}]}{[L^3T^{-1}][L]} = [M\,L^{-1}\,T^{-1}]$$

Assuming per unit length, as the data of l is provided in the question,

$$\mu = \frac{\pi\, r^4\,(P_1 - P_2)}{8Q} = \frac{\pi \times (0.5 \times 10^{-3})^4\,(200 - 150)10^3}{8 \times 4 \times 10^{-7}} = 0.3068 \times 10^{-2}$$

On taking extreme positive values of r, P_1 and P_2,

$$\mu = \frac{\pi\, r^4\,(P_1 - P_2)}{8Q} = \frac{\pi \times (0.51 \times 10^{-3})^4\,(203 - 152)10^3}{4 \times 8 \times 10^{-7}} = 0.3387 \times 10^{-2}$$

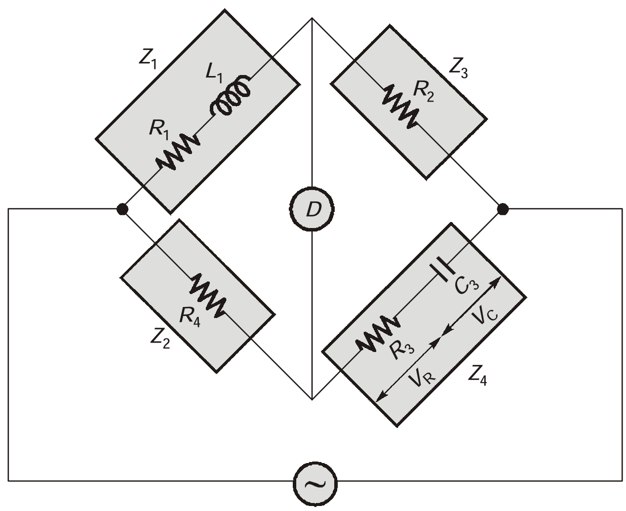
$$\text{Absolute error} = (0.3387 \times 10^{-2}) - (0.3068 \times 10^{-2})$$
$$= 0.0319 \times 10^{-2}$$

Q.40 In Hay's bridge, the four arms, namely, $R_1 - L_1$, R_2, $R_3 - C_3$ and R_4 are connected in clockwise order. Show that under balanced condition the quality factor Q of the coil is given by

$$Q = \frac{\text{Voltage across } C_3}{\text{Voltage across } R_3}$$

[8 marks : 2014]

Solution:



$$Q = \frac{\text{Voltage across } C_3}{\text{Voltage across } R_3}$$

as we known, $$Q = \frac{\omega L_1}{R_1} \quad \ldots(i)$$

From the bridge balanced condition,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\Rightarrow \quad (R_1 + j\omega L_1)\left(R_3 + \frac{1}{j\omega C_3}\right) = R_4 R_2$$

$$\Rightarrow \quad R_1 R_3 + j\omega L_1 R_3 + \frac{R_1}{j\omega C_3} + \frac{L_1}{C_3} = R_4 R_2$$

equating real and imaginary path,

$$R_1 R_3 + \frac{L_1}{C_3} = R_4 R_2 \quad \ldots(ii)$$

$$\omega L_1 R_3 - \frac{R_1}{\omega C_3} = 0$$

$$\Rightarrow \quad \omega L_1 R_3 = \frac{R_1}{\omega C_3}$$

$$\Rightarrow \quad \frac{\omega L_1}{R_1} = \frac{1}{\omega C_3 \times R_3} \quad \ldots(iii)$$

From equation (*i*) and (*iii*),

$$Q = \frac{\omega L_1}{R_1} = \frac{1/\omega C_3}{R_3} = \frac{1/C}{R_3} = \frac{1/C}{V_R}$$

5 Computer Fundamentals

Revised Syllabus of ESE: *Number systems, Boolean algebra (Part of Digital Electronics), arithmetic functions, Basic Architecture, Central Processing Unit, I/O and Memory Organisation; peripheral devices, data represenation and programming, basics of Operating system and networking, virtual memory, file systems; Elements of programming languages, typical examples.*

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1. Number System

Q.1 The binary representation of 100 110 is numerically equivalent to:
(a) The decimal representation of 46 (b) The octal representation of 46
(c) The octal representation of 26 (d) The decimal representation of 26

[2 marks : 2003]

Solution: (b)

$$(\underline{100}\ \underline{110})_2 = (4\,6)_8$$

100 110 is equivalent to octal representation of 46.

Q.2 Perform the following binary substraction and give the result in decimal value also:

11001 – 1100

[4 marks : 2017]

Solution:

Given: 11001 – 1100

First number in decimal $(11001)_2 = (27)_{10}$

Second number in decimal $(1100)_2 = (12)_{10}$

Their difference is, $27 - 12 = (15)_{10}$

Result in binary $(15)_{10} = (1111)_2$

Q.3 (i) How is division exactly done by ALU?
(ii) Prove with illustration that NAND is a "Universal GATE".
(iii) How will you implement a two-way switch using minimum number of logic gates?

[7 + 6 + 7 = 20 marks : 2018]

Solution:

(i) Division:

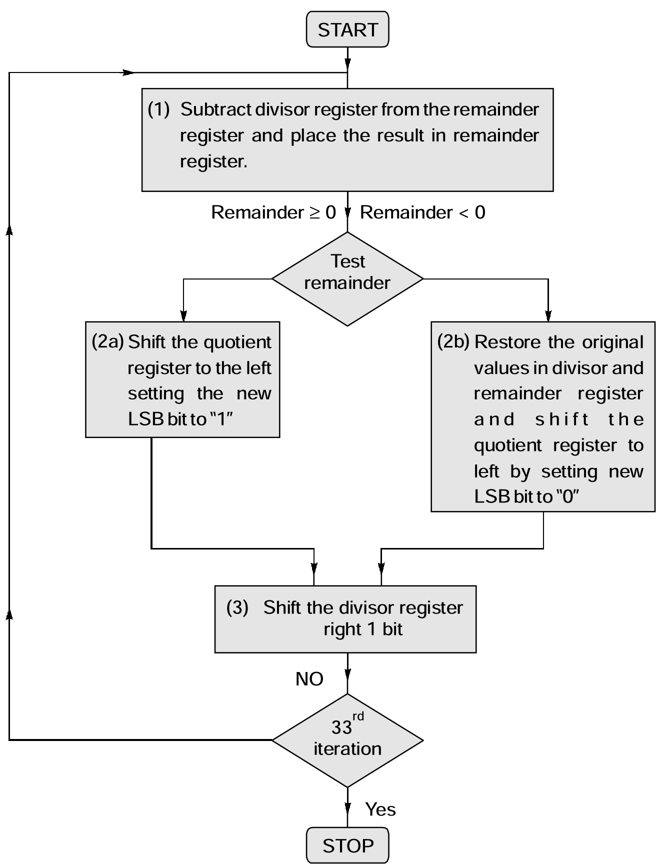
$$\text{Dividendend} = \underset{(Q)}{\text{Quotient}} * \text{Divisor} + \underset{(R)}{\text{Remainder}}$$

e.g.: $$7 \div 2 = \begin{Bmatrix} Q = 3 \\ R = 1 \end{Bmatrix}$$

0010 | 0000 011 1 Q : 00000011
(–) 0010
0001 1
(–) 001 0
0001 (R)

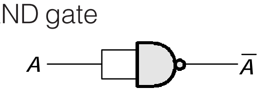
Dividend scanned from MSB to LSB in bitwise. If it is less than or equal to divisor then put 1' in quotient and subtract divisor from dividend (or) put '0' in quotient move to next bit scan of dividend.

32-bit Division: $\left[\frac{2n \text{ bit}}{n \text{ bit}}\right]$

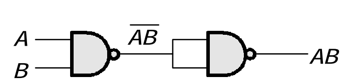


(ii) NAND gate is called as universal gate as it is possible to implement any logic function using it.

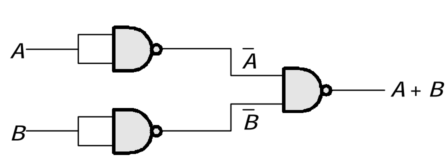
(a) Realization of NOT gate using NAND gate



(b) Realization of AND gate using NAND gate



(c) Realization of OR Gate using NAND gate

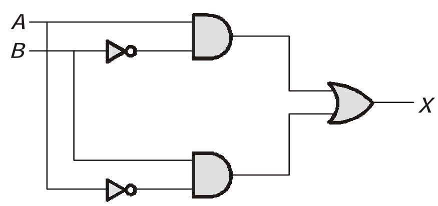


(iii) Two way switch using minimum number of logic gates.

$A = 0$ and $B = 1$ (or) $A = 1$ and $B = 0$

Therefore,

A	B	$X = A\bar{B} + \bar{A}B$
0	0	0
0	1	1
1	0	1
1	1	0



2. Boolean Algebra

Q.4 The Boolean expression $X = B + A\cdot\bar{B} + A\cdot B$ is equivalent to

(a) $A + B$ (b) $\bar{A}\cdot B$

(c) $\overline{A+B}$ (d) $A \times B$

[2 marks : 2005]

Solution: (a)

$$X = B + A\cdot\bar{B} + A\cdot B$$

$$= B + A(\bar{B} + B) = A + B$$

Q.5 The Boolean expression $ABC + \bar{A}\bar{B}C + AB\bar{C} + \bar{A}\bar{B}\bar{C}$ is equivalent to

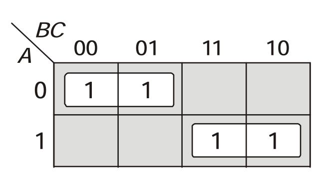
(a) OR GATE (b) AND GATE

(c) EX-NOR GATE (d) EX-OR GATE

[2 marks : 2007]

Solution: (c)

From K-map,



$$Y = \bar{A}\bar{B} + AB \equiv \text{EX-NOR Gate}$$

Q.6 The Boolean expression $ABCD + A\bar{B}CD + ABC\bar{D} + A\bar{B}C\bar{D}$ is equivalent to

(a) A (b) AC

(c) ABC (d) 1

[2 marks : 2008]