POSTAL Book Package 2021

Instrumentation Engineering

Objective Practice Sets

Communications

Contents

MADE EASY Publications

Fourier Analysis of Signals, Energy and Power Signals

Q.1 Let $x(t)$ be a periodic signal with fundamental period T and fourier series coefficient of Re$\{x(t)\}$ (where Re denotes the real part of signal) is

(a) $\frac{a_k + a_k^*}{2}$ (b) $\frac{a_k - a_{-k}^*}{2}$

(c) $\frac{a_k^* + a_{-k}}{2}$ (d) $\frac{a_k^* - a_k}{2}$

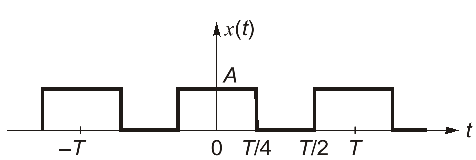
Q.2 A signal is such that $x(t) = x(t + T_0/2)$, it is also given that it is even in nature. The fourier series expansion has

(a) absent sine term
(b) absent cos term
(c) only odd harmonics
(d) odd term of cos as $\Sigma a_n \cos n\omega$

Q.3 A signal has Fourier series coefficients $C_n \Rightarrow C_{-1} = C_1 = 8,\ C_0 = 0,\ C_2 = C_{-2} = 2$ its power is

(a) 0 (b) 136
(c) 20 (d) 120

Q.4 Determine the fourier series coefficients for given periodic signal $x(t)$ is



(a) $\frac{A}{j2\pi k}\sin\left(\frac{\pi}{2}k\right)$ (b) $\frac{A}{\pi jk}\cos\left(\frac{\pi}{2}k\right)$

(c) $\frac{2A}{\pi k}\sin\left(\frac{\pi}{2}k\right)$ (d) $\frac{2A}{\pi k}\cos\left(\frac{\pi}{2}k\right)$

Q.5 Suppose we have given following information about a signal $x(t)$

1. $x(t)$ is real odd
2. $x(t)$ is periodic with $T = 2$
3. Fourier coefficients $C_n = 0,\ |n| > 1$
4. $\frac{1}{2}\int_0^2 \left|x(t)^2\right| dt = 1$

The signal that satisfy these conditions

(a) $\sqrt{2}\sin \pi t$ and unique
(b) $\sqrt{2}\sin \pi t$ but not unique
(c) $2 \sin \pi t$ and unique
(d) $2 \sin \pi t$ but not unique

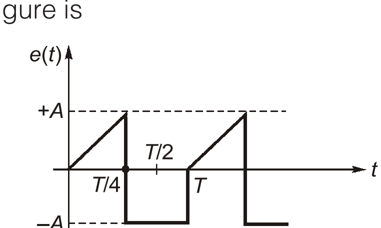
Q.6 The fourier series coefficients, of a periodic signal $x(t)$ expressed as $\sum_{k=-\infty}^{k=+\infty} a_k e^{j2\pi kt/T}$ are given by

$a_{-2} = 2 - j1$; $a_{-1} = 0.5 + j0.2$
$a_0 = j2$; $a_1 = 0.5 - j0.2$
$a_2 = 2 + j1$; and $a_k = 0$; for $|k| > 2$

which of the following is true.

(a) $x(t)$ has finite energy because only finitely many coefficients are non-zero
(b) $x(t)$ has zero average value because it is periodic
(c) the imaginary part of $x(t)$ is constant
(d) the real part of $x(t)$ is even

Q.7 The rms value of the periodic waveform $e(t)$ shown in figure is



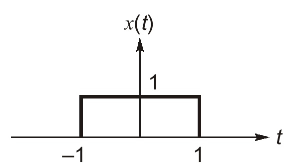
(a) $\sqrt{\frac{3}{2}}A$ (b) $\sqrt{\frac{2}{3}}A$

(c) $\sqrt{\frac{1}{3}}A$ (d) $\sqrt{\frac{5}{6}}A$

Q.16 The auto correlation function of a rectangular pulse of duration T is
(a) A rectangular pulse of duration T
(b) A rectangular pulse of duration $2T$
(c) A triangular pulse of duration T
(d) A triangular pulse of duration $2T$

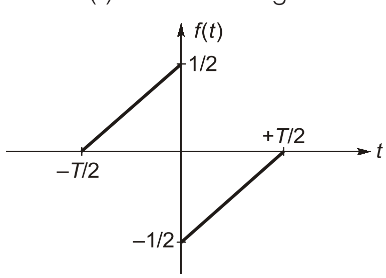
Q.17 The amplitude spectrum of Gaussian pulse is
(a) uniform (b) a sine function
(c) gaussian (d) an impulse function

Q.18 $x(t)$ is a positive rectangular pulse from $t = -1$ to $t = +1$ with unit height as shown in figure. The value of $\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$ (where $X(\omega)$ is Fourier transform of $x(t)$) is



(a) 2 (b) 2π
(c) 4π (d) 4

Q.19 A function $f(t)$ is shown in figure.



The Fourier transform $F(\omega)$ of $f(t)$ is
(a) real and even function of ω
(b) real and odd function of ω
(c) imaginary and odd function of ω
(d) imaginary and even function of ω

Q.20 A signal is represented by

$$x(t) = \begin{cases} 1 & |t| < 1 \\ 0 & |t| > 1 \end{cases}$$

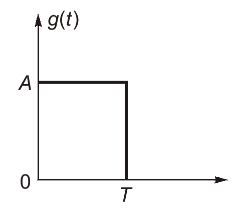
The fourier transform of the convolved signal $y(t) = x(2t) * x(t/2)$.

(a) $\frac{4}{\omega^2}\sin\left(\frac{\omega}{2}\right)$ (b) $\frac{4}{\omega^2}\sin\left(\frac{\omega}{2}\right)\sin(2\omega)$

(c) $\frac{4}{\omega^2}\sin 2\omega$ (d) $\frac{4}{\omega^2}\sin^2 \omega$

Q.21 A signum function is
(a) zero for t greater than zero
(b) zero for t less than zero
(c) unity for t greater than zero
(d) $2u(t) - 1$

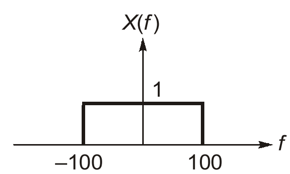
Q.22 The energy density spectrum $|G(f)|^2$ of a rectangular pulse shown in the given figure is

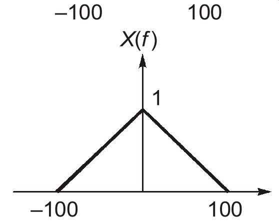


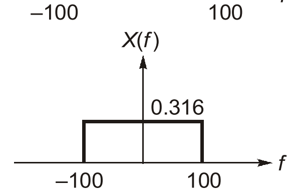
(a) $AT\left(\frac{\sin \pi f T}{\pi f T}\right)$ (b) $(AT)^2\left(\frac{\sin \pi f T}{\pi f T}\right)^2$

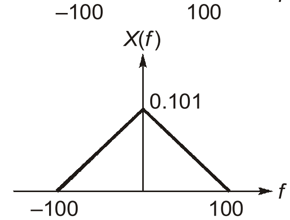
(c) $(AT)^2\left(\frac{\sin \pi f T}{\pi f T}\right)$ (d) $A^2\left(\frac{\sin \pi f T}{\pi f T}\right)$

Q.23 Frequency spectrum of signal $\frac{100}{\pi^2}\text{sinc}^2(100t)$ is

(a)


(b)


(c)


(d)


Q.24 A signal $x(t) = 6 \cos 10\pi t$ is sampled at the rate of 14 Hz. To recover the original signal, the cut-off frequency f_c of the ideal low-pass filter should be
(a) 5 Hz < f_c < 9 Hz (b) 9 Hz
(c) 10 Hz (d) 14 Hz

ANSWERS

1. (a)	2. (d)	3. (b)	4. (c)
5. (b)	6. (a)	7. (d)	8. (a)
9. (b)	10. (a)	11. (d)	12. (d)
13. (a)	14. (a)	15. (b)	16. (d)
17. (c)	18. (c)	19. (c)	20. (b)
21. (d)	22. (b)	23. (d)	24. (a)

EXPLANATIONS

1. (a)

$$x(t) \longleftrightarrow a_k$$
$$x(t) = \text{Re}(x(t)) + j\,\text{Img}(x(t))$$
$$x^*(t) = \text{Re}(x(t)) - j\,\text{Img}(x(t))$$
$$x(t) \longleftrightarrow a_k$$
$$x^*(t) \longleftrightarrow a_k^*$$
$$x(t) + x^*(t) \longleftrightarrow a_k + a_k^*$$
$$2\text{Re}(x(t)) \longleftrightarrow a_k + a_k^*$$
$$\text{Re}(x(t)) \longleftrightarrow \frac{a_k + a_k^*}{2}$$

2. (d)

$$x(t) = x\left(t + \frac{T_0}{2}\right),$$

where T_0 is fundamental period.
$\Rightarrow x(t)$ is half wave symmetric,
it consists of only odd harmonics. ...(1)
Also, $x(t)$ is even, thus contains only cosine terms ...(2)
$\therefore$ $x(t)$ contains only odd cosine terms.

$$x(t) = \sum_{n=0}^{\infty} a_n \cos n\omega,\ n = \text{odd}.$$

3. (b)

$$\text{Power} = \sum_{n=-\infty}^{\infty} |C_n| = \frac{1}{T}\int_{-T/2}^{T/2} |f(t)|^2\,dt$$
$$= |C_{-2}|^2 + |C_2|^2 + |C_{-1}|^2 + |C_{-1}|^2 + |C_0|^2$$
$$= 2^2 + 2^2 + 8^2 + 8^2 + 0^2 = 136$$

4. (c)

$$a_k = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cdot \cos k\omega_0 t\,dt$$
$$b_k = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cdot \sin k\omega_0 t\,dt$$
$$\Rightarrow \quad a_k = \frac{2}{T}\int_{-T/4}^{T/4} A\cos k\omega_0 t\,dt$$
$$= \frac{2A}{T}\cdot 2\int_{0}^{T/4} \cos k\omega_0 t\,dt$$
$$= \frac{4A}{T}\cdot\frac{1}{k\omega_0}\sin k\omega_0 t$$

Also,
$$\omega_0 = \frac{2\pi}{T}$$
$$\Rightarrow \quad a_k = \frac{4A}{T}\cdot\frac{1}{k\cdot\frac{2\pi}{T}}\sin\left(k\cdot\frac{2\pi}{T}\cdot\frac{T}{4}\right)$$
$$\Rightarrow \quad a_k = \frac{A}{\pi k}\sin\left(\frac{\pi}{2}k\right)$$
$$b_k = \frac{2}{T}\int_{-T/4}^{T/4} A\sin k_{\omega_0} t = 0$$
$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} A\,dt$$
$$a_0 = \frac{1}{T}\int_{-T/4}^{T/4} A\,dt = \frac{A}{T}\cdot\frac{t}{2} = \frac{A}{2}$$
$$x(t) = \frac{A}{2} + \sum_{n=0}^{\infty}\frac{2A}{\pi k}\sin\left(\frac{\pi}{2}k\right)$$

Fourier coefficient,
$$a_k = \frac{2A}{k\pi}\sin\left(\frac{\pi}{2}k\right)$$

5. (b)

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

since $c_n = 0$, $\quad |n| > 1, n > 1, n > 1 \,\&\, n < -1.$

$$x(t) = c_0 + c_1 e^{j\omega_0 t} + c_{-1}e^{-j\omega_0 t}$$

Also, $x(t)$ is periodic with period '2'.

$$x(t) \longleftrightarrow X(\omega)$$

$$x(2t) \longleftrightarrow \frac{1}{2}X\left(\frac{\omega}{2}\right)$$

$$x\left(\frac{1}{2}\right) \longleftrightarrow 2\,X(2\,\omega)$$

$$y(t) = x(2t) \otimes x\left(\frac{t}{2}\right) \longleftrightarrow \frac{1}{2}X\left(\frac{\omega}{2}\right)\cdot 2X(2\omega)$$

$$Y(\omega) = X\left(\frac{\omega}{2}\right)\cdot X(2\omega)$$

$$= 4\cdot\frac{\sin\frac{\omega}{2}}{\frac{\omega}{2}}\cdot\frac{\sin 2\omega}{2\omega}$$

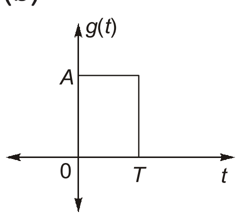
$$= \frac{4}{\omega^2}\sin\left(\frac{\omega}{2}\right)\cdot\sin(2\omega)$$

21. (d)

$$Sgn(t) = \begin{bmatrix} 1 & t>0 \\ -1 & t<0 \end{bmatrix}$$

$$Sgn(t) = 2u(t) - 1$$

22. (b)



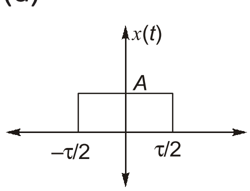
$\longleftrightarrow G(t)$

$$G(t) = \text{AT}\,\sin c\,(f\cdot T)$$

$$|G(t)|^2 = A^2T^2\sin^2 c(fT)$$

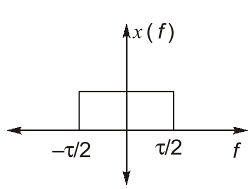
$$= A^2T^2\left(\frac{\sin(\pi fT)^2}{\pi fT}\right) \qquad \left(\sin cfT = \frac{\sin \pi fT}{\pi fT}\right)$$

23. (d)



$\longleftrightarrow A\tau\,\text{sinc}(f\tau)$

$A\tau\,\text{sinc}(t\tau) \longleftrightarrow$



Also:

$$x(t)\otimes h(t) \longleftrightarrow X(f)\cdot H(f)$$

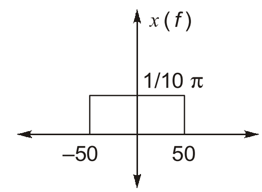
$$x(t)\cdot \text{h(t)} \longleftrightarrow X(f)\otimes H(f)$$

Hence,

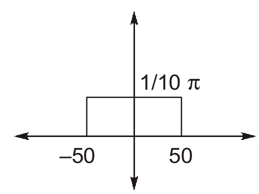
$$x(t) = \frac{100}{\pi^2}\sin^2 c(100t)$$

$$x(t) = \underbrace{\frac{10}{\pi}\text{sinc}(100t)}_{x_1(t)}\cdot\underbrace{\frac{10}{\pi}\text{sinc}(100t)}_{x(f)} x_2(t)$$

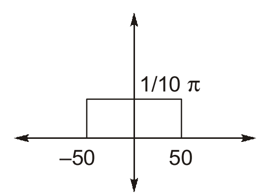
$\frac{10}{\pi}\text{sinc}(100t) \longleftrightarrow$



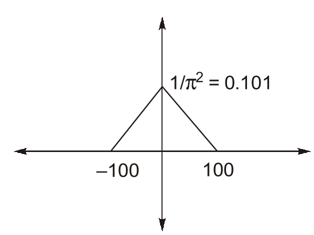
$X_1(f) \longleftrightarrow$



$X_2(f) \longleftrightarrow$



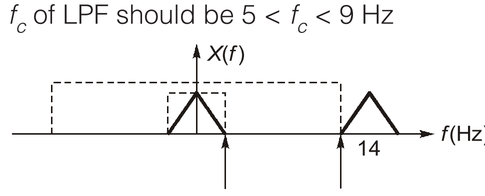
$X_1(f)\otimes X_2(f) \longleftrightarrow$



24. (a)

$$f_m = \frac{10\pi}{2\pi} = 5\text{ Hz}$$

f_c of LPF should be $5 < f_c < 9$ Hz



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