UPPSC-AE 2020

Uttar Pradesh Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Civil Engineering

Design of Concrete and Masonry Structures

Well Illustrated **Theory** *with*

Solved Examples and **Practice Questions**

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Design of Concrete and Masonry Structures

Contents

CHAPTER 6

Design for Torsion in Reinforced Concrete

6.1 Introduction

It is another important limit *state of collapse* just like shear and flexure. Torsion invariably occurs in most of the type of structures. In this chapter, we will define the two different types of torsion that exist in actual structures, the approach followed for the design of members subjected to combined bending/flexure, shear and torsion along with the concept of equivalent moment and equivalent shear.

6.2 Design for Torsion

Loads acting normal to the plane of bending in case of beams and slabs, gives rise to bending moments and shear force. However, when the loads act away from the plane of bending at an eccentricity then this will induce torsion in members. In reinforced concrete, the state of pure torsion rarely exists unlike the case of shafts where we encounter pure torsion. It generally occurs in combination with transverse shear and flexure. Behavior of reinforced concrete members under the combined influence of flexure, shear and torsion is quite complex owing to the fact that concrete is a non-homogeneous material. Presence of cracks in concrete members and composite nature of material i.e. concrete and steel adds to the complexity of torsion analysis. Research in this area is still going on. However, various design codes in the world describe very simplified procedure for torsion design of reinforced concrete members.

Torsional Moment v/s Flexural Moment

FLEXURAL MOMENT	TORSIONAL MOMENT
Flexural/bending moments are distributed among the connected members according to their distribution factors which are proportional to flexural stiffness $\frac{EI}{L}$.	Torsional moments are distributed among the connected members according to their distribution factors which are proportional to torsional stiffness $\frac{GJ}{L}$.

6.3 Mechanism of Torsion in Reinforced Concrete Structures

There are many mechanisms by which torsion can be induced in reinforced concrete members during load transfer stage (during the process of load transfer) in the structural system. Depending upon the situations which induce torsion in reinforced concrete, torsion can be classified as:

1. Equilibrium torsion (or primary torsion)
2. Compatibility torsion (or secondary torsion)

1. **Equilibrium Torsion:** This torsion gets induced in a reinforced concrete member due to eccentric loading w.r.t. shear centre of the cross section. Here, the equilibrium conditions are sufficient in itself to determine the twisting/torsional moments especially in **statically determinate structures**. This torsional component **must be considered in design (as per Cl. 41.4 of IS : 456-2000)** as it is a major component in statically determinate structures. Also, the ends of the member should be suitably designed to resist this type of torsion.

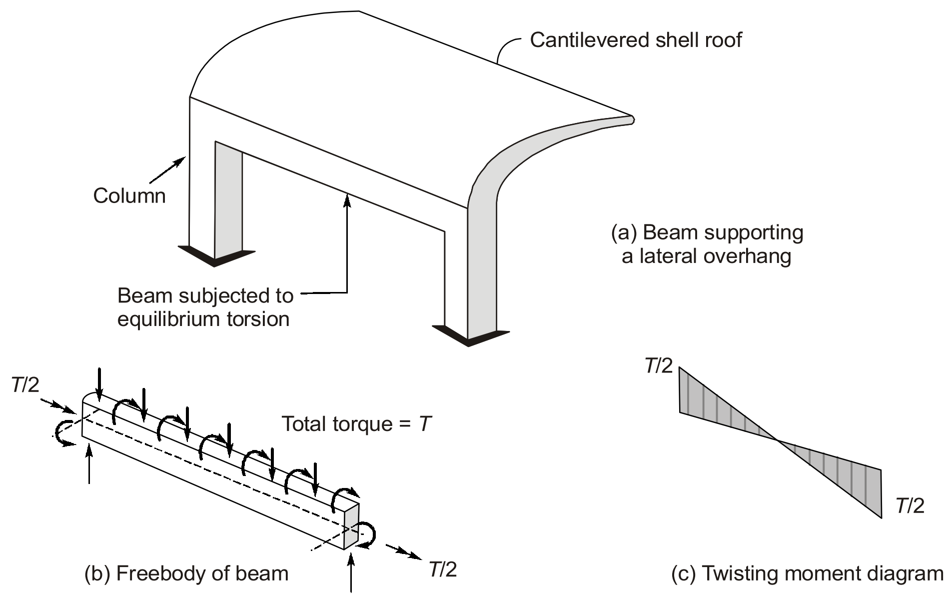


Fig. *Concept of Equilibrium Torsion*

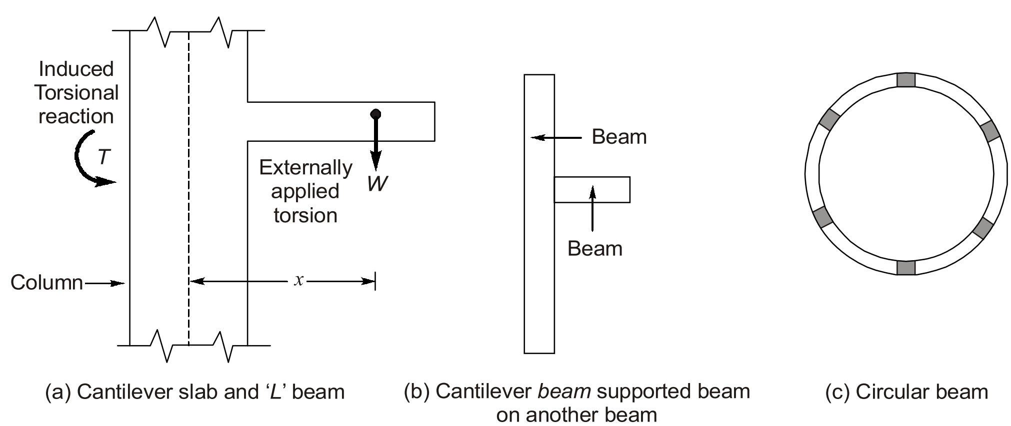


Fig. *Simplified explanation of equilibrium torsion*

2. **Compatibility Torsion:** This type of torsion arises due to need of the member to undergo a certain angle of twist to maintain the compatibility conditions. Here, the twisting moments developed are dependent on torsional stiffness of the member and these twisting moments are **statically indeterminate**, minor amounts of which **can be ignored in design due to multiple load paths available in statically indeterminate structures**.

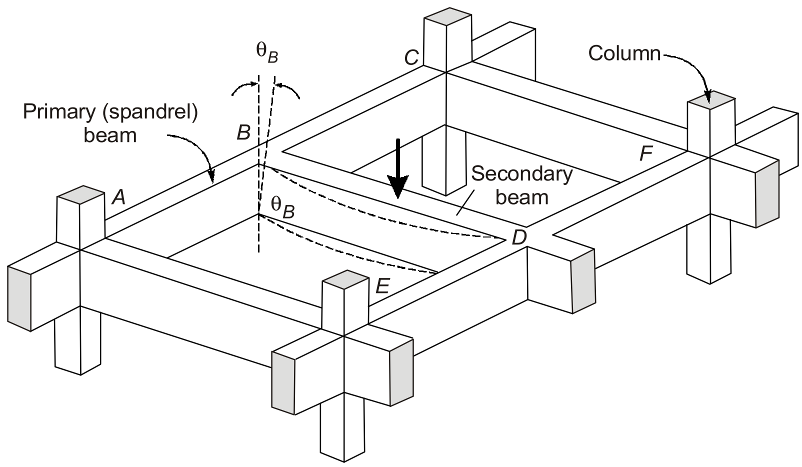


Fig. *Concept of Compatibility Torsion*

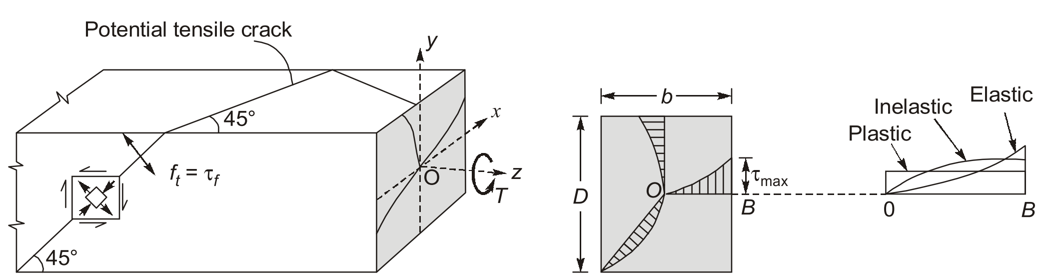
IS 456 : 2000 (Cl. 41.1) states that where the torsion can be eliminated by releasing the redundant restrains, there, no specific design for torsion is necessary provided torsional stiffness is not taken into the analysis of internal stresses.

Remember

- Just like *Rupture Moment* for flexural moment, twisting moment (or torsional moment) is associated with ***Cracking Torque*** which implies after the first time loading of plain concrete member, the cracks that develop prior to torsional cracking. A minimum torsional reinforcement is always provided in reinforced concrete members in order to control cracks and impart ductility to the member. Minimum stirrups requirement as per Cl. 26.5.1.6 of IS 456 : 2000 also reinforces the fact that some degree of torsional cracking can be controlled in concrete members due to compatibility torsion.

6.4 Plain Concrete Subjected to Torsion

From the principles of Solid Mechanics, it is known that torsion induces shear stresses and causes warping in non-circular sections. Figure shows the distribution of torsional shear stresses over a rectangular cross section which follows linear elastic behavior.



(a) Part section of beam **(b) torsional shear stress distribution** **(c) degrees of plastic behaviour**

Fig. *Distribution of Torsional Shear Stress in Elastic Rectangular Beam Section*

Maximum torsional shear stress occurs at the middle portion of the larger face of the section and is given by:

$$\tau_{tor,\,max} = \frac{T}{\alpha D b^2}$$

where T is the applied torsional moment, b and D are the cross section dimensions and α is a constant which depends on the ratio $\frac{D}{b}$.

Pure shear induces diagonal tensile and diagonal compressive stresses.

As shown in **figure**, principle tensile and compressive stress paths spiral around the beam in orthogonal directions at 45 degrees to the beam axis. When this diagonal tensile stress reaches the tensile strength of concrete then cracks start appearing on the section. Due to brittle nature of concrete, this crack penetrates rapidly in the inward direction from the exposed surface which nullifies the torsional resistance of the member.

Thus in plain concrete members, the diagonal torsional cracking leads to the failure of the entire section almost immediately.

The ultimate torsional resistance of the plain concrete can be assessed by measuring **cracking torque (T_{cr}).** The expression for the cracking torque (T_{cr}) can be computed from any of the following proposed theories viz.:

1. Elastic theory
2. Plastic theory
3. Skew bending theory
4. Theory of equivalent tube analogy

The cracking torque (T_{cr}) computed from any of the theory has to be verified and correlated experimentally with the actual tensile strength of concrete. **IS 456 : 2000** has adopted the **design shear strength of concrete (τ_c) (table 19 of IS 456 : 2000)** as the measure of tensile strength of concrete.

6.5 Torsionally Reinforced Concrete Subjected to Torsion

As discussed in the previous section, failure of plain concrete in torsion occurs due to the diagonal tensile stresses and thus to prevent this failure, steel should be provided in the form of spirals around the member in the direction of principle tensile stresses.

Thus t**o prevent a beam against torsional failure, torsional reinforcement should be provided in spirals along the direction of principal tensile stress which is in fact not a practical solution**. So, torsional reinforcement is provided in the form of longitudinal reinforcement (as longitudinal bars) and transverse reinforcement (as stirrups).

The twisting behaviour of torsionally reinforced concrete beam is similar to that of plain concrete beam until the formation of first crack (which corresponds to **torsional cracking moment T_{cr}**). After the occurrence of first crack, there is a large increase in twist at constant torque due to abrupt loss of torsional stiffness. After this the torsional behavior of reinforced concrete member depends on the amount of torsional reinforcement present.

Very small amount of torsional reinforcement may not be able to prevent the torsional cracks and thus no increase in strength is possible beyond T_{cr}. As the reinforcement is increased, torsional strength increases but this cannot be done indefinitely because ultimate failure occurs by crushing of concrete. Increasing **the torsional reinforcement increases the ductile behavior but this is felt at very large angle of twists.**

Consequences of Torsional Moment

1. Torsional moments are associated with shear stresses in beams since torsion causes diagonal tension thereby leading to spiral cracks.
2. Due to spiral cracks, reinforcement should be provided in the form of spirals along the direction of principal tensile stresses but this is often not possible. Thus the requirement of spiral reinforcement is converted to an equivalent longitudinal and transverse reinforcement. **Thus effect of torsion is split into equivalent shear and equivalent moment**.

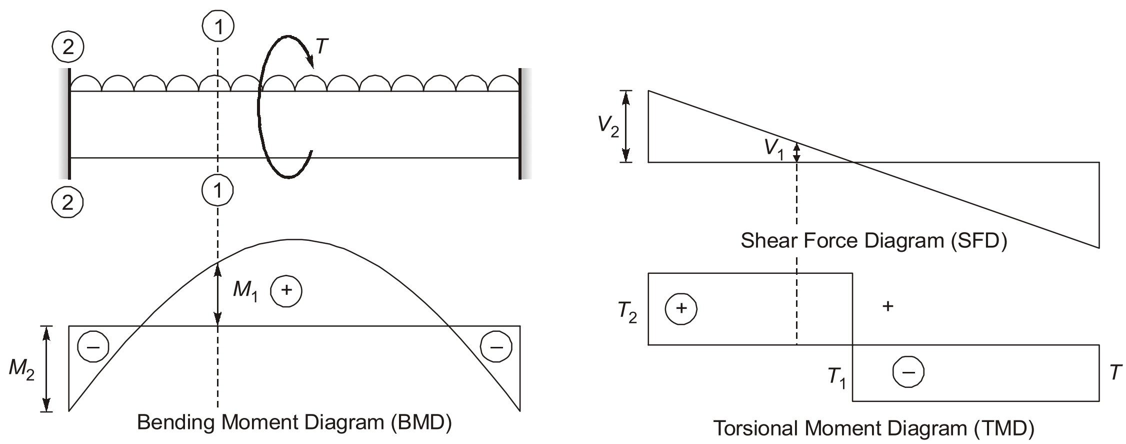


***Fig.** Variation of BM, SF and TM diagram in a fixed beam*

6.6 Analysis for Torsion

There are many methods available to understand the behavior of reinforced concrete members under torsion like **Skew Bending Theory**, **Space-Truss Analogy** etc.

6.7 Torsional Reinforcement

When torsional shear stress (τ_t) exceeds the design shear strength of concrete (τ_c) **(table 19 of IS 456 : 2000)**, then torsional reinforcement is required to be provided in concrete members. As stated earlier, normally torsional shear acts in association with flexural shear (V_u) and in that case **equivalent shear (V_e)** has to be considered as per **Cl. 41.3.1 of IS 456: 2000.**

The expression for equivalent shear (V_e) is given as:

$$\boxed{V_e = V_u + 1.6\left(\frac{T_u}{b}\right)}$$

REMEMBER:

> The flexural shear and torsional shear are additive only on one side of the beam and they act in opposite directions on the other side of the beam.

The equivalent nominal shear stress (τ_{ve}) is then given by:

$$\tau_{ve} = \frac{V_u + 1.6\left(\frac{T_u}{b}\right)}{bd}$$

If this equivalent nominal shear stress (τ_{ve}) exceeds the maximum shear strength of concrete ($\tau_{c\,max}$), then section has to be redesigned.

The strength of a member subjected to both torsion and flexure is described in terms of interaction of T_u/T_{uR} with M_u/M_{uR}. Where, T_{uR} and M_{uR} are respectively the strength of the member subjected to pure torsion and pure flexure.

Cl. 41.4.2 of IS 456 : 2000 gives recommendations for the design of concrete members subjected to both flexure and torsion. This recommendation is based on simplified **Skew Bending Theory**.

Accordingly, the torsional moment can be converted to an equivalent flexural moment as:

$$M_t = \frac{T_u}{1.7}\left(1+\frac{D}{b}\right)$$

Here the M_t so calculated is combined with the flexural moment (M_u) to give equivalent bending moments M_{e1} and M_{e2} as:

$$M_{e1} = M_t + M_u$$
$$M_{e2} = M_t - M_u$$

Reinforcement is designed to resist the equivalent bending moment M_{e1} and the corresponding required steel is provided in the flexural tension zone.

Now when $M_t > M_u$ i.e. $M_{e2} > 0$, then a reinforcement for this equivalent moment (M_{e2}) has also to be designed and is provided in flexural compression zone.

When $M_u = 0$ i.e. in case of **pure torsion**, then $M_{e1} = M_{e2} = M_t$ and equal longitudinal reinforcement has to be provided in the both flexural tension zone and flexural compression zone.

6.8 IS 456: 2000 Provisions for the Design of Reinforcement in Members Subjected to Torsion

1. **Cl. 41.4.3 of IS 456 : 2000** provides recommendations for the design of **2 legged, closed transverse stirrup reinforcement** the area of which is given by:

$$A_{sv} = \frac{T_u S_v}{0.87 f_y b_1 d_1} + \frac{V_u S_v}{2.5 d_1 (0.87 f_y)} = \frac{S_v}{0.87 f_y d_1}\left(\frac{\tau_u}{b_1}+\frac{V_u}{2.5}\right)$$

2. In addition to the above, **Cl. 41.4.3 of IS 456 : 2000** specifies limit on minimum area of transverse reinforcement also as:

$$A_{sv} \geq \frac{(\tau_{ve}-\tau_c)bS_v}{0.87 f_y} \quad \text{or} \quad \frac{A_{sv}}{bS_v} \geq \frac{(\tau_{ve}-\tau_c)}{0.87 f_y}$$

3. As per **Cl. 41.1 of IS 456 : 2000,** in structures where torsion is required to maintain the equilibrium, members shall be designed for torsion. However where torsion can be eliminated by releasing the redundant restraints, no specific design for torsion is necessary provided torsional stiffness is neglected in the calculation of internal forces. There, adequate control of any torsional cracking is being taken care of by the required nominal shear reinforcement.

4. **Cl. 41.2** of **IS 456 : 2000** states that sections located at less than distance '*d*' from the face of the support may be designed for the same torsion as computed at a distance *'d'*.

5. **Cl. 26.5.1.7a of IS 456 : 2000** specifies the distribution of torsional reinforcement in terms of maximum spacing of stirrups (s_v), in order to have sufficient post crack torsional resistance and to control crack widths.

$$s_v \leq \begin{cases} x_1 \\ \dfrac{x_1+y_1}{4} \\ 300\,\text{mm} \end{cases}$$

where, $x_1 = b_1 + 2$ (longitudinal bar dia/2) + 2 (stirrups dia/2)

$y_1 = d_1 + 2$ (longitudinal bar dia/2) + 2 (stirrups dia/2)

} shown in Figure

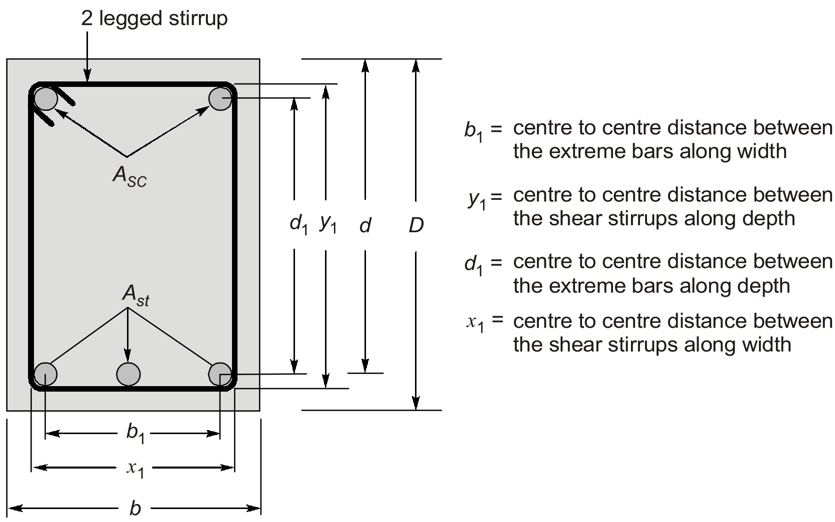


Fig. *Stirrups in beams*

6. **Cl. 26.5.1.7b of IS 456 : 2000** states that *longitudinal reinforcement shall be placed as close as possible to the corners of the cross section and in all cases, there shall be atleast one longitudinal bar at each corner of ties.*
7. **If depth of the member subjected to torsion exceeds 450 mm**, then additional longitudinal bars are required to be provided at the faces with total area not less than 0.1% of web area. These side face bars are to be distributed equally on each face at a spacing not exceeding 300 mm or web thickness, whichever is small.

Example - 6.1 At the limit state of collapse, an RC beam is subjected to flexural moment 200 kN-m, shear force 20 kN and torque 9 kN-m. The beam is 300 mm wide and has a gross depth of 425 mm, with an effective cover of 25 mm.

Determine (i) The equivalent shear force

(ii) The equivalent flexural moment for designing the longitudinal tension steel

Solution:

(i) The equivalent shear force, $$V_e = V + 1.6\frac{T_u}{b} = 20 + \frac{1.6 \times 9}{300 \times 10^{-3}} = 68 \text{ kN}$$

(ii) Since $\tau_{ve} < \tau_c$ so section will be designed for M_u only. (As per 41.3.2 and 41.3.3 of IS code, section should be design for M_{ue} only if $\tau_{ve} < \tau_c$.

Example - 6.2 Design a rectangular beam of size 350 × 750 mm which is acted upon by a factored twisting moment of 150 kNm in combination with an ultimate negative moment of 210 kNm and an ultimate shear force of 110 kN. Use M 30 grade of concrete and Fe415 steel.

Solution:

Given, b = 350 mm, D = 750 mm, f_{ck} = 30 N/mm^2

f_y = 415 N/mm^2, T_u = 150 kNm, M_u = 210 kNm

V_u = 110 kN

Let effective cover to the reinforcing bars = 50 mm

Thus effective depth, $d = 750 \text{ mm} - 50 \text{ mm} = 700 \text{ mm}$

Equivalent bending moment due to torsion:

$$M_t = \frac{T_u}{1.7}\left(1+\frac{D}{b}\right) = \frac{150}{1.7} \times (1 + 750/350) = 277.31 \text{ kNm}$$

Bending moment for design:

$$M_e = M_t \pm M_u$$
$$= 277.31 \pm 210 = 487.31 \text{ kNm} \quad \text{(flexural tension at bottom)}$$
$$= 67.31 \text{ kNm} \quad \text{(flexural compression at top)}$$

Design of bottom reinforcement:

$$R_1 = \frac{M_{e1}}{bd^2} = 487.31 \times 10^6/(350) \times (700)^2 = 2.84146 \text{ N/mm}^2$$

and $$\frac{M_{u\lim}}{bd^2} = 0.138 \times 30 = 4.14 \text{ N/mm}^2 > 2.84145 \text{ N/mm}^2$$

Thus $$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-4.598\frac{R_1}{f_{ck}}}\right] = 8.98798 \times 10^{-3}$$

$$p_t = 0.89\%$$
$$A_{st,\,reqd} = 2202.06 \text{ mm}^2$$

Provide 5-25 mm diameter bars at the bottom.

Thus $A_{st,\,provided} = 2454.37 \text{ mm}^2$

Design of top reinforcement:

$$R_2 = M_{e2}/bd^2 = 67.31 \times 10^6/(350) \times (700)^2 = 0.392478 \text{ N/mm}^2$$

$$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-4.598\frac{R_2}{f_{ck}}}\right] = 1.10398 \times 10^{-3}$$

$$p_t = 0.11\ \%$$

$$A_{st,\,reqd} = \frac{0.110398}{100}\times 350\times 700 = 270.48 \text{ mm}^2$$

Provide 2-16 mm diameter bars at the top.

Thus $A_{st,\,provided} = 402.12 \text{ mm}^2$

Side face reinforcement:

Since depth of the beam (D) is greater than 450 mm, side face reinforcement @ 0.1 % of beam cross sectional area, for torsion is required to be provided.

Thus torsional reinforcement on each face = 0.05 % = $\left(\frac{0.05}{100}\right) \times 350 \times 750 = 131.25 \text{ mm}^2$

Provide 2-10 mm diameter bars on each face.

Side face reinforcement provided on each face = 157.08 mm^2

Vertical spacing between the longitudinal bars should not exceed 300 mm.

Design of shear/transverse reinforcement:

Equivalent nominal shear stress $= \tau_{ve} = \dfrac{V_u + 1.6\dfrac{T_u}{b}}{bd} = \dfrac{110\times 10^3 + 1.6\left(\dfrac{150\times 10^6}{350}\right)}{350\times 700} = 3.24 \text{ N/mm}^2$

$\tau_{c,\,max}$ (for M30 concrete) $= 3.5 \text{ N/mm}^2 > 3.2478 \text{ N/mm}^2$ (OK)

Now for $p_t = \dfrac{2454.37\times 100}{350\times 700} = 1\ \%$ and M30 concrete, $\tau_c = 0.66 \text{ N/mm}^2$

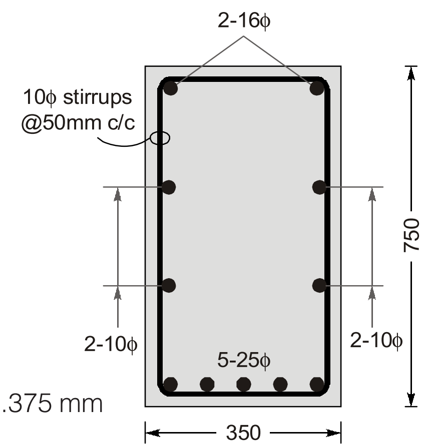
It can be seen that torsional shear stress (= 3.24 N/mm^2) is much higher than design shear strength of concrete (= 0.66 N/mm^2).

Using 2 legged 10 mm diameter bars as shear stirrups, A_{sv} = 2 × 78.54 mm^2 = 157 mm^2

Since effective cover assumed is 50 mm.

So d_1 = 750 mm – 2 × 50 mm = 650 mm

b_1 = 350 mm – 2 × 50 mm = 250 mm



$$s_v = \frac{0.87 f_y A_{sv}}{\frac{T_u}{b_1 d_1} + \frac{V_u}{2.5 d_1}} = \frac{0.87(415)157}{\frac{150\times10^6}{250\times650} + \frac{110\times10^3}{2.5\times650}}$$

$$= 57.21 \text{ mm}$$

Maximum shear spacing requirement:

$$s_v \le \begin{cases} x_1 = 250 + 25 + 10 = 285 \text{ mm} \\ \frac{x_1 + y_1}{4} = \frac{285 + (650 + 12.5 + 8 + 10)}{4} = 241.375 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

Provide 2 legged 10 mm diameter bars as shear stirrups @ 50 mm c/c.

Check for clear cover:

Clear cover = 50 – 10 – 14 mm = 26 mm > 20 mm (O.K.)

Student's Assignment

Q.1 An *RC* structural member rectangular in cross section of width *b* and depth *D* is subjected to a combined action of bending moment *M* and torsional moment *T*. The longitudinal reinforcement shall be designed for a moment M_e given by

(a) $M_e = M + \frac{T(1 + D/b)}{1.7}$

(b) $M_e = M + \frac{T(1 - D/b)}{1.7}$

(c) $M_e = \frac{T(1 + D/b)}{1.7}$

(d) $M_e = \frac{T(1 - b/D)}{1.7}$

Q.2 A beam of rectangular cross-section ($b \times d$) is subjected to a torque *T*. What is the maximum torsional stress induced in the beam ($b < d$ and α is a constant)?

(a) $\frac{T}{\alpha b^2 d}$ (b) $\frac{T}{\alpha b d^2}$

(c) $\frac{T}{\alpha b d}$ (d) $\frac{T}{bd}$

Q.3 Torsion resisting capacity of a given *RC* section

(a) decreases with decrease in stirrup spacing

(b) decreases with increase in longitudinal bars

(c) does not depend upon stirrup and longitudinal steels

(d) increases with the increase in stirrup and longitudinal steels

Q.4 Primary torsion occurs in the case of

(a) a simply supported, but laterally restrained beam subjected to eccentric loading along the span

(b) the edge beam of a building frame

(c) shells elastically restrained by edge beams

(d) a cantilever beam subjected to uniformly distributed load along the span

Q.5 As per IS 456 : 2000, which of the following type of torsion must be considered in the design of reinforced concrete members?

(a) Equilibrium torsion

(b) Compatibility torsion

(c) Both (a) and (b)

(d) No torsion need to be considered for design

Q.6 Side face reinforcement shall be provided in beams if:
(i) Beam depth exceeds 750 mm without torsion
(ii) Beam depth exceeds 450 mm without torsion
(iii) Beam depth exceeds 450 mm with torsion
(iv) Beam depth exceeds 750 mm with torsion
Among the above statements, true one(s) is/are:
(a) (i) and (iii) (b) (i) and (ii)
(c) (ii) and (iv) (d) (iii) and (iv)

Q.7 In a beam subjected to torsion, the equivalent torsion moment is less than the flexural moment. In this case
(a) steel is provided only on tension side
(b) steel is provided only on compression side
(c) steel is provided in both tension and compression side
(d) data insufficient

Q.8 On a 500 mm deep beam of 300 mm wide, subjected to a shear force of 150 kN and torsion 30 kN-m, equivalent shear is:
(a) 180 kN (b) 310 kN
(c) 246 kN (d) 210 kN

Q.9 Beams of grid floor are subjected to
(a) Primary torsion
(b) Secondary torsion
(c) Only bending moment
(d) Bending moment and shear force

Q.10 Torsion resisting capacity of a given reinforce concrete section
(a) decreases with decrease in stirrup spacing
(b) decreases with increase in longitudinal bars
(c) does not depend upon stirrup and longitudinal steels
(d) increases with the increase in stirrups and longitudinal steels

ANSWER KEY // STUDENT'S ASSIGNMENT

1. (a)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (a)
6. (a)	**7.** (a)	**8.** (b)	**9.** (b)	**10.** (d)

HINTS & SOLUTIONS // STUDENT'S ASSIGNMENT

1. (a)

The longitudinal reinforcement shall be designed to resist an equivalent bending moment

$$M_e = M + M_t$$

where $M_t = T\left(\frac{1+D/b}{1.7}\right)$

$\therefore$ $M_e = M + T\left(\frac{1+D/b}{1.7}\right)$

Equivalent shear,

$$V_e = V + 1.6\ T/b$$

2. (a)

As per IS code

$$Ve = V + \frac{1.6T}{B} = \frac{1.6T}{B}$$

$$\tau_v = \frac{Ve}{Bd} = \frac{1.6T}{B.Bd} = \frac{T}{\alpha B^2 d}$$

Here, $V = 0$

So, torsional shear stress for rectangular section

$$\tau = \frac{T}{\alpha b^2 d}$$

where, $\alpha = \frac{1}{1.6}$

3. (d)

The amount of torsion in a member depends upon the magnitude of the torsional stiffness of the member itself in relation to the stiffness of the interconnecting members. In reinforced concrete structures, the stiffness decreases considerably after the formation of cracks if the continuity at the joint are not considered in the design.

The presence of reinforcement in the form of longitudinal and transverse steel increases the torsional moment carrying capacity of beams.

4. (b)

See annex-D of IS 456 : 2000.

8. (b)

$$V_{ue} = V_u + 1.6\left(\frac{T_u}{b}\right)$$

$$= 150 + 1.6 \times \frac{30}{0.3} = 310 \text{ kN}$$

10. (d)

Clause 41.3 of IS 456: 2000, gives the expressions for equivalent shear and moment due to torsion. Thus both additional longitudinal steels and stirrups are required to resist torsion.

■■■■