POSTAL Book Package 2021

Computer Science & IT

Objective Practice Sets

Discrete and Engineering Mathematics

Contents

MADE EASY Publications

Note: This book contains copyright subject matter to MADE EASY Publications, New Delhi. No part of this book may be reproduced, stored in a retrieval system or transmitted in any form or by any means. Violators are liable to be legally prosecuted.

Probability

Q.1 There are four machines and it is known that exactly two of them are faultly. They are tested one by one in a random order till both the faulty machines are identified. The probability that only two tests are required?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

Q.2 Let A and B be any two arbitrary events, then, which one of the following is true?

(a) $P(A \cap B) = P(A)\, P(B)$

(b) $P(A \cup B) = P(A) + P(B)$

(c) $P(A|B) = P(A \cap B)/P(B)$

(d) $P(A \cup B) < P(A) + P(B)$

Q.3 Let $f(x)$ be the continuous probability density function of a random variable x, the probability that $a < x \leq b$, is

(a) $f(b - a)$ (b) $f(b) - f(a)$

(c) $\int_a^b f(x)dx$ (d) $\int_a^b xf(x)dx$

Q.4 A fair six sided die is thrown twice. If the sum of the face values of these two tosses is 5 then what is the probability that the face value of the first toss is less than that of second toss?

Q.5 In a certain year, there were exactly four fridays and exactly four mondays in January. On what day of the week did the 20th of the January fall that year (recall that January has 31 days)?

(a) Sunday

(b) Monday

(c) Wednesday

(d) Friday

Q.6 Candidates were asked to an interview with 3 pens each. Black, Blue, green and red were the permitted pen colours that candidate could bring. The probability that a candidate comes with all 3 pens having the same order is _______. (upto 1 decimal)

Q.7 Two dice are thrown simultaneously. The probability that at least one of them will have 6 facing up is

(a) $\frac{1}{36}$ (b) $\frac{1}{3}$

(c) $\frac{25}{36}$ (d) $\frac{11}{36}$

Q.8 If the coin is tossed for even number of times then find the probability?

(a) $\frac{2}{3}$

(b) $\frac{3}{4}$

(c) $\frac{4}{5}$

(d) None of these

Q.9 Manish and Rahul are family-members. They decide to go on trip so they decide to meet for planning between 1:00 pm to 2:00 pm on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will on that day is:

(a) $\frac{1}{4}$ (b) $\frac{1}{16}$

(c) $\frac{7}{16}$ (d) $\frac{9}{16}$

Q.10 Aishwarya studies either computer science or mathematics everyday. If she studies computer science on a day, then the probability that she studies mathematics the next day is 0.6. If she studies mathematics on a day, then the probability that she studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probablity that she studies computer science on Wednesday?

(a) 0.24 (b) 0.36
(c) 0.4 (d) 0.6

Q.11 Consider a large village, where only two newspapers P_1 and P_2 are available to the families. It is known that the proportion of families

1. not taking P_1 is 0.48
2. not taking P_2 is 0.58.
3. taking only P2 is 0.30.

The probability that a randomly chosen family from the village takes only P_1 is

(a) 0.24
(b) 0.28
(c) 0.40
(d) cannot be determined

Q.12 A determinant is chosen at random from the set of all determinants of order 2 with element 0 or 1 only. The probability of choosing a non-zero determinant is

(a) $\frac{3}{16}$ (b) $\frac{3}{8}$
(c) $\frac{1}{4}$ (d) None of these

Q.13 A and B are friends. They decide to meet between 1:00 pm and 2:00 pm on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is

(a) 1/4 (b) 1/16
(c) 7/16 (d) 9/16

Q.14 What is the probability that in a randomly chosen group of r people at least three people have the same birthday?

(a) $1-\frac{365.364...(365-r+1)}{365^r}$

(b) $1-\frac{365.364...(365-r+1)}{365^r}+{}^rC_2\cdot 365\cdot\frac{364.363...(364-(r-2)+1)}{364^{r-2}}$

(c) $1-\frac{365.364...(365-r+1)}{365^r}-{}^rC_2\cdot 365\cdot\frac{364.363...(364-(r-2)+1)}{364^{r-2}}$

(d) $\frac{365.364...(365-r+1)}{365^r}$

Q.15 A typical page in a book contains one typo per page. What is the probability that there are exactly 8 typos in a given 10-page chapter?

(a) $e^{-10}\cdot\frac{10^8}{8!}$ (b) $e^{-8}\frac{8^{10}}{10!}$
(c) $e^{-8}\frac{10^8}{8!}$ (d) none of these

Q.16 A six card hand is dealt from an ordinary deck of cards. Find the probability that there are 3 cards of one suit and 3 of another suit.

(a) $\frac{{}^{13}C_6}{{}^{52}C_6}$ (b) $\frac{({}^{13}C_3)^2}{{}^{52}C_6}$
(c) $\frac{2({}^{13}C_3)^2}{{}^{52}C_6}$ (d) $\frac{6({}^{13}C_3)^2}{{}^{52}C_6}$

Q.17 A bag contains 10 blue marbles, 20 green marbles and 30 red marbles. A marble is drawn from the bag, its colour recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same colour is

(a) $\frac{1}{36}$ (b) $\frac{1}{6}$
(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

Answers Probability

1.	(d)	2.	(c)	3.	(c)	4.	(0.5)	5.	(a)	6.	(0.2)	7.	(d)	8.	(a)	9.	(c)
10.	(c)	11.	(c)	12.	(a)	13.	(c)	14.	(c)	15.	(a)	16.	(d)	17.	(b)	18.	(b)
19.	(d)	20.	(b)	21.	(d)	22.	(d)	23.	(b)	24.	(c)	25.	(c)	26.	(d)	27.	(b)
28.	(d)	29.	(c)	30.	(b)	31.	(0.75)	32.	(a)	33.	(a)	34.	(0.4)	36.	(d)	37.	(a)
38.	(d)	39.	(c)	40.	(b)	41.	(b)	42.	(c)	43.	(b)	44.	(c)	45.	(b)	47.	(b)
48.	(a)	49.	(c)	50.	(b)	51.	(b)	52.	(a)	54.	(0.94)	55.	(c)	56.	(b)	57.	(d)
58.	(a)	59.	(c)	60.	(b)	61.	(b)	62.	(a)	63.	(a)	64.	(c)	65.	(b)	66.	(b)
67.	(b)	68.	(0.04)	69.	(b)	70.	(b)	71.	(0.1)	72.	(b)	73.	(c)	74.	(c)	75.	(a)
76.	(b)	77.	(d)	78.	(b)	79.	(0.22)	80.	(c)								

Explanations Probability

1. (d)

There are 4 machines M_1, M_2, M_3, M_4.

Here say M_3, M_4 are faulty.

So, we can select it either by M3, M4 or M4, M3 = 2 ways.

Now, among 4 machines, we can select 2 in (4) × (3) ways = 12 ways.

So, total probability that only 2 test cases required to get both machines are faulty is $\frac{2}{12} = \frac{1}{6}$

So, option (d) is correct.

2. (c)

(a) $P(A \cap B) = P(A)\,P(B)$ is false since this is true if and only if A and B are independent events.

(b) $P(A \cup B) = P(A) + P(B)$ is false since $P(A \cap B)$ is zero if and only if A and B are mutually exclusive.

(c) $P(A|B) = P(A \cap B)/P(B)$ is true.

(d) $P(A \cup B) < \text{P(A)} + P(B)$ is false.

Since $P(A \cup B) \le P(A) + P(B)$

3. (c)

If $f(x)$ is the continuous probability density function of a random variable X then,

$p(a < x \le b) = p(a \le x \le b)$

$$= \int_a^b f(x)dx$$

4. (0.5)

Total number of possible pairs = 36

$\{(a,b) | 1 \le a \le 6,\ 1 \le b \le 6\}$

(*i*) Sum of face values = 5

$\{(1,4), (2,3), (3,2), (4,1)\}$

(*ii*) First toss is less than that of second toss.

$\{(1,4), (2,3)\}$

$\therefore$ Probability $= \frac{2}{4} = \frac{1}{2}$

5. (a)

January has 31 days, no of complete weeks in Jan. $= \left(\frac{31}{7}\right) = 4$

Then remaining days 31 – 7(4) = 3 since mentioned there are exactly 4 mondays and 4 fridays then these monday and fridays are already covered in the 4 complete weeks. Hence, for these 3 days we need 5 consecutive days other than monday and friday.

The only 3 consecutive days other than monday and friday is:

29th Jan → Tuesday

30th Jan → Wednesday

31st Jan → Thursday

Then the 4 monday are:

28th Jan → Monday
21st Jan → Monday
14th Jan → Monday
7th Jan → Monday
Thus, 20th Jan → Sunday
Hence, option (a) is answer.

6. (0.2)
Probability of all 3 pens being same colour.
= number of ways of choosing 3 pens of same colour/Total number of ways of choosing 3 pens
= 4/[4 + (4 × 3) + C(4,3)] = 4/20 = 0.2

7. (d)
P(atleast one of dice will have 6 facing

$$= 1 - P(\text{none of dice have 6 facing up})$$

$$= 1-\left[\frac{5}{6}\times\frac{5}{6}\right] = 1-\frac{25}{36} = \frac{11}{36}$$

8. (a)
Coin is tossed n times:

$$\left.\begin{matrix} HTHT\cdots HH \\ \text{(or)} \\ THTH\cdots TT \end{matrix}\right\} n \text{ is even number}$$

$$\text{Probability } = \frac{1}{2^n}+\frac{1}{2^n} = \frac{1}{2^{n-1}}$$

Probability for even number of times

$$= 2 \text{ times} + 4 \text{ times} + \cdots$$

$$= \frac{1}{2^{2-1}}+\frac{1}{2^{4-1}}+\frac{1}{2^{6-1}}+\cdots$$

$$= \frac{1}{2^1}+\frac{1}{2^3}+\frac{1}{2^5}+\cdots$$

$$= \frac{1}{2}\left(1+\frac{1}{4}+\frac{1}{4^2}+\cdots\right)$$

$$= \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right) = \frac{2}{3}$$

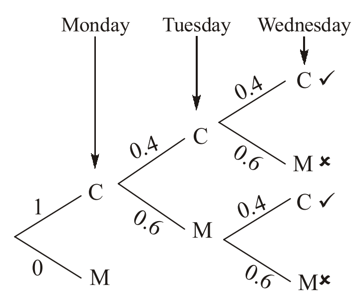
∴ Option (a) is correct.

9. (c)
Probability that one person meet on that day =
$\frac{15}{60} = \frac{1}{4}$

Prob. (failing to meet by both the persons) =
$\frac{3}{4}\times\frac{3}{4} = \frac{9}{16}$
Prob. (meet on that day by both the persons)

$$= 1-\frac{9}{16} = \frac{7}{16}$$

10. (c)
Let C denote computes science study and M denotes maths study. The tree diagram for the problem can be represented as shown below:

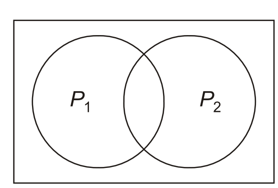


Now by rule of total probability we total up the desired branches (✓) and get the answer as shown below:
p(C on monday and C on wednesday)
= p(C on monday, C on tuesday and C on wednesday) + p(C on monday, M on tuesday and C on wednesday)

$$= 1 \times 0.4 \times 0.4 + 1 \times 0.6 \times 0.4$$

$$= 0.16 + 0.24 = 0.40$$

11. (c)



only P_1 = 0.30

Not P_1 = only $P_2 + \overline{P_1 \cup P_2}$ = 0.48

$\overline{P_1 \cup P_2}$ = 0.48 – 0.3 = 0.18

Not P_2 = only $P_1 + \overline{P_1 \cup P_2} = 0.58$

Only P_1 = 0.58 – 0.18 = 0.40
So, option (c) is correct.

12. (a)

With 0 and 1,

The no. of determinants possible $2^4 = 16$ as for every location of 0 and 1 there are 4 choices in total.

Now, there are only 3 determinants with positive values:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

So, the probability of choosing a non-zero determinant

$$\frac{\text{Total non-zero determinants possible}}{\text{Total determinants possible}} = \frac{3}{16}$$

13. (c)

Meeting occurs if the first person arrives between 1:00 and 1:45 and the second person arrives in the next 15 minutes or if both the persons arrive between 1:45 and 2:00.

Case 1:

45/60 are favourable cases and hence probability of first person arriving between 1:00 and 1:45 is 3/4.

Probability of second person arriving in the next 15 minutes =15/60=1/4

So, probability of one person arriving between 1:00 and 1:45 and meeting the other = 3/4 × 1/4 × 2 = 3/8 (2 for choosing the first arriving friend)

Case 2:

Both friends must arrive between 1:45 and 2:00.

Probability =1/4×1/4=1/16.

So, probability of a meet =3/8+1/16=7/16

Correct Answer: (c)

14. (c)

P(at least three people have the same birthday) = 1 – P (all have different b'days) – P(exactly two people have same b'day)

Now, P(all have different b'days)

$$= \frac{365.364...(365-r+1)}{365^r}$$

P(exactly two people have same b'day)

$$= {}^rC_2 \cdot 365 \cdot \frac{364.363...(364-(r-2)+1)}{364^{r-2}}$$

[${}^rC_2$ ways to choose who those two people with same b'day are, 365 ways to choose what the b'day is]

Now,

P (at least three people have the same birthday)

$$= 1 - \frac{365.364...(365-r+1)}{365^r} - {}^rC_2 \cdot 365 \cdot \frac{364.363...(364-(r-2)+1)}{364^{r-2}}$$

Which is option (c).

15. (a)

$$\text{Poisson distribution} = e^{-\lambda} \cdot \frac{\lambda^x}{x!}$$

Expected typos on one page is 1

$\Rightarrow$ Expected typos in 10 page is 10

$$\lambda = 10$$

$$\therefore \quad \text{Probability} = e^{-10} \cdot \frac{10^8}{8!}.$$

So option (a) is correct.

16. (d)

Two suits can be chosen: ${}^4C_2$ ways

3 cards can be picked from same suit: ${}^{13}C_3$ ways

$$\text{Probability} = \frac{{}^4C_2({}^{13}C_3 \cdot {}^{13}C_3)}{{}^{52}C_6}$$

$$= \frac{6 \cdot ({}^{13}C_3)^2}{{}^{52}C_6}$$

So option (d) is correct.

17. (b)

The given condition corresponds to sampling with replacement and with order.

No 2 marbles have the same color i.e. Drawn 3 different marble.

So total number of ways for picking 3 different marbles = 3! = 6.

Probability of getting blue, green, red in order

$$= \frac{10}{60} \times \frac{20}{60} \times \frac{30}{60} \times 6$$

[Since 6 ways to get the marbles]

$$= \frac{1}{6}$$