Electrical Engineering

Electromagnetic Theory

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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Electromagnetic Theory

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Contents

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Chapter 4

Time-Varying Electromagnetic Fields

4.1 Introduction

The basic relationship of electrostatic and the magnetostatic fields were obtained in the previous chapters, and we are now ready to discuss time-varying fields. The discussion will be precise, for vector analysis and vector calculus should now be more familiar tools; some of the relationships are unchanged, and most of the relationships are changed only slightly.

Two new concepts will be introduced: the electric field produced by a changing magnetic field and the magnetic field produced by a changing electric filed. The first of these concepts resulted from experimental research by Michael Faraday, and the second from theatrical efforts of James Clerk Maxwell.

4.2 Maxwell's Equations for Static EM Fields

Having derived Maxwell's four equations for static electromagnetic field, we put them together as in table 4.1.

Remarks	Differential or Point Form	Integral Form
Gauss's law	$\nabla \cdot \vec{D} = \rho_v$	$\oint_s \vec{D} \cdot d\vec{S} = \int_v \rho_v dv$
Nonexistence of magnetic monopole	$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot d\vec{S} = 0$
Conservativeness of electrostatic field	$\nabla \times \vec{E} = 0$	$\oint_L \vec{E} \cdot \vec{dl}$
Ampere's law	$\nabla \times \vec{H} = \vec{J}$	$\oint_L \vec{H} \cdot d\vec{l} = \int_v \vec{J}.d\vec{S}$

Table 4.1 : *Maxwell's Equations for Static EM Fields*

It is evident from table 4.1 that a vector field is defined completely by specifying its curl and divergence. A field can only be electric or magnetic if it satisfies the corresponding Maxwell's equations.

It should be noted that Maxwell's equations as in table 3.2 are only for static EM fields. As will be discussed later in chapter 4, the divergence equations will remain the same for time-varying EM fields but the curl equations will have to be modified.

4.3 Faraday's Law of Induction

In 1831 Michael Faraday performed experiments of check whether current is produced in a closed wire loop placed near a magnet, in analogy to dc currents producing magnetic fields. His experiment showed that this could not be done, but Faraday realized that a time-varying current in the loop was obtained while the magnet was being moved toward it or away from it. The law he formulated is known as Faraday's law of electromagnetic induction. It is perhaps the most important law of electromagnetism.

The phenomenon of electromagnetic induction has a simple physical interpretation. Two charged particles ("charges") at rest act on each other with a force given by Coulomb's law. Two charges moving with uniform velocities act on each other with additional force, the magnetic force. If a particle is accelerated, there is another additional force that it exerts on other charged particles, stationary or moving. As in the case of magnetic force, if only a pair of charges is considered, this additional force is much smaller than Coulomb's force. However, time-varying currents in conductors involve a vast number of accelerated charges, and produce effects significant enough to be easily measurable.

4.3.1 The Electromotive Force (EMF)

Faraday's original experiments consisted of a conducting loop through which he could impose a dc current via a switch. Another short circuited loop with no source attached was nearby, as shown in figure 4.1. When a dc current is passed through loop 1, no current flowed in loop 2. However, when the voltage was first applied to loop 1 by closing the switch, a transient current flowed in the opposite direction in loop 2.

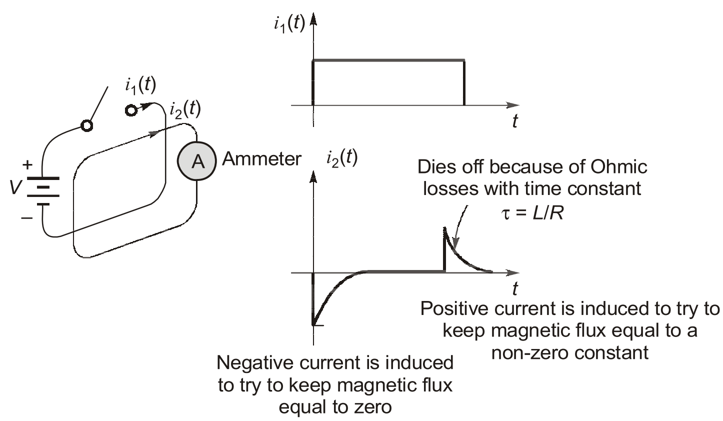


***Figure 4.1 :** Faraday's experiment*

When the switch was later opened, another transient current flowed in loop 2, this time in the same direction as the original current in loop 1. Currents are induced in loop 2 whenever a time varying magnetic flux due to loop 1 passes through it.

Faraday's law states that the line integral of the electric field around a closed loop equal the time rate of change of magnetic flux through the loop. The positive convention for flux is determined by the right-hand rule of curling the fingers on the right hand in the direction of traversal around the loop. The thumb then points in the direction of positive magnetic flux.

In general, a time varying magnetic flux can pass through a circuit due to its own or nearby time varying current or by the motion of the circuit through a magnetic field.

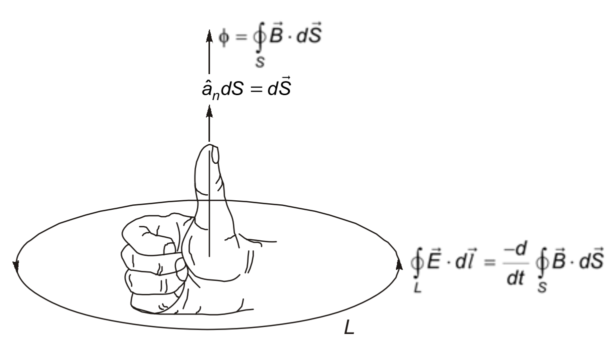


***Figure 4.2 :** Faraday's Law*

For any loop, as in figure 4.2, Faraday's law is

$$V_{EMF} = \oint_L \vec{E} \cdot d\vec{l} \tag{4.1 a}$$

$$= -\frac{d\phi}{dt} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{S} \tag{4.1 b}$$

where EMF is the electromotive force defined as the line integral of the electric field.

The above equation is called as Farayday's law of electromagnetic induction.

NOTE : The reference direction along the contour, by convention, is connected with the reference direction of the normal to the surface S spanning the contour by the right-hand rule.

Example 4.1 A time varying magnetic flux linking a coil is given by $\psi = \frac{1}{2}\alpha t^2$. If at time $t = 2$ s, the emf induced is 4 V, then the value of α is given by

(a) zero (b) -2 Wb/s² (c) 2 Wb/s² (d) 4 Wb/s²

Solution : (b)

$$V_{emf} = -\frac{d\psi}{dt} = \frac{d}{dt}\left[-\frac{1}{2}\alpha t^2\right] = -\frac{1}{2} \times 2 \times \alpha \times t = -\alpha t$$

Given, $-\alpha t = 4\,|_{t=2}$

⇒ $\alpha = -2$ Wb/s²

4.4 Transformer and Motional EMFs

The variation of flux with time as in eq. (4.1) may be caused in three ways:

1. By having a stationary loop in a time-varying B field.
2. By having a time-varying loop area in a static B field.
3. By having a time-varying loop area in a time-varying B field.

4.4.1 Stationary Loop in Time-Varying B Field (Transformer EMF)

This is the case portrayed in figure 4.3 where a stationary conducting loop is in a time varying magnetic field $\vec{B}$. Equation (4.1) becomes

$$V_{EMF} = \oint_L \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \quad ...(4.2)$$

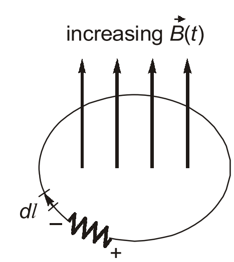


***Figure 4.3 :** Induced emf due to a stationary loop in a time varying B field.*

This emf induced by the time-varying current (producing the time-varying B field) in a stationary loop is often referred to as *transformer emf* in power analysis since it is due to transformer action. By applying Stokes's theorem to the middle term in eq. (4.2), we obtain

$$\oint_L \vec{E} \cdot d\vec{l} = \int_S \nabla \times \vec{E} \cdot d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \quad ...(4.3)$$

Which is equivalent to:

$$\int_S \left(\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t}\right) \cdot d\vec{S} = 0 \quad ...(4.4)$$

Student's Assignments 1

Q.1 In a certain medium, the ratio of magnitudes of displacement current density and conduction current density is given by 10^{-10}. The medium can be considered as
(a) Perfect dielectric
(b) Perfect conductor
(c) Lossy dielectric
(d) Lossy conductor

Q.2 Maxwell inserted the expression for displacement current J_d in Ampere's law to satisfy.
(a) Ampere's law for time varying case
(b) Faraday's law
(c) Gauss's law
(d) Equation of continuity

Q.3 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

	List-I		List-II
A.	Ampere's law	1.	$\nabla^2 V = \frac{-\rho_v}{\varepsilon}$
B.	Faraday's law	2.	$\nabla \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t}$
C.	Poisson's equation	3.	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
D.	Continuity equation	4.	$\nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	4	3	1	2
(c)	3	4	2	1
(d)	3	4	1	2

Q.4 A parallel plate capacitor has plate area A, separated by a distance d, and contains dielectric of permittivity ε. When a voltage $V_0 \sin \omega t$ is applied to its plate, the magnitude of displacement current density J_d and conduction current density J_c are
(a) $J_d > J_c$ (b) $J_d < J_c$
(c) $J_d = J_c$ (d) $J_d = 0$

Q.5 The following is not a Maxwell's equation
(a) Div $\vec{E} = \frac{\rho_v}{\varepsilon_0}$ (b) Div $\vec{H} = 0$
(c) Curl $\vec{E} = 0$ (d) Curl $\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Q.6 A loop is rotating about the y-axis in a magnetic field $\vec{B} = B_0 \sin\omega t\, \hat{a}_x$ Wb/m². The voltage induced in the loop is due to
(a) Motional emf
(b) Transformer emf
(c) A combination of motional and transformer emf
(d) None of these

Q.7 What is the generalized Maxwell's equation $\nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$ for free space?
(a) $\nabla \times \vec{H} = 0$ (b) $\nabla \times \vec{H} = \vec{J}_c$
(c) $\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$ (d) $\nabla \times \vec{H} = \vec{D}$

Q.8 The inconsistency of continuity equation for time varying fields was corrected by Maxwell and the correction applied was
(a) Ampere's law, $\frac{\partial \vec{D}}{\partial t}$
(b) Gauss's law, $\vec{J}$
(c) Faraday's law, $\frac{\partial \vec{B}}{\partial t}$
(d) Ampere's law, $\frac{\partial \vec{P}}{\partial t}$

■ ANSWERS

1.	(b)	**2.**	(d)	**3.**	(b)	**4.**	(c)	**5.**	(c)
6.	(c)	**7.**	(c)	**8.**	(a)				

Student's Assignments 2

Q.1 In a certain medium for which σ = 10 S/m and ε_r = 1 the electric field intensity is $E = 100 \sin 10^{10} t$ V/m.

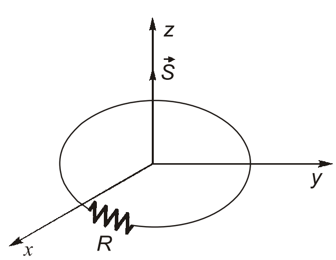
(i) Find conduction and displacement current densities.

(ii) The frequency at which they have equal magnitudes.

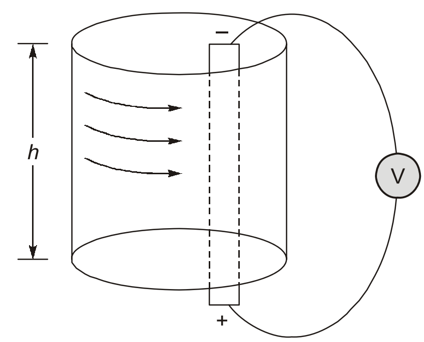
Q.2 The flux through each turn of 100-turn coil is $(t^3 - 2t)$ mWb, where t is in seconds. Find the value of induced emf at $t = 2$ s.

Q.3 A certain medium has a conductivity of 10^{-3} S/m and $\varepsilon_r = 2.5$. Find conduction current density J_c and displacement current density J_d if the electric field $E = 6 \times 10^{-6} \sin 9 \times 10^9 t$ (V/m).

Q.4 The circular loop conductor shown below lies in the $z = 0$ plane, has a radius of 10 cm and a resistance of 5 Ω. If the magnetic field is given by $\vec{B} = 0.2 \sin 10^3 t\, \hat{a}_z (T)$, calculate the value of current flowing through the loop.



Q.5 A conducting cylinder of radius 7 cm and height 15 cm rotates at 600 rev/min in a radial field $\vec{B} = 0.2\hat{a}_r$ T. Sliding contacts at the top and bottom connect to a voltmeter as shown below. Find the induced voltage.



■ ANSWERS

1. 180 GHz

2. –1 V

3. $J_c = 6 \times 10^{-9} \sin 9 \times 10^9 t$ (A/m^2),
$J_d = 1.2 \times 10^{-6} \cos 9 \times 10^9 t$ (A/m^2)

4. $-0.4\pi \cos 10^3 t$ (A)

5. $0.88(-\hat{a}_z)$ V/m

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