UPPSC-AE 2020

Uttar Pradesh
Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Civil Engineering

Fluid Mechanics and Machines including OCF and Hydro Power Engineering

Well Illustrated **Theory** *with*

Solved Examples and **Practice Questions**

MADE EASY Publications

Fluid Mechanics and Machines including OCF and Hydro Power Engineering

Contents

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CHAPTER 7

Laminar Flow

7.1 Introduction

- Laminar flow is a flow in which liquid moves in layers, one layer sliding over another layer. In this type of flow there is no mixing between different layers and hence shear force is exclusively due to viscosity.

7.2 Laminar Flow in Pipe

- In laminar flow, viscous forces predominates inertial forces.
- It has been found that, in a pipe flow, flow is always laminar if $R_e\left(\text{Reynolds no} = \frac{Vd}{\nu}\right) < 2000$

  where, V = Velocity of flow
  d = Diameter of pipe
  ν = Kinematic viscosity
- Flow generally becomes turbulent when $R_e < 4000$. But the value of Reynold's number at which flow changes from laminar to turbulent is not well defined.
- In between (i.e. generally between $R_e = 2000$ to 4000) flow is in transition (sometimes laminar sometimes turbulent).
- **For flow between parallel plates:**
  $R_e < 1000$, laminar
  $1000 < R_e < 2000$, transition
  $R_e > 2000$, turbulent
- **For flow between open channel:**
  $R_e < 500$, laminar
  $R_e > 2000$, turbulent in between transition
- **For flow through soil:**
  $R_e < 1$, laminar
  $1 < R_e < 2$, transition
  $R_e > 2$, turbulent

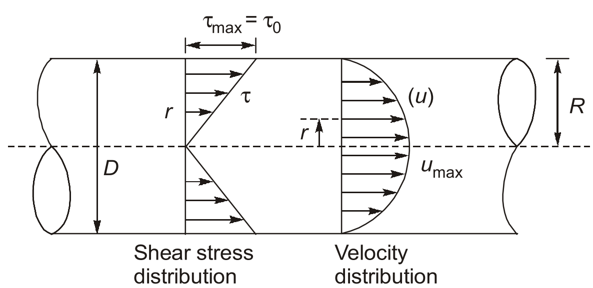
Basic Equations

The basic equations which govern the motion of viscous and incompressible fluid in laminar flow are Navier stoke's equations. For a 2-dimensional, steady and uniform flow, relation between shear and pressure gradient $\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$ i.e. pressure gradient in the direction of flow is equal to the shear stress gradient in the normal direction.

NOTE

- Flow in the pipe does not get established from the starting itself. The length of pipe from the entry in which flow is fully established and thereafter remains constant is called establishment length. For laminar flow this length is (0.05 R_e) D and for turbulent flow it is (25 – 40) D.

7.3 Laminar Flow Through Circular Pipe



7.3.1 Shear Stress and Its Distribution

Linear variation zero at centre and maximum at boundary.

i.e.

$$\tau = \tau_0 \cdot \frac{r}{R}$$

$$\tau = -\frac{dp}{dx} \cdot \frac{r}{2}$$

At boundary,

$$\tau_0 = \tau_{max} = -\frac{dp}{dx} \cdot \frac{R}{2}$$

Note:

$$-\frac{dp}{dx} = \frac{\gamma \cdot h_L}{L}$$

∴

$$\frac{2 \cdot \tau_0}{R} = \frac{\gamma \cdot h_L}{L} = \frac{\gamma}{L} \cdot \frac{fLV^2}{D \times 2g}$$

$$\frac{4\tau_0}{D} = \frac{\rho g f V^2}{D \times 2g}$$

$$8\,\tau_0 = \rho f \cdot V^2 \qquad (V = V_{av})$$

∴

$$\frac{\tau_0}{\rho} = \frac{f}{8} \cdot V_{av}^2$$

$$\frac{\tau_0}{\rho} = \frac{N}{m^2 \times kg/m^3} = m^2/s^2$$

$$\sqrt{\frac{\tau_0}{\rho}} = \text{Unit of velocity}$$

$$\sqrt{\frac{\tau_0}{\rho}} = u_* = \text{shear velocity}$$

$$\therefore \quad \frac{V_{av}}{u_*} = \sqrt{\frac{8}{f}}$$

$$u_* = \sqrt{\frac{f}{8}} \cdot V_{av}$$

Valid for both laminar and turbulent flow.

7.3.2 Velocity Distribution

$$u = \frac{1}{4\mu} \cdot \left(-\frac{dp}{dx}\right) \cdot (R^2 - r^2)$$

At $\gamma = 0$

$$u = u_{max}$$

$$\therefore \quad u_{max} = \frac{1}{4\mu} \cdot \left(-\frac{dp}{dx}\right) \cdot R^2$$

$$\therefore \quad u = u_{max} \cdot \left(1 - \frac{r^2}{R^2}\right)$$

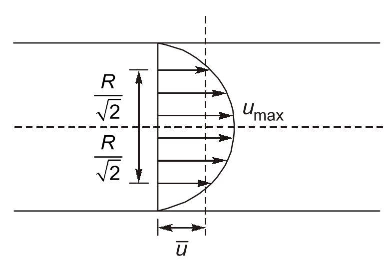
$$\bar{u} \text{ (average velocity)} = \frac{1}{8\mu} \cdot \left(-\frac{dp}{dx}\right) \cdot R^2$$

$$\therefore \quad \frac{u_{max}}{\bar{u}} = 2$$

At, $u = \bar{u}$

$$u_{max} \cdot \left(1 - \frac{r^2}{R^2}\right) = \frac{u_{max}}{2}$$

$$\frac{r^2}{R^2} = \frac{1}{2}$$



$$r = \pm\frac{R}{\sqrt{2}} = \pm 0.707 \text{ R}$$

- Hence, velocity distribution is parabolic, mean velocity occurs at 0.707 R from centre of pipe and is equal to half of the maximum velocity which occurs at centre.

7.3.3 Head Loss

$$h_L = \frac{32\mu \bar{u} L}{\gamma \cdot D^2} \quad \text{(Hagen Poiseuille equation)} \qquad ...(1)$$

$$h_L = \frac{128\mu \cdot QL}{\pi\gamma \cdot D^4}$$

$$\therefore \quad h_L \propto \frac{1}{D^4}$$

Now, $$h_L = \frac{fLV^2}{D \times 2g} \qquad ...(2)$$

Equating equation (1) and equation (2)

$$\therefore \quad \frac{fLV^2}{D \times 2g} = \frac{32\mu \bar{u} L}{\gamma \cdot D^2}$$

$$f = \frac{32\mu \cdot V \cdot L \times 2g \times D}{\rho g D^2 \times LV^2} = \frac{64}{\left(\frac{\rho \cdot VD}{\mu}\right)} = \frac{64}{R_e}$$

$\therefore$ Friction factor for laminar flow through circular pipe is $\frac{64}{R_e}$.

7.3.4 Power (P)

In laminar flow the power required to overcome the frictional resistance,

$$P = \gamma \cdot Q\, h_L = (p_1 - p_2)Q$$

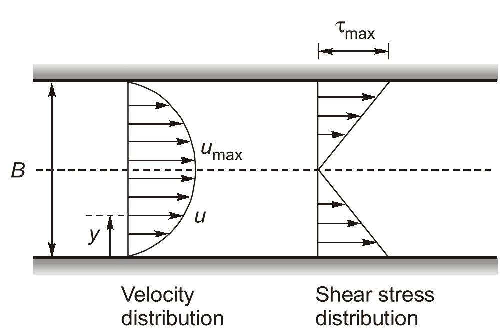
p_1 and p_2 are pressure at section (1) and (2).

NOTE

$$\tau_0 = -\frac{dp}{dx} \cdot \frac{R}{2} = \frac{\gamma h_L}{L} \cdot \frac{D}{4} = \frac{\gamma \cdot \frac{32\mu \bar{u} L}{\gamma D^2}}{L} \frac{D}{4}$$

$$\tau_0 = \frac{8\mu \bar{u}}{D}$$

7.4 Flow Between Two Stationary Parallel Plates



7.4.1 Velocity Distribution

$$u = \frac{1}{2\mu} \cdot \left(-\frac{dp}{dx}\right) \cdot (By - y^2)$$

$$u_{max}\left(\text{at } y = \frac{B}{2}\right) = \frac{1}{8\mu} \cdot \left(-\frac{dp}{dx}\right) \cdot B^2$$

$$\bar{u} = u_{av}$$

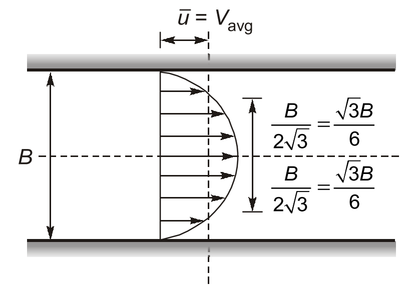
$$= \frac{1}{12\mu} \cdot \left(-\frac{dp}{dx}\right) \cdot B^2$$

$$\frac{u_{max}}{\bar{u}} = \frac{3}{2}$$

Point where $u = \bar{u}$

$$y = \frac{B}{2} \pm \frac{B}{2\sqrt{3}}$$

i.e. $\left(\text{At } \frac{B}{2\sqrt{3}} \text{ from centre}\right)$



7.4.2 Shear Stress Distribution

$$\tau = \frac{1}{2}\cdot\left(-\frac{dp}{dx}\right)\cdot(B-2y)$$

$$\therefore \quad \tau\,(\text{at boundary}) = \tau_0$$

$$= \frac{1}{2}\cdot\left(-\frac{dp}{dx}\right)\cdot B$$

$$\tau_0 = \frac{6\mu\bar{u}}{B}$$

7.4.3 Head Loss (h_L)

$$h_L = \frac{12\mu\bar{u}L}{\gamma\cdot B^2}$$

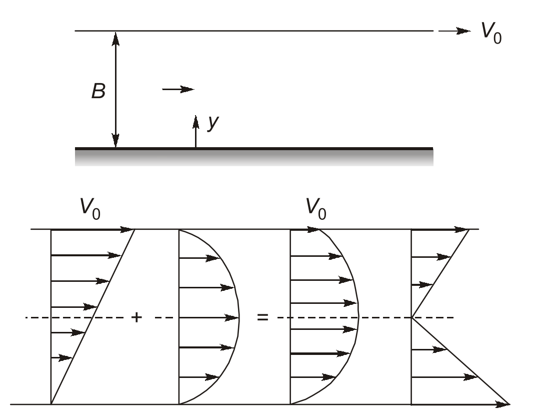
$$Q = \bar{u}\times(B\times 1)$$

$$h_L = \frac{12\mu Q\cdot L}{\gamma\cdot B^3}$$

$$\therefore \quad h_L \propto \frac{1}{B^3}$$

7.5 Couette Flow

When one plate is stationary and other is moving with velocity V_0 the flow is called couette flow.



$$u = \frac{V_0 y}{B} + \frac{1}{2\mu}\left(\frac{-dP}{dx}\right)(By - y^2)$$

7.6 Kinetic Energy Correction Factor (α)

For laminar flow in circular pipe - (2.0)

For laminar flow between parallel plates = 1.543

For turbulent flow in pipe in case of power law = $\frac{4}{3}$

For turbulent flow in pipe in case of logarithmic law - 1.03 to 1.06.

7.7 Momentum Correction Factor

For laminar flow in circular pipe = $\frac{4}{3}$

For turbulent flow through circular pipe = 1.015

For laminar flow between parallel plate = 1.2

Example - 7.1 Which of the following represents the Darcy's friction factor in terms of Reynold's number (Re) for the laminar flow in circular pipes

(a) 16/Re (b) 32/Re
(c) 64/Re (d) None of these

Ans.(c)

Example - 7.2 For a laminar flow through a channel, Reynolds number is given by 1500, what is the friction factor?

(a) 0.1 (b) 0.043
(c) 0.086 (d) 0.0054

Ans.(b)

$$f = \frac{64}{\text{Re}}$$

Given Re = 1500 (< 2000)
Hence, flow through channel is laminar

$\therefore$ $$f = \frac{64}{\text{Re}} = \frac{64}{1500} = 0.043$$

Example - 7.3 A circular pipe of radius R carries a laminar flow of a fluid. The average velocity is indicated as the local velocity at what radial distance measured from the centre?

(a) 0.50 R (b) 0.71 R
(c) 0.67 R (d) 0.29 R

Ans.(b)

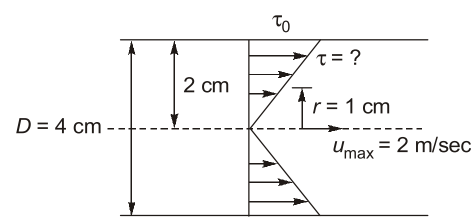
At $$r = \pm\frac{R}{\sqrt{2}} \quad (u = \bar{u})$$

Example - 7.4 In a 4 cm diameter pipeline carrying laminar flow of a liquid with μ = 1.6 centipoise, the velocity at the axis is 2 m/sec. What is the shear stress midway between the well and the axis?

(a) 0.01 N/m² (b) 0.0125 N/m²
(c) 0.0175 N/m² (d) 0.02 N/m²

Ans. (*)

$$\tau_0 = \text{Boundary shear stress} = \frac{8\mu\bar{u}}{D}$$



Given

$$u_{max} = 2 \text{ m/sec}$$
$$\mu = 1.6 \text{ centipoise}$$
$$= \frac{1.6\times10^{-2}}{10} = 1.6\times10^{-3} \text{ N-s/m}^2$$
$$\bar{u} = \frac{u_{max}}{2} = \frac{2}{2} = 1 \text{ m/sec}$$

∴
$$\tau_0 = \frac{8\mu\bar{u}}{D} = \frac{8\times1.6\times10^{-3}\times1}{4\times10^{-2}} = 0.32 \text{ N/m}^2$$

Now,
$$\tau = \tau_0\cdot\frac{r}{R} = 0.32\times\frac{1}{2} = 0.16 \text{ N/m}^2$$

Example - 7.5 The highest velocity (in cm/sec) for flow of water of viscosity 0.01 poise to be laminar in a 6 mm pipe is

(a) $\frac{100}{3}$ cm/sec (b) $\frac{125}{3}$ cm/sec
(c) 50 cm/sec (d) 200 cm/sec

Ans. (a)

For flow to be laminar

$$(Re) \le 2000$$
$$\frac{\rho VD}{\mu} \le 2000$$
$$\frac{1000\times V\times6\times10^{-3}}{\left(\frac{0.01}{10}\right)} \le 2000$$
$$V \le \frac{1}{3} \text{ m/sec}$$
$$V \le \frac{100}{3} \text{ cm/sec}$$

∴
$$V_{max} \le \frac{100}{3} \text{ cm/sec}$$

Example - 7.6 If the velocity profile in laminar flow is parabolic, then the shear stress profile must be

(a) A hyperbola
(b) A parabola
(c) A straight line
(d) An ellipse

Solution: (c)

Example - 7.7 The pressure drop in a 30 cm diameter horizontal pipe is 60 kPa in a distance of 15 m. The well shear stress (in kPa) is ________.

(a) 0.1
(b) 0.2
(c) 0.3
(d) 0.4

Solution: (c)

$$\text{Diameter of pipe} = 30 \text{ cm}$$

$$\therefore \quad R = 15 \text{ cm}$$

$$\tau_0 = -\left(\frac{dp}{dx}\right)\cdot\frac{R}{2}$$

$$= \left(\frac{60}{15}\right)\times\frac{0.15}{2}$$

$$\tau_0 = 0.3 \text{ kPa}$$

Student's Assignments

Q.1 The velocity at which the laminar flow ceases, is known as
(a) Approach velocity
(b) Lower critical velocity
(c) Higher critical velocity
(d) None of these

Q.2 The ratio of maximum velocity to average velocity for steady flow between fixed parallel plates is
(a) $\frac{2}{3}$
(b) $\frac{4}{3}$
(c) $\frac{3}{2}$
(d) 2

Q.3 The shear stress distribution for a flowing fluid in between parallel plates, both at rest is
(a) constant over the cross-section
(b) parabolic distribution across the section
(c) zero at the mid plane and varies linearly with distance from mid plane
(d) zero at plates and increases linearly to mid point

Q.4 Laminar developed flow at an average velocity of 5 m/s occurs in a pipe of 10 cm radius. The velocity at 5 cm radius is ________.
(a) 7.5 m/s
(b) 10 m/s
(c) 2.5 m/s
(d) 5 m/s

Q.5 In a two-dimensional flow of a viscous fluid couette flow is defined for
(a) pressure gradient driven laminar flow between fixed parallel plates
(b) pressure gradient driven laminar flow through non-circular duct

(c) pressure gradient driven laminar flow through pipe
(d) laminar flow between a fixed and a moving plate

Q.6 The ratio of average velocity to maximum velocity for steady laminar flow in circular pipe is
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$
(c) $\frac{3}{2}$ (d) 2

Q.7 In a laminar flow between two static parallel plates, the velocity plates, the velocity at mid point is found to be 2.0 m/sec. If the space between the plates is 10 cm, then the discharge per unit width (in $m^3/s/m$) will be
(a) 0.01 (b) 0.02
(c) 0.13 (d) 0.20

Q.8 The laminar flow is characterized by
(a) irregular motion of fluid particles
(b) fluid particles moving in layers parallel to the boundary surface
(c) high Reynolds number of flow
(d) existence of eddies

Q.9 Equations govern the motion of incompressible fluid of laminar flow are
(a) Euler's equations
(b) Navier Stoke's equation
(c) Bernoulli's equation
(d) Hagen-Poiseuille equation

Q.10 The velocity distribution for laminar flow through a circular tube
(a) is constant over the cross-section
(b) varies linearly from zero at well to maximum at centre
(c) varies parabolically with maximum at the centre
(d) varies logarithmically

Q.11 Flow commences between two parallel plates with the upper plate moving in the direction of flow, while the other plate is stationary. The resulting flow between parallel plates is called
(a) creep flow (b) couette flow
(c) plug flow (d) stokes flow

Q.12 A pipe of 20 cm diameter and 30 km length transports oil from a tanker to the shore with a velocity of 0.318 m/s. The flow is laminar. If μ = 0.1 N-s/m^2, the power required for the flow would be
(a) 9.25 kW (b) 8.36 kW
(c) 7.63 kW (d) 10.13 kW

Q.13 In a laminar flow through pipe, the point of maximum instability exists at a distance of y from the wall which is
(a) $\frac{3}{2}$ of pipe radius R
(b) $\frac{2}{3}$ of pipe radius R
(c) $\frac{1}{2}$ of pipe radius R
(d) $\frac{1}{3}$ of pipe radius R

Q.14 The shear stress τ_o for steady, fully developed flow inside a uniform horizontal pipes with coefficient of friction 'f', density ρ and velocity V is given by
(a) $\frac{f\rho V^2}{2}$ (b) $\frac{f\rho^2 V}{2}$
(c) $\frac{\rho^2 V}{2f}$ (d) $\frac{\rho V^2}{2f}$

Q.15 An oil of viscosity 8 poise flows between two parallel fixed plates, which are kept at a distance of 30 mm apart. If the drop of pressure for a length for 1 m is 0.3×10^4 N/m^2 and width of plate is 500 mm, the rate of oil flow between the plates will be
(a) 4.2×10^{-3} m^3/s
(b) 5.4×10^{-3} m^3/s
(c) 6.6×10^{-3} m^3/s
(d) 7.8×10^{-3} m^3/s

Q.16 An oil flows through a pipe at a velocity of 1.0 m/s. The pipe is 45 m long and has 150 mm diameter. What is the head loss due to friction. If ρ = 869 kg/m^3 and kg/m^3 and μ = 0.0814 kg/m-sec?

(a) 0.61 m (b) 0.51 m
(c) 0.41 m (d) 0.31 m

Q.17 Flow takes place at Reynolds number of 1500 in two different pipes with relative roughness of 0.001 and 0.002. The friction factor
(a) will be higher in case of pipe with relative roughness of 0.001
(b) will be higher in case of pipe with relative roughness of 0.002
(c) will be same in both pipes
(d) in the two pipes cannot be compared on the basis of data given

Q.18 The pressure drop for laminar flow of a liquid in a smooth pipe at normal temprature and pressure is
(a) directly proportional to density
(b) inversely proportional to density
(c) independent of density
(d) proportional to $(\text{density})^{3/4}$

Q.19 The relation $\left(\frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial y}\right)$ is valid for laminar flow between two plates when
(a) both the plates are stationary
(b) both the plates are moving
(c) one plate is moving and the other one is stationary
(d) any of the above cases is considered

Q.20 The velocity distribution across a section of two fixed llrl plates having viscous flow is given by

(a) $u = \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)(t^2 - y^2)$

(b) $u = \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\cdot(ty - y^2)$

(c) $u = \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\cdot(y - ty)$

(d) $u = \frac{-1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\cdot(t - y^2)$

where, t = Distance between two plates,
y = measured distance from the lower plate.

ANSWER KEY — STUDENT'S ASSIGNMENTS

1. (d)	**2.** (c)	**3.** (c)	**4.** (a)	**5.** (d)
6. (a)	**7.** (c)	**8.** (b)	**9.** (b)	**10.** (c)
11. (b)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (a)
16. (a)	**17.** (c)	**18.** (c)	**19.** (d)	**20.** (b)

HINTS & SOLUTIONS — STUDENT'S ASSIGNMENTS

1. (d)

The velocity at which the flow changes from the laminar to turbulent for the case of a given fluid at a given temperature and in a given pipe is known as critical velocity.

4. (a)

For laminar flow between circular pipe

$$\bar{u} = 5 \text{ m/sec}$$
$$R = 10 \text{ cm}$$
At, $\gamma = 5$ cm; $u = ?$
$$u = u_{max}\cdot\left(1 - \frac{r^2}{R^2}\right)$$
$$\frac{u_{max}}{\bar{u}} = 2$$
$$u_{max} = 2 \times 5 = 10 \text{ m/sec}$$
$$\therefore \quad u = 10\left(1 - \left(\frac{5}{10}\right)^2\right) = 10 \times \frac{3}{4}$$
$$u = 7.5 \text{ m/sec}$$

7. (c)

The average velocity $= \frac{2}{3} \times 2 = \frac{4}{3}$ m/sec

∴ The discharge per unit width

$$= \frac{4}{3} \times 0.1 = 0.13 \text{ m}^3\text{/s/m}$$

12. (c)

Given, dia. of pipe, D = 20 cm
Length of pipe, L = 30 km
Velocity of flow $(\bar{u})$ = 0.18 m/s
$$\mu = 0.1 \text{ N-s/m}^2$$

Pumping power required would be

$$(P_1 - P_2)Q = \gamma Q h_L$$

$$= \gamma Q\left(\frac{32\mu\bar{u}L}{\gamma D^2}\right)$$

$$= \frac{\pi}{4}\times D^2 \times \bar{u}\,\frac{32\mu\bar{u}L}{D^2}$$

$$= \frac{\pi}{4}\times 32\times 0.1\times(0.318)^2\times 30\times 10^3$$

$$= 7624.56 \text{ Watt} = 7.624 \text{ kW}$$

$$\simeq 7.63 \text{ kW}$$

15. (a)

For flow between two parallel fixed plate

$$\bar{u} = \frac{1}{12\mu}\left(\frac{-dP}{dx}\right)t^2,$$

t = distance between plate; = 30 mm

width of plate, B = 500 mm = 0.5 m

Given, μ = 8 poise, = 0.8 N-s/m^2

$$\bar{u} = \frac{1}{12\times 0.8}\left(\frac{0.3\times 10^4}{1}\right)\times(0.03)^2 = 0.281 \text{ m/s}$$

Now,

$$Q = \bar{u}\times(B\times t)$$

$$= 0.281\times(0.03\times 0.5)$$

$$= 4.2\times 10^{-3} \text{ m}^3\text{/s}$$

16. (a)

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{869\times 1\times 0.15}{0.0814}$$

$$= 1601.35 < 2000$$

$\Rightarrow$ Flow is laminar

$$\text{Head loss, } h_L = \frac{32\mu\bar{u}L}{\gamma D^2} = \frac{32\times 0.0814\times 1\times 45}{869\times 9.81\times(0.15)^2}$$

$$= 0.61 \text{ m}$$

17. (c)

As Reynolds number is less than 2000 so flow is laminar therefore the friction will be same for both pipe, friction factor,

$$f = \frac{64}{\text{Re}} \text{ for laminar flow.}$$

18. (c)

$$\text{Pressure drop, } \Delta P = \frac{32\mu\bar{u}L}{D^2}$$

From above expression it is clear that pressure is independent of density.

■■■■