POSTAL Study Package 2021

Production and Industrial Engineering

Objective Practice Sets

Operations Research and Operations Management

Contents

MADE EASY Publications

2 CHAPTER Queuing Theory

Q.1 In the notation (*a*/*b*/*c*) : (*d*/*e*/*f*) for summarizing the characteristics of queuing situation, the letters 'b' and 'd' stands respectively for
(a) Service time distribution and queue discipline.
(b) Number of servers and size of calling source.
(c) Number of servers and queue discipline.
(d) Service time distribution and maximum number allowed in system.

Q.2 In a queuing problem, if the arrivals are completely random, then the probability distribution of number of arrivals in a given time follows :
(a) Poisson distribution
(b) Normal distribution
(c) Binomial distribution
(d) Exponential distribution

Q.3 In the queuing theory, if the arrival in a single server model follows Poisson distribution, the time between arrivals will follows a
(a) Gamma distribution
(b) Exponential distribution
(c) Binomial distribution
(d) Weibull distribution

Q.4 In the Kendall's notation for representing queuing models the first position represents
(a) Probability law of the arrival
(b) Probability law for the service
(c) Number of channels
(d) Capacity of the system

Q.5 Which one of the following statements is correct? Queuing theory is applied best in situations where
(a) Arrival rate of customers equal to service rate.
(b) Average service time is greater than average arrival time.
(c) There is only one channel of arrival at random and the service time is constant.
(d) The arrival and service rate can not be analyzed through any standard statistical distribution.

Q.6 The cost of providing service in a queuing system increases with
(a) Increased mean time in the queue
(b) Increased arrival rate
(c) Decreased mean time in the queue
(d) Decreased arrival rate

Q.7 Consider two queuing disciplines in a single server queue. Case 1 has a first come first served discipline and Case 2 has a last come first served discipline. If the average waiting time in the two cases are W_1 and W_2 respectively, then which one of the following inferences would be true?
(a) $W_1 > W_2$
(b) $W_1 < W_2$
(c) $W_1 = W_2$
(d) Data insufficient to draw any tangible inference

Q.8 The *M*/*M*/*S* queue configuration allows for :
(a) General service time
(b) A single server
(c) Multiple servers
(d) Constant service time

Q.9 The utilization factor for a system is defined as:
(a) the average time a customer spends in the system.
(b) the mean number of arrivals per period divided by the mean number of customers served per period.
(c) the percent idle.
(d) the average time a customer spends waiting in the queue.

distribution for arrival rate and exponential distribution for service time. Calculate the average number of customers in the system

(a) 9 (b) 10
(c) 12 (d) None of the above

Q.22 In Q.21, average number of customers in the queue is

(a) 4.5 (b) 8.1
(c) 2.5 (d) None of the above

Q.23 In Q.21, average time a customer spends in the system is (in minutes)

(a) 9 (b) 10
(c) 5 (d) None of the above

Q.24 In Q.21, average time a customer waits before being served is (in minutes) :

(a) 5 (b) 9
(c) 4.5 (d) 10

Q.25 Customers arrive at a one window drive-in bank according to a Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in front of the window, including that for the serviced car can accomodate a maximum of three cars. Other cars can wait outside this space.
What is the probability that an arriving customer can drive directly to the space in front of the window?

Q.26 In Q.25, what is the probability that an arriving customer will have to wait outside the indicated space?

Q.27 In Q.25, how long (in minutes) is an arriving customer expected to wait before starting service?

Q.28 In Q.25, how many car spaces should be provided in front of the window so that all the arriving customers can wait in front of the window at least 20% of the time?

Q.29 Customers arrive at the first class ticket counter of a theatre at the rate of 12 per hour. There is one clerk serving the customers at the rate of 30 per hours. What is the probability that there is no customer in the counter.

(a) 0.4 (b) 0.6
(c) 0.5 (d) None of the above

Q.30 In Q.29, what is the probability that there are more than 2 customers in the counter?

(a) 0.064 (b) 0.16
(c) 0.4 (d) None of the above

Q.31 In Q.29, what is the probability that there is no customer waiting to be served?

(a) 0.60 (b) 0.24
(c) 0.84 (d) None of the above

Q.32 In Q.29, what is the probability that a customer is being served and nobody is waiting?

(a) 0.24 (b) 0.84
(c) 0.60 (d) None of the above

Q.33 Data have been accumulated at a banking facility regarding the waiting time for delivery of trucks to be loaded. The data show that the average arrival rate for the trucks at the loading dock is 21 hours. The average time to load a truck, using 2 loaders is 10 minutes so that service rate is 3 trucks/hour. Drivers are paid Rs. 4 per hour and truck utilisation is valued at Rs. 3 per hour.
The total cost/hr is

(a) 14 (b) 7
(c) 28 (d) None of the above

Q.34 In Q.33, if the management is considering hiring another loader at Rs. 5 per hour to increase the service rate to 4 trucks per hour. The new total cost/hr is

(a) 14 (b) 7
(c) 12 (d) None of the above

Q.35 The tool room company's quality control department is manned by a single clerk who takes an average of 5 minutes in checking parts of each of the machine coming for inspection. The machines arrives once in every 8 minutes on the average. One hour of the machine is valued at Rs. 15 and the clerk's time is valued at Rs. 4 per hour. What are the average hourly queuing system costs associated with the quality control department?

Q.36 Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be exponentially distributed with mean of 4 min. What is the probability that it will take him more than 10

Answers Queuing Theory

1. (a)	2. (a)	3. (b)	4. (a)	5. (c)	6. (c)	7. (c)	8. (c)
9. (b)	10. (c)	11. (c)	12. (c)	13. (b)	14. (d)	15. (d)	16. (b)
17. (b)	18. (a)	19. (c)	20. (b)	21. (a)	22. (b)	23. (c)	24. (c)
25. (0.42)	26. (0.58)	27. (25)	28. (1)	29. (b)	30. (a)	31. (c)	32. (a)
33. (a)	34. (c)	35. (29)	36. (0.1889)	37. (0.3679)	38. (0.4823)	39. (10)	40. (b)
41. (a)	42. (1.33)	43. (10)	44. (1.28)	45. (58)	46. (c)	47. (0.167)	48. (0.5)
49. (1)	50. (a)						

Explanations Queuing Theory

1. (a)

Kandal and Lee Representation

$(a / b / c) : (d / e / f)$

where
- a = Probability distribution of arrival rate
- b = Probability distribution of service rate
- c = Number of servers
- d = Queue disciplines
- e = Maximum number of customers allowed in system
- f = Size of calling population

2. (a)

Number of arrival per unit time is estimated by Poisson's distribution.

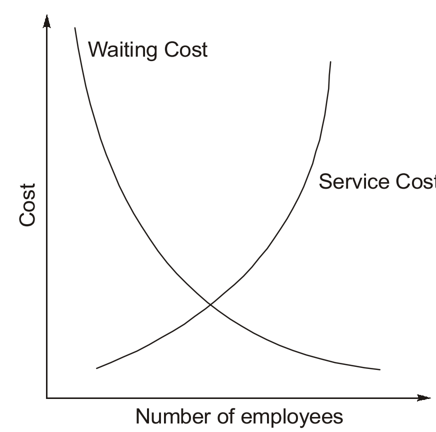
3. (b)

If arrivals in a single server model follows Poisson distribution, the time between arrivals will follow exponential distribution.

5. (c)

Queuing theory is applied best in situation where there is only one channel of arrival at random and the service time is constant.

6. (c)



As number of employees will increase, service cost will increase but waiting cost will decrease (i.e., mean time in the queue decreased).

7. (c)

Average waiting time in the system in both the cases will be same.

$$W_1 = W_2 = W_S = \frac{1}{\mu - \lambda}$$

where μ = service rate and λ = arrival rate

8. (c)

$M/M/S \Rightarrow$ Multiple servers

27. (25)

Average waiting time of a customer in the queue,

$$W_q = \frac{L_q}{\lambda} = \frac{\left(\frac{\rho\lambda}{\mu-\lambda}\right)}{\lambda}$$

$$\Rightarrow \quad W_q = \frac{\rho}{\mu-\lambda} = \frac{\left(\frac{10}{12}\right)}{(12-10)}$$

$$= \frac{10}{12\times 2} = \frac{5}{12} \text{ hours}$$

$$\Rightarrow \quad W_q = \frac{5}{12}\times 60 \text{ minutes}$$

$$\Rightarrow \quad W_q = 25 \text{ minutes}$$

28. (1)

$$P_0 = \left(1-\frac{\lambda}{\mu}\right) = \left(1-\frac{10}{12}\right) = 0.16$$

$$P_1 = \frac{\lambda}{\mu}P_0 = \left(\frac{10}{12}\right)\times 0.16 = 0.13$$

∴ Probability that there will be no car or one car in the space

= 0.16 + 0.13 = 0.29

Since, it is more than 20%, there whould be at least one car space in front of the window.

29. (b)

$$\lambda = 12/\text{hr}, \mu = 30/\text{hr}$$

$$P_0 = \left(1-\frac{\lambda}{\mu}\right) = \left(1-\frac{12}{30}\right)$$

$$= (1-0.4)$$

$$\Rightarrow \quad P_0 = 0.6$$

30. (a)

Probability that there are more than two customers in the counter

$$= P_3 + P_4 + P_5 + ...$$

$$= 1-(P_0 + P_1 + P_2)$$

$$= 1-\left[P_0\left(1+\frac{\lambda}{\mu}+\frac{\lambda^2}{\mu^2}\right)\right]$$

$$= 1-\left[0.6\left(1+\frac{12}{30}+\frac{144}{900}\right)\right]$$

$$= 0.064$$

31. (c)

Probability that there is no customer waiting to be served

= Probability that there is no customer waiting to be served

$$= P_0 + P_1 = 0.6+0.6\left(\frac{12}{30}\right) = 0.84$$

32. (a)

Probability that a customer is being served and nobody is waiting

$$= P_1 = P_0\times\frac{\lambda}{\mu}$$

$$\Rightarrow \quad P_1 = 0.6\left(\frac{12}{30}\right)$$

$$\Rightarrow \quad P_1 = 0.24$$

33. (a)

Given: $\lambda = 2/\text{hr}, \mu = 3/\text{hr}$

Average length of the system

$$L_s = \frac{\lambda}{\mu-\lambda} = \frac{2}{3-2} = \frac{2}{1} = 2$$

∴ Total cost/hr = 2 × {Rs. (4 + 3)} = Rs. 14

34. (c)

$$\lambda = 2 \text{ per hour}, \mu = 4 \text{ per hour}$$

New average length of the system

$$L'_s = \frac{\lambda}{\mu-\lambda} = \frac{2}{(4-2)} = \frac{2}{2} = 1$$

∴ Total new cost/hour = Rs. [1 × (3 + 4) + 5]

= Rs. 12

35. (29)

$$\lambda = \frac{60}{8} = 7.5/\text{hr}$$

$$\mu = \frac{60}{5} = 12/\text{hr}$$

Average waiting time in the system

$$W_s = \frac{1}{\mu-\lambda} = \frac{1}{12-7.5} \text{ hr}$$

$$= \frac{1}{4.5}\text{hr} = \frac{2}{9}\text{ hr}$$

Average queuing cost/machine

$$= \text{Rs.}\left(15\times\frac{2}{9}\right) = \text{Rs. }\frac{10}{3}$$

∴ Average queuing cost/hr

$$= \text{Rs. }\frac{10}{3}\times 7.5 = \text{Rs. } 25$$

Average cost of clerk/hr = Rs. 4