POSTAL Book Package 2021

Electrical Engineering

Conventional Practice Sets

Signals & Systems

Contents

MADE EASY Publications

4 CHAPTER Sampling Theorem

Q.1 **Explain how an analog signal can be constructed from its samples. What are the conditions for faithful reconstruction? Give any one reconstruction circuit.**

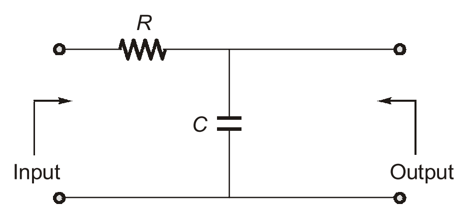
Solution:

An analog signal can be reconstructed back from samples using interpolation, which is a method of connecting sample points to achieve an analog signal. Interpolation can be done through zero order hold circuit connecting samples by a straight line or higher order polynomials like a low pass filter can be used for interpolation.

Conditions for faithful reconstruction:

(i) The sampling rate must be greater than the Nyquist rate.

(ii) The sampled signal must be passed through an ideal low pass filter.



A simple reconstruction circuit

Q.2 **A continuous time signal is given by:**

$$x(t) = A\cos(8000\pi t)\cos(2000\pi t)$$

The signal is sampled with a sample period of 3×10^{-4} seconds. Can we recover the signal from the sampled version using an appropriate low pass filter?

Solution:

$$x(t) = A\cos(8000\,\pi t)\cos(2000\,\pi t)$$

$$= \frac{A}{2}[\cos 10000\pi t + \cos 6000\pi t]$$

Maximum frequency component in the signal,

$$f_m = \frac{10000\pi}{2\pi} = 5000 \text{ Hz}$$

So, minimum sampling frequency required

$$f_s = 2 f_m = 10000 \text{ Hz}$$

So, maximum allowable sampling period,

$$T_s = \frac{1}{f_s} = \frac{1}{10000} \text{sec} = 0.1 \times 10^{-4} \text{ sec.}$$

The sampling period given in question is greater than the maximum allowable sampling period. Thus, it cannot be recovered using a low pass filter.

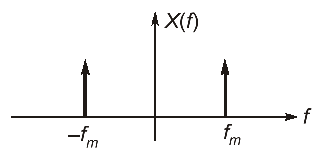
Q.3 A sinusoid $x(t)$ of unknown frequency is sampled by an impulse train of period 20 ms. The resulting sample train is next applied to an ideal low-pass filter with a cut-off at 25 Hz. The filter output is seen to be a sinusoid of frequency 20 Hz. Find the component of frequency of $x(t)$ which is less than 100 Hz.

Solution:

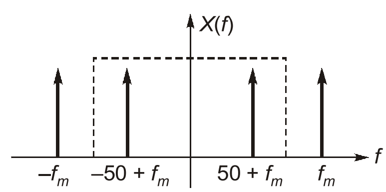
Given, impulse train of period 20 ms.

Then, Sampling frequency $= \dfrac{1}{20 \times 10^{-3}} = 50 \text{ Hz}$

If the input signal $x(t) = \cos\omega_m(t)$ having spectrum,



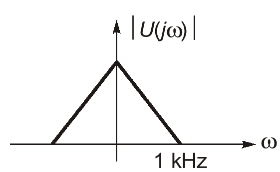
The filtered out sinusoidal signal has 20 Hz frequency the sampling must be under sampling.
The output signal which is an under sampled signal with sampling frequency 50 Hz is



and $50 - f_m = 20 \text{ Hz}$

$f_m = 30 \text{ Hz}$

Q.4 The frequency spectrum of a signal is shown in the figure. If this is ideally sampled at intervals of 1 ms, then find the frequency spectrum of the sampled signal.



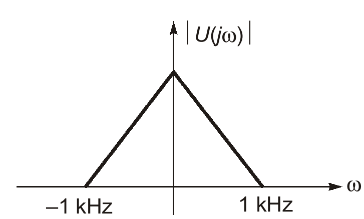
Solution:

Given that, sampling interval = 1 msec

i.e. $T_s = 1 \text{ msec} = 10^{-3} \text{ sec}$

Therefore sampling frequency,

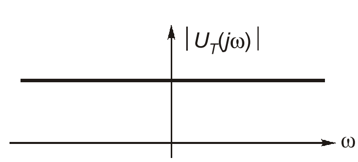
$$f_s = \frac{1}{T_s} = \frac{1}{10^{-3}} = 1 \text{ kHz}$$



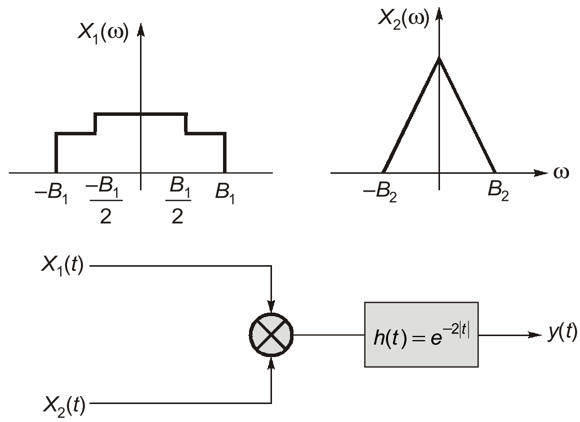
After sampling new signal in frequency domain,

$$U_T(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} U(f - nf_s)$$

∴ Spectrum of sampled signal will be



Q.5 Let $x_1(t) \leftrightarrow X_1(\omega)$ and $x_2(t) \leftrightarrow X_2(\omega)$ be two signals whose Fourier Transforms are as shown in the figure below. In the figure, $h(t) = e^{-2|t|}$ denotes the impulse response.



For the system shown above, find the minimum sampling rate required to sample $y(t)$, so that $y(t)$ can be uniquely reconstructed from its samples.

Solution:

Given that, Bandwidth of $X_1(\omega) = B_1$

Bandwidth of $X_2(\omega) = B_2$

System has $h(t) = e^{-2|t|}$ and input to the system is $x_1(t) \cdot x_2(t)$

The bandwidth of $x_1(t) \cdot x_2(t)$ is $B_1 + B_2$.

The bandwidth of output will be $B_1 + B_2$.

So sampling rate will be $2(B_1 + B_2)$.

Q.6 A signal $x(t) = 1 + \cos 100\pi t$ is sampled with sampling interval of 0.02 second. Can the original signal be recovered from these samples?

Solution:

Maximum frequency in $x(t)$, $f_m = \frac{100\pi}{2\pi} = 50$ Hz

$\therefore$ Minimum sampling frequency required,

$= 2 f_m = 100$ Hz

$\therefore$ Maximum allowable sampling period,

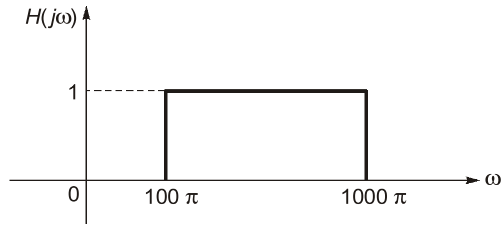
$= \frac{1}{100}$ sec $= 0.01$ sec

The sampling interval given in the question is greater than this value. Thus the original signal cannot be recovered.

Q.7 A Sawtooth wave signal having trigonometric Fourier series is given by, $x(t) = \frac{A}{2} - \frac{A}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\omega_0 t$,

where 'ω_0' is the angular frequency, $A = 1$ volt and $T = 10$ m-sec.

The signal $x(t)$ is applied to an ideal BPF whose response is shown below:



Calculate the output $y(t)$ of the filter?

Solution:

Given that,

$$A = 1\text{ V},\ T = 10\text{ msec} = 10 \times 10^{-3} = \frac{1}{100}\text{sec}$$

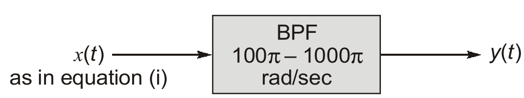
$$\therefore \qquad \omega_0 = \frac{2\pi}{T} = 200\,\pi\text{ rad/sec}$$

Since,
$$x(t) = \frac{A}{2} - \frac{A}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\omega_0 t$$

Putting all above values in (i) we get,

$$x(t) = \frac{1}{2} - \frac{1}{\pi}\left[\sin 200\pi t + \frac{1}{2}\sin 400\pi t + \frac{1}{3}\sin 600\pi t + \frac{1}{4}\sin 800\pi t + \frac{1}{5}\sin 1000\pi t + \frac{1}{6}\sin 1200\pi t + \frac{1}{7}\sin 1400\pi t + \dots\dots\right] \qquad \text{...(i)}$$

Since BPF having cut-off frequencies $\omega_{c1} = 100\pi$ and ω_{c2} 1000π rad/sec and $x(t)$ signal is applied to the BPF, so only those frequency components which comes in the ranges will passed through it. For this we have,

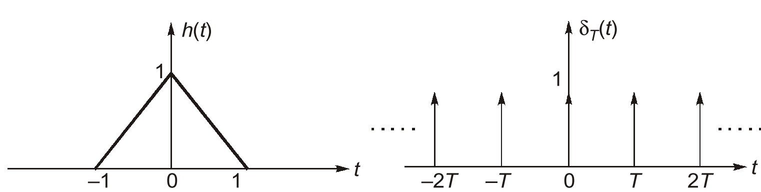


So,
$$y(t) = -\frac{1}{\pi}\left[\sin 200\pi t + \frac{1}{2}\sin 400\pi t + \frac{1}{3}\sin 600\pi t + \frac{1}{4}\sin 800\pi t\right]$$

Q.8 Let $h(t)$ be the triangular pulse shown in figure and let $x(t)$ be the unit impulse train expressed as:

$$x(t) = \delta_T(t) = \sum_{n=-\infty}^{\infty}\delta(t - nT)$$

Determine and sketch $y(t) = h(t) * x(t)$ for the following values of T: (a) $T = 3$, (b) $T = 2$, (c) $T = 1.5$.



Solution:

Using equation and, we obtain,
$$y(t) = h(t) * d_T(t) = h(t) * \left[\sum_{n=-\infty}^{\infty}\delta(t - nT)\right]$$

$$= \sum_{n=-\infty}^{\infty} h(t) * \delta(t - nT) = \sum_{n=-\infty}^{\infty} h(t - nT)$$

(a) For $T = 3$, equation becomes

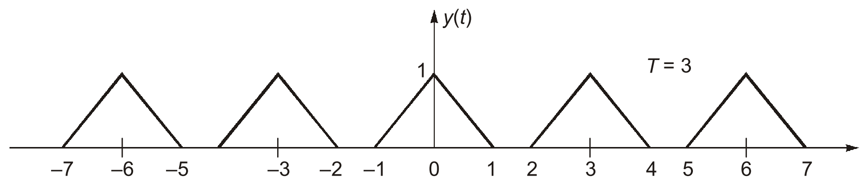
$$y(t) = \sum_{n=-\infty}^{\infty} h(t-3n)$$

which is sketched in figure (a).

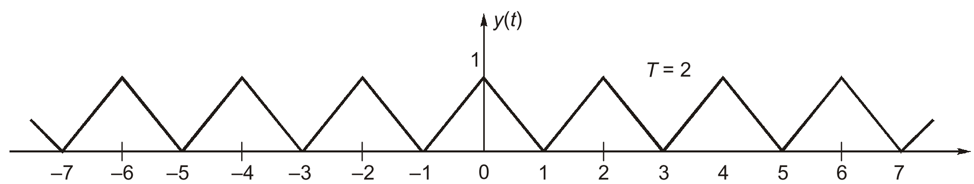
(b) For $T = 2$, equation becomes

$$y(t) = \sum_{n=-\infty}^{\infty} h(t-2n)$$

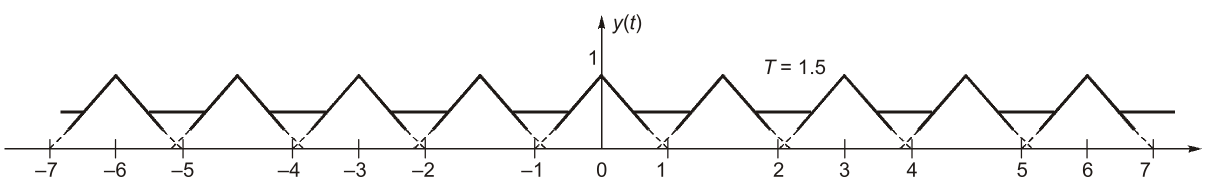
which is sketched in figure (b).



(a)



(b)



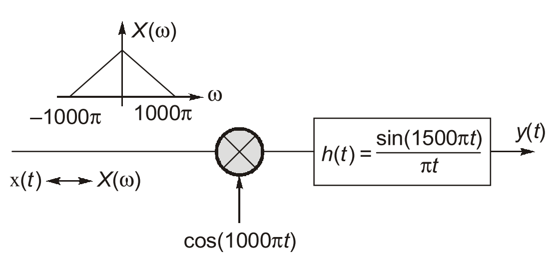
(c)

(c) For $T = 1.5$ equation becomes

$$y(t) = \sum_{n=-\infty}^{\infty} h(t-1.5n)$$

which is sketched in figure (c). Note that when T < 2, the triangular pulses are no longer separated and they overlap.

Q.9 The output $y(t)$ of the following system is to be sampled, so as to reconstruct it from its samples uniquely. Find the minimum sampling rate.



Solution: