POSTAL Book Package 2021

Instrumentation Engineering

Objective Practice Sets

Signals and Systems

Contents

MADE EASY Publications

3 CHAPTER Continuous Time Fourier Series

Q.1 Match **List-I** with **List-II** and select the correct answer using the codes given below the Lists :

List-I	List-II
A. $f(t) = -f(-t)$	1. Exponential form of Fourier series
B. $\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$	2. Fourier transform
C. $\int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$	3. Convolution integral
D. $\int_0^l f_1(t)f_2(t-\tau)\,dt$	4. z-transform
	5. Odd function wave symmetry

Codes :

	A	B	C	D
(a)	5	1	2	3
(b)	2	1	5	3
(c)	5	4	2	1
(d)	4	5	1	2

Q.2 If $f(t) = -f(-t)$ and $f(t)$ satisfy the Dirichlet's conditions, then $f(t)$ can be expanded in a Fourier series containing
(a) only sine terms
(b) only cosine terms
(c) cosine terms and a constant term
(d) sine terms and a constant term

Q.3 Consider the following statements related to Fourier series of a periodic waveform:
1. It expresses the given periodic waveform as a combination of DC component, sine and cosine waveforms of different harmonic frequencies.
2. The amplitude spectrum is discrete.
3. The evaluation of Fourier coefficients gets simplified if waveform symmetries are used.
4. The amplitude spectrum is continuous.

Which of the above statements are correct?
(a) 1, 2 and 4
(b) 2, 3 and 4
(c) 1, 3 and 4
(d) 1, 2 and 3

Q.4 The trigonometric Fourier series expansion of an even function that is also half wave symmetric shall contain
(a) odd harmonics of sine terms only
(b) both sine and cosine terms
(c) only odd harmonics of cosine terms.
(d) only cosine terms

Q.5 A waveform that is neither EVEN nor ODD but has half-wave symmetry can be expressed in terms of Fourier series expansion as sum of
(a) odd harmonics of both sine and cosine terms
(b) even harmonics of both sine and cosine terms
(c) odd harmonics of sine terms
(d) even harmonics of cosine terms.

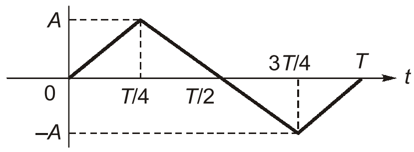
Q.6 A trigonometric series has
(a) single sided spectrum
(b) double sided spectrum
(c) may have both
(d) none

Q.7 The frequency spectrum of a single non-recurring pulse will be
(a) a line spectrum containing all harmonics with adjacent lines very closely spaced
(b) a single line
(c) a continuous frequency spectrum defined by Fourier integral rather than a discrete line spectrum defined by Fourier series
(d) none of these.

Q.8 **Assertion (A):** A periodic function satisfying Dirichlet's condition can be expanded into a Fourier series.
Reason (R): A Fourier series is a summation of weighted sine and cosine waves of the fundamental frequency and its harmonics.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q.9 Which of the following periodic waveforms will have only odd harmonics of sinusoidal waveforms?

1.


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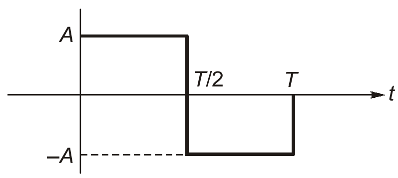

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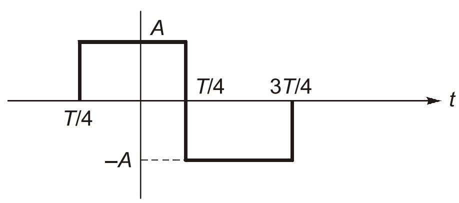

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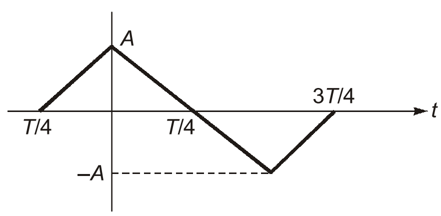


Select the correct answer using the codes given below:

Codes:
(a) 1 and 2 (b) 1 and 3
(c) 1 and 4 (d) 2 and 4

Q.10 The dc component of the function $\pi |\sin t|$ is
(a) 0 (b) 1
(c) 2 (d) $2/\pi$

Q.11 Match **List-I (Properties)** with **List-II (Characteristics of the trigonometric form)** in regard to Fourier series of periodic $f(t)$ and select the correct answer using the codes given below the lists:

	List-I		**List-II**
A.	$f(t) + f(-t) = 0$	1.	Even harmonics can exist
B.	$f(t) - f(-t) = 0$	2.	Odd harmonics can exist
C.	$f(t) + f(t - T/2) = 0$	3.	The dc and cosine terms can exist
D.	$f(t) - f(t - T/2) = 0$	4.	Sine terms can exist
		5.	Cosine terms of even harmonics can exist

Codes:

	A	B	C	D
(a)	4	5	3	1
(b)	3	4	1	2
(c)	5	4	2	3
(d)	4	3	2	1

Q.12 **Statement (I):** Dirichlet's conditions restrict the periodic signal $x(t)$, to be represented by Fourier series, to have only finite number of maxima and minima.
Statement (II): $x(t)$ should possess only a finite number of discontinuities.
(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is **NOT** the correct explanation of Statement (I)
(c) Statement (I) is true but Statement (II) is false
(d) Statement (I) is false but Statement (II) is true

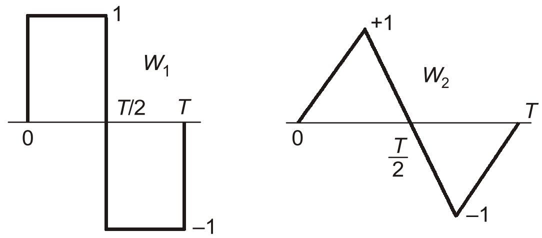
Q.13 A periodic voltage having the Fourier series $v(t) = 1 + 4 \sin \omega t + 2 \cos \omega t$ volts is applied across a one-ohm resistor. The power dissipated in the 1-ohm resistor is
(a) 1 W (b) 11 W
(c) 21 W (d) 24.5 W

Q.14 The Fourier series expansion of a real periodic signal with fundamental frequency f_0 is given by

$$t \to g_p(t) \sum_{n=-\infty}^{\infty} c_n e^{j2\pi f_0 t}.$$

It is given that $c_3 = 3 + j5$. Then c_{-3} is
(a) $5 + j3$ (b) $-3 - j5$
(c) $-5 + j3$ (d) $3 - j5$

Q.15 Consider the following statements regarding the fundamental component $f_1(t)$ of an arbitrary periodic signal $f(t)$:
1. the amplitude of $f_1(t)$ to exceed the peak value of $f(t)$
2. $f_1(t)$ to be identically zero for a non-zero $f(t)$.
3. the effective value of $f_1(t)$ to exceed the effective value of $f(t)$.



(a) $|n^{-3}|$ and $|n^{-2}|$
(b) $|n^{-2}|$ and $|n^{-3}|$
(c) $|n^{-1}|$ and $|n^{-2}|$
(d) $|n^{-4}|$ and $|n^{-2}|$

Q.26 Let $x(t)$ be a periodic function with period $T = 10$. The Fourier series coefficients for this series are denoted by a_k, that is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

The same function $x(t)$ can also be considered as a periodic function with period $T' = 40$. Let b_k be the Fourier series coefficients when period is taken as T'.

If $\sum_{k=-\infty}^{\infty} |a_k| = 16$, then $\sum_{k=-\infty}^{\infty} |b_k|$ is equal to

(a) 256
(b) 64
(c) 16
(d) 4

Q.27 Let $x(t) = \sin^2 t$ be represented as the complex Fourier series representation i.e.

$$\sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} \quad \text{where, } \omega_0 = \frac{2\pi}{T_0},$$

T_0 is fundamental period of $x(t)$

The value of complex Fourier coefficient C_1 for $x(t)$ is ________.

Q.28 A periodic sequence with period $N = 4$ is defined as $x[n] = \left[\underset{\uparrow}{0}, 1, 2, 3\right]$ for one period. It has discrete time Fourier series coefficient C_k. The value of C_2 is ________.

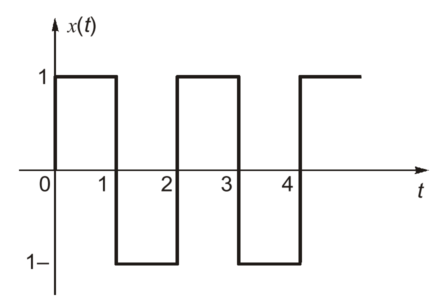
Q.29 Fourier series of the periodic function (period 2p) defined by

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

is $\frac{\pi}{4} + \sum_{n=1}^{\infty}\left[\frac{1}{\pi n^2}[\cos(n\pi) - 1]\cos(nx) - \frac{1}{n}\cos(n\pi)\sin(nx)\right]$.

By putting $x = \pi$ in the above, one can deduce that the sum of the series $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$ is ________.

Q.30 Consider the periodic square wave in the figure shown.



The ratio of the power in the 7th harmonic to the power in the 5th harmonic for this waveform is closest in value to ________.

■■■■

Answers Continuous Time Fourier Series

1. (a)	2. (a)	3. (d)	4. (c)	5. (a)	6. (a)	7. (c)	8. (b)	9. (a)
10. (c)	11. (d)	12. (b)	13. (b)	14. (d)	15. (a)	16. (b)	17. (b)	18. (c)
19. (d)	20. (a)	21. (b)	22. (b)	23. (b)	24. (a)	25. (c)	26. (c)	27. (–0.25)
28. (–0.5)	29. (1.23)	30. (0.51)						

Explanations Continuous Time Fourier Series

1. (a)

$f(t) = -f(-t) \rightarrow$ for odd function
$f(t) = f(-t) \rightarrow$ for even function

Fourier transform, $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$

Convolution $= \int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau$

2. (a)

For even function, $b_n = 0$
For odd function, a_0, $a_n = 0$
Hence, here odd function $\therefore$ only sine terms.

3. (d)

Statement 1 gives the Trigonometric form of periodic Fourier series statement 3 is correct.
Fourier series is periodic and continuous $\xrightarrow{\text{Spectrum}}$ discrete and aperiodic.

Periodic in one domain $\rightleftharpoons$ discrete in other domain.

Continuous in one domain $\rightleftharpoons$ aperiodic in other domain.

4. (c)

For even and half wave symmetry,
$b_n = 0$ for all n
$a_n = 0$ when n = even
$\therefore$ only odd cosine terms exits.
For odd and half wave symmetry
$a_n = 0$ for all n,
$b_n = 0$ for n = even
$\therefore$ only odd sine terms exits.

5. (a)

Half wave symmetry : a_n, $b_n = 0$ for n = even
$\therefore$ odd cosine and odd sine terms exits.

6. (a)

Trigonometric series has single sided spectrum.

7. (c)

Discreteness of a signal in one domain leads to periodicity in second domain and vice-versa. So, signal is non-periodic, and continuous in other domain.

8. (b)

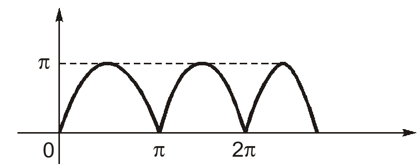
Both the statements are correct individually but not the correct explanation.

9. (a)

From the given waveforms:
(1) and (2) $\rightarrow$ odd and Half wave symmetry
(3) and (4) $\rightarrow$ even and Half wave symmetry
If function is odd – only size terms
function have half wave symmetry $\rightarrow$ even harmonics are zero.
Hence, option (a).

10. (c)

$\pi|\sin t|$



$\therefore$ DC component

$$= \frac{1}{T}\int_0^T f(t)dt = \frac{1}{T}\pi\int_0^\pi \sin t\, dt$$

$$= \frac{\pi}{T}(-\cos t)_0^\pi = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$$

11. (d)

For odd function $\rightarrow$ only sine terms
$$f(t) = -f(-t)$$
For even function: $f(t) = f(-t) \rightarrow$ DC + cosine terms.
For half wave symmetry: a_n, $b_n = 0$ for n = even
$$f(t) = -f(t \pm T/2)$$

12. (b)

Dirichlet's conditions for convergence of Fourier series of $x(t)$.

(i) over any period, $x(t)$ must be integrable
$$\int_T x(t)\,dt < \infty$$
(ii) There are no more than a finite number of minima and maxima during any single period of signal.
(iii) In any finite interval of time, there are only a finite number of discontinuities.

13. (b)

$$v(t) = 1 + 4\sin\omega t + 2\cos\omega t$$

Power dissipated = $\dfrac{V_{rms}^2}{R}$; $R = 1\ \Omega$

Here, $V_{rms} = \sqrt{1+\dfrac{4^2}{2}+\dfrac{2^2}{2}} = \sqrt{11}$

$\therefore$ P_{diss} = 11 Watt

14. (d)

c_3 and c_{-3} are complex conjugate.
$C_{-n} = C_n^*$ property.