POSTAL Book Package 2021

CIVIL ENGINEERING

Strength of Materials

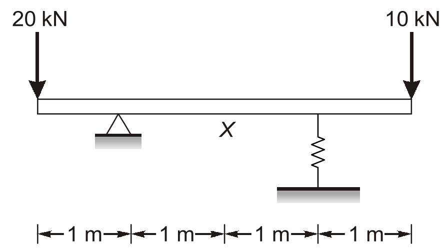
Conventional Practice Sets

Contents

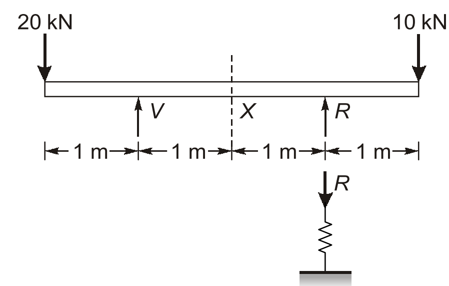
MADE EASY Publications

3 CHAPTER Shear Force and Bending Moment

Q.1 Calculate the bending moment (in kNm units) at the mid span location X in the beam with overhangs shown below.



Solution:



$$R + V = 30 \quad \ldots(i)$$

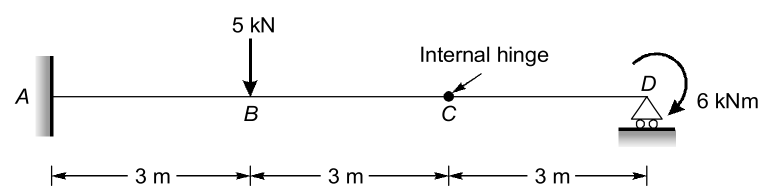
Now, $V \times 2 - 20 \times 3 + 10 \times 1 = 0$

or $V = \dfrac{60 - 10}{2} = 25\text{kN}$

$\therefore$ $R = 5$ kN

BM at $X = 25 \times 1 - 20 \times 2 = -15$ kNm

Q.2 Find the support reactions of the beam shown in the figure. The beam has an internal hinge at C.



Solution:

Let V_A and V_D be the support reactions at A & D respectively and M_A be the bending moment at A.

We know, $\Sigma F_V = 0$

$\Rightarrow$ $V_A + V_D = 5$ kN

Taking moment about hinge

$$M_C = 0$$

$\Rightarrow$ $V_D \times 3 = 6$

$\Rightarrow$ $V_D = 2$ kN ($\uparrow$)

$\therefore$ $V_A = (5 - 2) = 3$ kN($\uparrow$)

Taking moment about C from LHS;

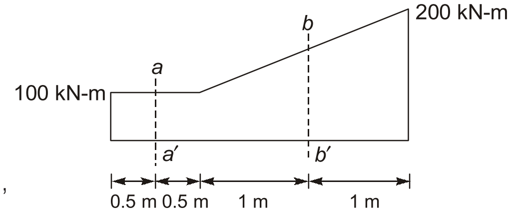
$$V_A \times 6 + M_A = 5 \times 3$$

$\Rightarrow$ $3 \times 6 + M_A = 15$

$\Rightarrow$ $M_A = -3$ kNm

Q.3 The bending moment diagram for a beam is given below:

Find the shear force at section *aa'* and *bb'* (in kN).



Solution:

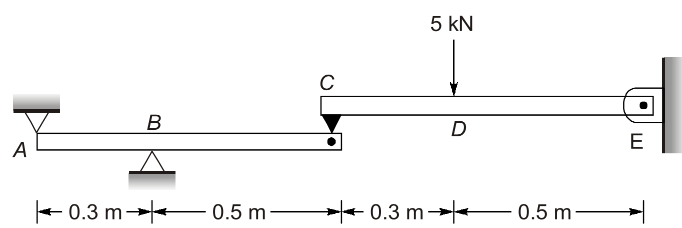
Since $$V = \frac{dM}{dx}$$ from BMD slope at *aa'*=0,

The bending moment to the left as well as right of section *aa*' is constant which means shear force is zero at *aa*'.

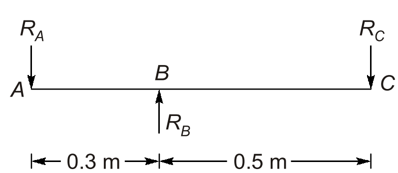
Slope at *bb*'

$$\text{Shear force at bb'} = \frac{200-100}{2} = 50 \text{ kN}$$

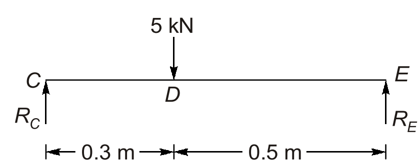
Q.4 Draw the free body diagram for lower and upper parts of the system. Calculate all reactions for the figure shown below.



Solution:



FBD-1



FBD-2

From FBD-2

In equilibrium condition

$$\Sigma M_C = 0$$

$$\Rightarrow \quad 5 \times 0.3 - R_E \times 0.80 = 0$$

$$\Rightarrow \quad R_E = 1.875 \text{ kN}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad R_C + R_E = 5$$

$$\Rightarrow \quad R_C = 5 - 1.875 = 3.125 \text{ kN}$$

From FBD-1

$$\Sigma M_B = 0$$

$$\Rightarrow \quad R_A \times 0.30 - R_C \times 0.50 = 0$$

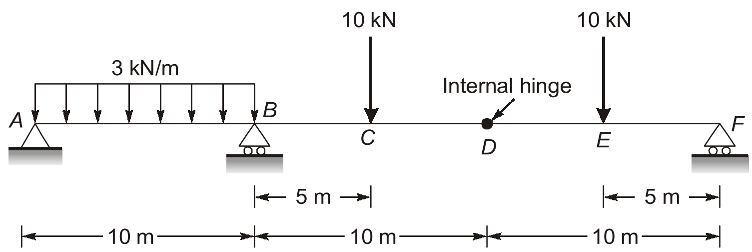
$$\Rightarrow \quad R_A = \frac{3.125 \times 0.50}{0.30} = 5.208 \text{ kN}$$

$$\Sigma F_y = 0$$

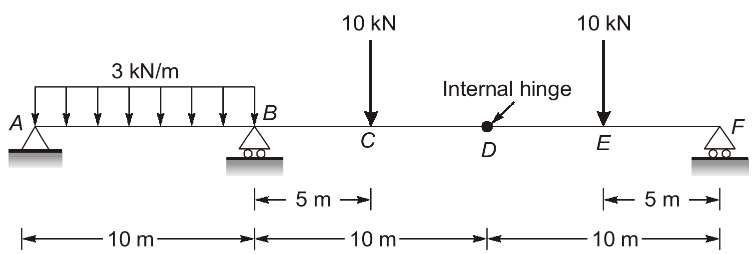
$$\Rightarrow \quad R_B = R_A + R_C$$

$$= 5.208 + 3.125 = 8.33 \text{ kN}$$

Q.5 Draw the bending moment and shear force diagrams of the following beam as shown in the figure. The beam has an internal hinge at D.



Solution:


10 kN
10 kN
3 kN/m
Internal hinge
A
B
C
D
E
F
5 m
5 m
10 m
10 m
10 m

Shear force diagram:

Let V_A, V_B and V_F be the support reactions

$$\therefore \quad M_D = 0$$

Taking moment from RHS,

$$V_F \times 10 = 10 \times 5$$

$$\Rightarrow \quad V_F = 5 \text{ kN} (\uparrow)$$

Similarly taking moment from LHS,

$$V_A \times 20 + V_B \times 10 = 3 \times 10 \times 15 + 10 \times 5$$

$$\Rightarrow \quad 20\, V_A + 10\, V_B = 500 \text{ kN}$$

$$\Rightarrow \quad 2V_A + V_B = 50 \quad \text{...(i)}$$

For equilibrium of beam, $\Sigma F_y = 0$

$$\Rightarrow \quad V_A + V_B + V_F - 30 - 10 - 10 = 0$$

$$\Rightarrow \quad V_A + V_B + V_F = 50$$

$$\Rightarrow \quad V_A + V_B = 45 \text{ kN} \quad \text{...(ii)}$$

Solving eq. (i) and (ii), we get

$$V_A = 5 \text{ kN} (\uparrow), V_B = 40 \text{ kN} (\uparrow)$$

For portion AB; $\quad SF_x = V_A - 3x = 5 - 3x$

At $x = 0$, $\quad (SF)_A = 5 \text{ kN} (\uparrow)$

At $x = 10$ m, $\quad (SF)_B = 5 - 3 \times 10 = -25 \text{ kN}$

For $(SF)_x = 0$

$$\Rightarrow \quad 5 = 3x$$

$$\Rightarrow \quad x = 1.67 \text{ m}$$

For portion BD;

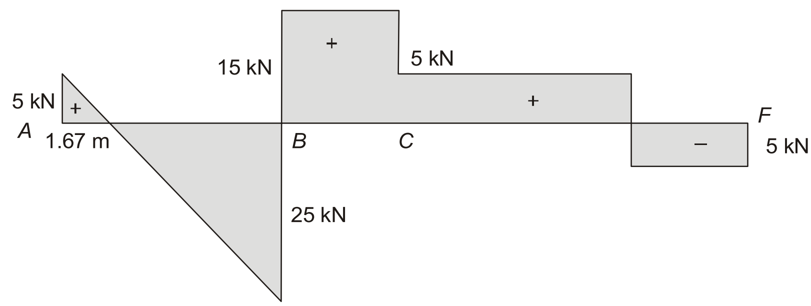
$$(SF)_D = V_A - 3 \times 10 + V_B - 10 = 45 - 40 = 5 \text{ kN}$$

$$(SF)_B = -25 \text{ kN} + 40 = 15 \text{ kN}$$

For portion DF;

$$(SF)_E = (SF)_D - 10 = (5 - 10) = -5 \text{ kN}$$

$$(SF)_F = -5 \text{ kN}$$



For portion AB; $(0 < x \leq 10 \text{ m})$ $\quad M_x = V_A x - \frac{3x^2}{2} = 5x - 1.5x^2$

At $x = 0$, $M_A = 0$

and $\quad x = B = 10 \text{ m} = 50 - 1.5 \times 100 = -100 \text{ kNm}$

For maximum value, $\quad \frac{dM_x}{dx} = -5 - 3x$

$$x = \frac{5}{3} = 1.67 \text{ m}$$

$$M_x = 0 = x(5 - 1.5x)$$

$$x = \frac{5}{1.5} = 3.33 \text{ m}$$

$$M_{max} = 5 \times 1.67 - 1.5 \times (1.67 \times 1.67) = 4.167 \text{ kNm}$$

For portion BC, $(0 < x < 5 \text{ m})$ $\quad M_x = 5(x + 10) + 40x - 30(5 + x)$

At $x = 0$ $\quad M_B = 50 - 30 \times 5 = -100 \text{ kNm}$

At $x = 5$ $\quad M_C = 5 \times 15 + 40 \times 5 - 30 \times 10 = 75 + 200 - 300 = -25 \text{ kNm}$

For portion CD; $(0 < x < 5)$ $\quad M_x = 5(x + 15) + 40(x + 5) - 30(x + 10) - 10 \times x$

At $x = 0$ $\quad M_C = -25 \text{ kNm}$

At $x = 5$ $\quad M_D = 100 + 400 - 450 - 10 \times 5 = 0$

From RHS of the beam;

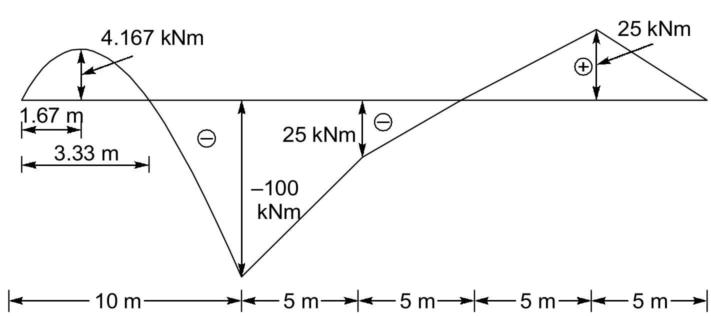
Portion FD; $0 < x < 5$ $\quad M_x = 5x$

At $x = 0$, $\quad M_F = 0$

At $x = 5$ m, $\quad M_F = 25 \text{ kNm}$

Portion ED; $0 < x < 5$ $\quad M_x = 5(x + 5) - 10x$

At $x = 0$ M_D $\quad = 50 - 50 = 0$



Q.6 A three-span continuous beam has an internal hinge at *B*. Section *B* is at the mid-span of *AC*. Section *E* is at the mid-span of *CG*. The 20 kN load is applied at section *B* whereas 10 kN loads are applied at sections *D* and *F* as shown in the figure. Span *GH* is subjected to uniformly distributed load of magnitude 5 kN/m. For the loading shown, shear force immediate to the right of section *E* is 9.84 kN upwards and the hogging moment at section *E* is 10.31 kN-m. [*CD* = *DE* = *EF* = *FG* = 1 m]