Civil Engineering

Structural Analysis

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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Structural Analysis

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Contents

Structural Analysis

Chapter 4

Arches 146

Chapter 5

External Equilibrium of Structures 176

Chapter 6

Methods of Indeterminate Analysis : Basic Methods....................................186

Chapter 7

Energy Methods of Analysis 240

Chapter 01

Stability and Indeterminacy

1.1 Support System

1.1.1 2-D Supports

(a) Fixed Support

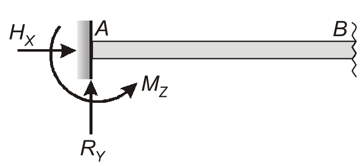


***Fig. 1.1 (i)** Number of reaction = 3*

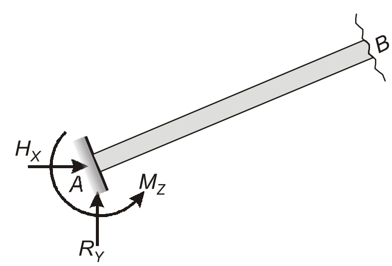


***Fig. 1.1 (ii)** Number of reactions = 3*

At 2-D fixed support, there can be three reactions:

(*i*) one vertical reaction (R_y)

(*ii*) one horizontal reaction (H_x)

(*iii*) one moment reaction (M_z)

(b) Hinge Support

Hinge support is represented by the symbol △.

***Fig. 1.2 (i)** Number of reactions = 2*

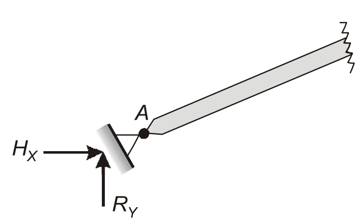


***Fig. 1.2 (ii)** Number of reactions = 2*

At hinged support, there can be two reactions:

(*i*) one horizontal reaction (H_x)

(*ii*) one vertical reaction (R_y)

(c) **Roller Support**

Roller support is represented by the symbol  or .

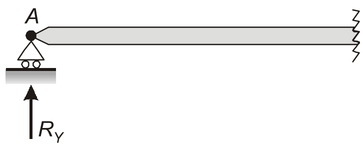


Fig. 1.3 (i) Number of reactions = 1

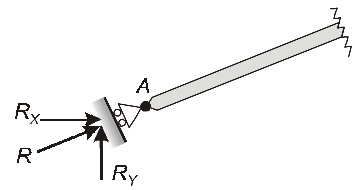


Fig. 1.3 (ii) Number of reactions = 1

At roller support there can be only one externally independent reaction which is normal to the contact surface.

(d) **Guided Roller Support**

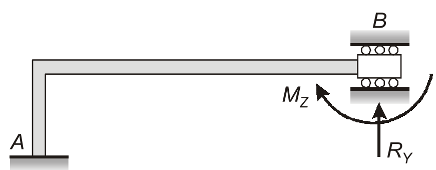


Fig. 1.4 Number of reactions = 2

At guided roller supports there can be two reactions:

(*i*) one vertical reaction (R_y) (*ii*) one moment reaction (M_z)

1.1.2 2-D Internal Joints

(a) **Internal Hinge**

At internal hinge bending moment will be zero.

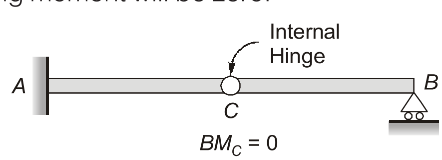


Fig. 1.5

NOTE: An internal hinge provides one additional equilibrium equation for structures.

(b) **Internal Roller**

At internal roller either axially force or shear force will be zero.

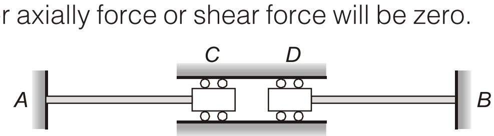


Fig. 1.6

In fig. 1.6, axially force at *C* and *D* is zero.

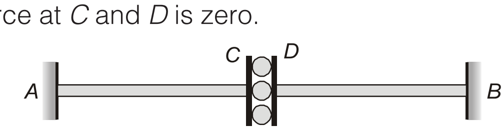


Fig. 1.7

In fig. 1.7, shear force at *C* and *D* will be zero i.e., $S_C = S_D = 0$

(c) Internal Link

If any member is connected by hinges at its end and subjected to no external loading in between then it can be termed as internal link and carry axial force only.

Here *BC* is a link, link *BC* carry only axial force

Also $BM_B = 0$ and $BM_C = 0$

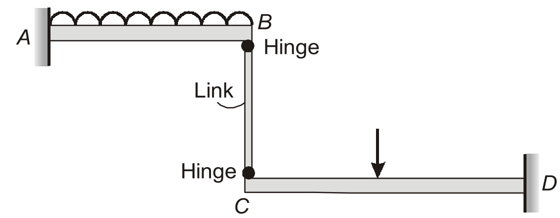


Fig. 1.8

NOTE: Internal release also provides additional equation for analysis of structure.

1.1.3 3-D Supports

(a) Fixed Support

At 3-D fixed support there can be six reactions:

(*i*) three reactions R_x, R_y and R_z

(*ii*) three moment reactions M_x, M_y and M_z

The fixed support are also called ***Built-in support***.

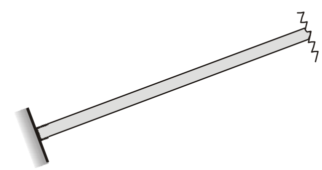


Fig. 1.9: *Number of reactions = 6*

(b) 3-D Hinged Support

At 3-D hinged support there can be three reactions

(*i*) R_x

(*ii*) R_y

(*ii*) R_z

The 3-D hinged support is also called **'ball and socket joint'**.

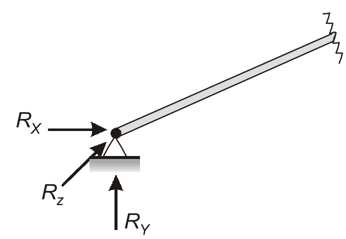


Fig. 1.10 *Number of reactions = 3*

(c) Roller Support

At 3-D roller support there can be only one externally independent reaction which is perpendicular to the contact surface

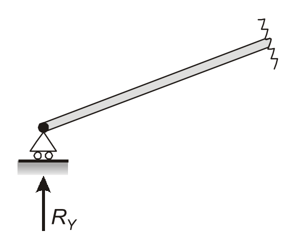


Fig. 1.11 *Number of reactions = 1*

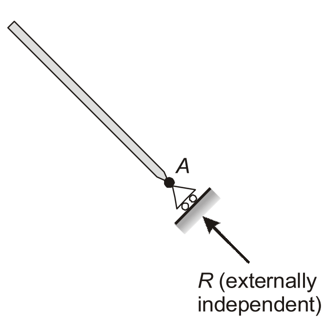


Fig. 1.12 (i)

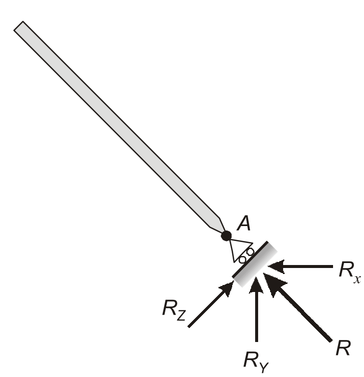


Fig. 1.12 (ii)

in figure 1.12.(ii), reactions at roller support *A*, R_x, R_y and R_z are externally dependent reactions which depends on reaction *R*.

1.2 Structure

1.2.1 Elements of Structure

Some of the major elements of structure by which structures are fabricated are as follows:

(a) Beams: Beams are structural members which is predominantly subjected to bending. On the basic of support system beams can be classified as:

(*i*) Simply supported beam

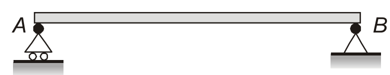


Fig. 1.13

(*ii*) Cantilever beam

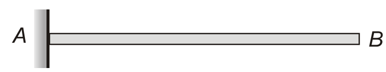


Fig. 1.14

(*iii*) Propped cantilever

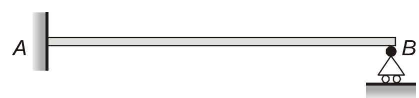


Fig. 1.15

(*iv*) Fixed beam

Fig. 1.16

(*v*) Continuous beam

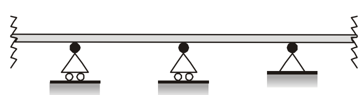


Fig. 1.17

(b) Columns: A column is a vertical compression member which is slender and straight. Generally columns are subjected to axial compression and bending moment as shown in figure.

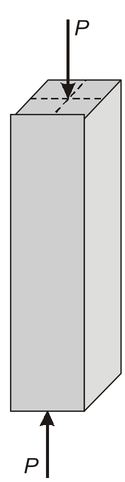


Fig. 1.18 (i)

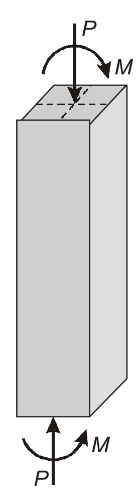


Fig. 1.18 (ii)

(c) Tie Members: Tie members are tension members of trusses and frame, which are subjected to axial tensile force. (Figure : 1.19)

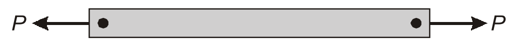


Fig. 1.19 *Tie Rod*

1.2.2 Types of Structures

(a) Trusses: A truss is constructed from pin jointed slender members, usually arranged in triangular manner. In trusses, loads are applied on joints due to which each member of truss subjected to only axial forces i.e., either axial compression or axial tension. Generally trusses are used when span of structure is large.

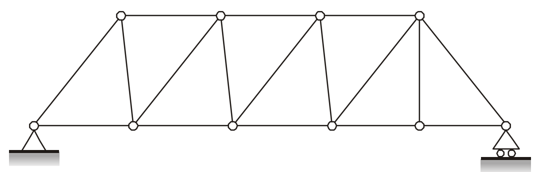


Fig. 1.20 *Truss*

(b) Frames: A frame is constructed from either pin jointed or fixed jointed beam and columns. Generally loads are applied on beams and this loading causes axial force, shear force and bending to the members of frame.

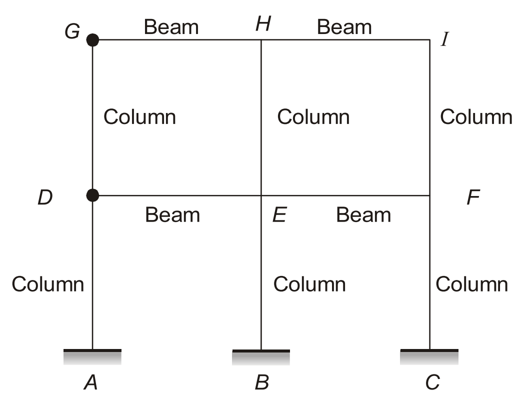


Fig. 1.21 *Frames*

(c) Arches: Arches are used in bridges, dome roof, auditorium, where span of structures are relatively more due to external loading, Arch can be subjected to axial compression, shear force or bending moment.

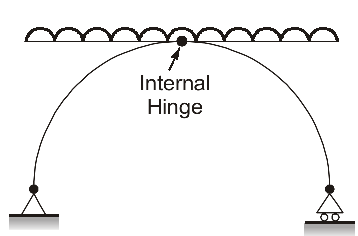


Fig. 1.22 ***(i)*** *Three Hinge Arch*

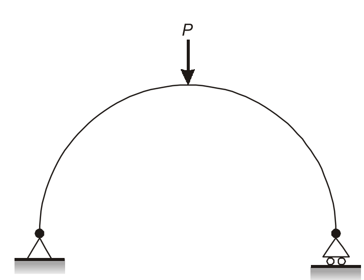


Fig. 1.22 ***(ii)*** *Two Hinge Arch*

(d) Cables: Cables are used to support long span bridges. Cables are flexible members and due to external loading it is subjected to axial tension only.

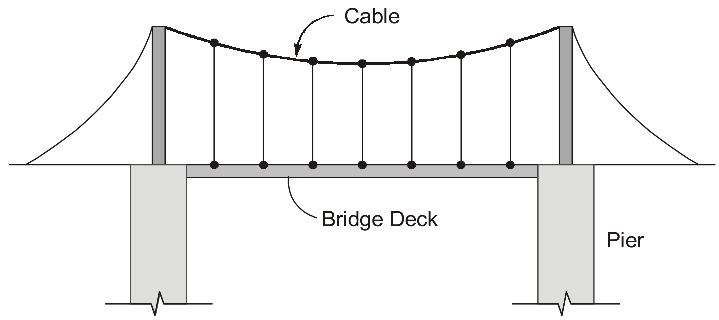


Fig. 1.23 *Cable and Bridge*

1.3 Types of Loading

(a) Point load: A point load is considered to be acting at a point. It is also called concentrated load. In actual practice point loads are distributed load which are distributed over very small area.

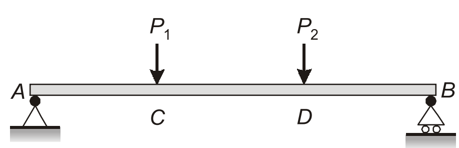


Fig. 1.24 *Point Load*

(b) Distributed loads: Distributed loads are those loads, which acts over some measurable area. Distributed loads are measured by the intensity of loading per unit length along the beam.

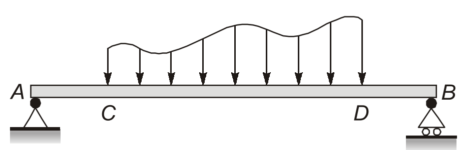


Fig. 1.25 *Distributed Loads*

(c) Uniformly distributed loads: Uniformly distributed loads are those distributed loads which have uniform intensity of loading over the area.

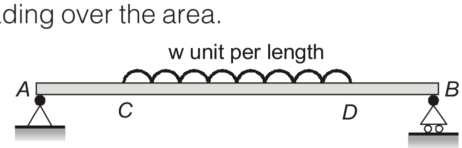


Fig. 1.26 *Uniformly Distributed Loads*

(d) Uniformly varying loads : A uniformly varying load, commonly abbreviated as UVL, is the one in which the intensity of loading varies from one end to other. For example, intensity is zero at one end and w at other end.

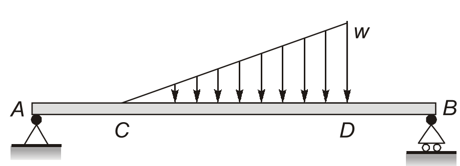


Fig. 1.27 *Uniformly Varying Loads*

(e) Couple : A system of forces with resultant moment, but no resultant force is called couple. It is statically equivalent to force times the offset distance.

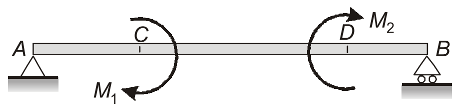


Fig. 1.28 *Couple*

1.4 Stability of Structures

Structural stability is the major concern of the structural designer. To ensure the stability, a structure must have enough support reaction along with proper arrangement of members. The overall stability of structures can be divided into

(i) External stability (ii) Internal stability

1.4.1 External Stability

(a) 2-D Structures: For stability of 2-D structures there should be no rigid body movement of structure due to loading so, it should have support in x-direction, y-direction and no rotation in x-y plane. So there should be enough reactions to restrain the rigid body motion.

For stability of 2-D structures, following three conditions of static equilibrium should be satisfied.

(i) $\Sigma F_x = 0$ (To prevent Δ_x)

(ii) $\Sigma F_y = 0$ (To prevent Δ_y)

(iii) $\Sigma M_z = 0$ (To prevent θ_z)

For stability in 2-D structures following conditions also be satisfied:

(i) There should be minimum three number of externally independent support reaction.

(ii) All reactions should not be parallel, otherwise linearly unstability will set up.

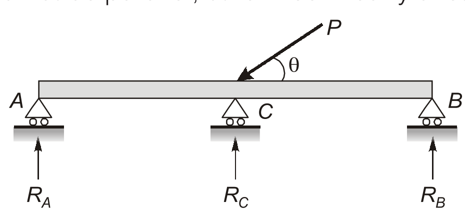


Fig. 1.29 *Unstable*

(iii) All reactions should not be linearly concurrent otherwise rotational unstability will setup.

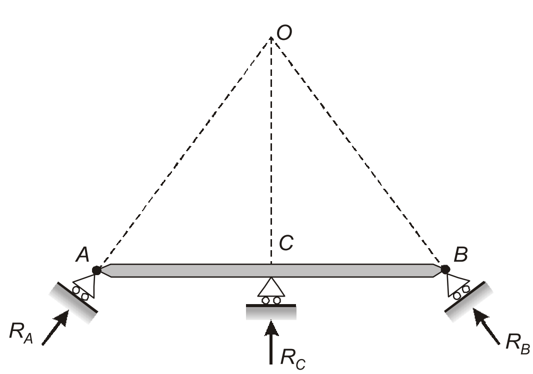


Fig. 1.30 (i) *Unstable*

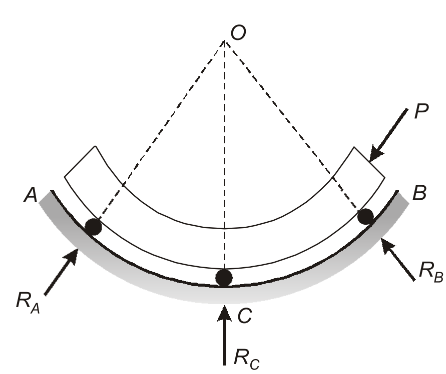


Fig. 1.30 (ii) *Unstable*

(*iv*) Reactions should be non-trival i.e. there should be enough magnitude and enough difference between them.

(b) 3-D Structures: In case of 3-D structures, there should be a minimum of six independent external reactions to prevent rigid body displacement of structure. The displacement to be prevented are: Δ_x, Δ_y, Δ_z, θ_x, θ_y and θ_z. Therefore, there will be six equation of static equilibrium.

(*i*) $\Sigma F_x = 0$ (*ii*) $\Sigma F_y = 0$ (*iii*) $\Sigma F_z = 0$

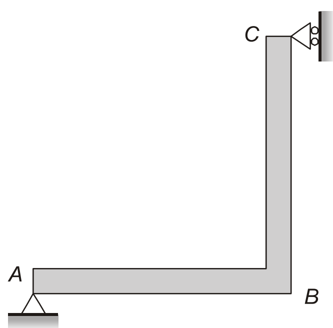
(*iv*) $\Sigma M_x = 0$ (*v*) $\Sigma M_y = 0$ (*vi*) $\Sigma M_y = 0$

For stability in 3-D structures, all the reactions should be non-coplanar, non-concurrent and non-parallel.

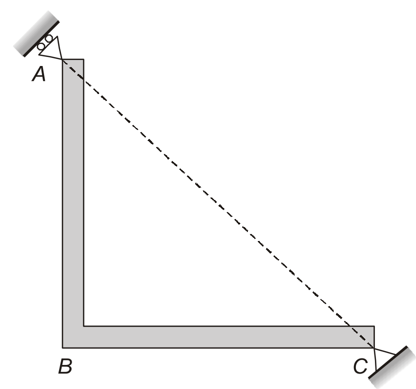
Remember: If a structure is constructed from elastic members then small elastic displacement may be permitted but small rigid body displacement will not be permitted.

Example 1.1 Which one of the following structures is stable?

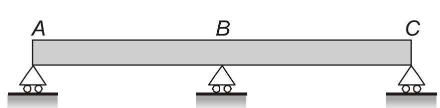
(a)



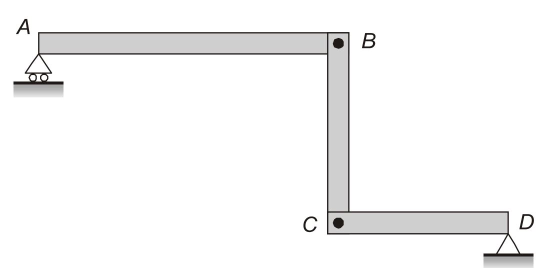
(b)



(c)

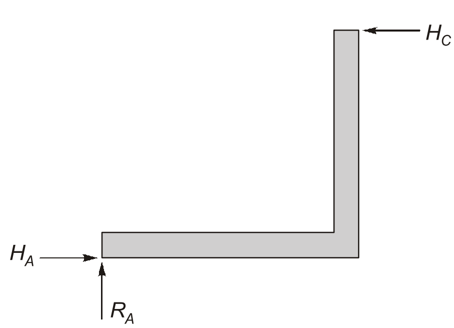


(d)

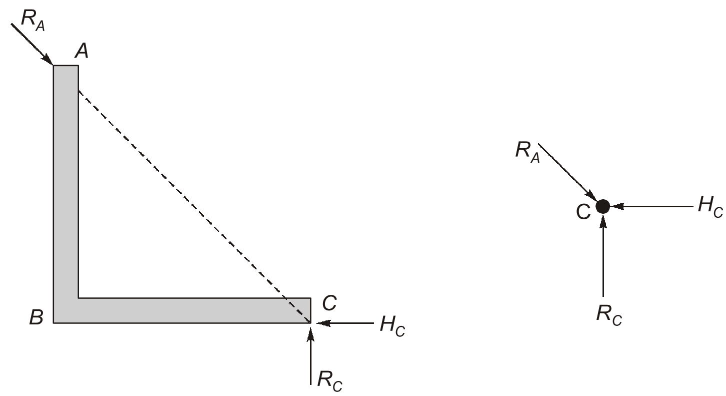


***Ans.* (a)**

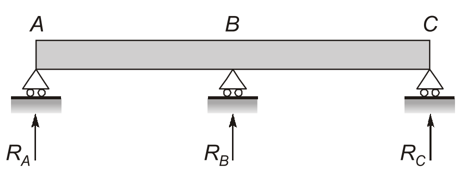
Member (a) is stable, since reactions are non-parallel and non-concurrent.



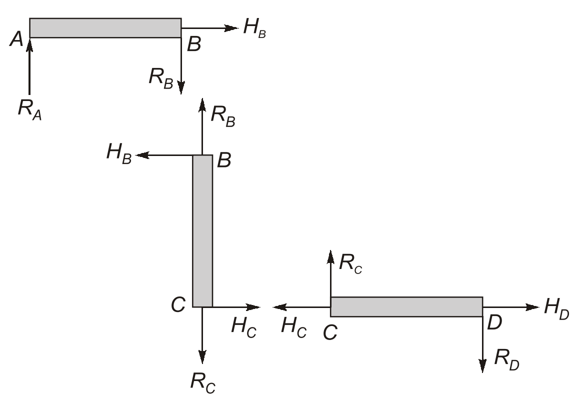
Member (b) is unstable since all the reactions are concurrent at *C*.



Beam (c) is unstable, since all three reactions are parallel.



Structure (d) is unstable, since the member *AB* can move horizontally without any restrain. i.e. $\Sigma F_x \neq 0$



1.4.2 Internal Stability

For the internal stability, no part of the structure can move rigidly relative to the other part so that geometry of the structure is preserved, however small elastic deformations are permitted. To preserve geometry, enough number of members and their adequate arrangement is required. For the geometric stability, there should not be any condition of mechanism. Mechanism is formed when there are three collinear hinges, hence to preserve geometric stability there should not be three collinear hinges.

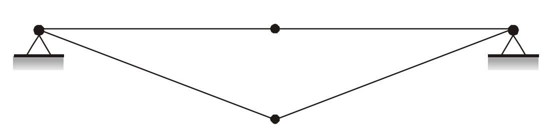


Fig. 1.31

For 2-D truss the minimum number of members needed for geometric stability are:

$$m = 2j - 3$$

and for 3-D truss,

$$m = 3j - 6$$

where, j = Number of joint in truss

m = Member required for geometrical stability.

All the members should be arranged in such a way that truss can be divided into triangular blocks. i.e. no rectangular or polygonal blocks.

Hence, for overall geometrical stability of truss:

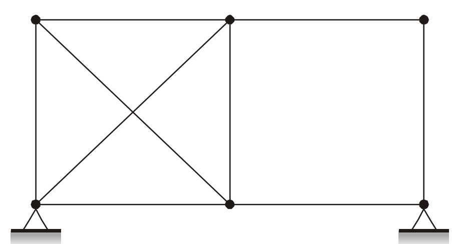
(*i*) Minimum number of member should be present

$m = 2j - 3$ (2-D truss)

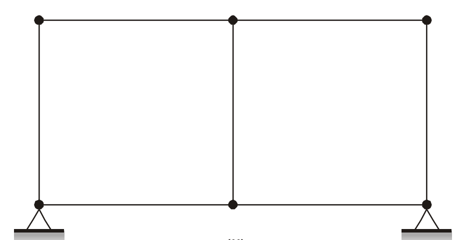
and $m = 3j - 6$ (3-D truss)

(*ii*) There should be no condition of mechanism i.e. no three collinear hinges.

Example 1.2 Check geometrical stability for given trusses.



(*i*)



(*ii*)

Solution:

(*i*) In case (*i*), arrangement of members is not adequate, hence right panel is unstable and left panel is over stiff. For geometric stability, all panels of truss should be stable so given truss is geometrical unstable.

For right panel: $j = 4$

Number of member present, $m = 4$

But minimum number of member needed $= 2j - 3 = 2 \times 4 - 3 = 5$

Hence Right panel is deficient.

For left panel: $j = 4$

Number of member present, $m = 6$

But minimum number of member needed $= 2j - 3 = 2 \times 4 - 3 = 5$

Hence left panel is over stiff.

(*ii*) $j = 6$

Number of members present, $m = 7$

But minimum number of member needed $= 2j - 3 = 2 \times 6 - 3 = 9$

Hence, above truss is geometrically unstable and it can be called 'deficient structure'.

$\therefore$ Number of deficiency $= 2$